The lattice as a supersymmetry breaking regulator and applications of invariant regulators

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GB, F. Bruckmann, J. M. Pawlowski arXiv:0807.1110; arXiv:0810.3547
F. Synatschke, GB, H. Gies, A. Wipf arXiv:0809.4396

Introduction

- supersymmetry is an important ingredient of many theories beyond the standard model
- the analysis of the quantum nature needs nonperturbative methods
- to extend the successful story of the lattice calculations to SUSY theories a discretisation compatible with supersymmetry must be found
- however, on the lattice: ~~Poincaré-invariance $\Rightarrow$ SUSY~~
- more precisely: ~~Leibniz-rule $\Rightarrow$ SUSY invariance of the action~~

"Solutions":
- use partial realisation of supersymmetry (e. g. Nicolai-improvement) ~~reflection-positivity~~
- use lattice perturbation theory to ensure the correct continuum limit
- reduce violation with nonlocal lattice operators

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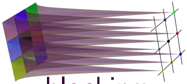
The approach of Ginsparg and Wilson



continuum action —— blocking / RG step ——→ lattice action

symmetry $\{\gamma_5, \mathcal{D}\} = 0$ —— imply ——→ relation $\{\gamma_5, \mathcal{D}\} = a\mathcal{D}\gamma_5\mathcal{D}$ defines a "symmetry" $\bar{\gamma}_{5,\mathrm{def}}\mathcal{D} + \mathcal{D}\gamma_{5,\mathrm{def}} = 0$

relation ←— condition for generation —— symmetry

- "perfect" lattice action: blocking of the continuum theory (correct continuum limit)
- "perfect" lattice symmetry: symmetry of a blocked action

The blocking transformation

1. averaging of the continuum field $\varphi(x)$ around lattice point $x_n = an$:

$$\Phi_n[\varphi] := \int dx\, f(x - x_n)\varphi(x)$$

2. define a blocked lattice action $S[\phi]$ depending on lattice fields ϕ_n for a given continuum action $S_{\mathrm{cl}}[\varphi]$

$$e^{-S[\phi]} := \frac{1}{\mathcal{N}} \int d\varphi\, e^{-\frac{1}{2}(\phi-\Phi[\varphi])_n \alpha_{nm} (\phi-\Phi[\varphi])_m}\, e^{-S_{\mathrm{cl}}[\varphi]}$$

- simple interpretation if $f(x - x_n) \to \delta(x - x_n)$ and $\alpha \to \infty$ as $a \to 0$ since $S \to S_{\mathrm{cl}}$; more generally

$$\int d\phi\, e^{-S[\phi]+J\phi} = e^{\frac{1}{2}J\alpha^{-1}J} \int d\varphi\, e^{-S_{\mathrm{cl}}[\varphi]+J\Phi[\varphi]}$$ perfect!

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A lattice symmetry

- continuum action invariant under infinitesimal continuum symmetry transformations:
  $S_{\mathrm{cl}}[\varphi + \delta\varphi] = S_{\mathrm{cl}}[(1 + \varepsilon \tilde{M})^{ij} \varphi^j] = S_{\mathrm{cl}}[\varphi]$

3. translate continuum symmetry transformations $\tilde{M}$ into naive lattice transformations M:

$$\Phi^i_n[\tilde{M}\varphi] = \int dx \; f_n(x) \; \tilde{M}^{ij} \varphi^j(x) = M^{ij}_{nm} \Phi^j_m[\varphi]$$

- can not be found for every $\tilde{M}$ and f $\hookrightarrow$ additional constraint

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Inherited symmetry of the blocked action

$$e^{-S[\phi]} = \frac{1}{\mathcal{N}} \int d\varphi \; e^{-S_{\rm cl}[\varphi]} \; e^{-\frac{1}{2}(\phi - \Phi[\varphi])\alpha(\phi - \Phi[\varphi])}$$

