Confinement in Polyakov gauge

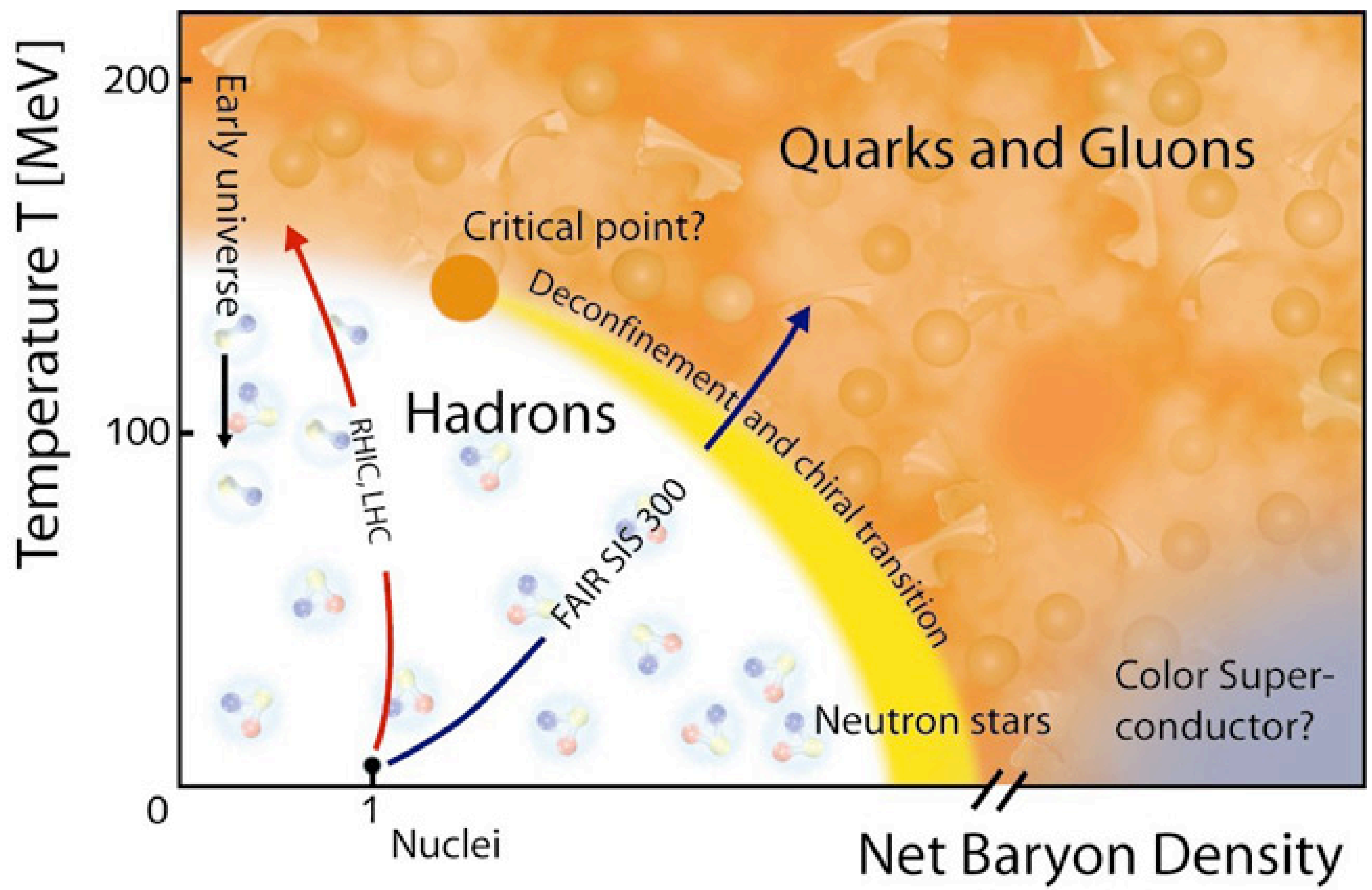
Florian Marhauser

arXiv:0812.1144

HGSFP

QCD Phase Diagram

- chiral vs. deconfinement phase transition
- finite density
- critical point
- ...



Confinement Order Parameter

$$\phi(\vec{x}) = \langle L(\vec{x}) \rangle = \frac{1}{N_c} Tr_c \left\langle \mathcal{P} \exp\left(ig \int_0^\beta d\tau \;\; A_0(\vec{x}, \tau) \right) \right\rangle$$

puts a static quark into the theory

$$\frac{\partial \Psi}{\partial \tau} = igA_0\Psi \Rightarrow \Psi(\vec{x}, \tau) = \mathcal{P} \exp\left(ig \int_0^\tau d\tau' \;\; A_0 \right) \Psi(\vec{x}, 0)$$

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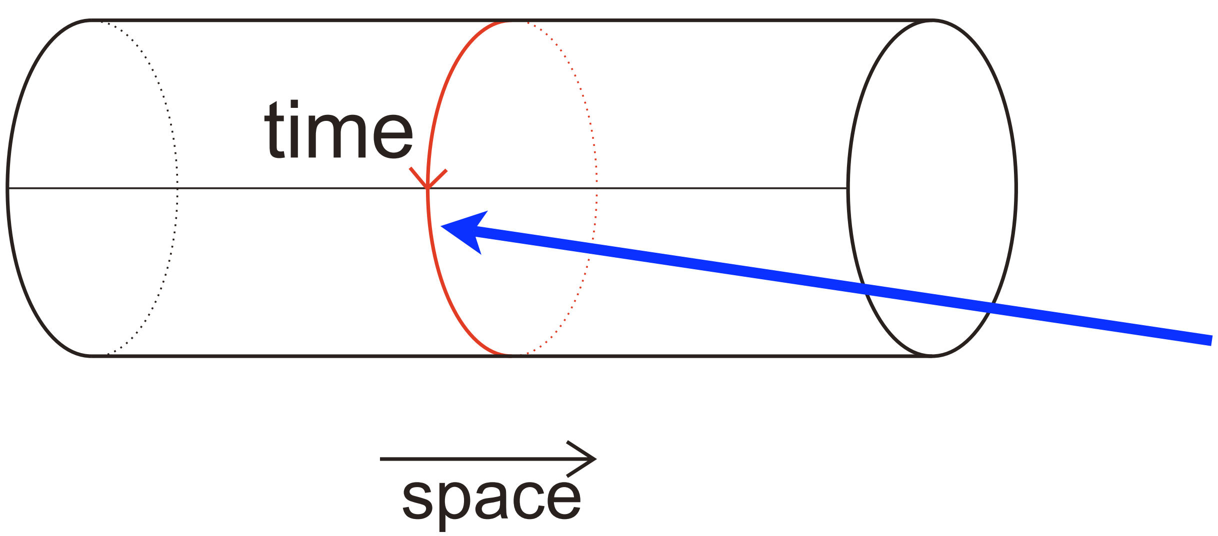
quark propagating in time



$$j^\mu = \delta^{\mu 0} \int_0^\beta d\tau \delta(x - x(\tau))$$

Polyakov Loop

$$\phi \propto \mathsf{Tr}_c \langle \mathcal{P}\ e^{ig\int d^4x\ \ A_\mu(x)\ \ j^\mu(x)} \rangle = \exp\left(-\beta F_q\right)$$

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Energy of an
infinitely heavy quark

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Energy of an infinitely heavy quark

Confinement	Deconfinement
$F_q \to \infty \leftrightarrow \phi = 0$	$F_q < \infty \leftrightarrow \phi \neq 0$

Perturbation Theory $\longrightarrow$ Expansion around 0 $\longrightarrow$ $F_q < \infty$

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F_q: Energy of an infinitely heavy quark

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Perturbation Theory $\longrightarrow$ Expansion around 0 $\longrightarrow$ $F_q < \infty$

Gauge group	Universality class	PT order
SU(2)	Ising	2nd
SU(3)	Potts	1st

Order Parameter reformulation

order parameter: $\phi = \langle L[A_0(x)] \rangle$

with the Jensen inequality $L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle$
and properties of the Polyakov loop,

we can show

$$T < T_c: \qquad L[\langle A_0 \rangle] = 0 \quad \Leftrightarrow \quad \tfrac{1}{2} g\beta \langle A_0(\vec{x}) \rangle = \tfrac{\pi}{2}\,,$$
$$T > T_c: \qquad L[\langle A_0 \rangle] \neq 0 \quad \Leftrightarrow \quad \tfrac{1}{2} g\beta \langle A_0(\vec{x}) \rangle < \tfrac{\pi}{2}$$

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$\Rightarrow$ $L[\langle A_0(x) \rangle]$ also serves as an order parameter

Gauge

$$\phi(\vec{x}) = \langle L(\vec{x}) \rangle = \frac{1}{N_c} Tr_c \left\langle \mathcal{P} \exp\left(ig \int_0^\beta d\tau \ A_0(\vec{x},\tau) \right) \right\rangle$$

Polyakov gauge in SU(2):

$$\boxed{A_0(x) = A_0^c(\vec{x})\sigma^c}$$

Pauli matrix

time-independent, removes path ordering

$$L(\vec{x}) = \frac{1}{N_c} Tr_c \left[e^{ig\beta A_0^c(\vec{x})\sigma^c} \right] = \cos\left[\frac{g}{2} \beta A_0^c(\vec{x}) \right]$$

