



Yang-Mills thermodynamics

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R. Hofmann

ITP-Universität Heidelberg, IPS-KIT

 $= \operatorname{tr} \frac{1}{2} \int_{0}^{r} d\tau \int d^{3}x F_{\mu\nu} F_{\mu\nu}$

motivation

- Andrei Linde (1980): "Infrared Problem in the Thermodynamics of the Yang-Mills Gas"
 - soft magnetic sector screened weakly in perturbation theory (infrared instability)
 - no "convergence" of series since kinetic and interaction energies comparable in this sector
- issue of finite-volume constraints in lattice gauge theory
 - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

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nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst & RH (2004), RH (2005-2007), Giacosa & RH (2006), Schwarz, Giacosa & RH (2007), Ludescher & RH (2008), Falquez, Baumbach & RH (2010- 2011), RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)]

thermal ground state at high temperature:

- Euclidean action:

$$S = {{\rm tr} \over 2} \int_0^\beta d au \int d^3x \, F_{\mu
u} F_{\mu
u} \,,$$
 ($eta \equiv 1/T$)
where $F_{\mu
u} \equiv \partial_\mu A_
u - \partial_
u A_\mu - ig[A_\mu, A_
u]$ [Schafer et Shuryak (1996)

 - (anti)selfdual gauge fields: [(anti)calorons]

$$e F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}] \text{ [Schafer et Shuryak (19)]}$$

fields:
$$F_{\mu\nu}[A] = \pm \tilde{F}_{\mu\nu}[A] \Rightarrow \theta_{\mu\nu}[A] \stackrel{\checkmark}{=} 0.$$

 $(A_{\mu} \text{ periodic})$

field configs. stabilized by gauge-field winding: $\partial \mathbf{R}^4 = S_3 \rightarrow SU(2) = S_3$

- in particular: (anti)calorons of winding number unity

Calorons of top. charge unity (selfdual field configs. on $S_1 imes R_3$): (singular g

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Harrington-Shepard (1977): (trivial holonomy)

$$\begin{split} A_{\mu} &= \bar{\eta}^{a}_{\mu\nu} t_{a} \partial_{\nu} \log \Pi(\tau, r) \\ \text{with} \quad \Pi &= \begin{cases} \left(1 + \frac{1}{3} \frac{s}{\beta}\right) + \frac{\rho^{2}}{x^{2}} & (|x| \ll \beta) \\ 1 + \frac{s}{r} & (r \gg \beta) \end{cases} \\ \text{and} \quad s &\equiv \frac{\pi \rho^{2}}{\beta} \,, \quad \beta &\equiv \frac{1}{T} \,. \end{split}$$

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 $\Rightarrow F_{\mu\nu} \text{ that of singular-gauge instanton with } \rho'^2 = \frac{\rho^2}{1 + \frac{1}{3}\frac{s}{\beta}} (|x| \ll \beta)$ (action: $S_c = \frac{8\pi^2}{g^2} \int_{S_3^\delta} d\Sigma_{\mu} K_{\mu} = \frac{8\pi^2}{g^2}$ localised about instanton center in $S_1 \times \mathbf{R}_3$)

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$$E_{i}^{a} = B_{i}^{a} = s \frac{\delta_{i}^{a} - 3 \hat{x}^{a} \hat{x}^{i}}{r^{3}} \quad (r \gg s) \,.$$

(static selfdual dipole-field with dipole moment: $p_i^a = s \, \delta_i^a$)

Nahm (1983), Lee-Lu-Kraan-van-Baal (1998): (nontrivial holonomy) - M

- M and A of finite mass and extent:

$$m_M = 4\pi u \,, m_A = 4\pi \left(\frac{2\pi}{\beta} - u\right)$$

(action density on spatial slice)



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(M-A separation, caloron center)

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- locus of action within $S_3^\delta \ (\delta \to 0)$
- trivial-holonomy limit:
 M massless, A still massive, stable

[Herbst & RH (2004)]

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \operatorname{tr} \int d^3x \int d\rho \, t^a \, F_{\mu\nu}(\tau,\vec{0}) \, \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \, F_{\mu\nu}(\tau,\vec{x}) \, \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of family of phases, where

$$\left\{ (\tau, \vec{0}), (\tau, \vec{x}) \right\} \equiv \mathcal{P} \exp \left[i \int_{(\tau, \vec{0})}^{(\tau, \vec{x})} dz_{\mu} A_{\mu}(z) \right] \quad \text{and}$$
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- $\{\hat{\phi}^a\}$ sharply dominated by cut-off for ho integration

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ

- no explicit eta dependence in ϕ field dynamics (caloron action!)
- absorb eta dependence of operator D into potential V

(BPS and EL yield:



(Euclidean time dependence of HS (anti)caloron centers coarse-grains into a time dependence of ϕ which can be made trivial by singular but admissible gauge trafo.)

