Line of constant physics in QCD

Szabolcs Borsányi
(Wuppertal)

<u>based on unpublished work by</u>

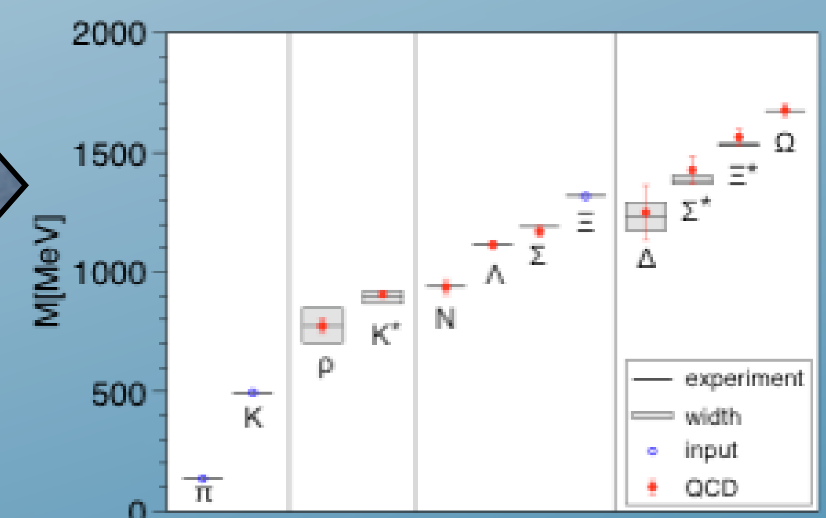
Yasumichi Aoki
Stephan Dürr
Zoltán Fodor
Antal Jakovác
Sándor Katz
Stephan Krieg
Kálmán Szabó
(and myself, of course)

Lattice field theory -- ideally

Few easily accessible experimental data

m_π = 135 MeV
m_K = 495 MeV
m_Ξ=1317 MeV

Standard model’s prediction

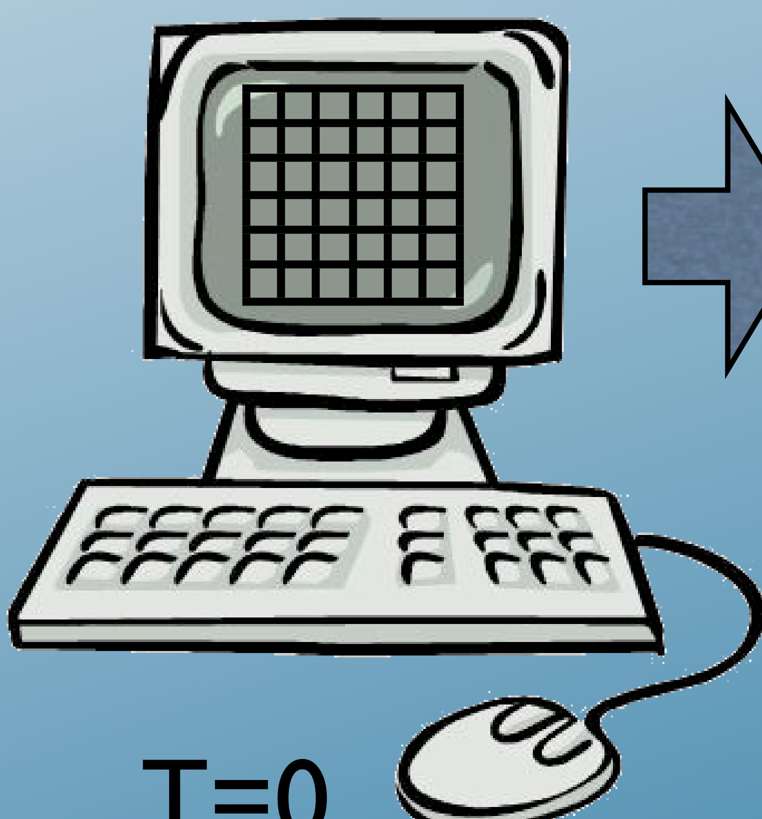


http://www.bmw.uni-wuppertal.de/

But the action needs bare parameters

Physical input:

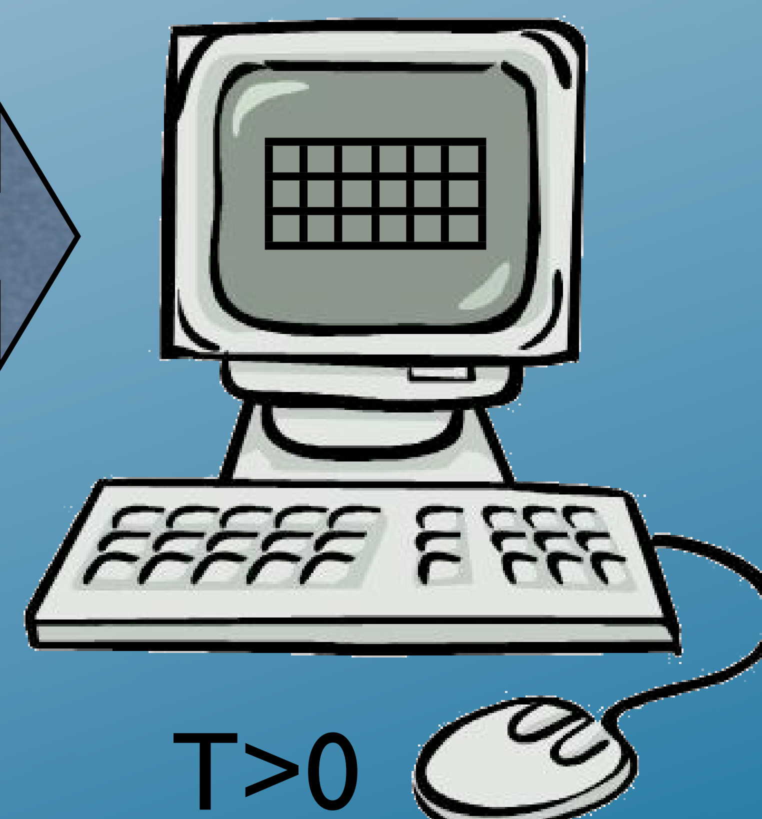
m_π = 135 MeV
m_K = 495 MeV
f_K=155.5 MeV



T=0

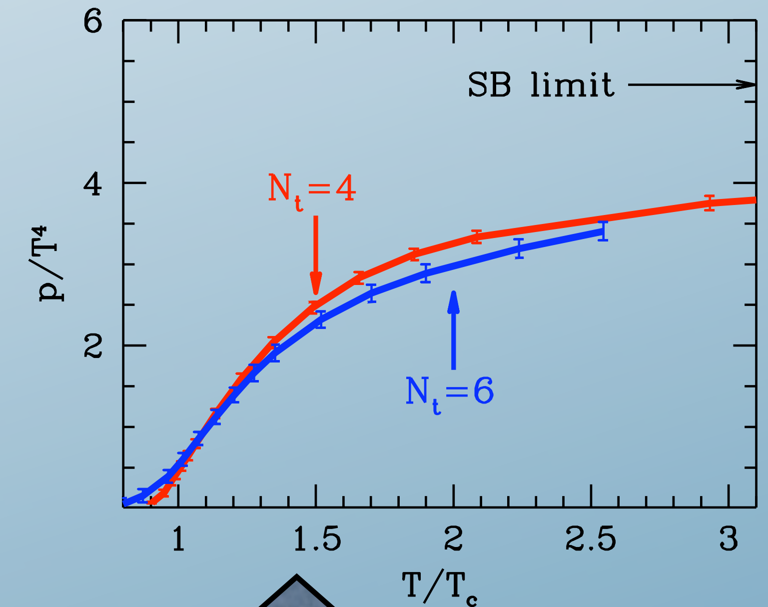
a: β,m_s, m_{ud}

Lattice QCD thermodynamics



T>0

We have to take a continuum limit, too



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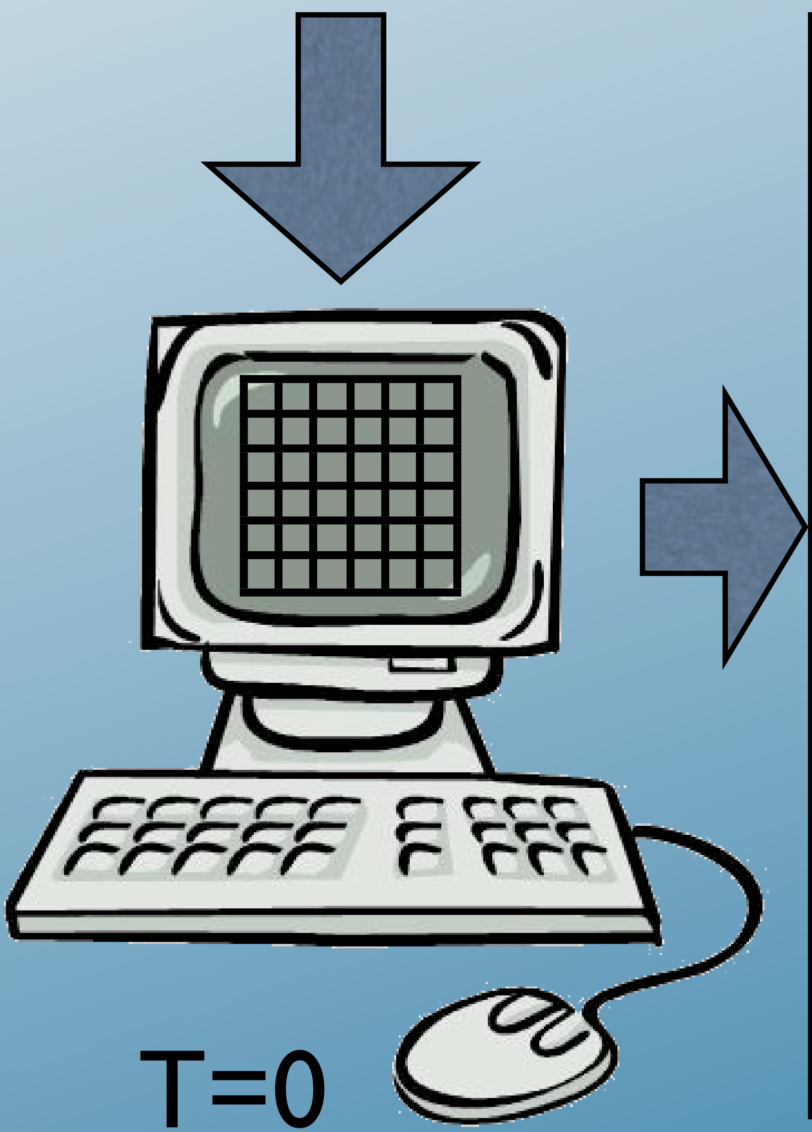
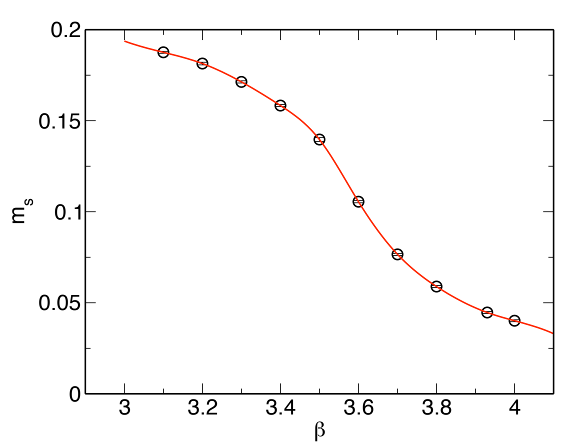


Line of constant physics

a_1: β,m_s, m_{ud}
a_2: β,m_s, m_{ud}
a_3: β,m_s, m_{ud}



T=0

T>0

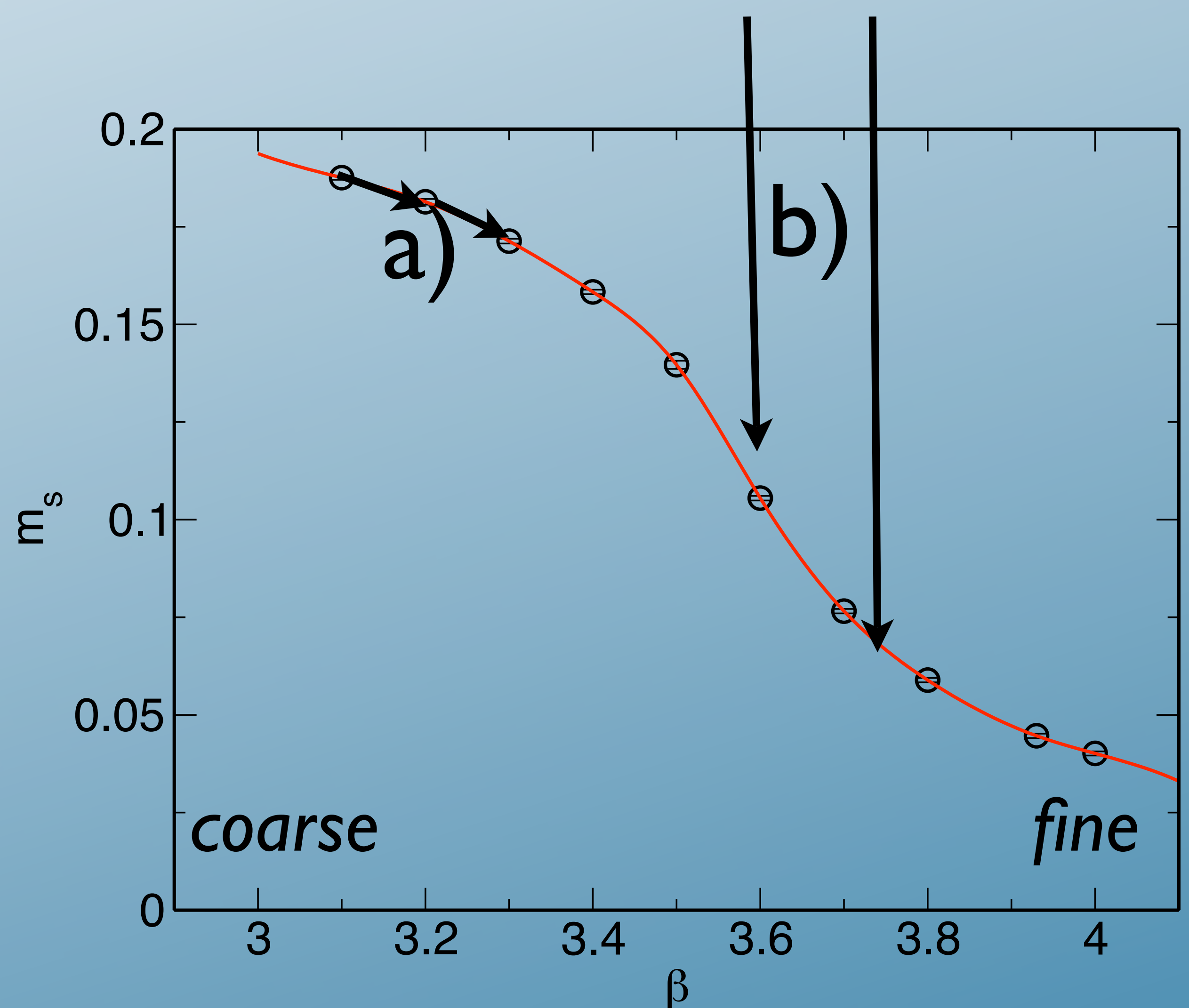
LCP in use

QCD pressure
(no direct measurement)

$$\frac{p}{T^4} = N_t^4 \left[\frac{1}{N_t N_s^3} \log Z(N_s, N_t; \beta, m_q) - \frac{1}{N_{t0} N_{s0}^3} \log Z(N_{s0}, N_{t0}; \beta, m_q) \right]$$

$$\frac{p}{T^4} = N_t^4 \int_{(\beta_0, m_{q0})}^{(\beta, m_q)} d(\beta, m_q) \left[\frac{1}{N_t N_s^3} \begin{pmatrix} \partial \log Z / \partial \beta \\ \partial \log Z / \partial m_q \end{pmatrix} - \frac{1}{N_{t0} N_{s0}^3} \begin{pmatrix} \partial \log Z_0 / \partial \beta \\ \partial \log Z_0 / \partial m_q \end{pmatrix} \right]$$