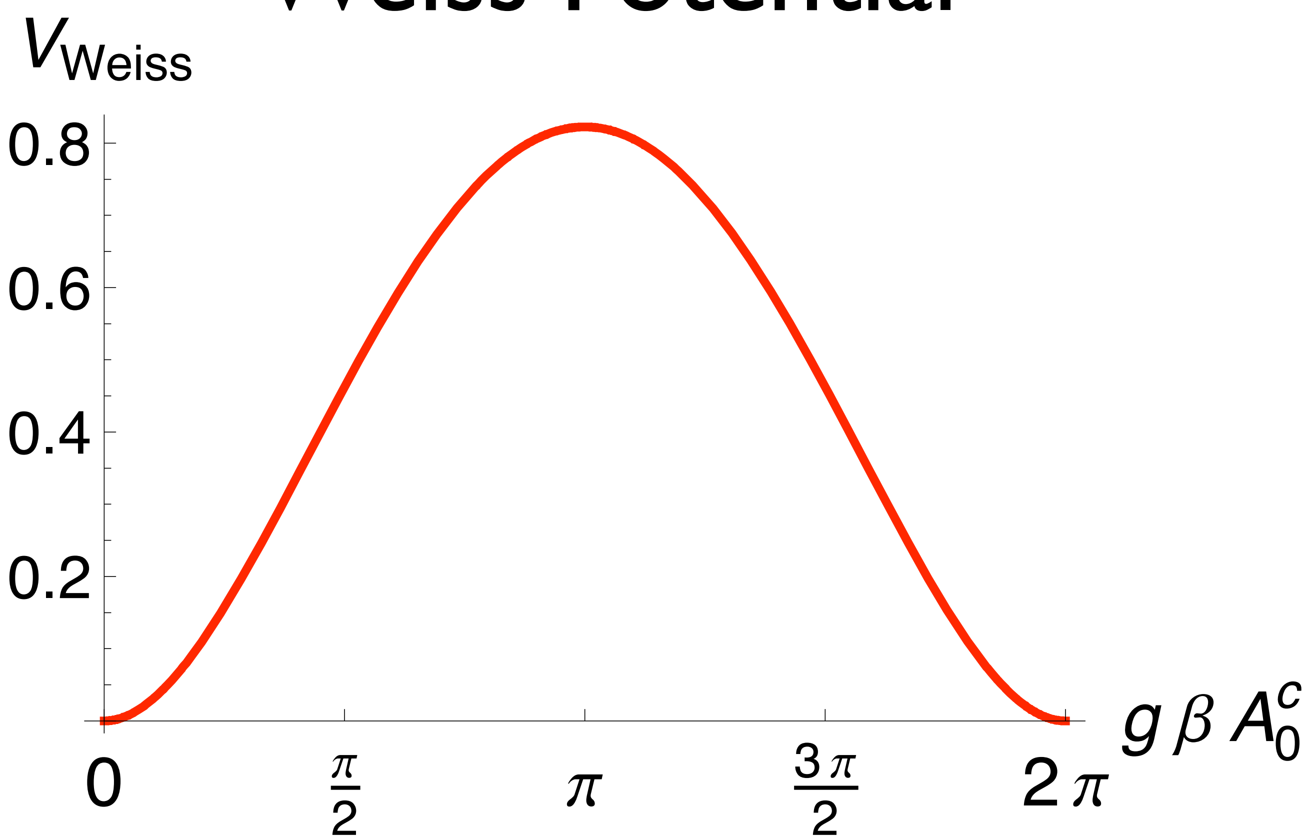
compute $L[\langle A_0 \rangle]$ in polyakov gauge via $V_{\text{eff}}(\langle A_0 \rangle)$

Perturbative Treatment (1-loop)