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$$M^{ij}_{nm}\phi^j_m \frac{\delta S}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S}{\delta \phi^j_m}\frac{\delta S}{\delta \phi^i_n} - \frac{\delta^2 S}{\delta \phi^j_m \delta \phi^i_n}\right) + (\mathrm{STr} M - \mathrm{STr}\tilde{M})$$

- $(\phi - \Phi)$ replaced by $\frac{\delta}{\delta\phi}$ and α^{-1}
- $\mathrm{STr}\,\tilde{M}$ infinitesimal change of the measure $\rightarrow$ anomaly

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Symmetry relation for the lattice action

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- quadratic action, $S = \frac{1}{2}\phi K \phi$:
  $M^T K + (M^T K)^T = K^T\left[(M\alpha^{-1})^T + M\alpha^{-1}\right] K$ or:

$$M_{\mathrm{def}}{}^T K + K^T M_{\mathrm{def}} = 0; \quad M_{\mathrm{def}} = M(\mathbb{1} - \alpha^{-1} K)$$

- conditions for M_{def} to define a deformed symmetry
  1. M_{def} local
  2. M_{def} approaches continuum counterpart (excludes $M_{\mathrm{def}} = 0$)

$\Rightarrow$ restricts possible choices of α and K

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Ginsparg-Wilson relation

$$\alpha_{nm} = \frac{1}{a}\delta_{nm}$$
$$\{\gamma_5, \mathcal{D}\} = a\mathcal{D}\gamma_5\mathcal{D}$$

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$$\bar{\gamma}_{5,\mathrm{def}}\mathcal{D} + \mathcal{D}\gamma_{5,\mathrm{def}} = 0;\ \gamma_{5,\mathrm{def}} = \gamma_5(1 - a\mathcal{D}),\ \bar{\gamma}_{5,\mathrm{def}} = (1 - \mathcal{D}a)\gamma_5$$

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GW: excludes Wilson fermions

$\Rightarrow$ restricts possible choices of α and K

Solution of the additional constraint for SUSY

$$\int dx\, f(x-an)\, \tilde{M}^{ij}\varphi^j(x) = M^{ij}_{nm}\Phi^j_m[\varphi] = M^{ij}_{nm}\int dx\, f(x-am)\, \varphi^j(x)$$

- trivial if $\tilde{M}^{ij}$ merely acts on multiplet index j; but for SUSY derivative operators in the continuum transformations
- must hold for all φ; in Fourier space

$$[\nabla(p_k) - ip_k]f(p_k) = 0$$

for $p_k = \frac{2\pi}{L}k$, $k \in \mathbb{Z}$ and $\nabla(p + \frac{2\pi}{a}) = \nabla(p)$

- solutions: nonlocal SLAC-derivative; otherwise effective cutoff below $\frac{2\pi}{a}$ is introduced by $f(p)$

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The naive infinitesimal translations are generated by a nonlocal derivative operator! Can M_{def} be local?

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Setting for supersymmetric quantum mechanics

- transformations in the continuum,
  $\varphi^i(x) = (\chi(x), F(x), \psi(x), \bar{\psi}(x))$:

$$\delta\chi = -\bar{\varepsilon}\psi + \varepsilon\bar{\psi} \quad \delta F = -\bar{\varepsilon}\partial\psi - \varepsilon\partial\bar{\psi}$$
$$\delta\psi = -\varepsilon\partial\chi - \varepsilon F \quad \delta\bar{\psi} = \bar{\varepsilon}\partial\tilde{\varphi} - \bar{\varepsilon}F$$

- naive transformations on the lattice, $\phi_n^i = (\chi_n, F_n, \psi_n, \bar{\psi}_n)$:

$$\delta\begin{pmatrix}\chi\\ F\\ \psi\\ \bar{\psi}\end{pmatrix} = \begin{pmatrix} 0 & 0 & -\bar{\varepsilon} & \varepsilon \\ 0 & 0 & -\bar{\varepsilon}\nabla & -\varepsilon\nabla \\ -\varepsilon\nabla & -\varepsilon & 0 & 0 \\ \bar{\varepsilon}\nabla & -\bar{\varepsilon} & 0 & 0 \end{pmatrix}\begin{pmatrix}\chi\\ F\\ \psi\\ \bar{\psi}\end{pmatrix} = (\varepsilon M + \bar{\varepsilon}\bar{M})\phi$$