line integral



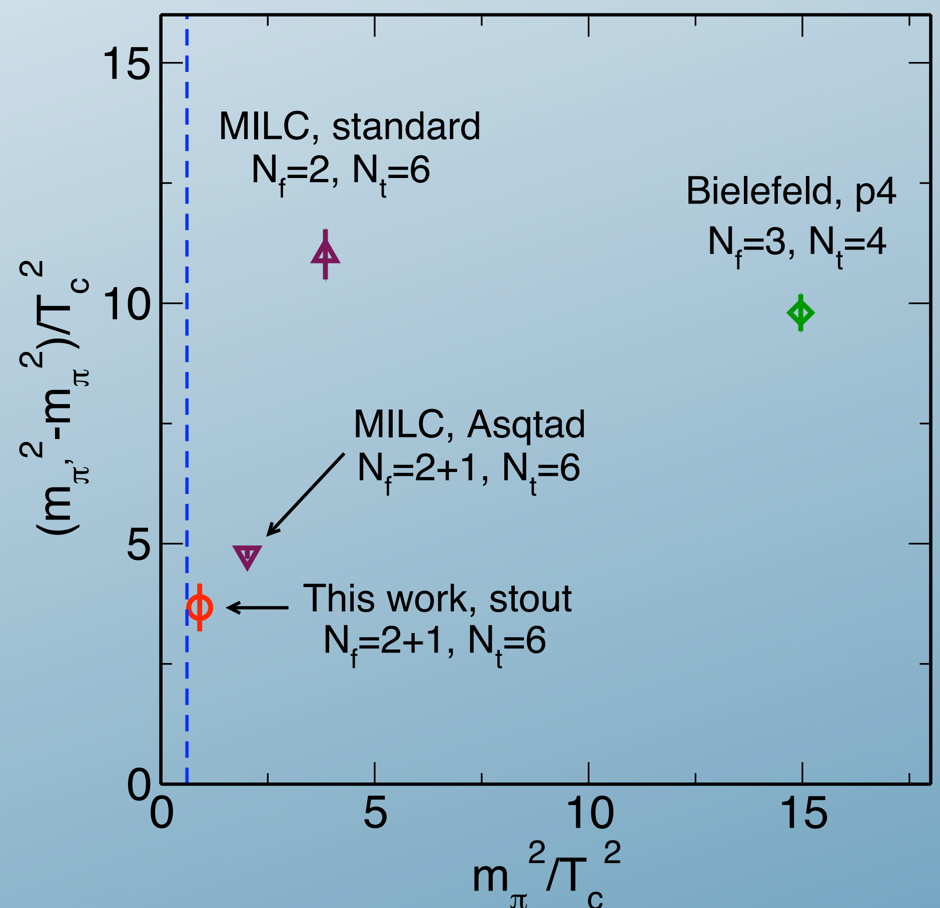
beta → -log(a) → log(T)

a) along the LCP
(assumes that the pressure is known at a reference temperature)

b) down from quenched
(a lot more simulation points, but much less statistics are required)

Action, simulation, ...

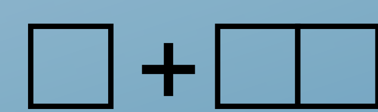
Gauge: Symanzik improved action
Fermion: Stout-improved staggered

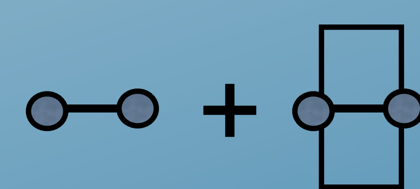


small taste violation, fast

$$S_g = \sum_x \frac{\beta}{3}\left(c_0 \sum_{\mu>\nu} W^{1\times1}_{\mu,\nu}(x) + c_1 \sum_{\mu\neq\nu} W^{1\times2}_{\mu,\nu}(x)\right),$$

$$S_f = \sum_{x,y}\{\overline{\eta}_{ud}(x)[\not{D}(U^{stout})_{xy} + m_{ud}\delta_{x,y}]^{-1/2}\eta_{ud}(y)$$
$$+\, \overline{\eta}_s(x)[\not{D}(U^{stout})_{xy} + m_s\delta_{x,y}]^{-1/4}\eta_s(y)\},$$

$S_g =$ 

$S_f =$ 

stout smearing parameters $\rho=0.15$ $N_{smr}=2$

(oversimplified)

[Morningstar, Peardon PRD69, 054501

Updating: Rational Hybrid Monte Carlo
+ improvemens

[Clark, Joo,Kennedy, hep-lat/0209035]

Machines

Blue Gene/P,
total sustained performance for QCD:
Jülich Supercomputing Centre: 82.5 Teraflops,
IDRIS/CNRS: 51,5 Teraflops

CPU and GPU clusters,
Bergische Universität Wuppertal
and at CNRS Marseille
31 Teraflops (sustained for QCD)

A recent tendency: QCD on GPUs

x 300

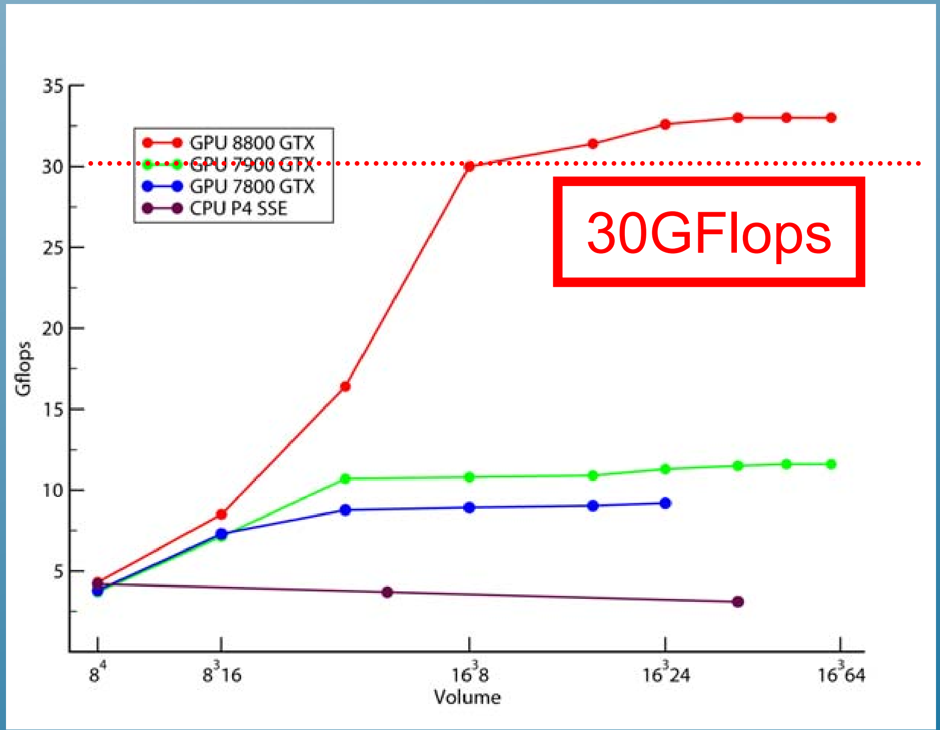
○ "**Lattice QCD as a video game**",
G.I.Egri, Z.Fodor, S.D.Katz, D.Nogradi,
K.K.Szabo, hep-lat/0611022.

Simulation codes are ported to Graphical processors (OpenGL / Cuda)

(> 700 Gflop)
(> 110 GB/sec)

Our staggered code:
10-30 Gflop sustained

Currently: max size/card is $24^3 48$
This limit will soon be broken
- new tesla cards,
- communication support in our code



Quark masses

1. light quarks: $m_{ud}(m_s)$

a) leading order χpt:

$$(m_{\bar{s}s}/m_K)^2 \sim m_s/(m_{ud}+m_s)$$

b) keep this constant.

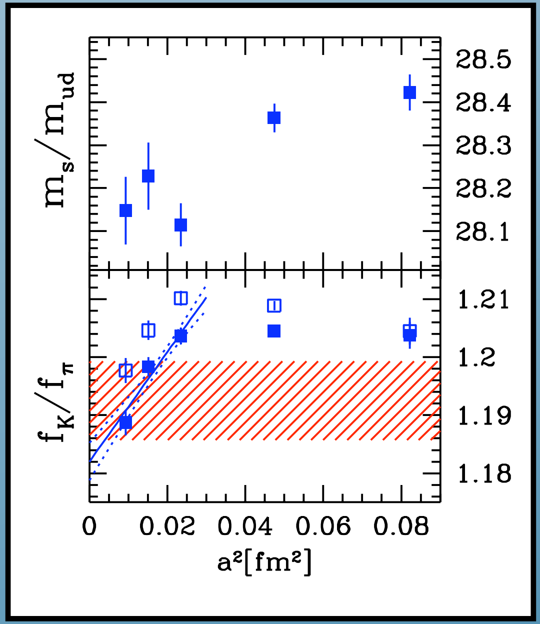
$$m_s/m_{ud} = 10$$

[Cheng et al PRD77,014511]

$$m_s/m_{ud} = 27.3$$

[Aoki et al Nature 443,675]

c) Find final m_s/m_{ud} by interpolation



2. strange quark $m_s(\beta)$

trial runs + χpt

$$m_{\bar{s}s} = \sqrt{2m_K^2 - m_\pi^2} = 686\ \mathrm{MeV}$$

$$\left(r^2 \frac{\mathrm{d}V_{\bar{q}q}(r)}{\mathrm{d}r}\right)_{r=r_0} = 1.65,$$

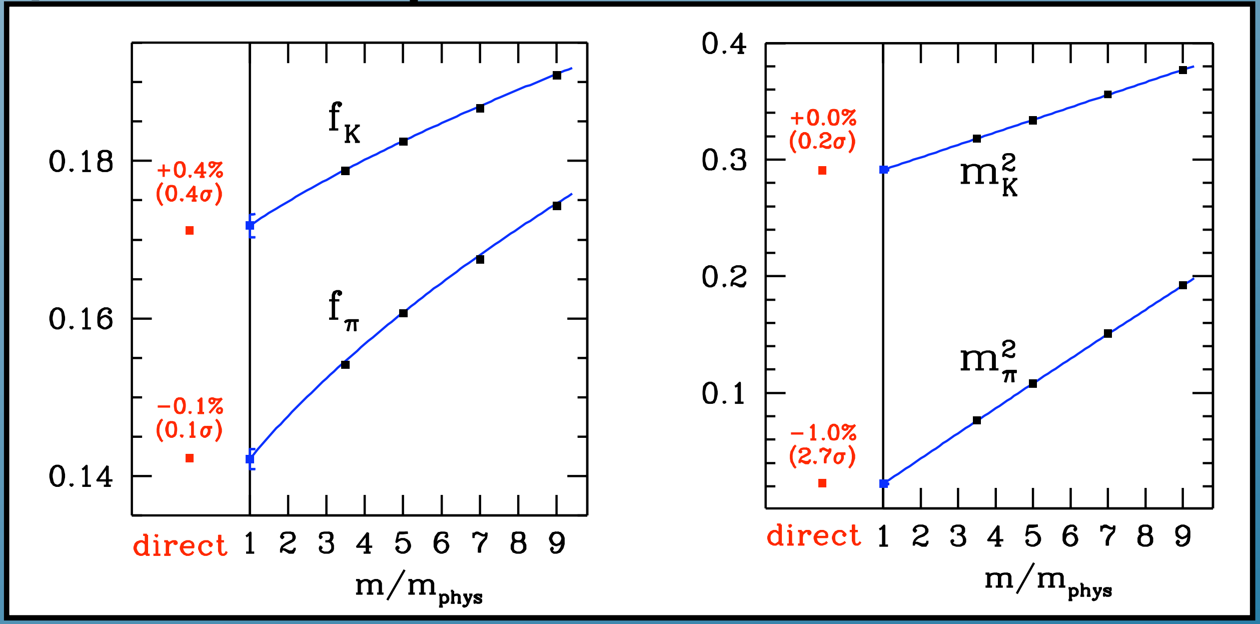
$$m_{\bar{s}s} r_0 = 1.59,$$

[Cheng et al PRD77,014511]

m_K/f_K=135/159.8

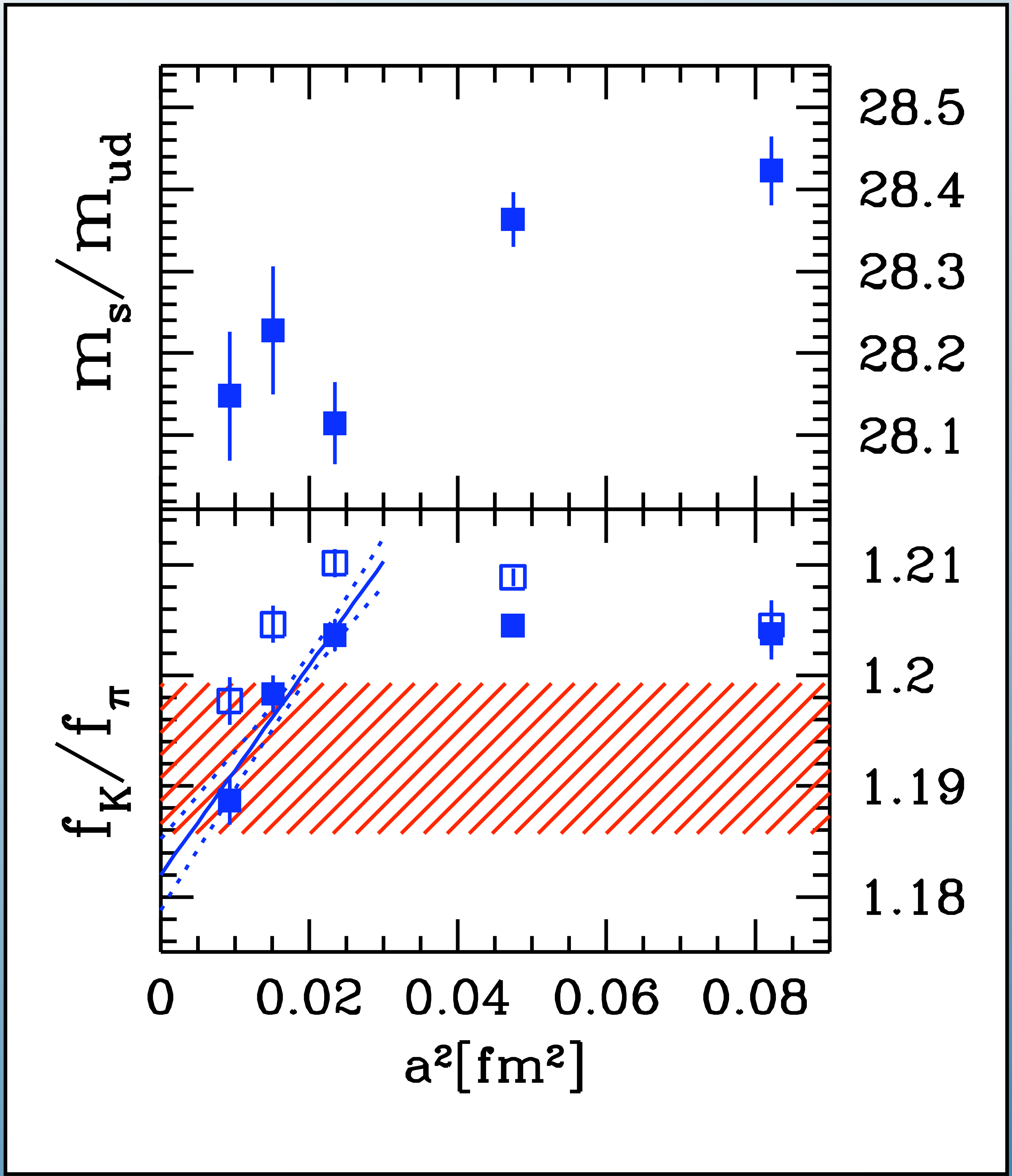
[Aoki et al Nature 443,675]

[Aoki *et al* hep-lat/0609068]



[Aoki et al. in prep]