Potential is periodic with period 2π

$$L(\vec{x}) = \cos\left[\frac{g}{2}\beta A_0^c(\vec{x})\right]$$

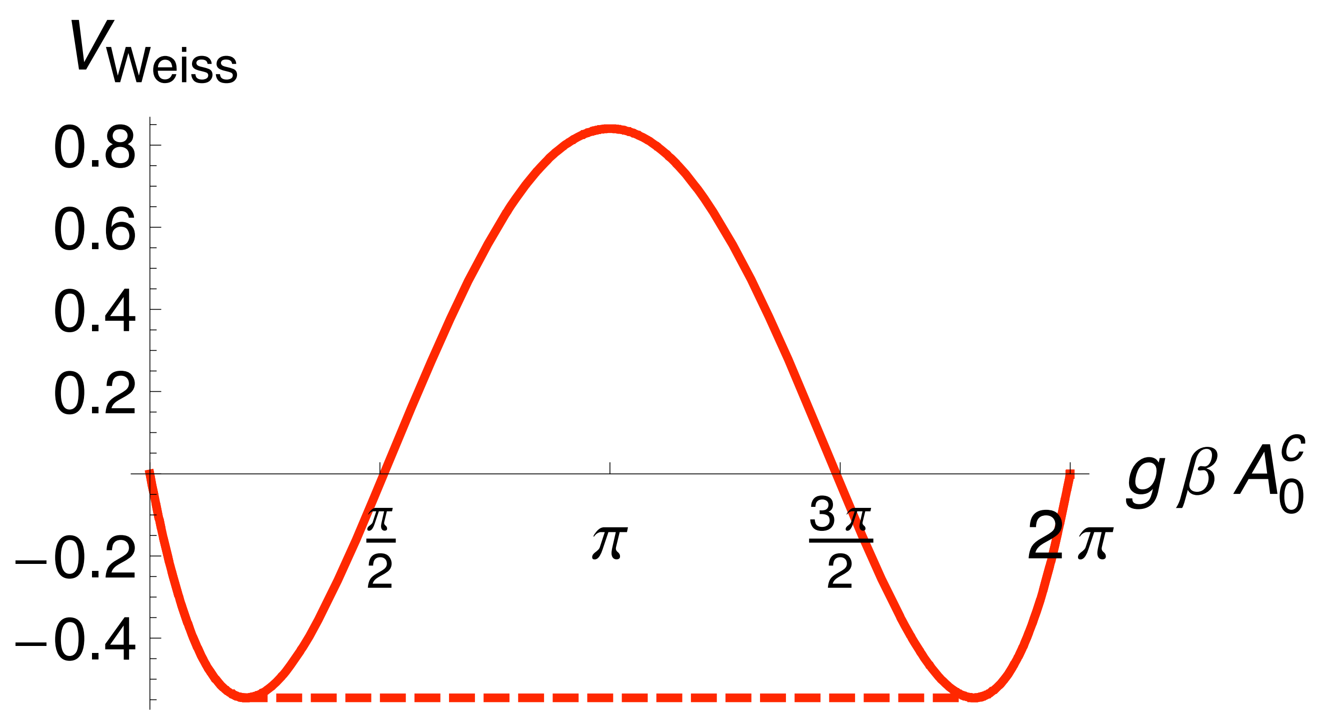
Weiss Potential



deconfining

$$\langle \frac{g\beta}{2} A_0^c \rangle = \{0, \pi\} \to L = 1$$

non-pert. Potential



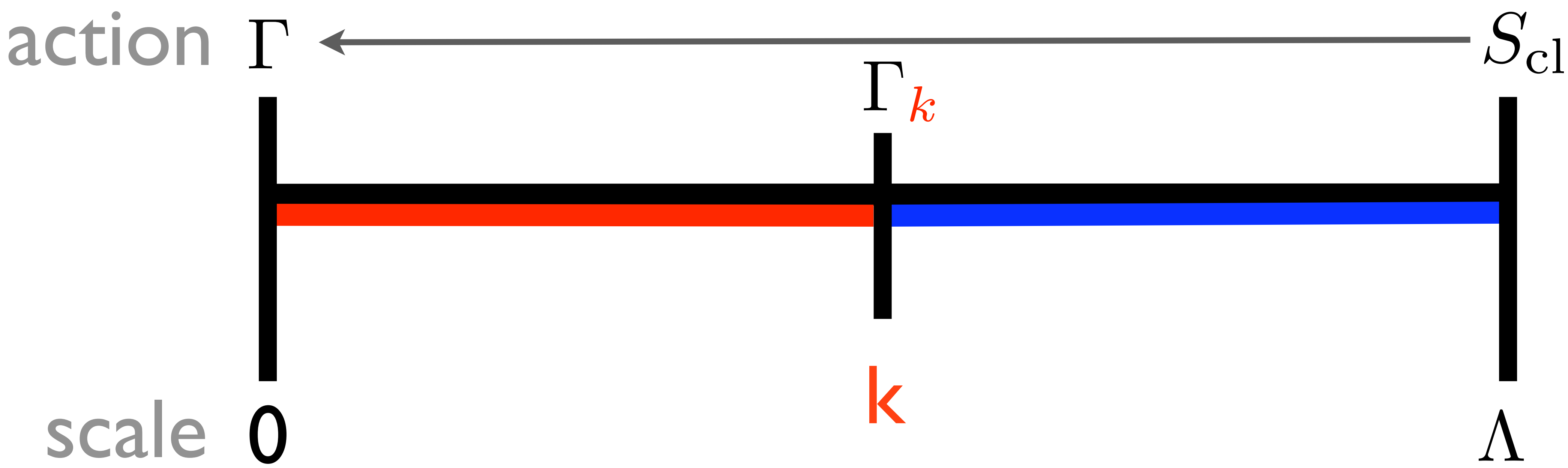
minimum away from edge

smoothly ➡ 2nd order PT

Non-Perturbative Treatment

FRG Method: scale dependent effective action Γ_k

include quantum fluctuations by lowering scale



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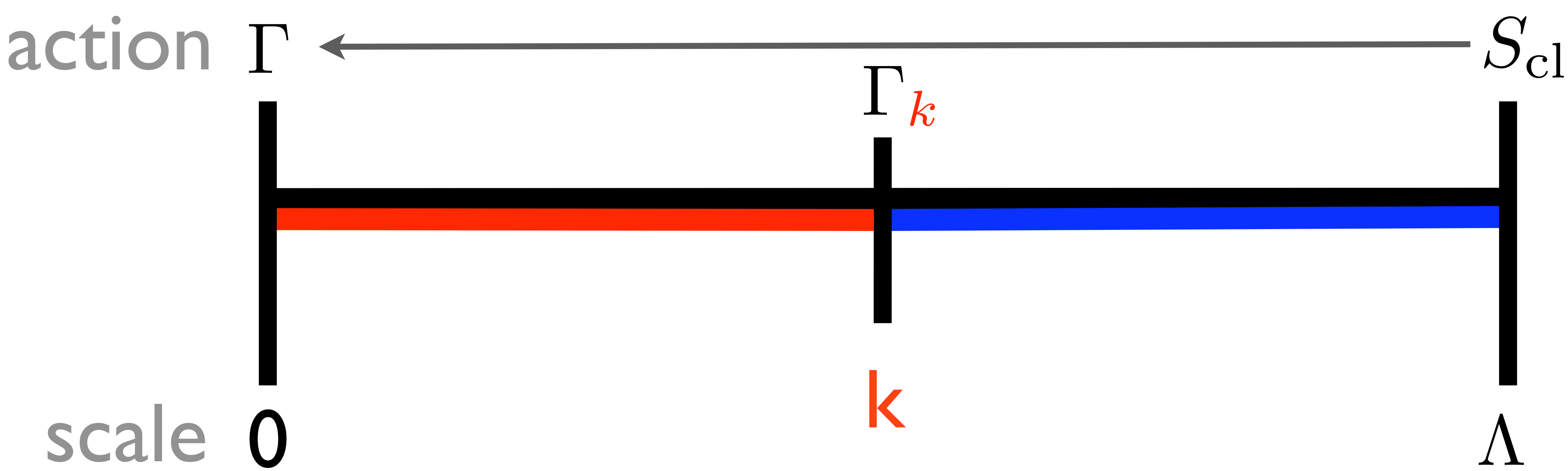

flow equation for effective action („Wetterich‘s equation“)

$$k\partial_k\Gamma_k = \partial_t\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k}\partial_t R_k\right]$$

Parametrisation

- $L(\vec{x}) = \cos\left[\frac{g}{2}\beta A_0^c(\vec{x})\right]$ focus on A_0^c
- effective potential sufficient to generate confinement

$$\boxed{\Gamma_k = \int d\tau \int d^3x \left\{ \frac{1}{2} A_0^c \partial^2 A_0^c + V_k(A_0^c) \right\} + \Gamma_\psi}$$

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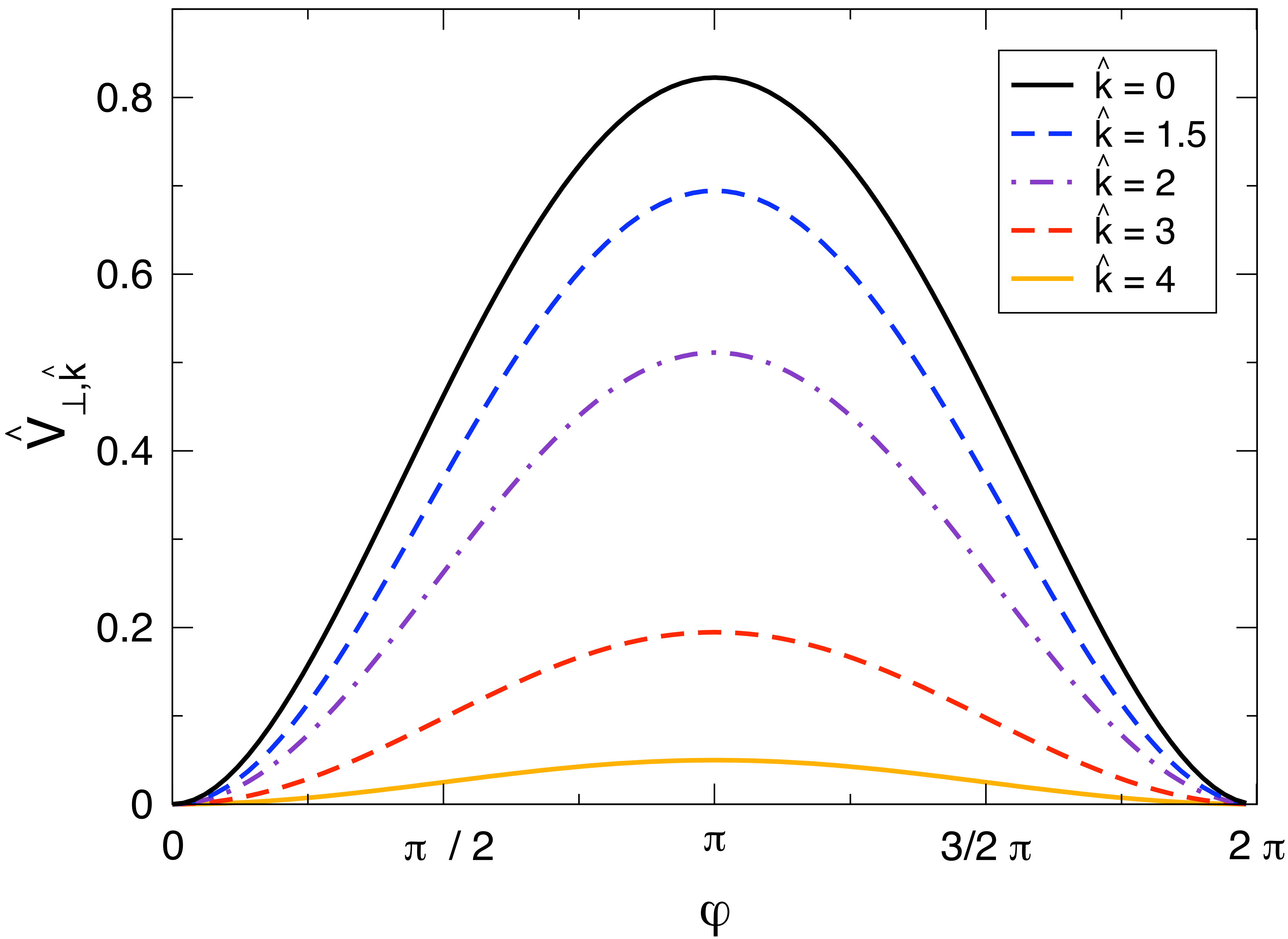
- spatial gluons integrated out