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dependence of operator
$$D$$
 into potential V
EL yield: $\frac{\partial V(|\phi|^2)}{\partial |\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2} \Longrightarrow$
 $V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}$ (Yang-Mills scale constant of integr.)
 $|\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}$ and

- BPS equation:

$$\partial_\tau \phi = \pm 2i \Lambda^3 t_3 \phi^{-1} = \pm i V^{1/2}(\phi)$$

no **additive** ambiguity in V !

effective action (deconfining phase), thermal ground state

$$\mathcal{L}_{\rm eff}[a_{\mu}] = \operatorname{tr} \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + (D_{\mu}\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

(i) perturbative renormalizability (G^2 highest power in effect. action, propagating part of a_{μ} adiabatic excitation of thermal ground state) (iil) ϕ 's inertness – no higher dim., mixed operators to mediate 4-momentum transfer between ϕ and a_{μ} (iii) gauge invariance [see also RH (2016)]

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effective YM equation $D_{\mu}G_{\mu\nu} = ie[\phi, D_{\nu}\phi]$ has ground-state solution:

$$a_{\mu}^{\rm gs} = \mp \delta_{\mu 4} \frac{2\pi}{e\beta} t_3 \qquad (D_{\nu}\phi \equiv G_{\mu\nu} \equiv 0)$$

(centers of HS (anti)calorons packed densely, static peripheries overlap to form $a_{\mu}^{
m gs}$)

$$\implies P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T \,.$$

interacting small and transient-holonomy (anti)calorons, (collapsing monopoleantimonopole pairs)

(vanishing entropy density of ground state!)

adjoint Higgs mechanism (deconfining phase)



- no off-shell propagation of massive modes (otherwise: momentum transfer to ϕ !)





anatomy of caloron, inferred after spatial coarse-graining:



defining Yang-Mills action: classical, Euclidean gauge-field theory on $S_1 \times \mathbf{R}_3$

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small-holonomy (anti)calorons of action ħ constitute effective thermal ground state, mediate interactions (vertices) between effectively propagating modes (BE distributed QF – massiv; low-frequency waves, high-frequency BE distr. QF massless) [Kaviani & RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)] defining Yang-Mills action: classical, Euclidean gauge-field theory on $S_1 \times \mathbf{R}_3$

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kinematic constraints in (totally fixed) unitary-Coulomb gauge imply that radiative corrections are extremely well controlled

[Schwarz, Giacosa, & RH (2006), Ludescher & RH (2008)]

expansion of thermodyn. quantities into **1PI loops** probably terminates at finite order, say, pressure



real-world implications

electric-magnetically dual interpretation of U(1) charge:

if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc] then **electric-magnetically dual** interpretation required: in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar} \,,$$

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But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e} \,.$$

SU(2) to be interpreted in an electric-magnetically dual way.
 (e.g., magnetic monopole <-> electric monopole, etc.)

electric/magnetic dipole density (permittivity/permeability of vacuum): [temperature a fictitious quantity]

$$|\mathbf{D}_e| = rac{2s}{V_{
m cg}} \propto T^{1/2}$$

external electric field strength (plane wave):

$$\rho_{\rm gs} = 4\pi T \Lambda^3 = \rho_{\rm EM} = \epsilon_0 \mathbf{E}_e^2 \Rightarrow |\mathbf{E}_e| \propto T^{1/2}$$

$$\epsilon_0 \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} \neq f(T)$$

similarly for magnetic permeability $\,\mu_{0}$.

[Grandou & RH (2015)]

evidences for $SU(2)_{\rm CMB}$ ($\Lambda_{\rm CMB} \sim 10^{-4} \, {\rm eV}$): photon at tree level



- CMB angular spectrum vs. Gunn-Peterson trough (quasars) inferred early re-ionisation of intergalactic medium (z=11 vs. z=6 discrepancy), non-conformal T - a relation at late times

[Becker et al. (2001), WMAP coll. (2004), Planck coll. (2013)]



evidences for $SU(2)_{\rm CMB}$ ($\Lambda_{\rm CMB} \sim 10^{-4} \, {\rm eV}$): one-loop polarization

- **spectral blackbody anomaly:** max. gap in Rayleigh-Jeans reg. at $T \sim 5 \text{ K}$, massless mode – transverse polarizations [Schwarz, Giacosa & RH (2006), Ludescher & RH (2008), Falquez, RH & Baumbach (2010,2011)]



evidences for $SU(2)_{\rm CMB}$ ($\Lambda_{\rm CMB} \sim 10^{-4} \, {\rm eV}$): one-loop polarization

- integral blackbody anomaly:

difference $\delta \rho$ between energy density of SU(2) and U(1), massless mode – transverse polarizations



evidences for $SU(2)_{\rm CMB}$ ($\Lambda_{\rm CMB} \sim 10^{-4} \, {\rm eV}$): one-loop polarization

 low-momentum support of magnetic branches (dual interpretation) massless mode – longitudinal polarization



summary

- alternative to high-T perturbation theory: caloron induced dynamical gauge SB by thermal ground state
- effective thermal quantum field theory for deconfining phase of SU(2) YM
- effective coupling evolution: caloron action \hbar ,
 - caloron mediation of effective vertices,
 - e-m dual interpretation
- effective radiative corrections: extremely well controlled
- SU(2) photons: tree-level and one-loop polarization anomalies \rightarrow CMB anomalies
 - cosmic radiobackground
 - quasar vs CMB wrt reionization,
 - spectral & integral BB anomalies
 - (CMB at large angles)
 - → extragalactic magnetic fields

Theory:



(1st ed. World Scientific, 2011; 2nd ed. World Scientific, June 2016)

Cosmological applications (CMB photons):

F. Giacosa and RH, Eur. Phys. J. C (2005);
F. Giacosa, RH, M. Neubert, JHEP (2008);
M. Szopa, RH, JCAP (2008);
RH, Annalen d. Physik (2009);
RH, Nature Physics (2013);
RH, Annalen d. Physik (2015);
T. Grandou & RH, Adv. Math. Phys. (2015);
RH, Entropy (2016)

Thank you !