Quark masses



simulations

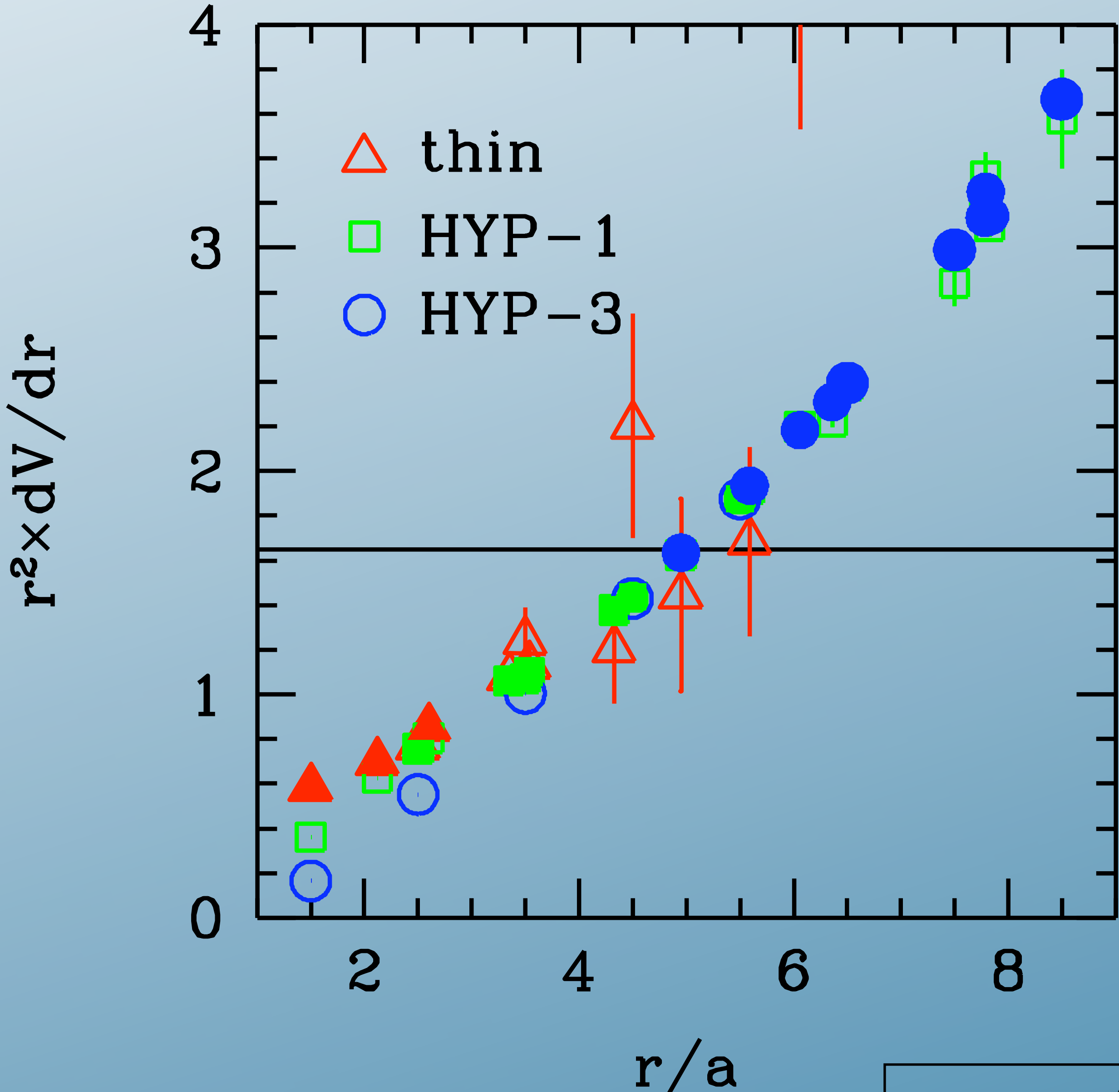
with $m_{ud}/m_{ud,LCP}$=1..10

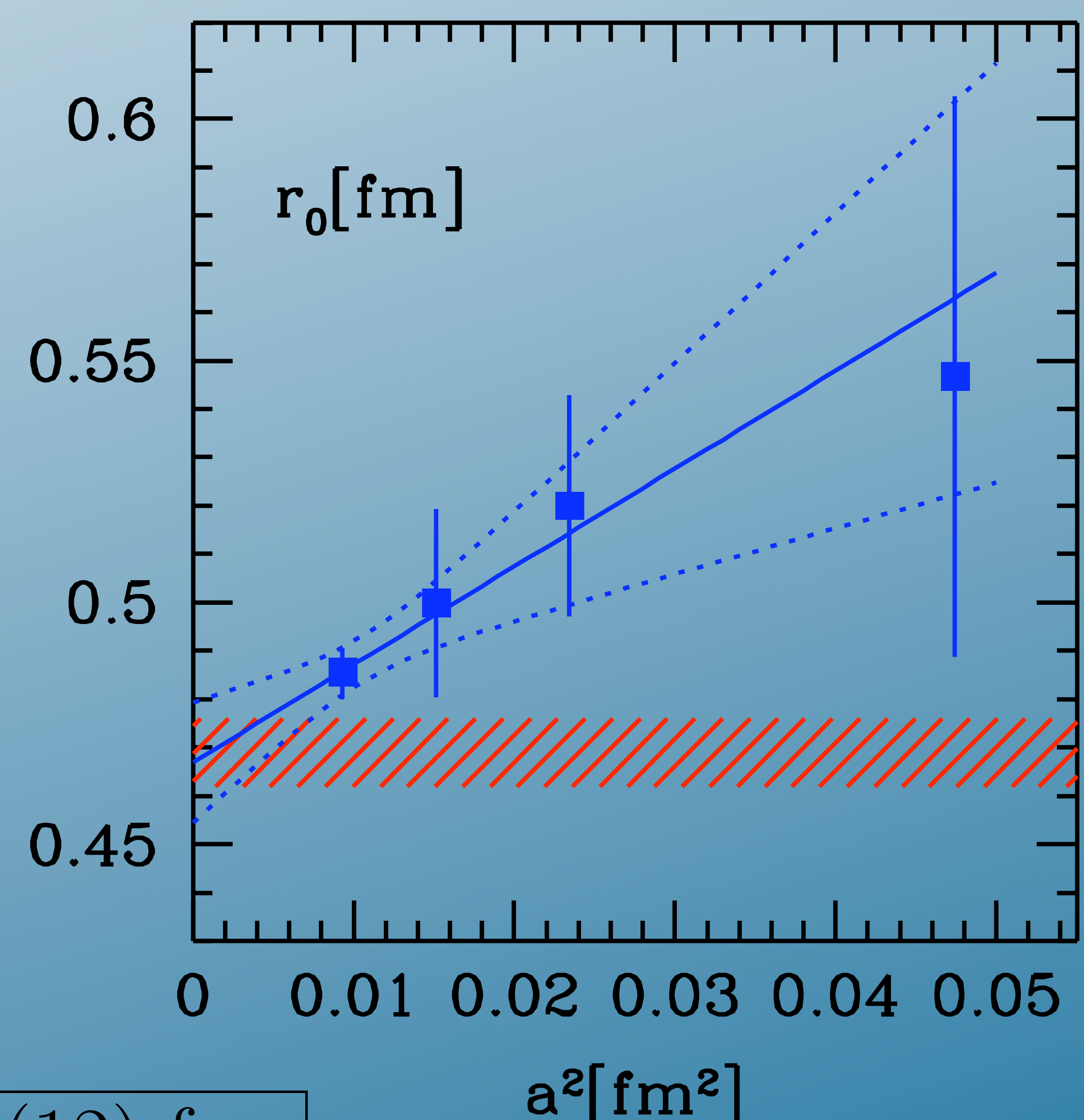
$$\boxed{m_s/m_{ud} = 28.15(8)}$$

$$f_K/f_\pi = 1.182(3)$$

[Aoki et al. in prep]

r_0 from the kaon condensate



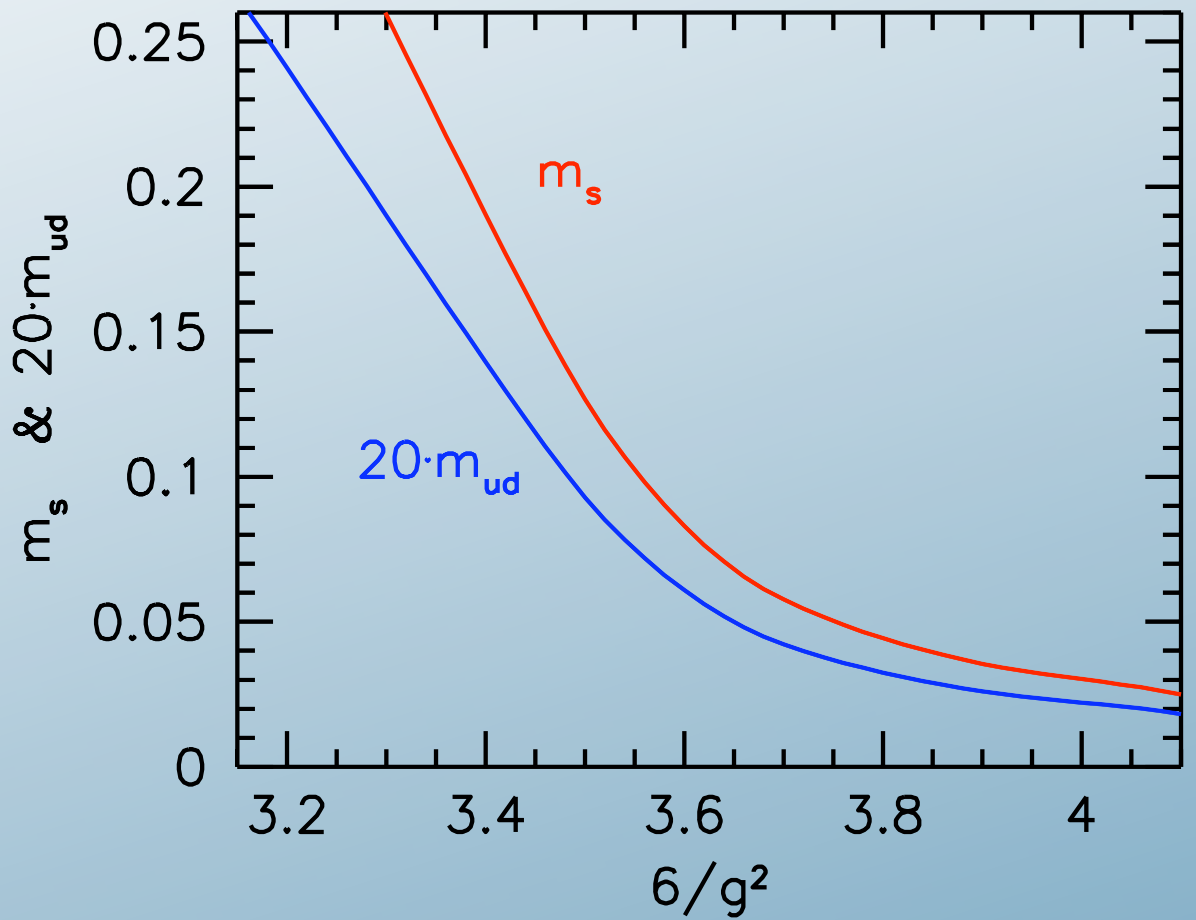


$$r_0 = 0.467(12)\ \text{fm}.$$

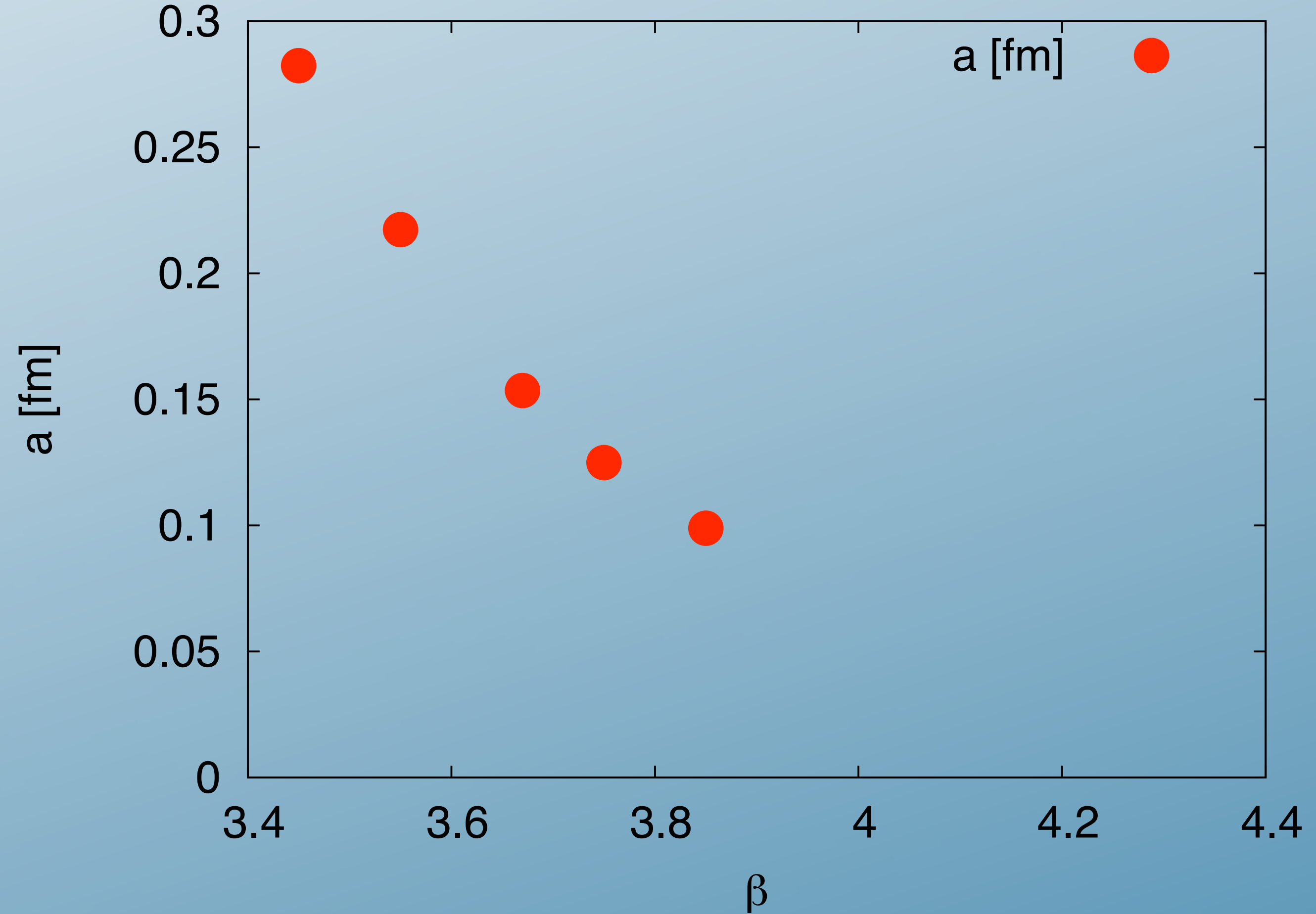
charmonium mass splitting: r_0=0.469(7) fm

[Aoki et al. in prep]

LCP up to a<0.1 fm



[Aoki et al Nature 443,675]



[Aoki et al. in prep]

How to proceed?

Smaller lattice spacing requires bigger (numerical) lattice size so that the used scales (m_K,f_K,m_π) fit into the box.