$$V_k(A_0^c) = V_{\perp,k}(A_0^c) + \Delta V_k(A_0^c)$$

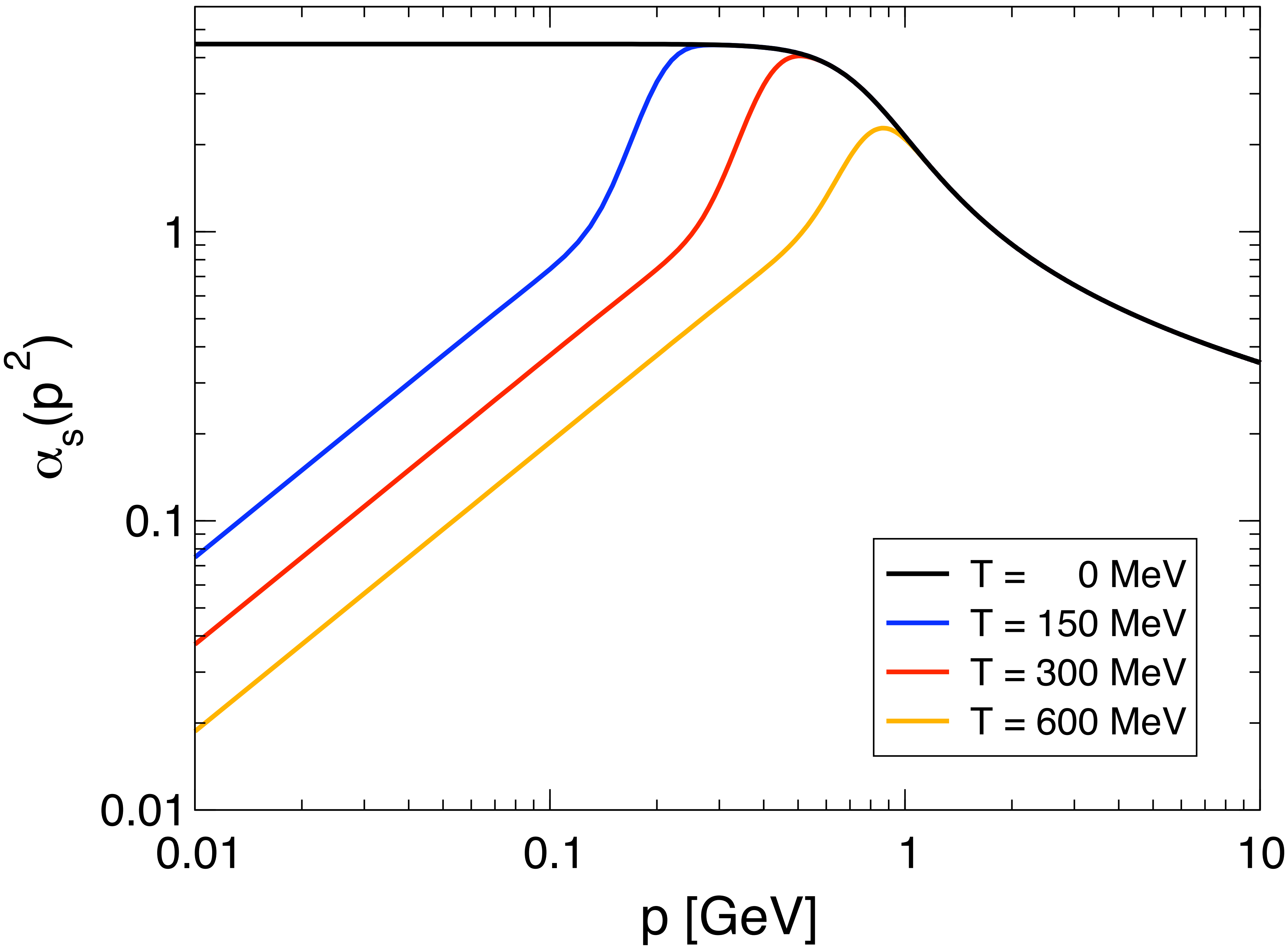
analytical result

Flow of spatial gluons



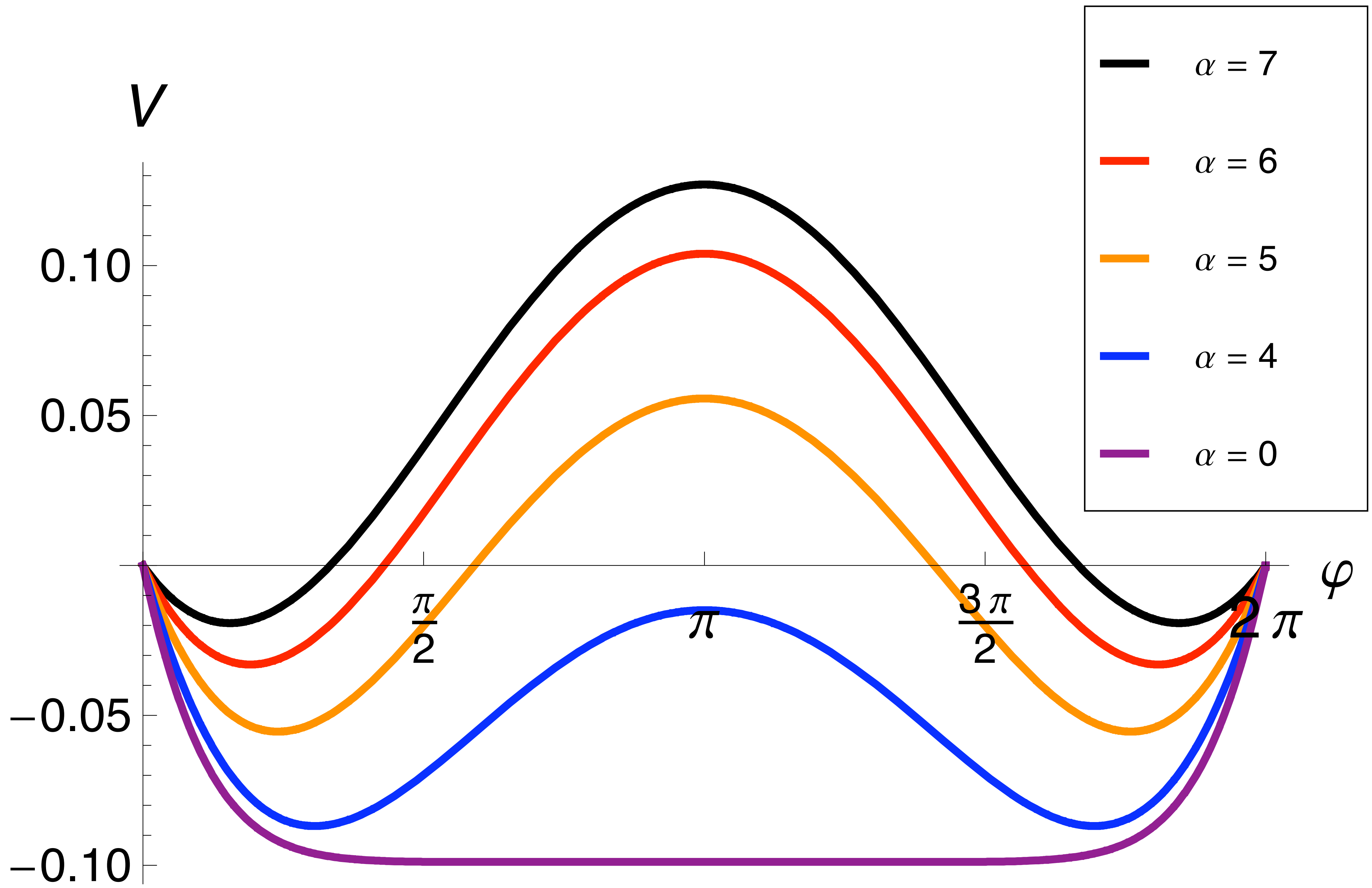
Coupling

for small momenta $\alpha_s \propto p\beta$



in flow identify k = p

Flow of V_{eff}

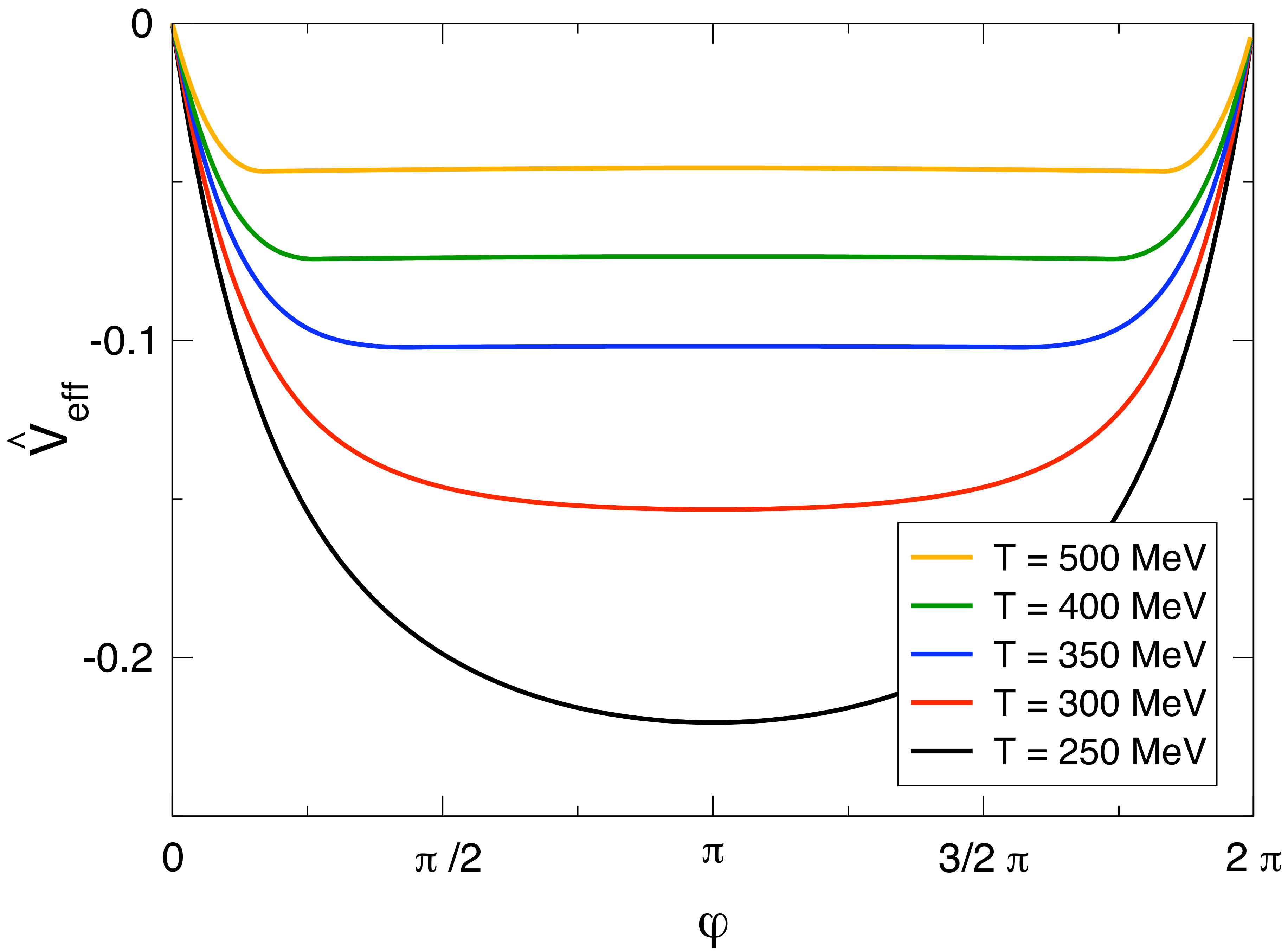


$$\alpha = k\beta$$
$$\varphi = g\beta A_0^c$$

expectation value $\langle A_0^c \rangle$ from $0 = \left.\frac{\partial V(A_0^c)}{\partial A_0^c}\right|_{A_0^c = \langle A_0^c \rangle}$