∇ solution of additional constraint (SLAC-derivative)

Setting for supersymmetric quantum mechanics

- invariant quadratic action in the continuum:

$$S_{\mathrm{cl}} = \int dx \left[\frac{1}{2}(\partial_x \chi) + \bar{\psi}\partial_x\psi - \frac{1}{2}F^2 + \bar{\psi}W'(\chi)\psi - FW(\chi) \right]$$
$$= \int dx \left[\frac{1}{2}(\partial_x \chi) + \bar{\psi}\partial_x\psi - \frac{1}{2}F^2 + m\bar{\psi}\psi - mF\chi \right]$$

- ansatz for the lattice action $S = \frac{1}{2}\phi K \phi$:

$$\frac{K_{ij}}{a} = \begin{pmatrix} -\Box_{nm} & -m_{b,nm} & 0 & 0 \\ -m_{b,nm} & -I_{nm} & 0 & 0 \\ 0 & 0 & 0 & (\hat{\nabla} - m_f)_{nm} \\ 0 & 0 & (\hat{\nabla} + m_f)_{nm} & 0 \end{pmatrix}$$

I, $\Box$, m_b, m_f symmetric; $\hat{\nabla}$ antisymmetric
translation invariance: all circulant matrices ($\rightarrow$ commute)

Solutions for a quadratic action

- solve $M_{\mathrm{def}}^T K + K^T M_{\mathrm{def}} = 0$ with $M_{\mathrm{def}} = M(\mathbb{1} - \alpha^{-1} K)$
- $\alpha \sim \delta_{nm}$ (as for overlap) $\rightarrow$ nonlocal action

$$a(\alpha^{-1})_{nm} = \begin{pmatrix} a_2 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 0 & 0 & 0 & -a_1 \\ 0 & 0 & a_1 & 0 \end{pmatrix} \delta_{nm};\quad \begin{aligned} &\hat{\nabla} + m_f = \frac{\nabla + m_b}{1 + a_0 + a_1 mb + (a_1 + a_2 mb)\nabla} \\ &-\Box + m_b^2 = \frac{-\nabla^2 + m_b^2}{1 + a_0 - a_2 \nabla^2} \\ &I = \mathbb{1} \end{aligned}$$

- local actions (e. g. symmetric derivative) $\rightarrow$ generically nonlocal M_{def}
- M_{def} and K local $\Leftrightarrow$

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Beyond the quadratic action

- final goal: construct a supersymmetric local interacting lattice action
- the given relation extends beyond the quadratic case
- it connects different orders of the field $\rightarrow$ generically nonpolynomial solutions
- not unexpected since blocked action is comparable to the effective action
- under special conditions a truncation can be achieved

Remarks

- symmetry of a continuum action implies the fulfilment of certain relations for the lattice action which ensure a symmetric continuum limit and define deformed lattice symmetry operators
- requirement: definition of a naive lattice transformation by the "averaged" continuum symmetry transformation (additional constraint) $\hookrightarrow$ SLAC-derivative for SUSY
- severe restriction: M_{def} and the action must be local; can be fulfilled under special conditions
- although the relation couples different orders of the fields, even for interacting theories a polynomial solution can be achieved
- still much work to be done; but nonperturbative arguments are needed to ensure SUSY on the lattice
- alternative: use a different nonperturbative approach to verify lattice results

ERG calculations in supersymmetric theories

- SUSY GW relation corresponds to a modified Slavnov-Taylor-identity due to a non-invariant regulator [1]

$$e^{-(\Gamma_k[\phi]+\Delta S_k[\phi])} = \int d\varphi e^{-S[\varphi]-\Delta S_k[\varphi]+\frac{\delta(\Gamma_k[\phi]+\Delta S_k[\phi])}{\delta\phi}(\varphi-\phi)}$$