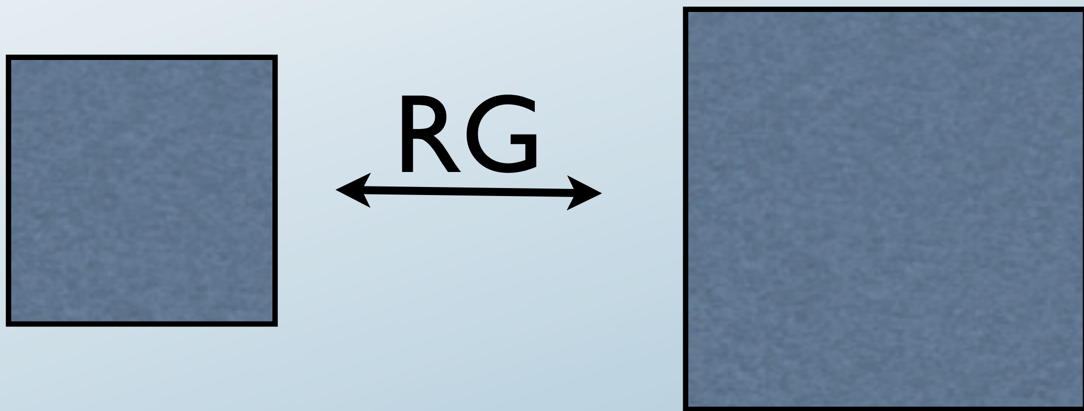
Using $N_f = 3$

The beta function and the mass renormalisation is N_f dependent, but mass independent in the continuum limit

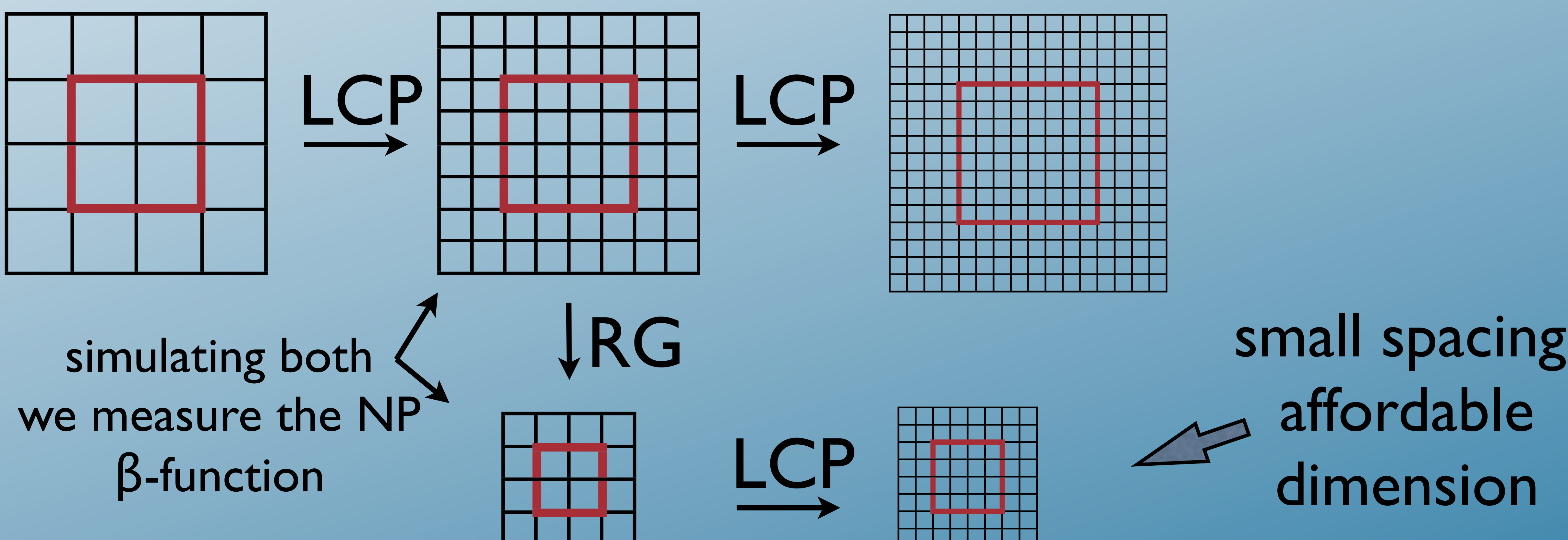
We set $m_q := m_s, N_f := 3$.
But what is the physics to keep fixed along the LCP?

1) Calculate the continuum limit of
   m_{PS} and f_{PS} along the 2+1 flavour LCP
2) These (unphysical) values define the 3 flavour LCP
3) We extract $m_u = m_q/28.15$, $m_s = m_q$
4) Check LCP with further 2+1 flavour simulations

RG on the lattice: Step scaling



Renormaliation group:
RG flow equations connect the two continuum systems with very different physics



.. if we made the RG step in the continuum limit

This scheme has been previosly used to measure the running coupling.

[Lüscher, Weisz, Wolff NPB359,221]
[Lüscher, Sommer, Weisz, Wolff NPB413,481]

LCP from *a*-halving

From the known bit of 2+1f LCP: N_s=4,6,8,10,12

We do N_f=3 simulations with fixed physical volume:

$f_{PS}L$=1.3 (201Mev)

The $m_{PS}L$ is not constant, but has a continuum limit

$m_{PS}L$=4.9 (758Mev)

Then we double N_s, keeping $m_{PS}L$ and $f_{PS}L$ fixed.
This involves a search in the *(beta,*m_q*)* space.
This way we arrive at beta=4.057; a=0.064 fm
After this point:

a) matching m_q*(beta)* to perturbative running
b) continuing with m_{PS}\`=1.66 $m_{PS}$ ; $m_q$\`/$m_q \rightarrow$ *1.64*

Checks: m_q(beta) from a) vs b)

b) pushing the LCP with heavier “pion”

We know LCP up to beta<=4.057

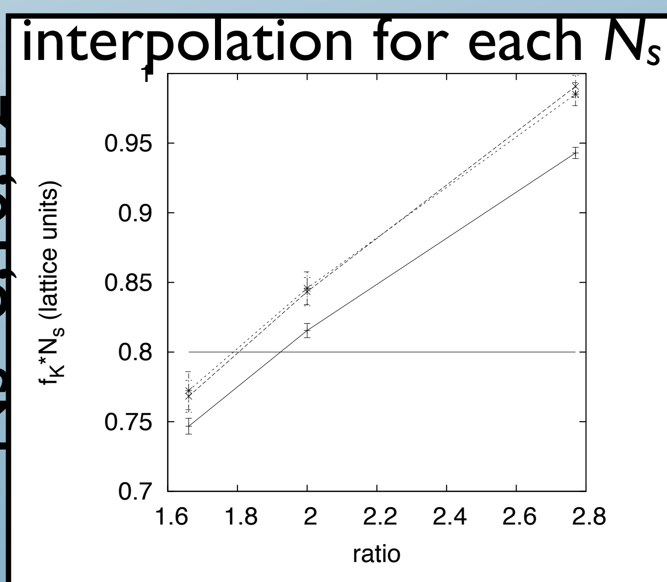
We had $m_{PS}L$=4.9 at N=20.

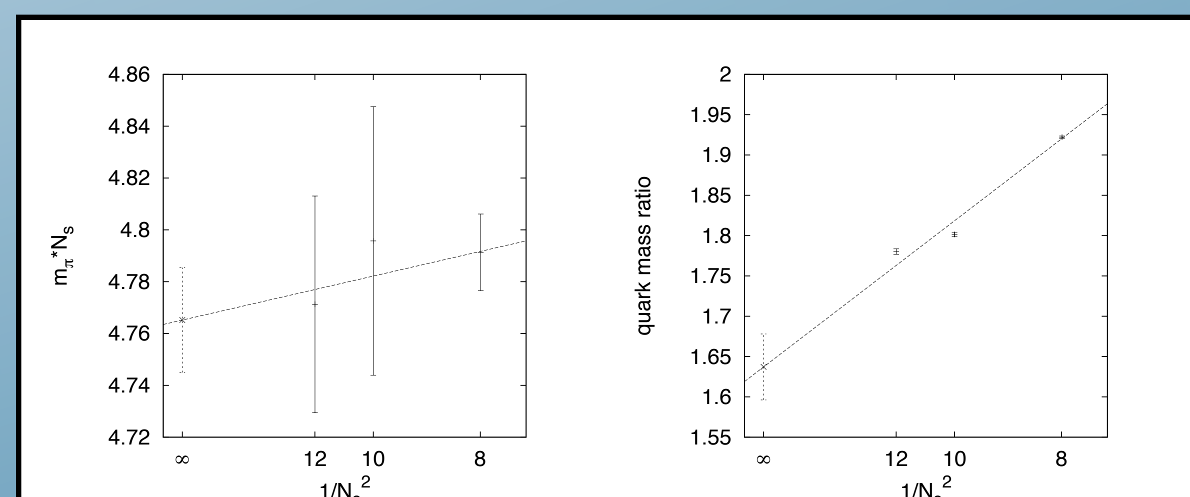
New lattice: L’=L/1.667, but with m_{PS}’L’=4.9

($f_{PS}L$’=0.8, properly scaled)

First we search m_q’/m_q so that $f_{PS}L$’ is kept fixed.

Ns=8,10,12



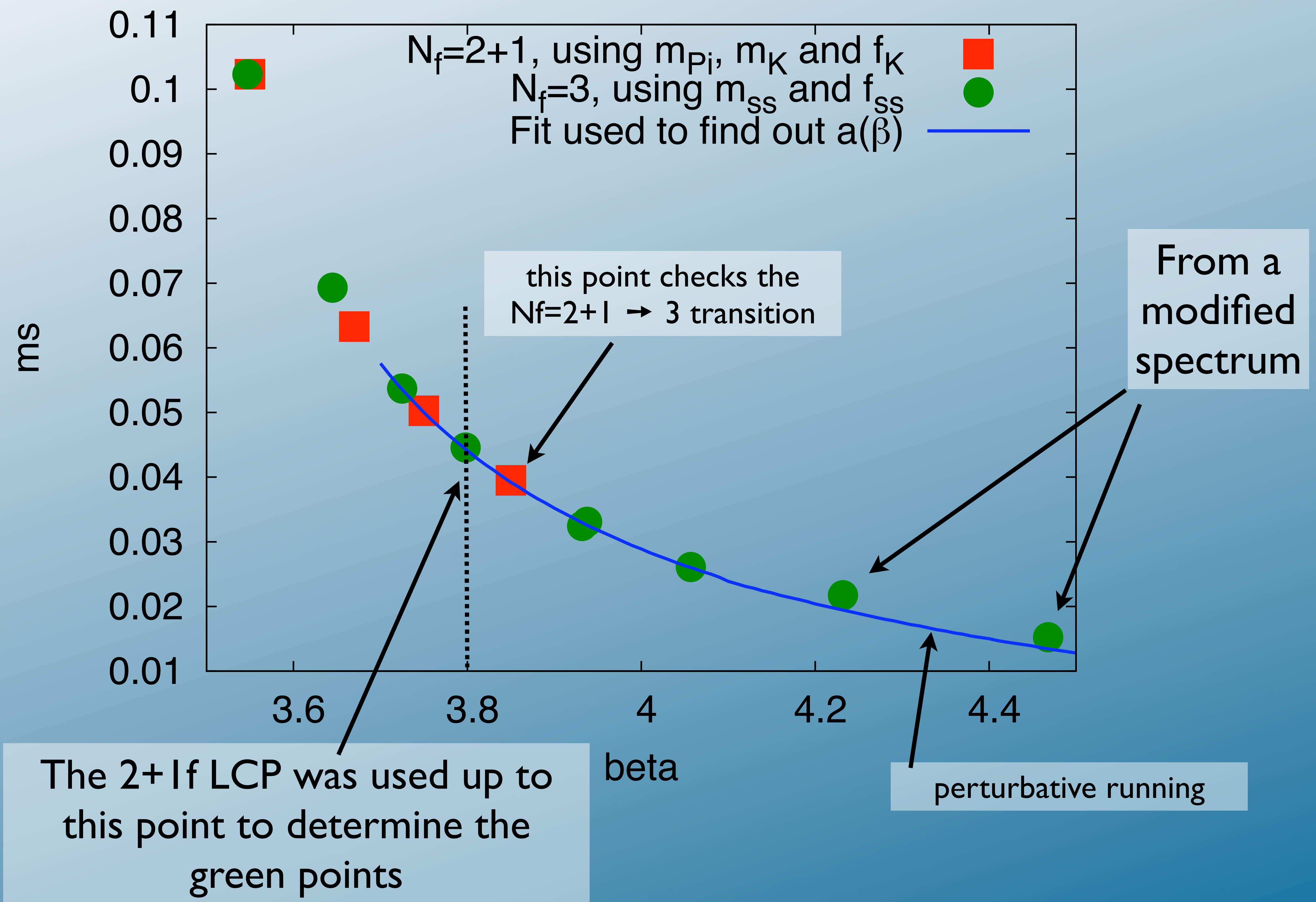


m_q`/m_q → 1.64

Then we search beta and m_q’ for the N=16,20 lattices.

m_q is then scaled back to the physical quark mass

LCP from *a*-halving



But,

what is *a(beta)*,
down to arbitrarily small lattice spacings?
Can we reproduce the perturbative runnnig?
(as opposed to matching)
From which point on, is the running perturbative?

To answer these questions
we'll need a new dimensionless observable.

Alpha collaboration (quenched): g_{SF}
alternatively: <u>coupling constant from the Wilson loop</u>

Wilson loop scheme

[Itou&Kurachi Lattice08]

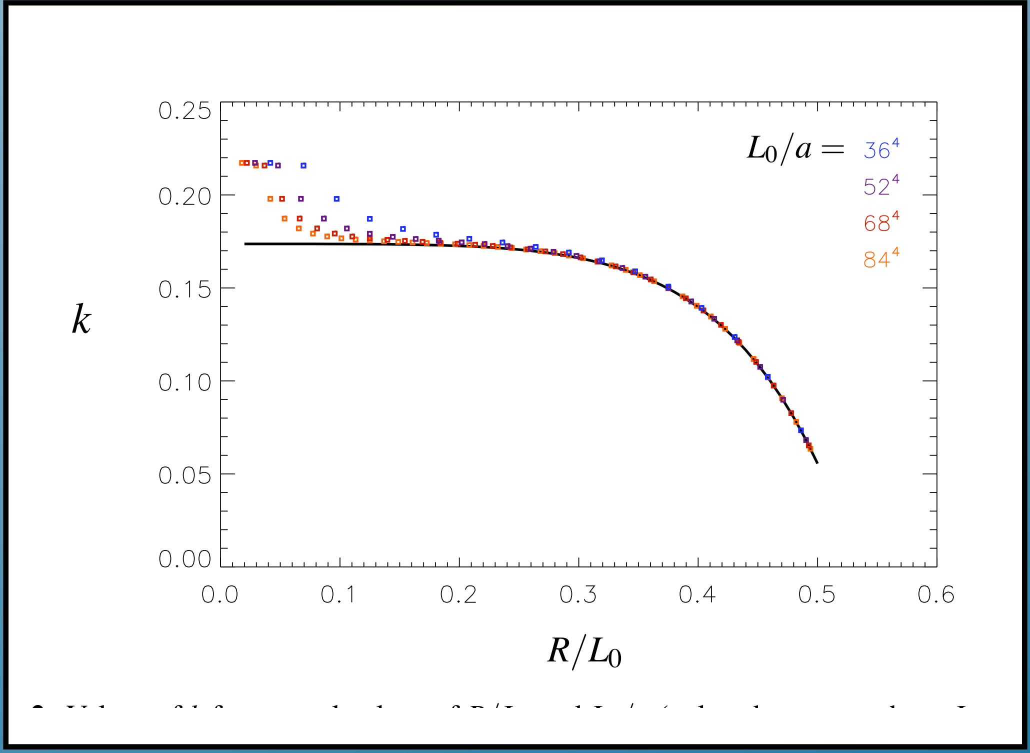
If we find an observable with $A^{\rm tree} = k g_0^2.$

then we evaluate non-perturbatively and define $g^2(\mu) = \dfrac{A^{\rm NP}(\mu)}{k}.$

Our choice: $-R^2 \dfrac{\partial^2}{\partial R \partial T} \ln \langle W(R,T;L_0)\rangle^{\rm tree}\Big|_{T=R} = k g_0^2,$

with
$$k = -R^2 \frac{\partial^2}{\partial R \partial T}\left[\frac{4}{(2\pi)^4} \sum_{n_0,n_1,n_2,n_3(\neq 0)} \left(\frac{\sin(\frac{\pi n_0 T}{L_0})}{n_0}\right)^2 \frac{e^{i\frac{2\pi n_1 R}{L_0}}}{n_0^2+\vec{n}^2} \right]_{T=R}$$
$$+ \text{zero mode contribution}.$$

[Coste et al NPB262,67]