Effective Potential

$$L[\langle A_0^c \rangle] = \cos\left[\frac{g\beta}{2}\langle A_0^c \rangle\right]$$



phase transition

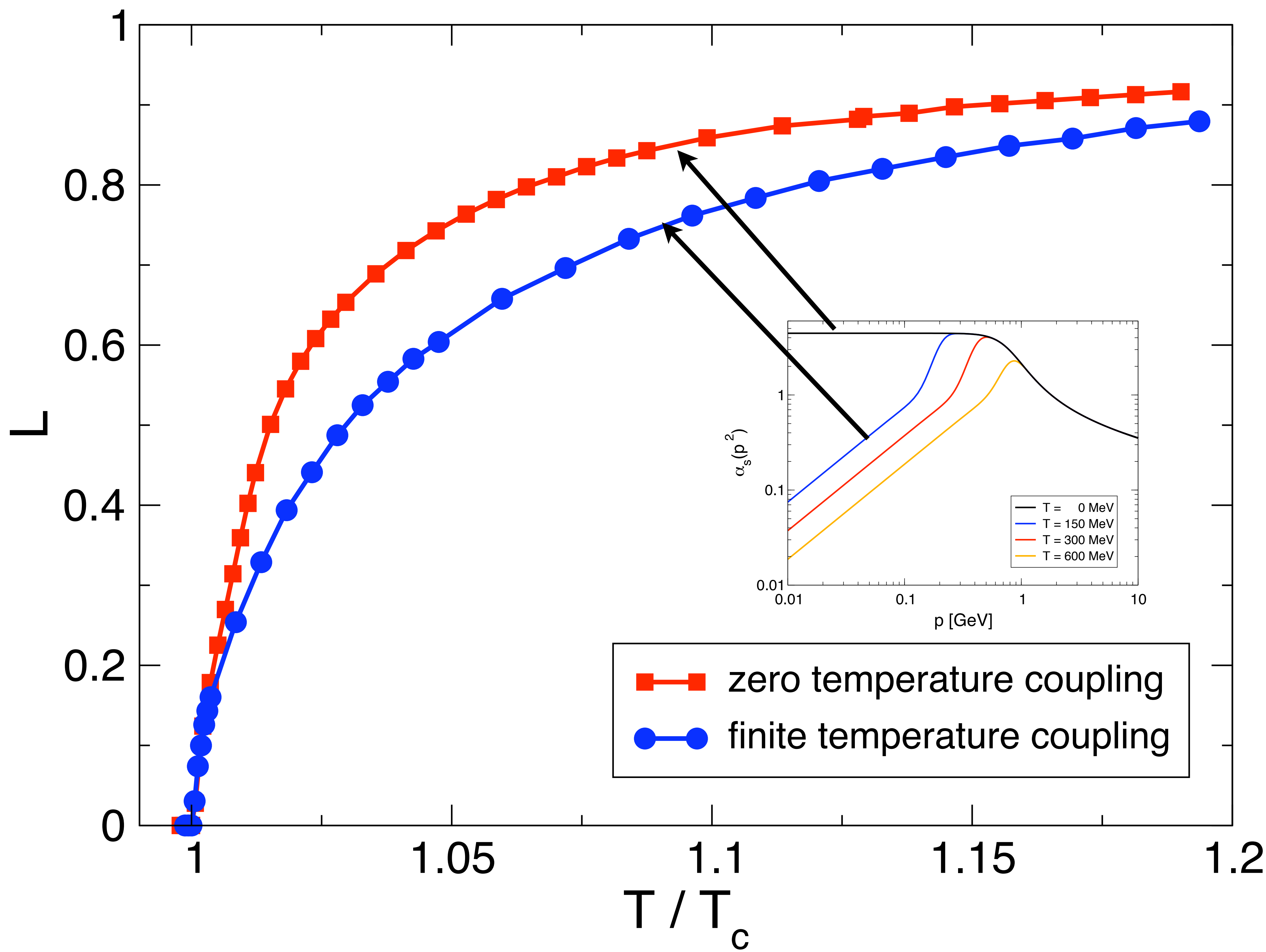
$$T_c = 305^{+40}_{-55}\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.69^{+.09}_{-.12}$$