- complicate equations; better choose an invariant regulator
- supersymmetric regulator in terms of D and $\bar{D}$ (superspace)

$$\{M, D\} = \{M, \bar{D}\} = \{\bar{M}, D\} = \{\bar{M}, \bar{D}\} = 0$$

- flow equation for Γ [2] ($R_k(p)|_{p^2/k^2\to 0} > 0$; $R_k(p)|_{k^2/p^2\to 0} = 0$; $R_k(p)|_{k\to\Lambda\to\infty} \to \infty$):

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left[\Gamma_k^{(2)} + R_k\right]^{-1} \partial_k R_k\right\}; \quad \begin{matrix}\Gamma_{k\to\Lambda} \to S \\ \Gamma_{k\to 0} \to \Gamma\end{matrix}$$

[1] U. Ellwanger, *Phys. Lett.* **B335** (1994) 364–370

[2] C. Wetterich, *Phys. Lett.* **B301** (1993) 90–94

ERG calculations in supersymmetric theories

- SUSY GW relation corresponds to a modified Slavnov-Taylor-identity due to a non-invariant regulator [1]

$$e^{-(\Gamma_k[\phi]+\Delta S_k[\phi])} = \int d\varphi e^{-S[\varphi]-\Delta S_k[\varphi]+\frac{\delta(\Gamma_k[\phi]+\Delta S_k[\phi])}{\delta\phi}(\varphi-\phi)}$$

- complicate equations; better choose an invariant regulator
- supersymmetric regulator in terms of D and $\bar{D}$ (superspace)

$$\{M, D\} = \{M, \bar{D}\} = \{\bar{M}, D\} = \{\bar{M}, \bar{D}\} = 0$$

- flow equation for Γ [2] ($R_k(p)|_{p^2/k^2\to 0} > 0$; $R_k(p)|_{k^2/p^2\to 0} = 0$; $R_k(p)|_{k\to\Lambda\to\infty} \to \infty$):

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left[\Gamma_k^{(2)} + R_k\right]^{-1} \partial_k R_k\right\}; \quad \begin{matrix}\Gamma_{k\to\Lambda} \to S \\ \Gamma_{k\to 0} \to \Gamma\end{matrix}$$

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SUSY QM

[F. Synatschke et al.; arXiv:0809.4396]

Setting

- action in superspace

$$S = \int d\tau \left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}F^2 - i\bar{\psi}\dot{\psi} + iFW'(\varphi) - i\bar{\psi}W''(\varphi)\psi \right)$$

first term in convariant derivative expansion (SUSY) of Γ_k: W_k

- regulator term

$$\Delta S_k = \frac{1}{2}\int d\tau d\theta d\bar{\theta}\, \Phi(\tau,\theta,\bar{\theta}) R_k(D,\bar{D}) \Phi(\tau,\theta,\bar{\theta})$$
$$= \frac{1}{2}\int \frac{dp}{2\pi} d\theta d\bar{\theta}\, \Phi(-p,\theta,\bar{\theta})(ir_1(p) + r_2(p) D\bar{D})\Phi(p,\theta,\bar{\theta})$$

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$$\partial_k W_k(\phi) = \frac{1}{2}\int \frac{dp}{2\pi} \frac{\partial_k r_1}{p^2 + (r_1 + W_k''(\varphi))^2}$$

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Methods

- different regulators $\hookrightarrow$ small difference
- polynomial truncations $\hookrightarrow$ only for convex potentials

$$W_k(\varphi) = \sum_{n=1}^{N} \frac{\tilde{a}_n(k)}{n} \big(\varphi - \varphi_0(k)\big)^n$$

  compared with solution of the partial differential equation for $W(\varphi, k)$
- wave function renormalisation $\hookrightarrow$ improvement

$$\Gamma_k = \int d\tau d\theta d\bar{\theta} \left(\frac{1}{2} \underbrace{\mathcal{Z}_k(\Phi) D\bar{D} \mathcal{Z}_k(\Phi)}_{\mathcal{Z}'_k(\varphi)^2(F^2+\dot{\varphi}^2)+\dots} + iW_k(\Phi) \right)$$