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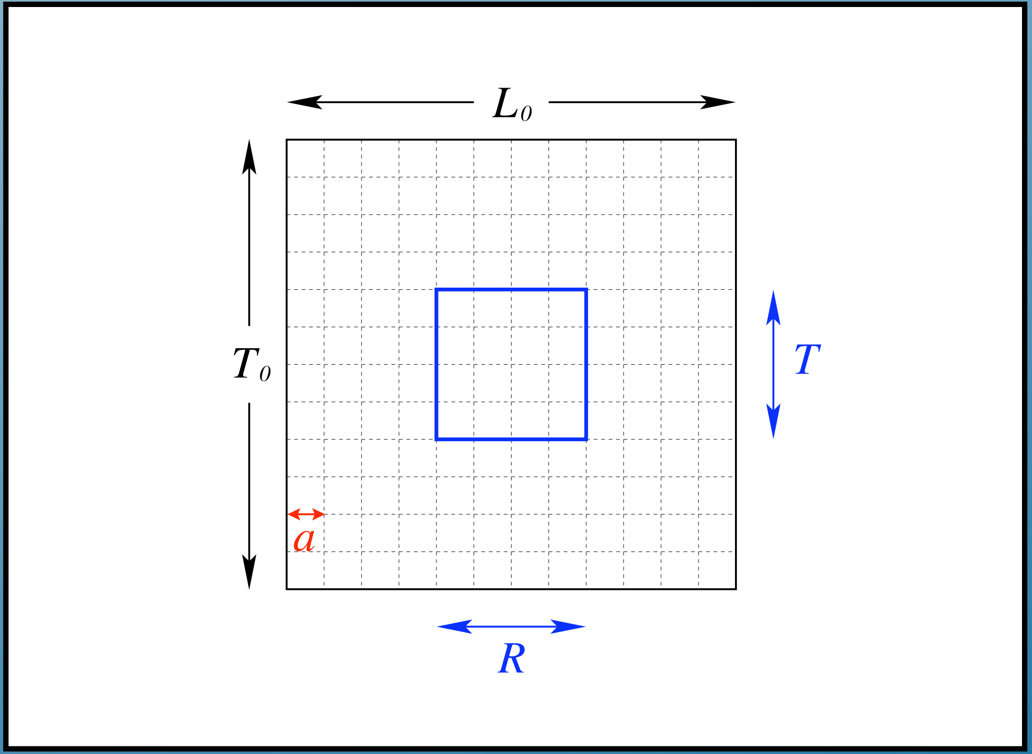
[Coste et al NPB262,67]

This defines a coupling ~ *Creutz ratio*

$$\chi(\hat{R}+1/2, \hat{T}+1/2; L_0/a) = -\ln \left(\frac{W(\hat{R}+1, \hat{T}+1; L_0/a)\, W(\hat{R}, \hat{T}; L_0/a)}{W(\hat{R}+1, \hat{T}; L_0/a)\, W(\hat{R}, \hat{T}+1; L_0/a)} \right),$$

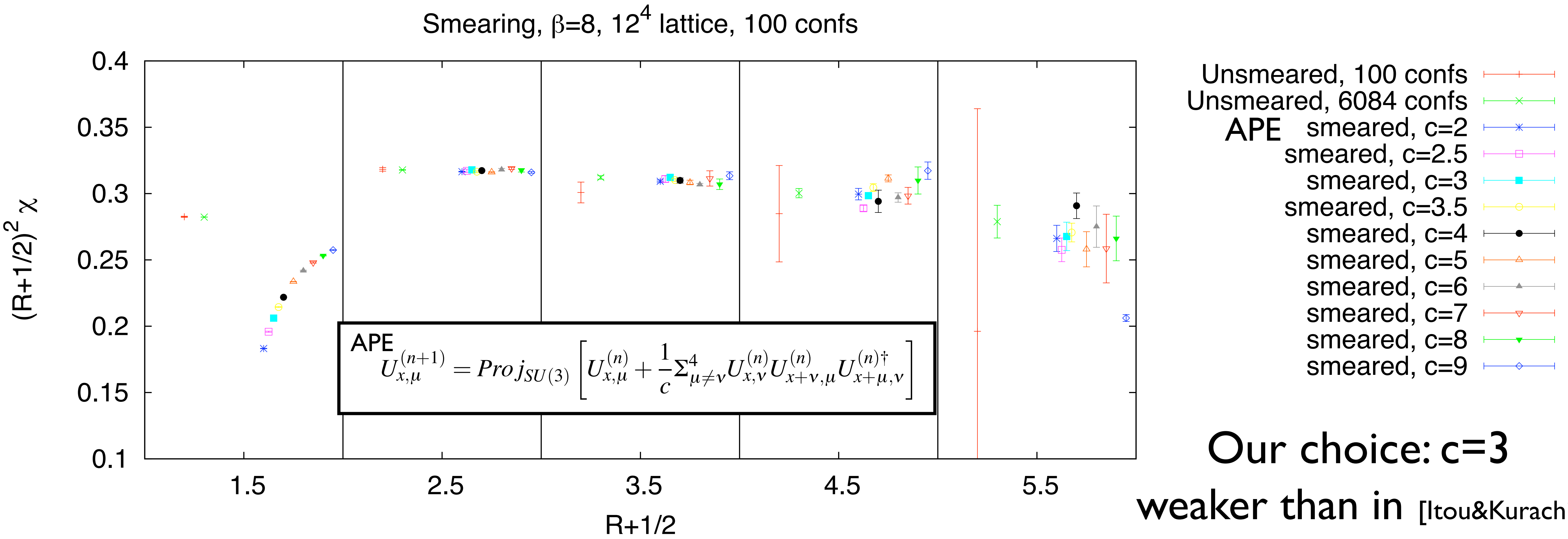


$$g_w^2\left(L_0, \frac{R+a/2}{L_0}, \frac{a}{L_0}\right) = (\hat{R}+1/2)^2 \cdot \chi(\hat{R}+1/2; L_0/a)/k$$

renormalisation scale

How to measure it at a desired μ?

The Creutz ratio is not sensitive to smearing, discretisation effects enter at a^2 order.



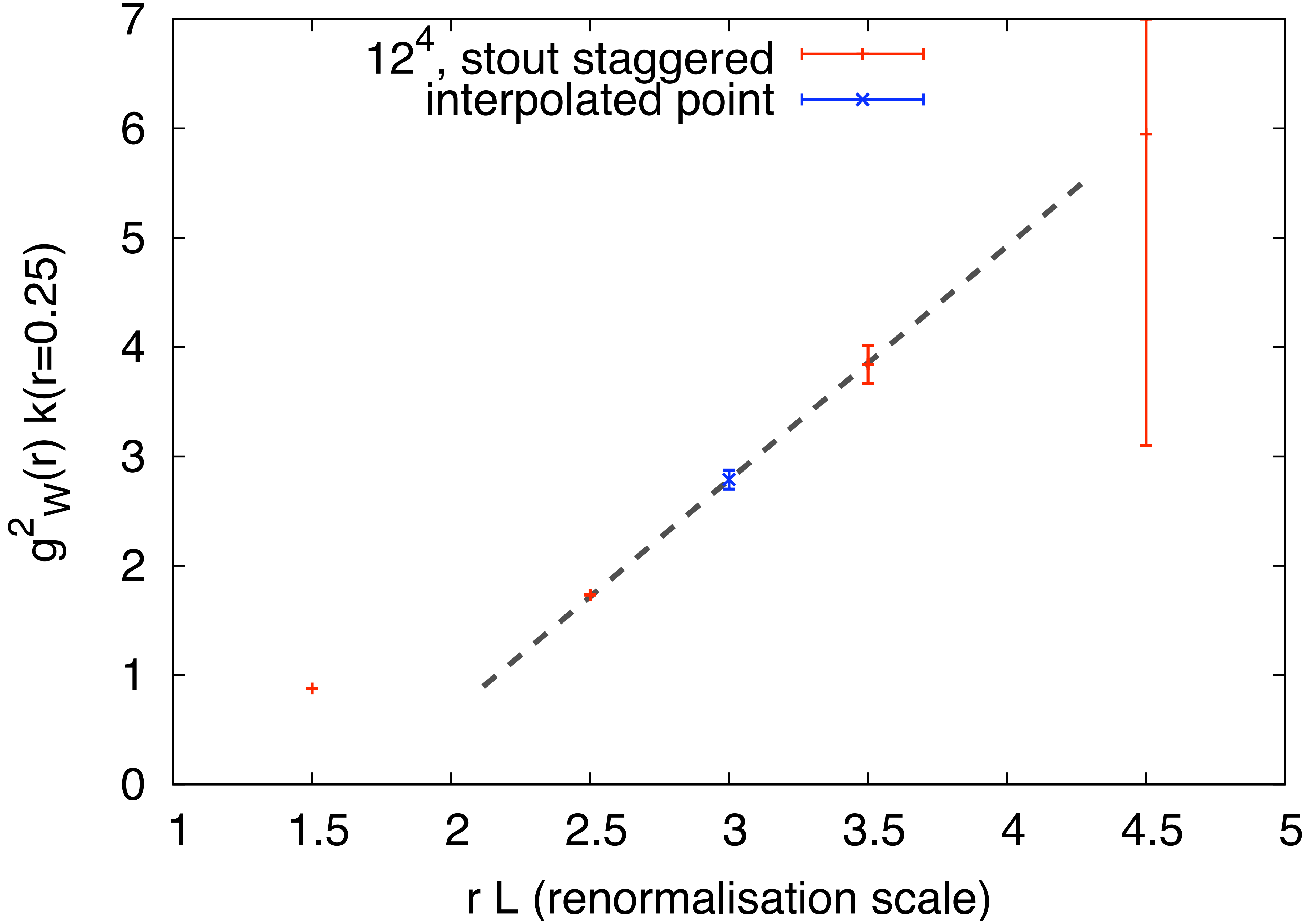
Our choice: c=3

weaker than in [Itou&Kurachi]

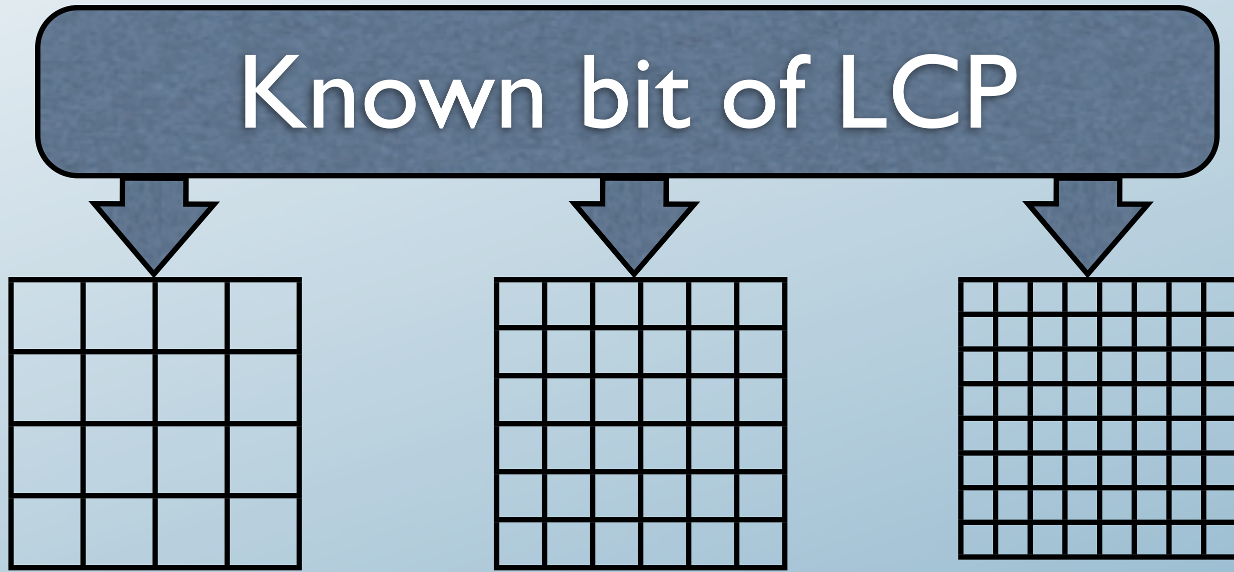
renormalisation scale in discrete points $R=1/a\mu$

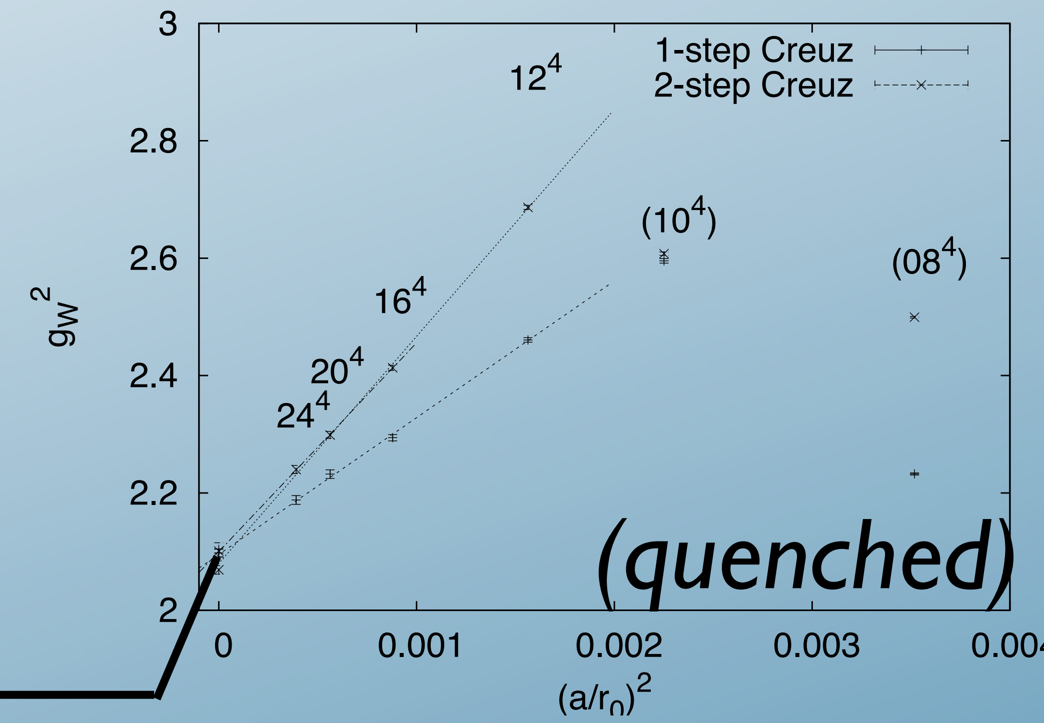
we interpolate get to the desired scale

Analogous plot with st. fermions



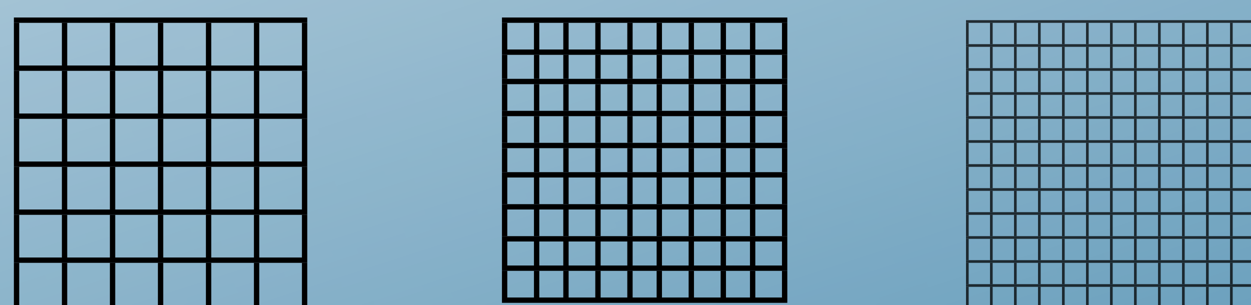
How to take the continuum limit ?