for comparison

$$\frac{T_{c,lattice}}{\sqrt{\sigma}} = 0.71$$

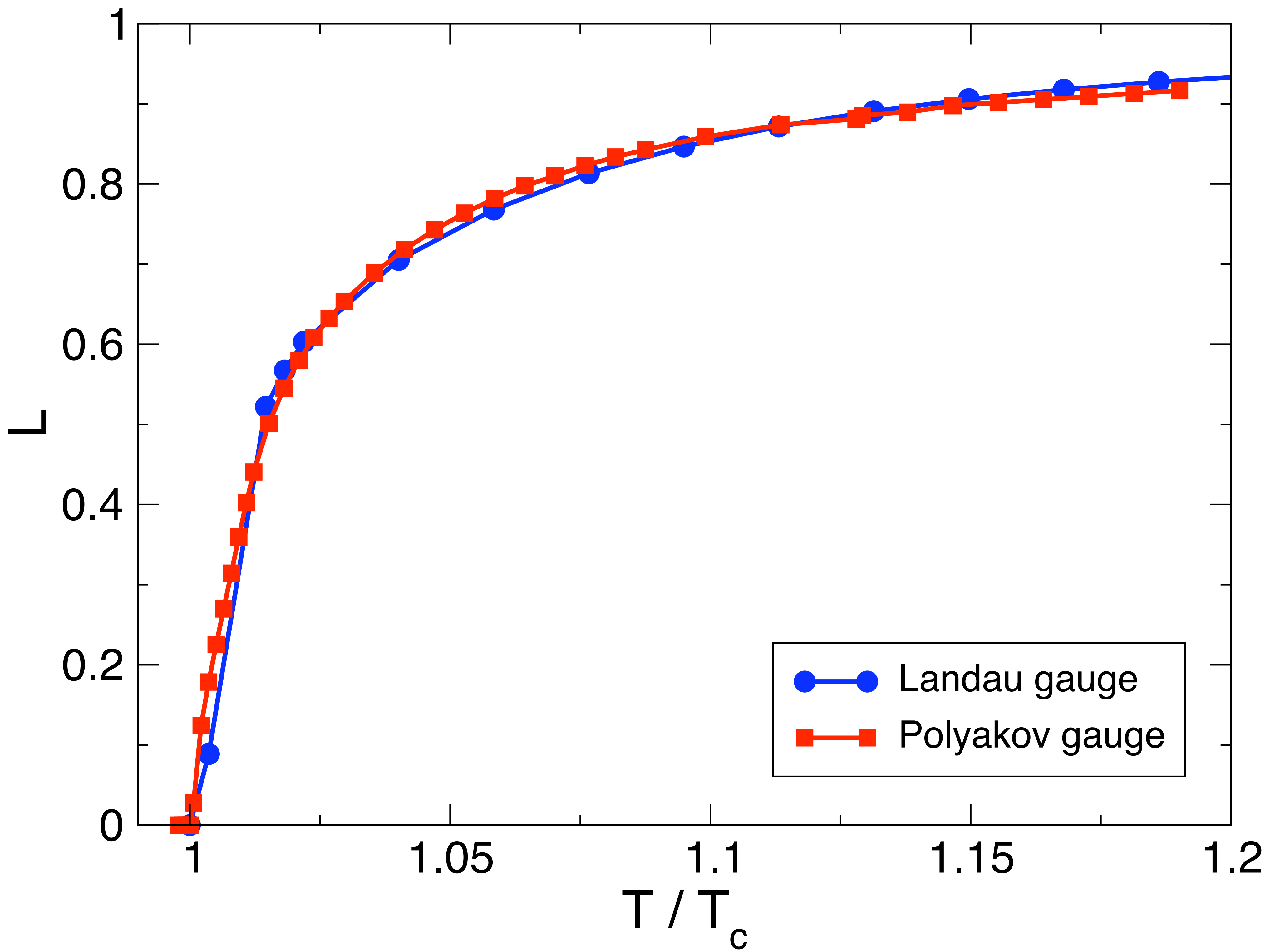
Phase Diagram



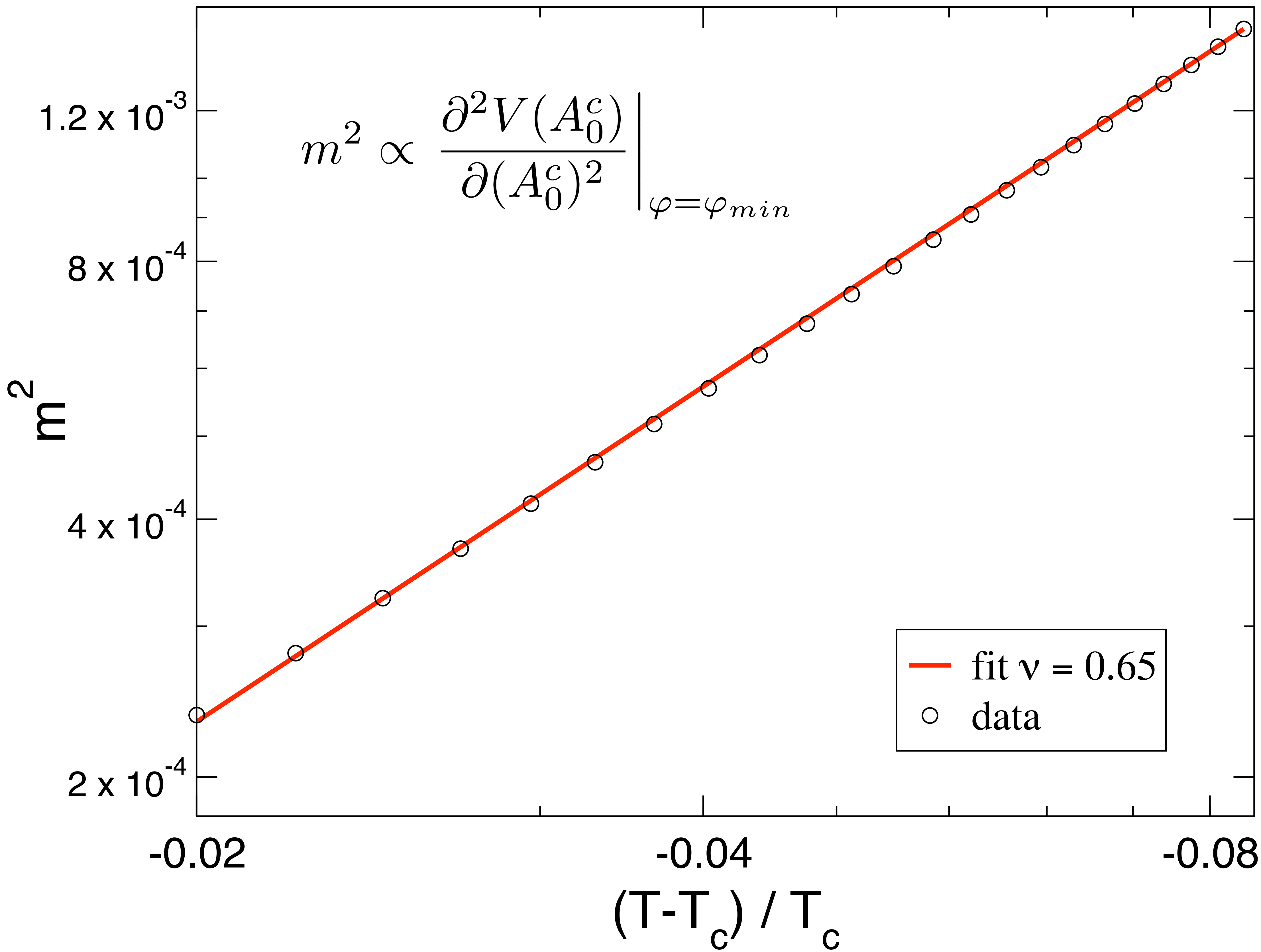
Phase Diagram

polyakov gauge vs. landau gauge

Landau gauge: Braun, Gies, Pawlowski, arXiv: 0708.2413



Critical Exponent ν



Outlook - SU(3)

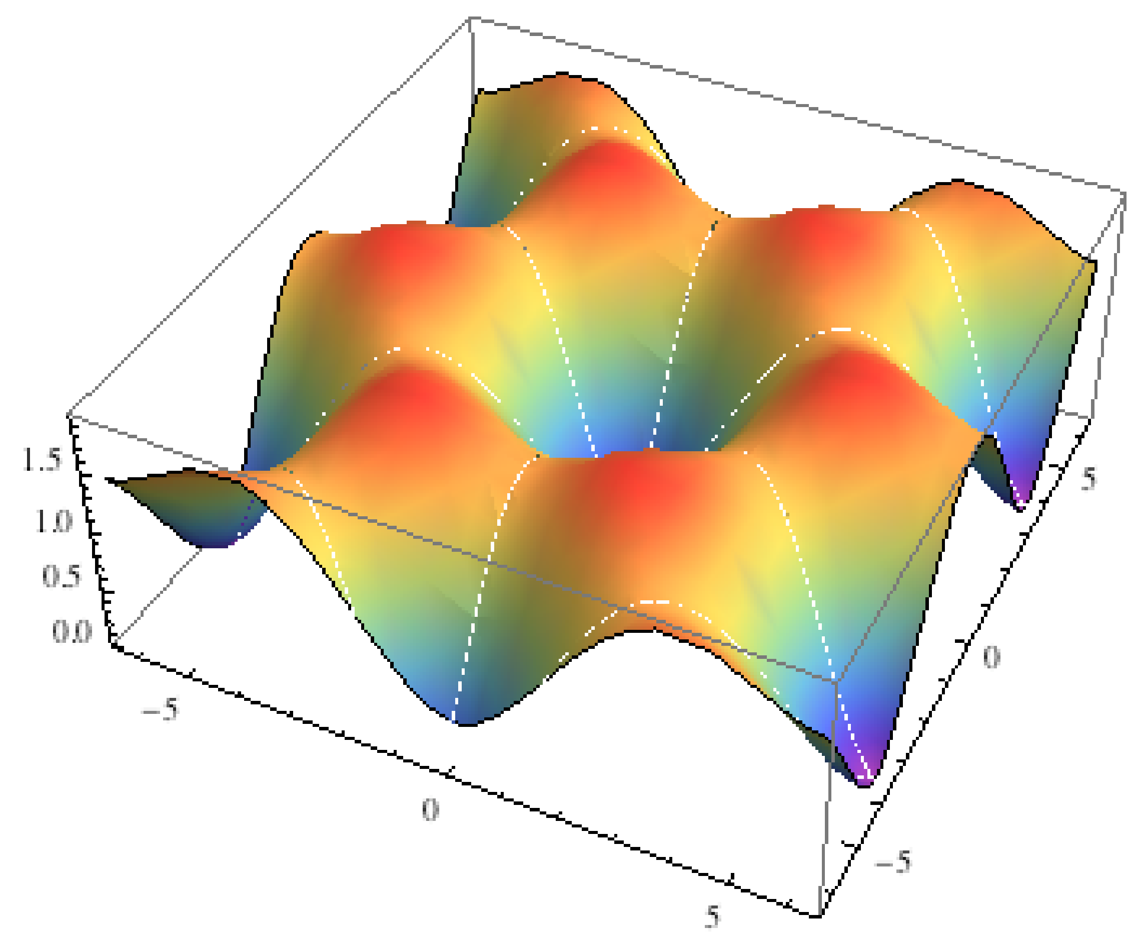
FM, J.Pawlowski, work in progress

2 instead of 1 variable

need 2D grid

- no conceptual problems
- numerically demanding

various methods for solving
however, no full solution **yet**



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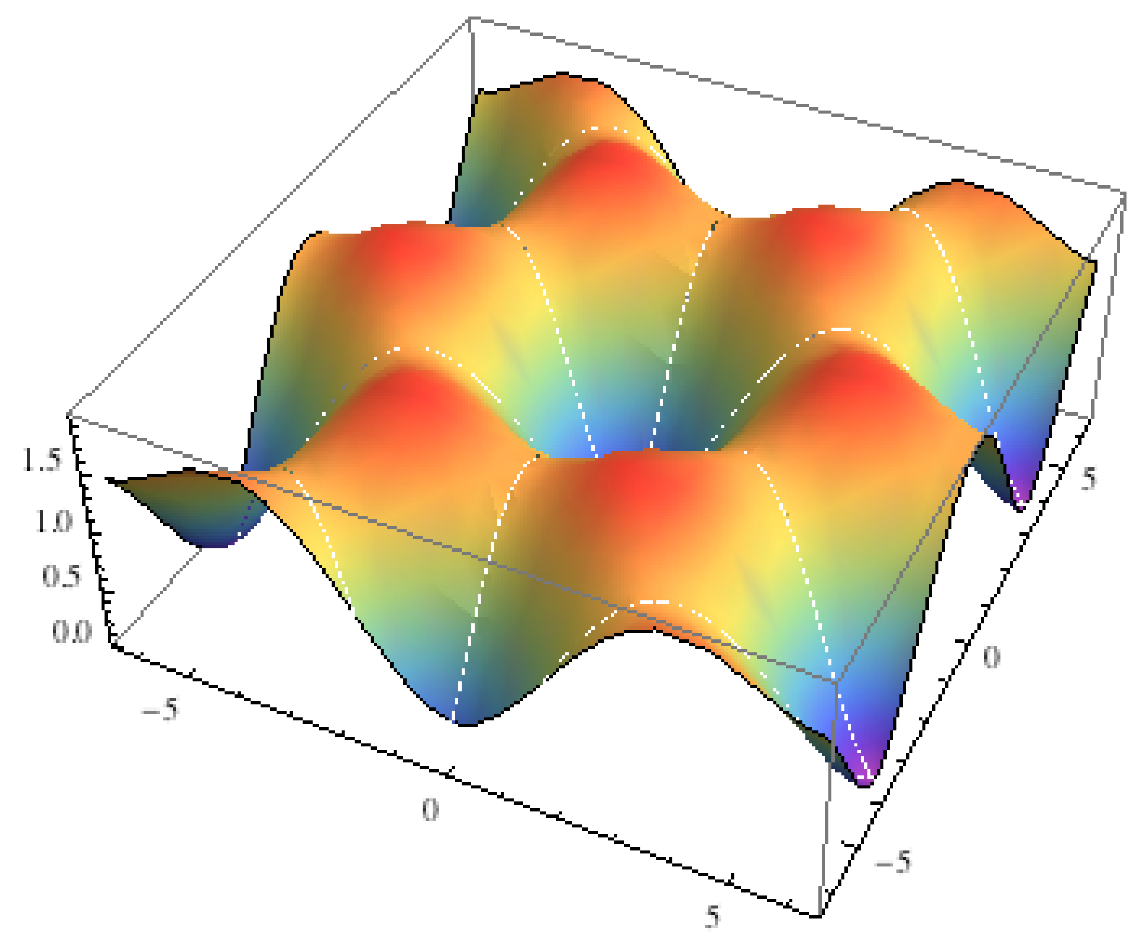