(flow of $\mathcal{Z}_k$ from F^2 term)

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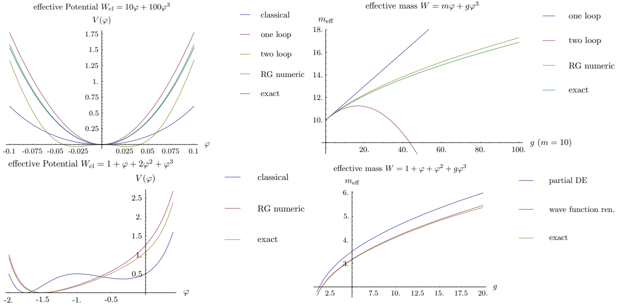
Results

- on-shell Potential: $\frac{1}{2}F^2 + iFW' \longrightarrow \frac{1}{2}W'^2$ $(\partial_\varphi^2 V|_{\varphi_{\min}}$: effective mass)



- no SUSY breaking $\Rightarrow$ ground state energy $= 0$; resonable results
- SUSY truncation: F on the same level as $\dot{\varphi}$; but $F^3 \ldots$ contribute to lowest order on-shell derivative expansion

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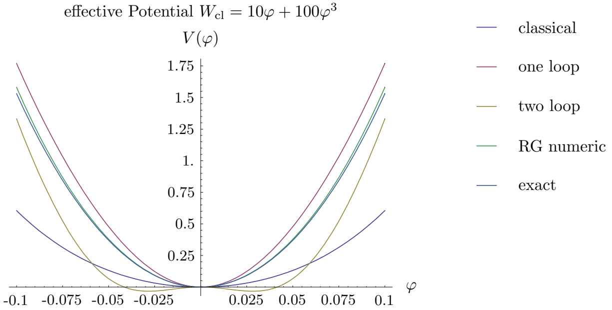

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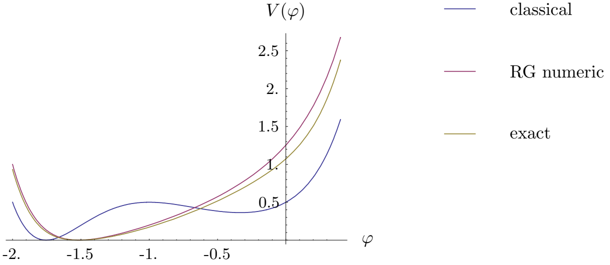
effective Potential $W_{\text{cl}} = 1 + \varphi + 2\varphi^2 + \varphi^3$



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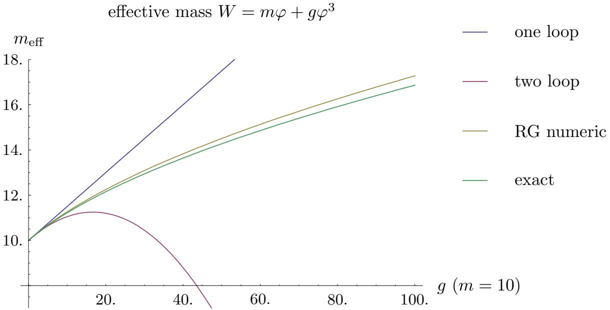

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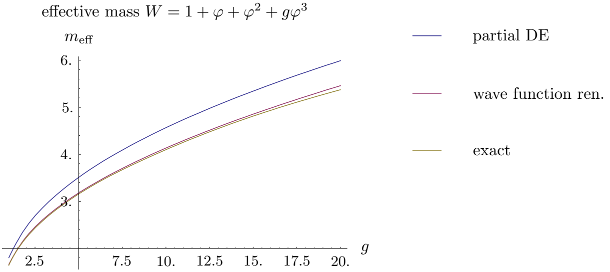