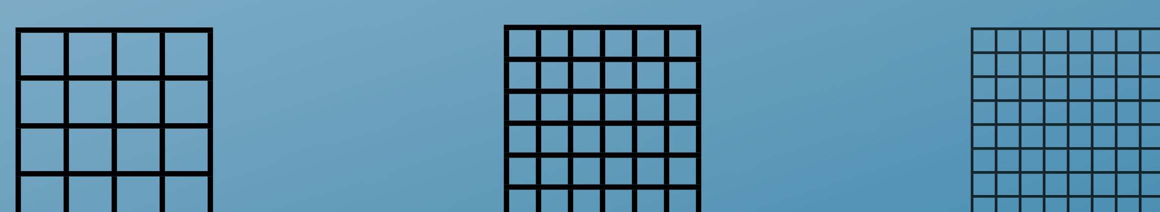
g_{cont}

Find bare parameters so that these big lattices have g_{cont}



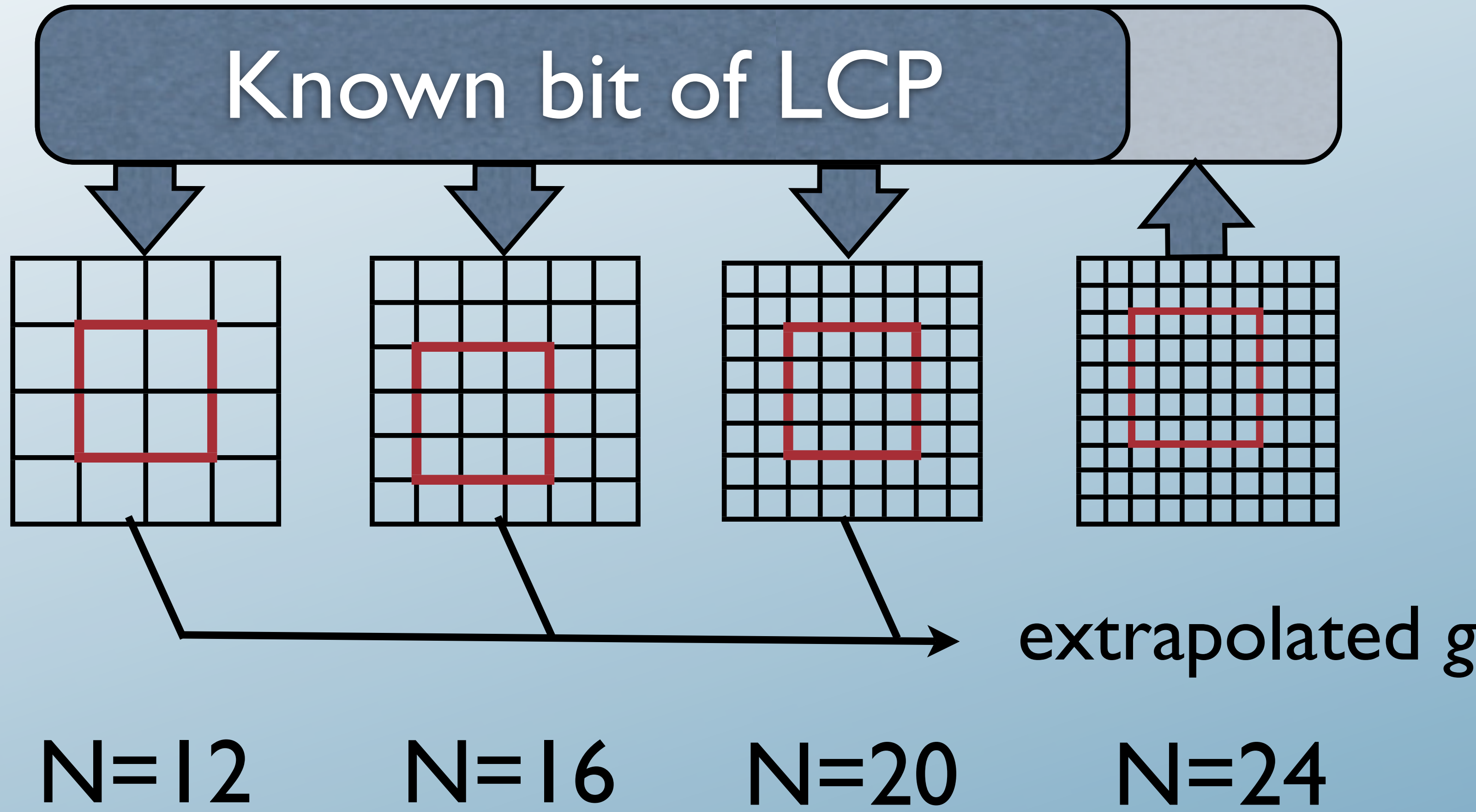
Room for improvement:
big lattices should have a g extrapolated to their own lattice spacing, not to zero.