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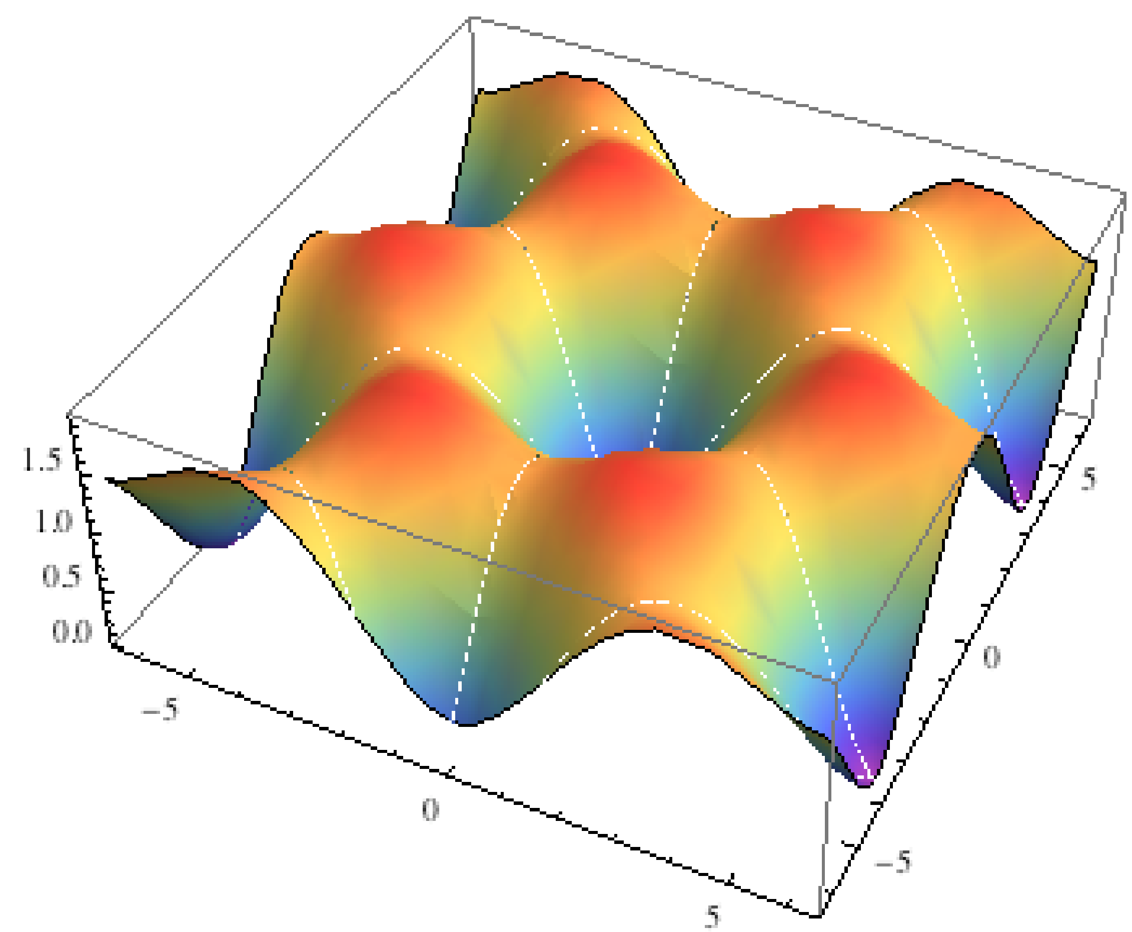
Yes we can!



Outlook - SU(3)

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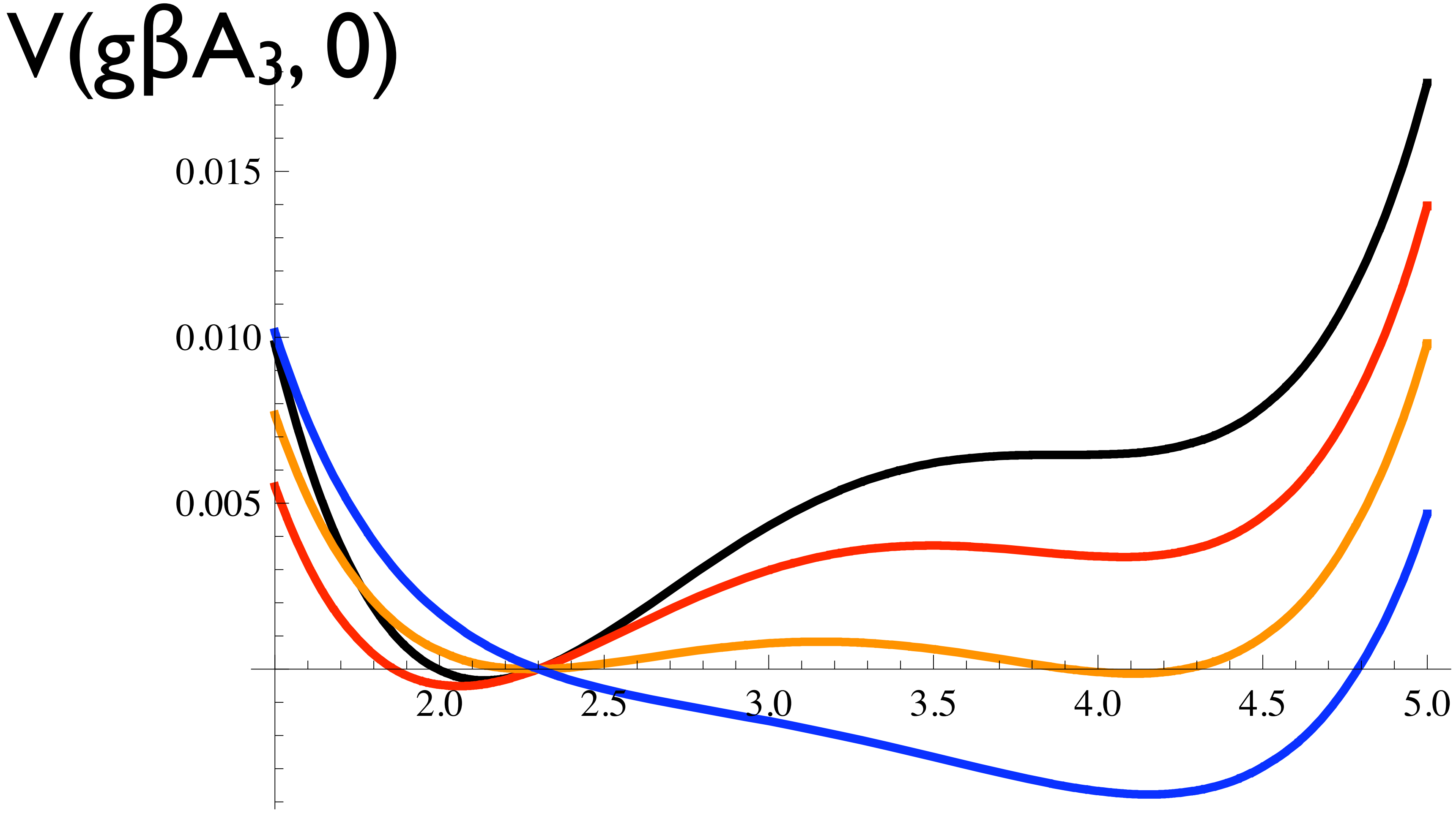
phase transition - preliminary