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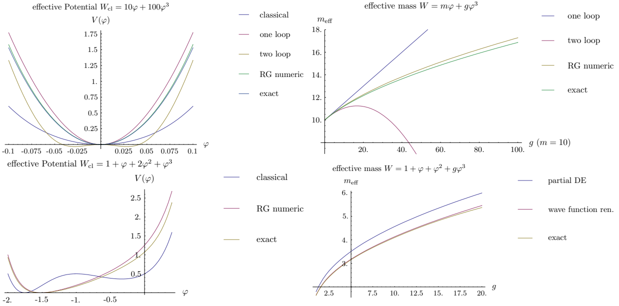

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2D N=2 Wess-Zumino-Model [Chr. Wozars talk]

$$S = \underbrace{\int d^2x d\theta_1 d\theta_2 d\bar{\theta}_1 d\bar{\theta}_2 \Phi\bar{\Phi}}_{FF^* + |\partial\varphi|^2 + \dots} + \left(\int d^2x d\theta_1 d\theta_2 W(\Phi) + \text{c.c.} \right)$$

- nonrenormalisation theorem: cancellation of fermionic and bosonic loops

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  $\Rightarrow$ only K renormalised
- higher terms of covariant derivative expansion important (truncation difficult)
- cancellations lead to better perturbative results (comparison perturbation theory / lattice results $\hookrightarrow$ Chr. Wozars talk)

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Outlook: 2D N=1 WZ-Model

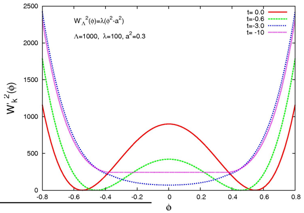
[F. Synatschke et al. in preparation]

- dimensional reduction: SUSY QM ⇒ "only" additional momentum / space integration
- different structure[3]: phase transition from unbroken to broken SUSY for a finite a in $W(\phi) = \lambda(\frac{\phi^3}{3} - a^2\phi)$
- divergent integrals ⇒ either cutoff-dependence or renormalisation

difficulties:

- lattice: Pfaffian
- ERG: ϕ dimensionless

preliminary:
(F.Synatschke)



[3] E. Witten, *Nucl. Phys.* **B202** (1982) 253

Conclusions

Lattice

- lattice calculations can be useful for SUSY
- standard methods fail
- exist strategies to reduce problem
- final solution: GW relation
- interacting case: hard to find a solution

ERG

- possible to get good results from RG flow calculations
- no explicit SUSY breaking
- truncation possible problem: higher derivative and F terms needed especially for models with nonren. theorem

- best: both methods should agree $\Rightarrow$ truncation and discretisation errors are under control
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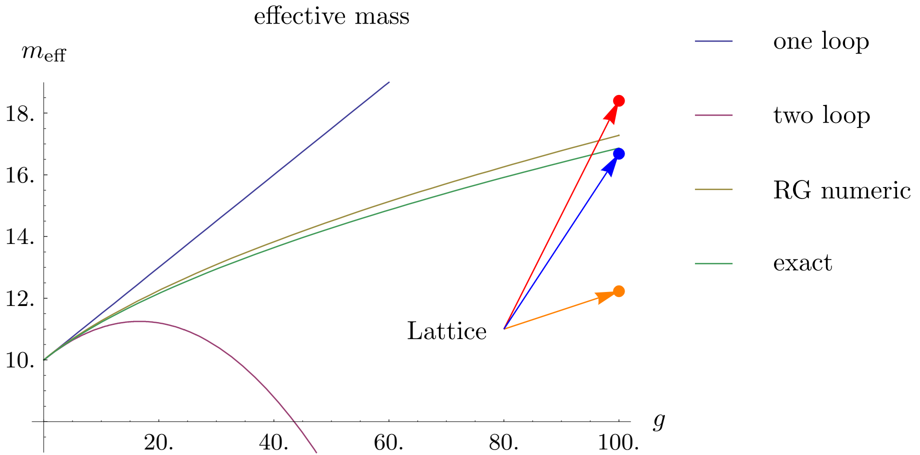
effective mass
m_{eff}
18.
16.
14.
12.
10.
20.
40.
60.
80.
100.
g
Lattice
one loop
two loop
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