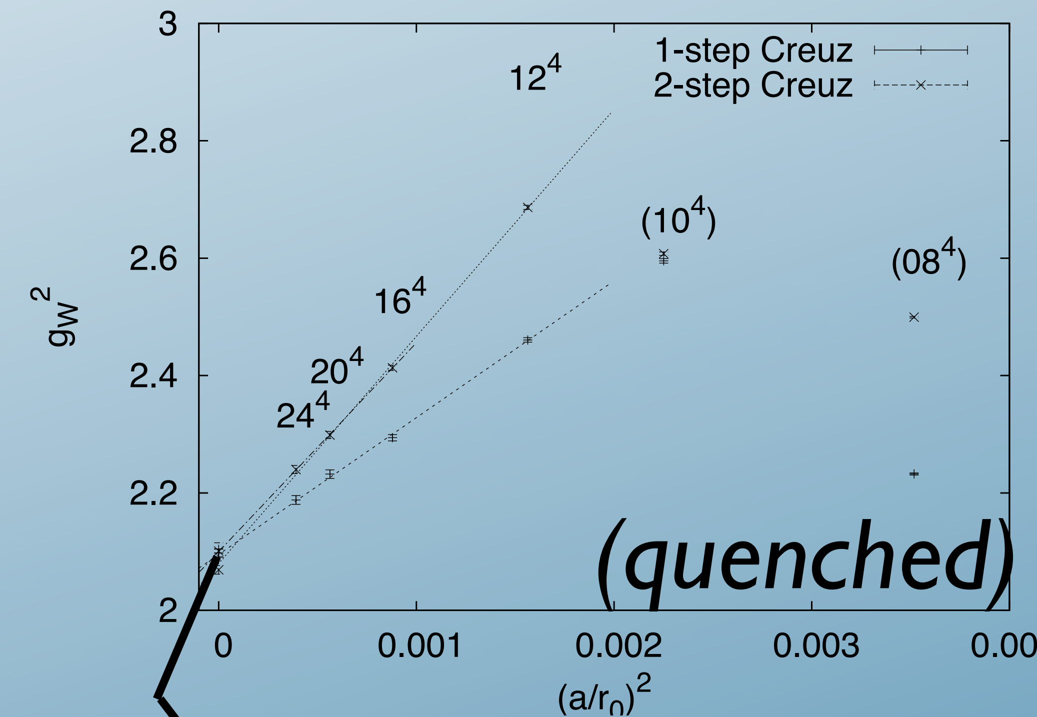
Change renormalisation scale / box size



Use this in step 2

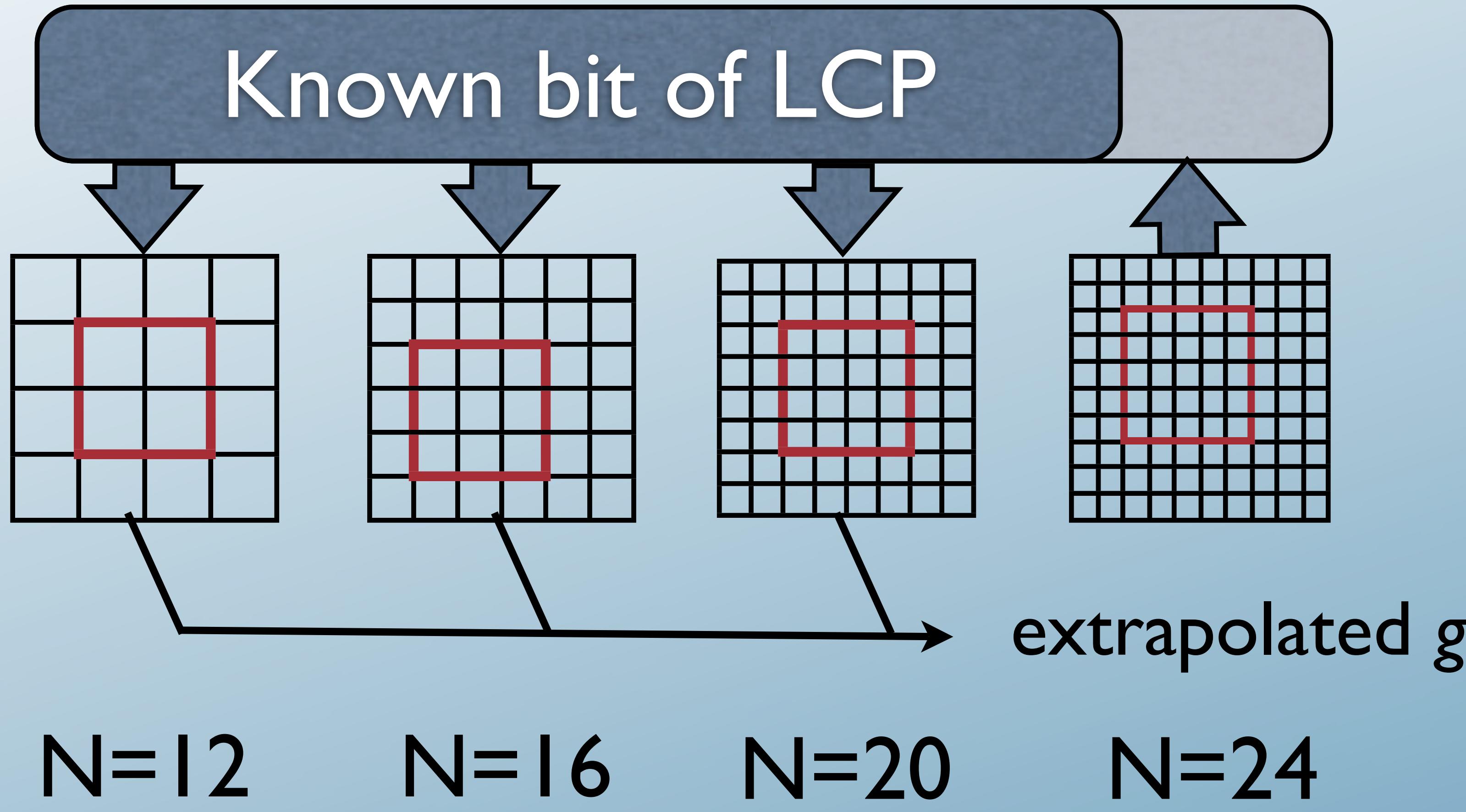
Our scheme

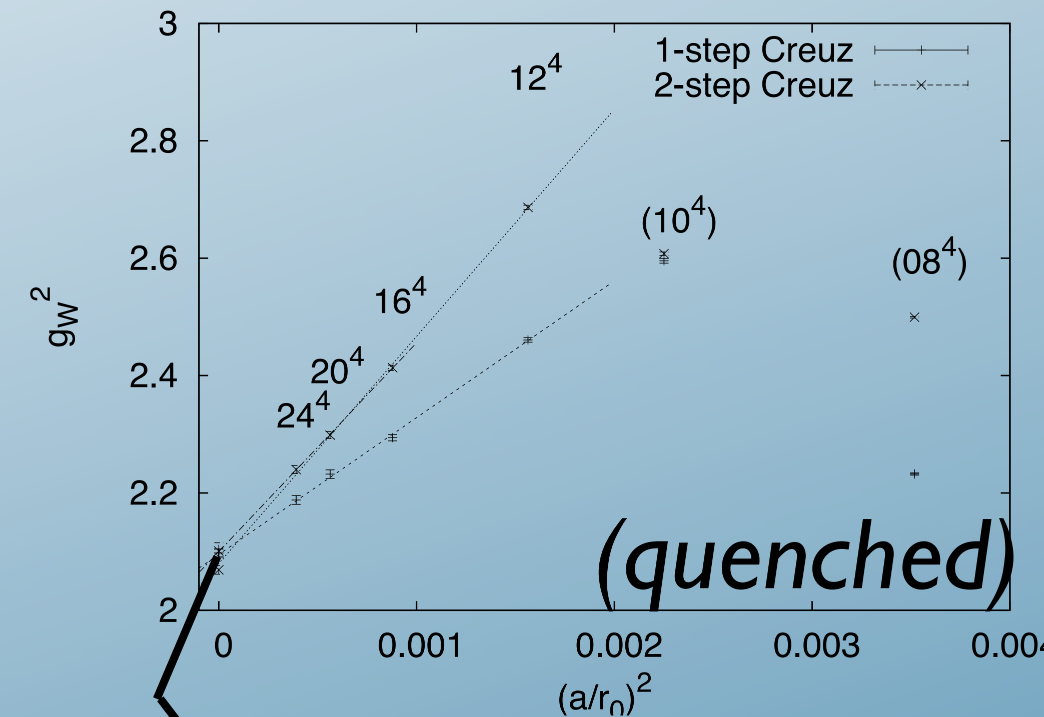




g_{cont} is used for the beta function only

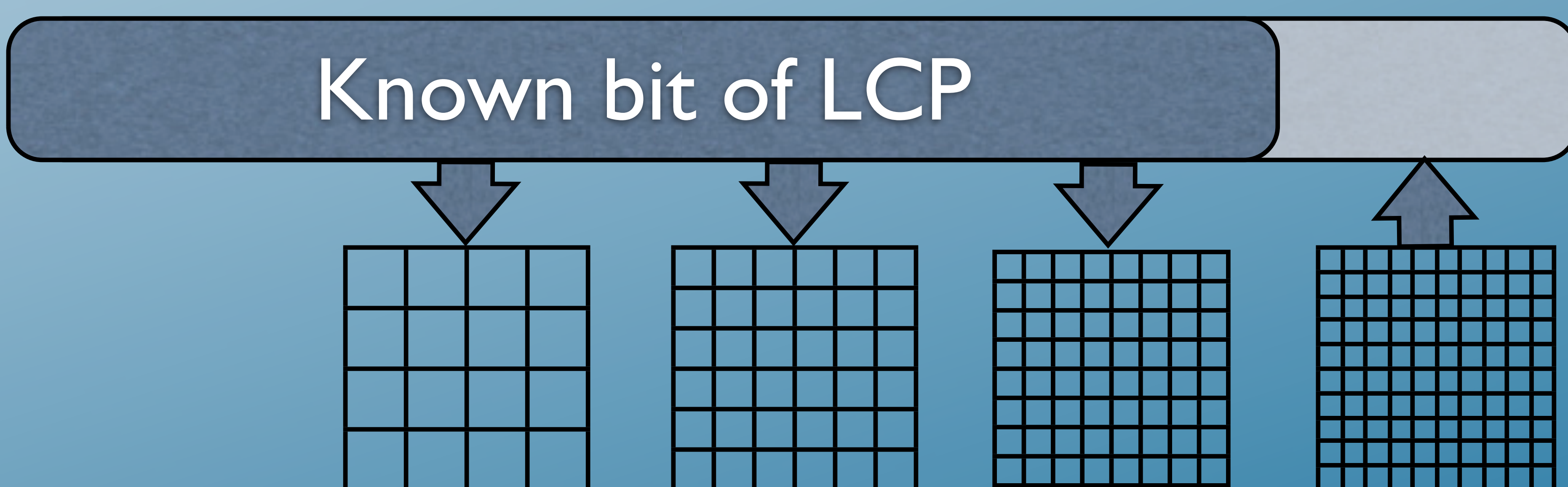
Our scheme





g_{cont} is used for the beta function only

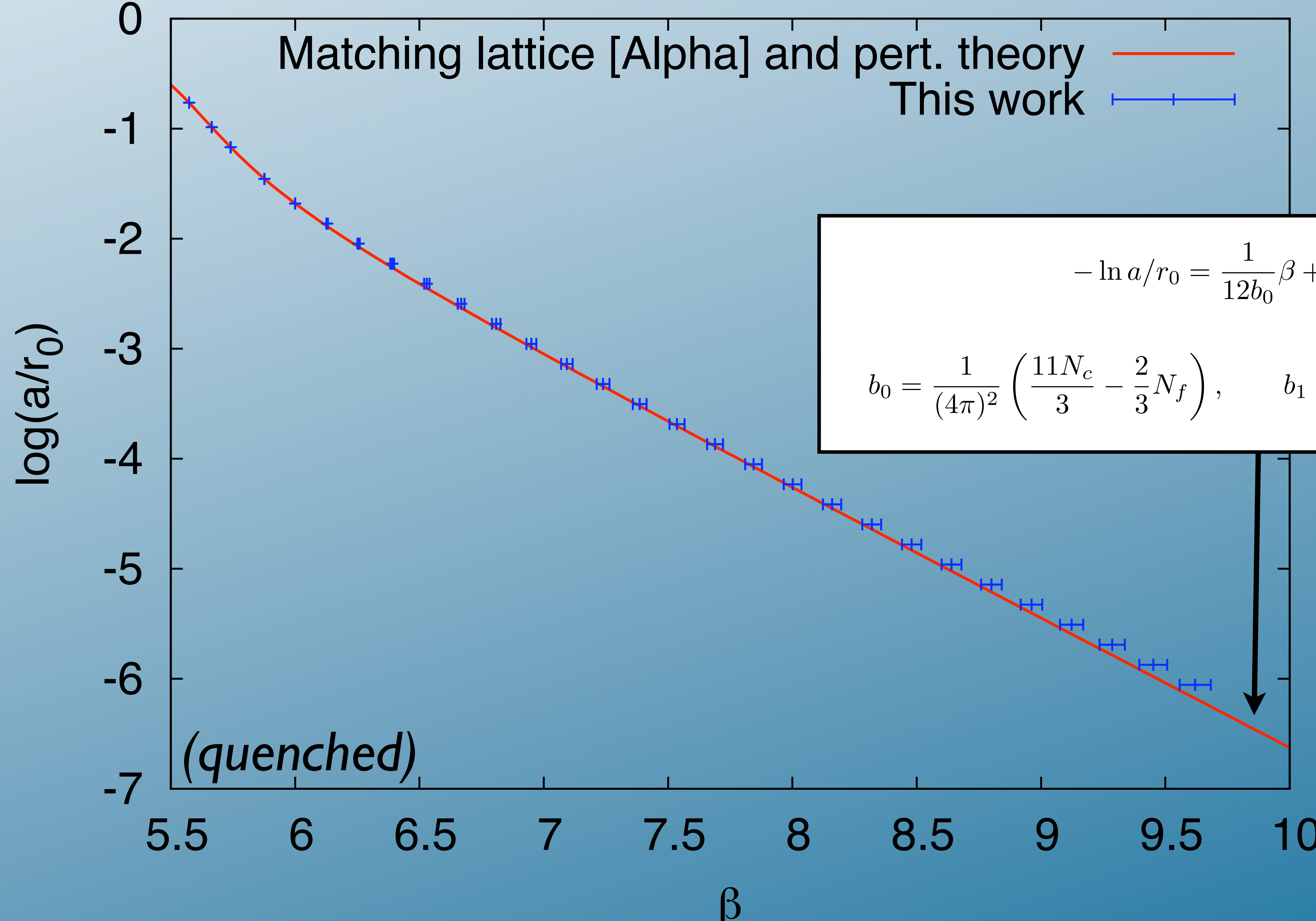
Next step: interpolate LCP to find beta(a) at geometrically suitable cut-offs



Can this scheme work?

over several orders of magnitude in *a*

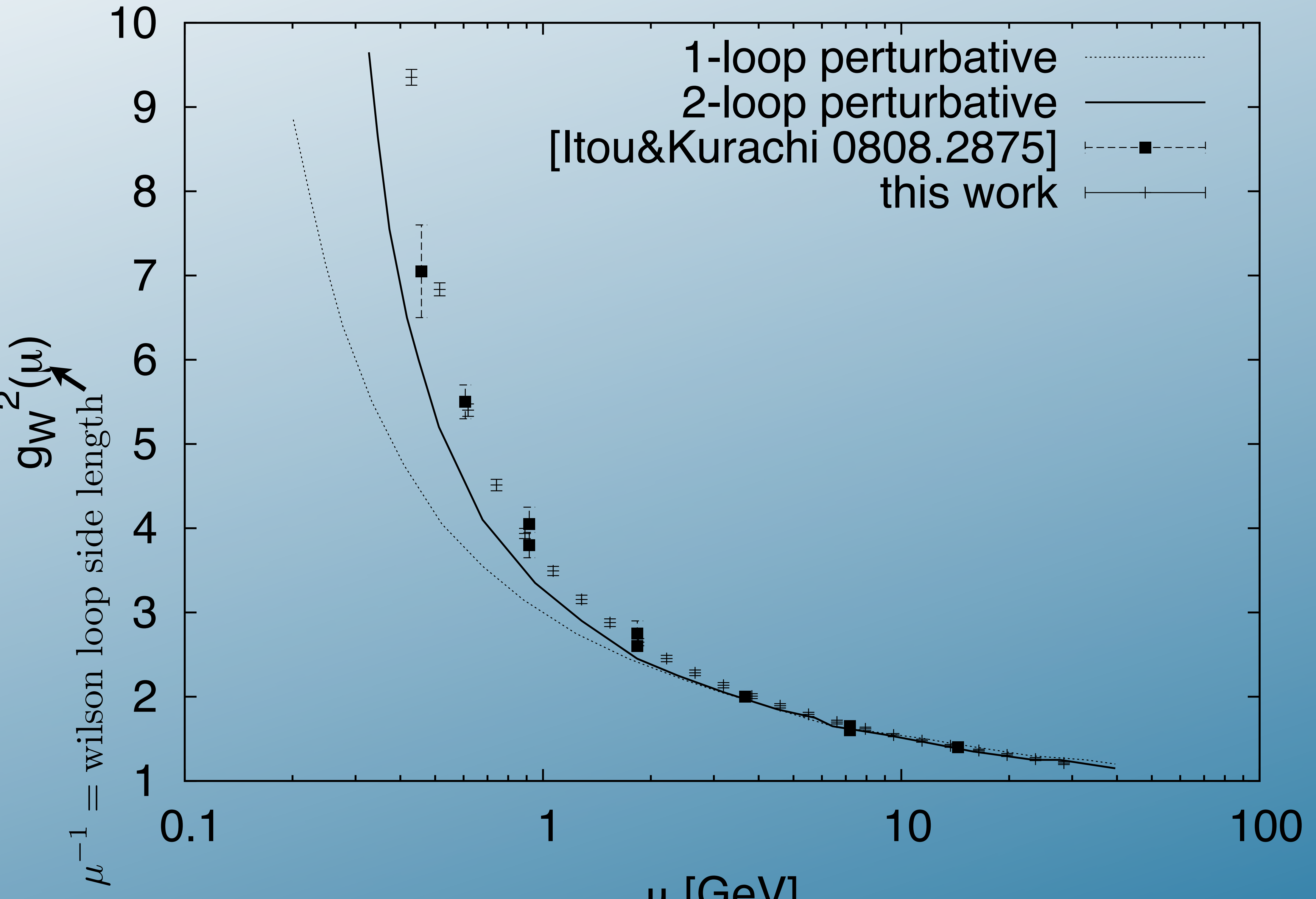
Quenched QCD: LCP is known, it is a safe testing ground



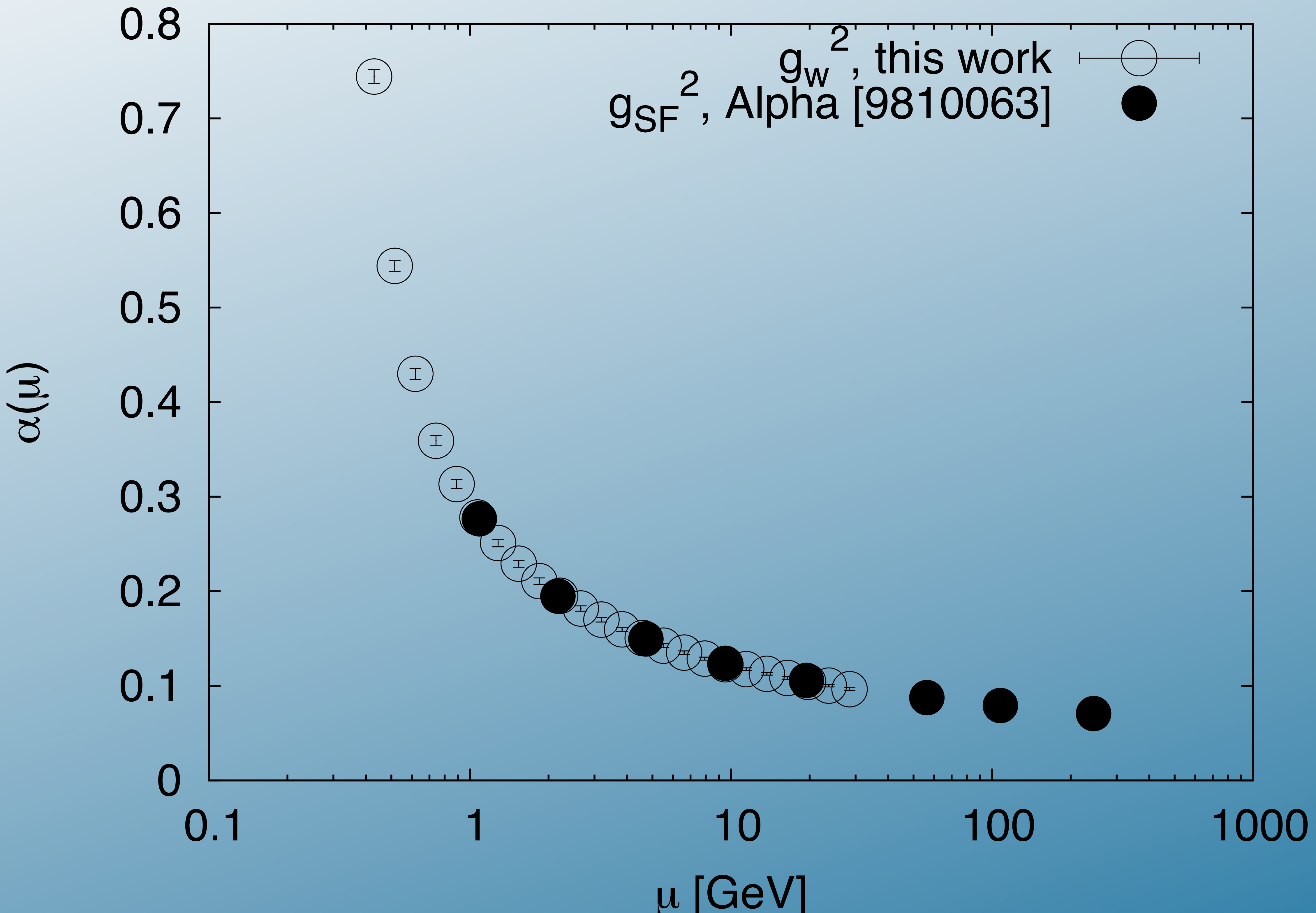
$$-\ln a/r_0 = \frac{1}{12b_0}\beta + \frac{b_1}{2b_0^2}\ln\frac{\beta}{6b_0},$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11N_c}{3} - \frac{2}{3}N_f\right), \qquad b_1 = \frac{1}{(4\pi)^2}\left(\frac{102N_c}{3} - \frac{38}{3}N_f\right).$$

Running coupling

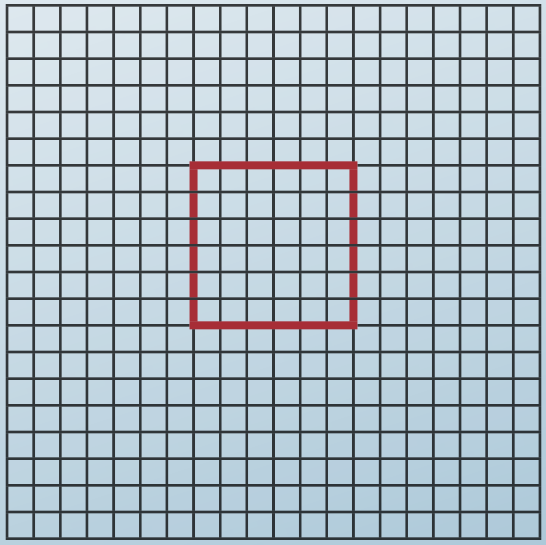


How does it relate to g_{SF} ?



$\alpha_{\overline{\mathrm{MS}}} = \alpha + k_1 \alpha^2 + \ldots, \qquad k_1 = 1.25563(4),$ [Lüscher, Sommer, Weisz, Wolff NPB413,481]

Remark: scale is independent of L



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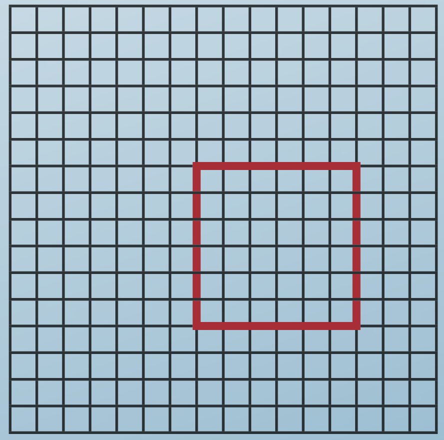
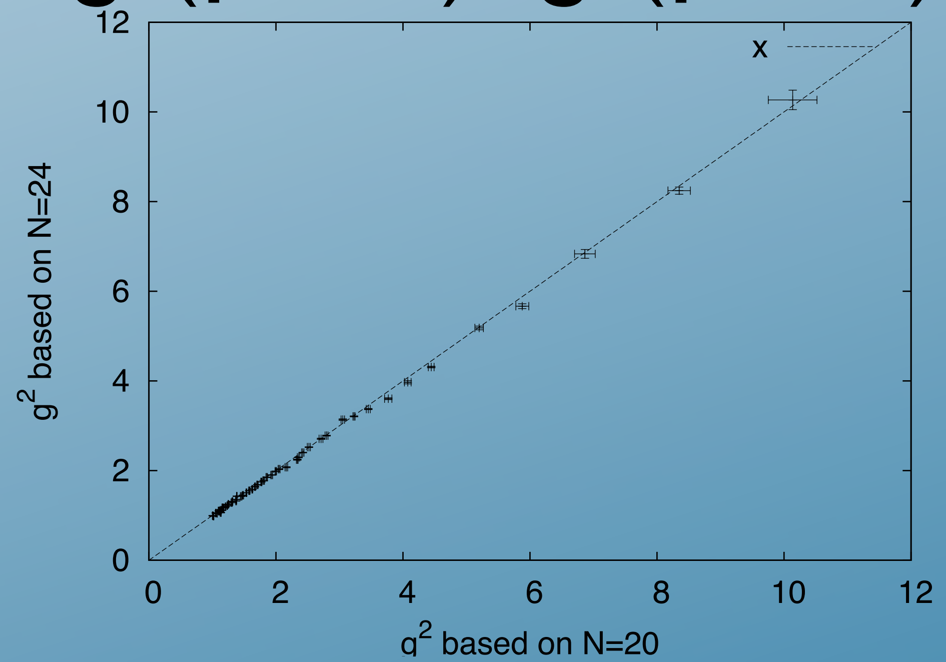
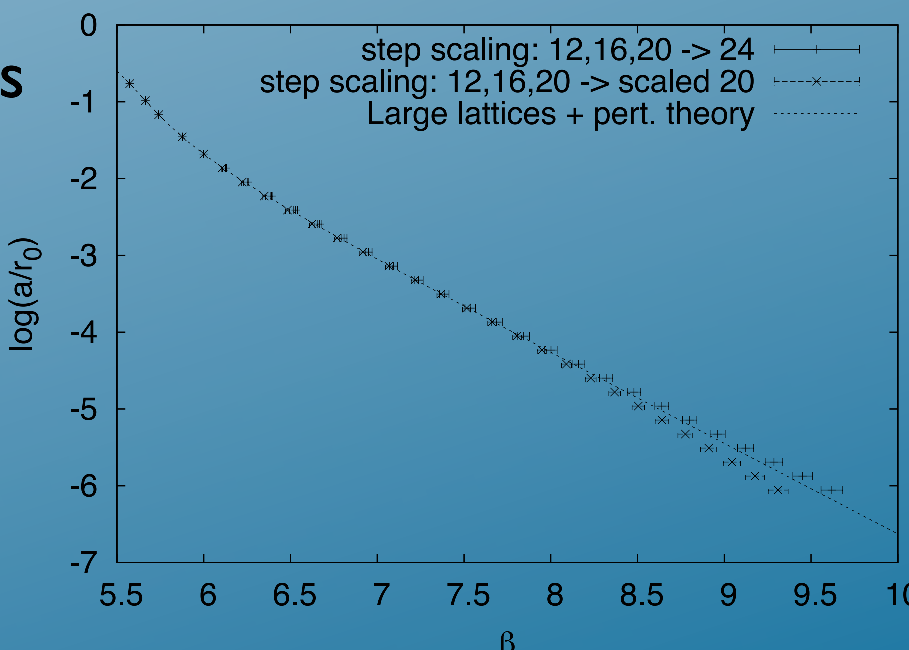


The renormalisation scale is not set by the box size,
but the size of the Wilson loop.

$g^2(\mu,20^2) \sim g^2(\mu,24^2)$



replace 24^4 by 20^4
in the previous analysis



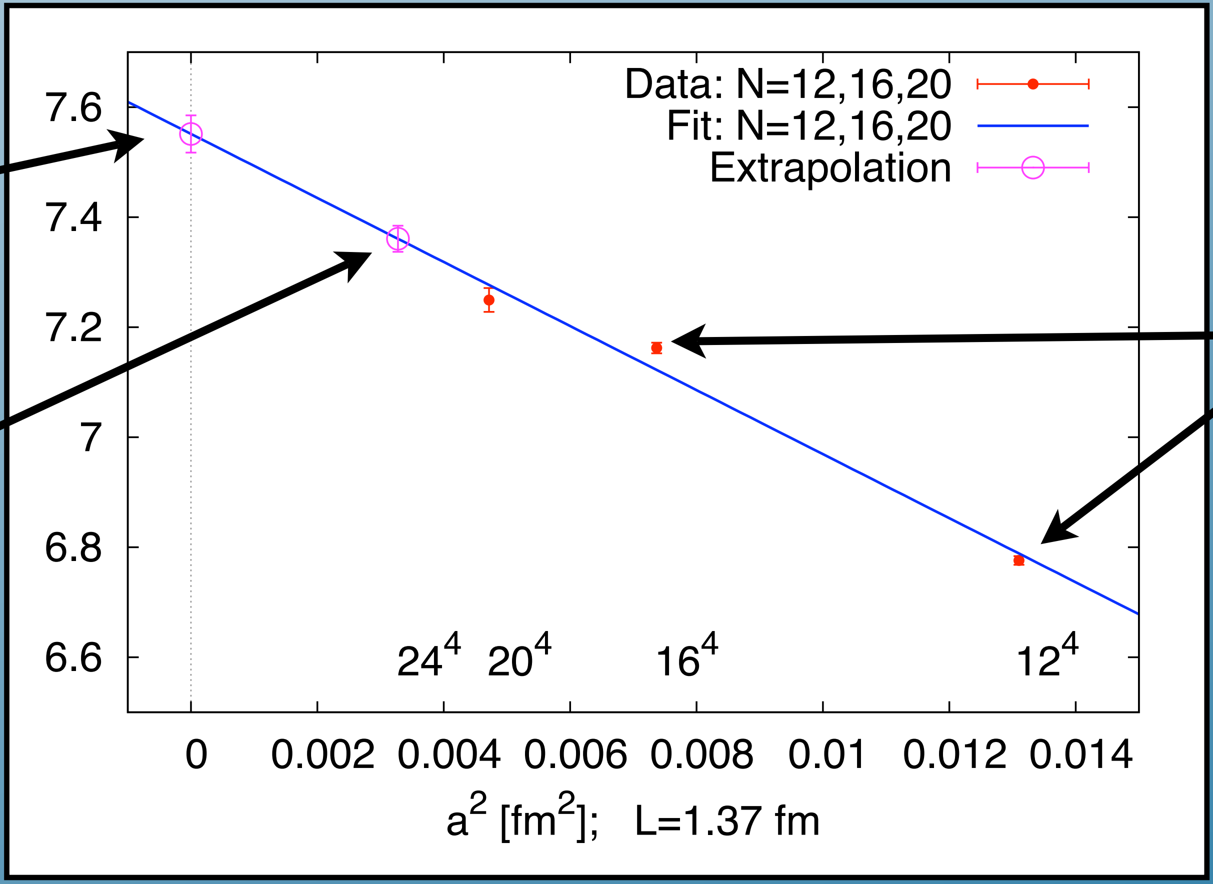
The same for the unquenched model:

We know "a" and search for a mathing "beta".

We use: m_q(beta) as determined previously;
we keep the volume fixed with N=12,16,20;
we determine the coupling constant;
we extrapolate g^2 to N=24, search for beta to hit this g^2;

we use this point for $g^2(L)$

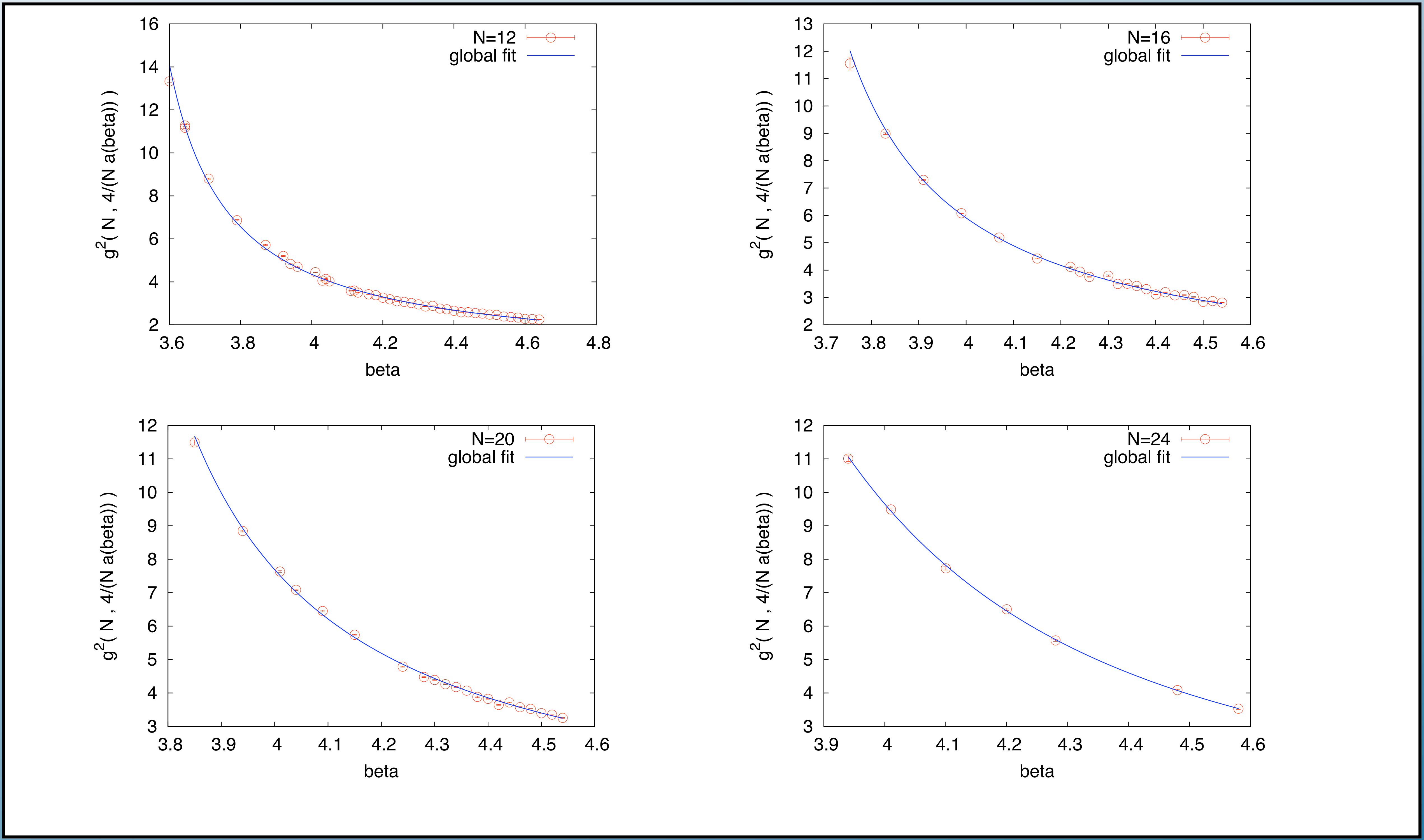
we use this point for *beta(L/24)*



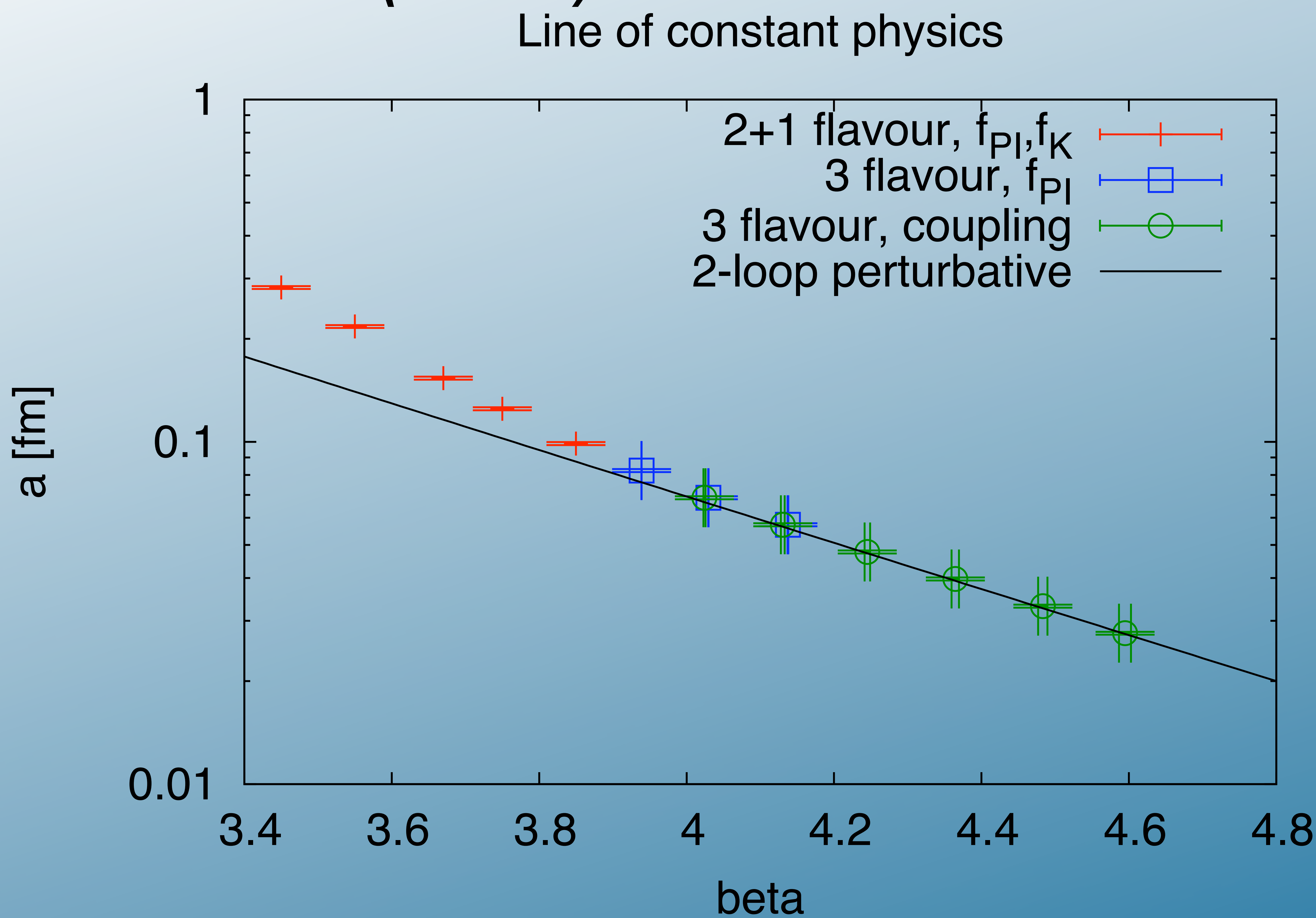
these points are based on an interpolated *beta(a)*

In fact, we do no searching at all

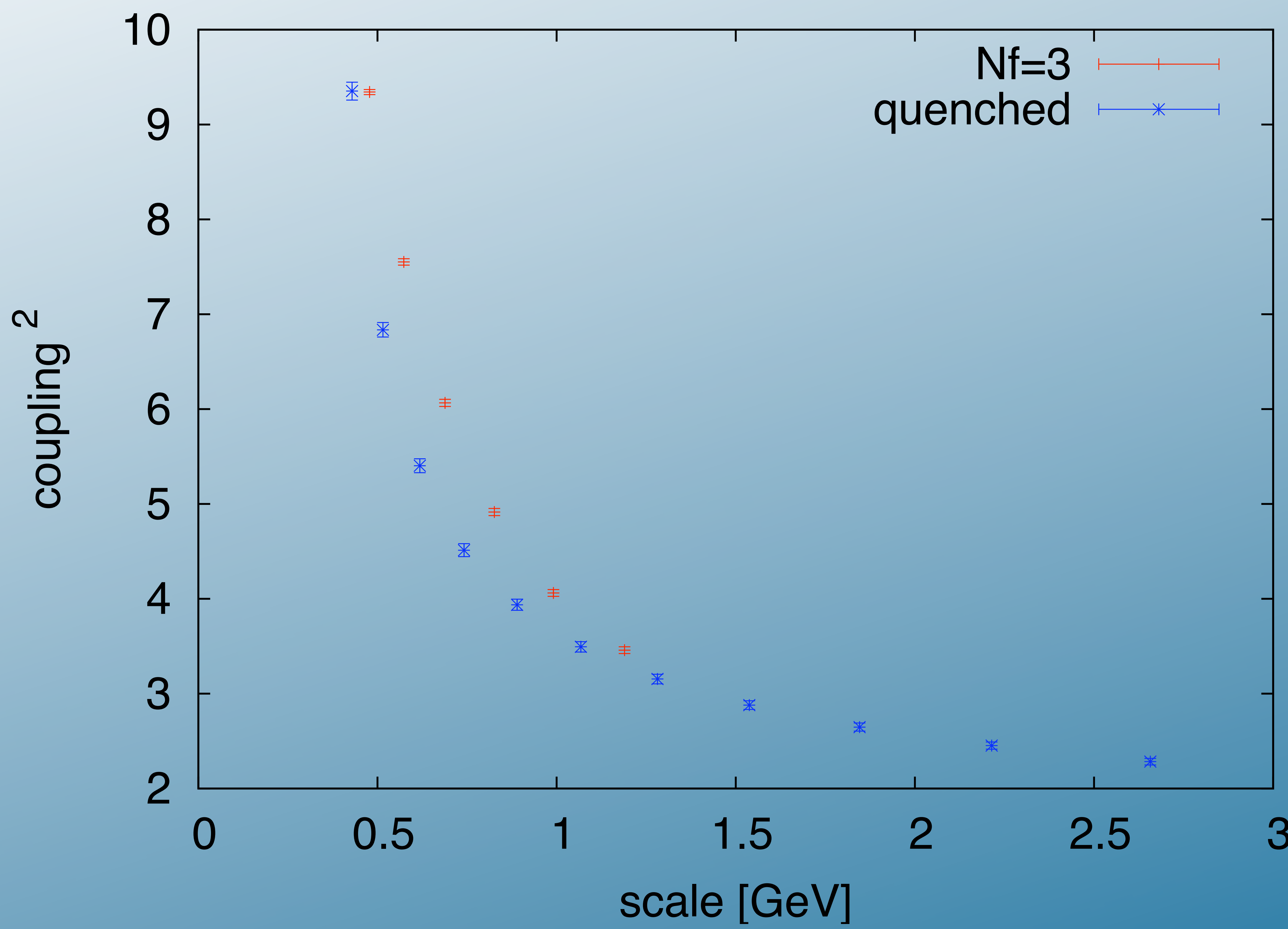
global fits for better project scheduling



Result: *a(beta)*



Running coupling:



Outlook

The LCP (for our action) is know to arbitrary UV scales.
What can we do with our new LCP?

Lattice QCD thermodynamics:

EOS with physical quark masses, Nt=8+

Nt=4,6 has been known... [Aoki *et al* JHEP 0601:089,2006.] [Cheng et al PRD77,014511]

Todo: What is the impact of charm?

$\text{Lambda}_{\text{MSbar}}$ could be calculated without doing lattice perturbation theory (coupling is measured).