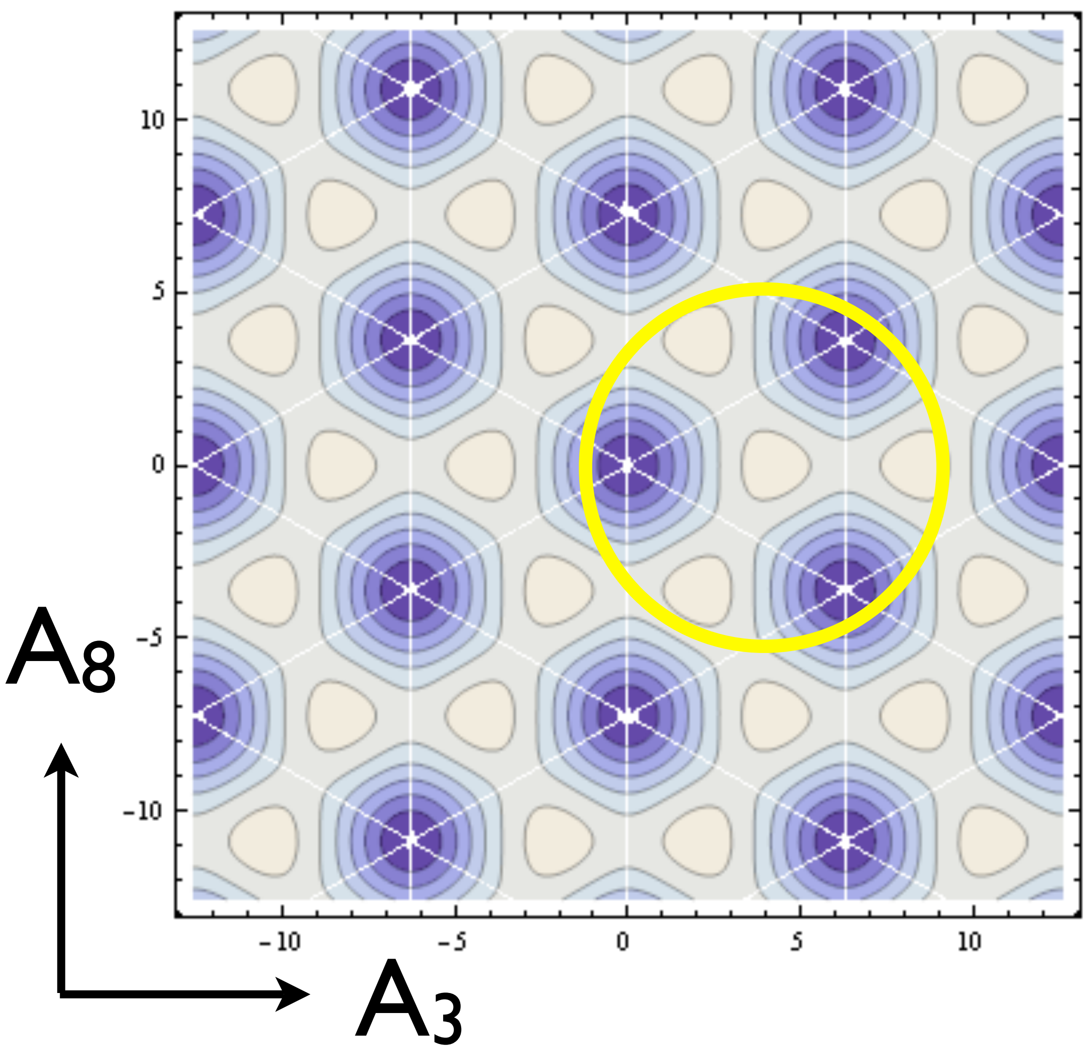
1^{st} order

phase transition between
200-300 MeV

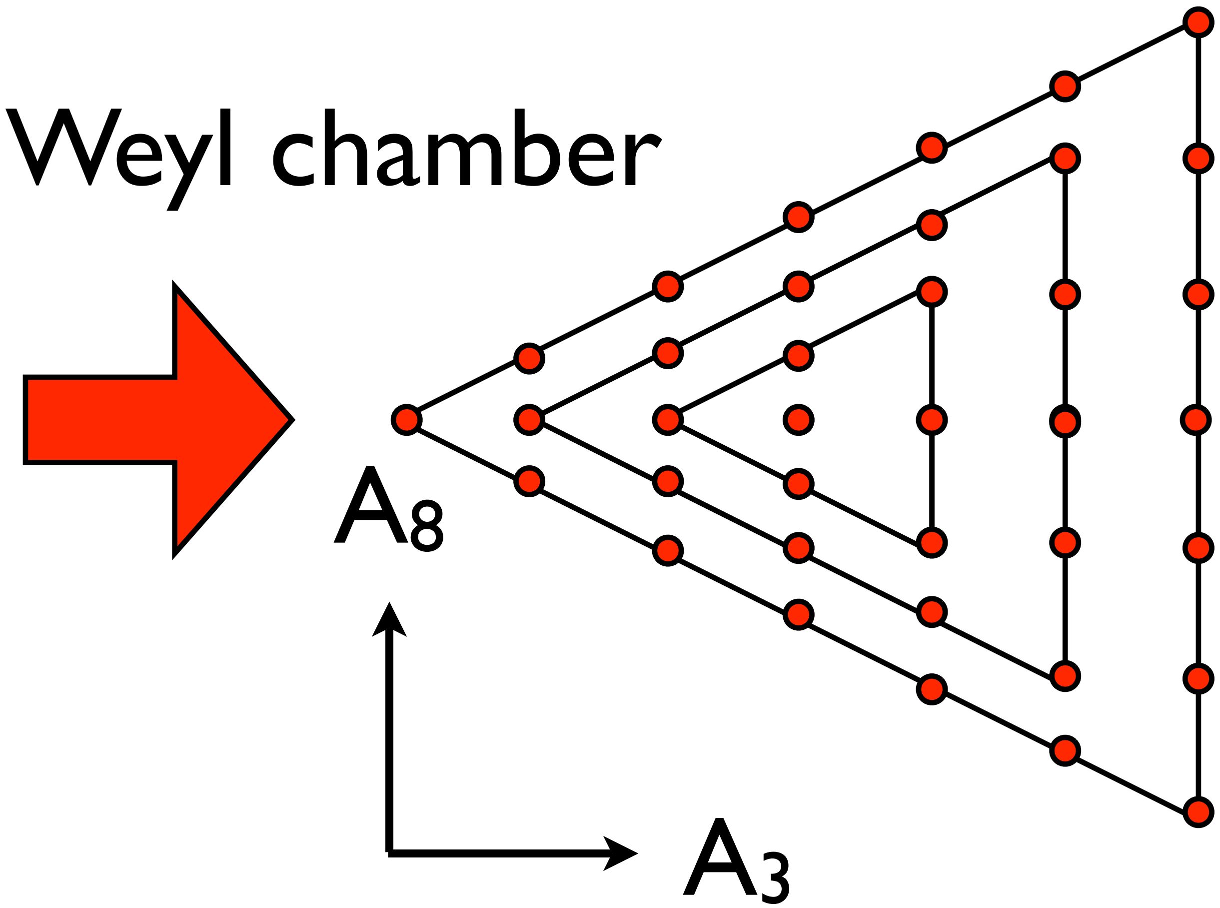
Outlook - SU(3)

FM, J.Pawlowski, work in progress

key ingredient I: adequate grid



Weyl chamber



full information in one Weyl chamber

key ingredient II: advantageous rewriting of the equations

Thank you for your attention

Backup slides

order parameter reformulation

center symmetric phase $\langle L \rangle = 0$ write $L = \langle L \rangle + \delta L$

$$\frac{1}{2} g\beta \langle A_0 \rangle = \langle \arccos L \rangle = \arccos \langle L \rangle - \frac{1}{\sqrt{1 - \langle L \rangle^2}} \langle \delta L \rangle + O\left(\langle \delta L^2 \rangle\right)$$

center trafo $L \to ZL \Rightarrow \delta L \to Z\delta L$

it follows $\langle \delta L^{2n+1} \rangle = Z \langle \delta L^{2n+1} \rangle = 0$

all even powers vanish because arccos is an odd funtion

$$\frac{1}{2} g\beta \langle A_0 \rangle = \arccos \langle L \rangle = \frac{\pi}{2}$$

center broken phase $\langle L \rangle > 0$

$$L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle \quad \Rightarrow \quad \langle L \rangle > 0 \Rightarrow \frac{1}{2} g\beta A_0 < \frac{\pi}{2}$$

gauge fixing

time-independence and rotation into Cartan

$$\boxed{\partial_0 \mathrm{tr}\, \sigma_3 A_0 = 0\,, \qquad \mathrm{tr}\,(\sigma_1 \pm i\sigma_2) A_0 = 0\,,}$$

residual gauge fixing

$$\partial_1 \int dx_0\, \mathrm{tr}\, \sigma_3 A_1 = 0\,,$$

$$\partial_2 \int dx_0 dx_1\, \mathrm{tr}\, \sigma_3 A_2 = 0\,,$$

$$\partial_3 \int dx_0 dx_1 dx_2\, \mathrm{tr}\, \sigma_3 A_3 = 0$$

Faddeev-Popov determinant

$$\Delta_{FP}[A] = (2T)^2 \left[\prod_x \sin^2\left(\frac{g A_0^3(\vec{x})}{2T} \right) \right]$$

cancels longitudinal gauge fields in the static approximation

integrating out

no backreaction of temporal on spatial gauge fields

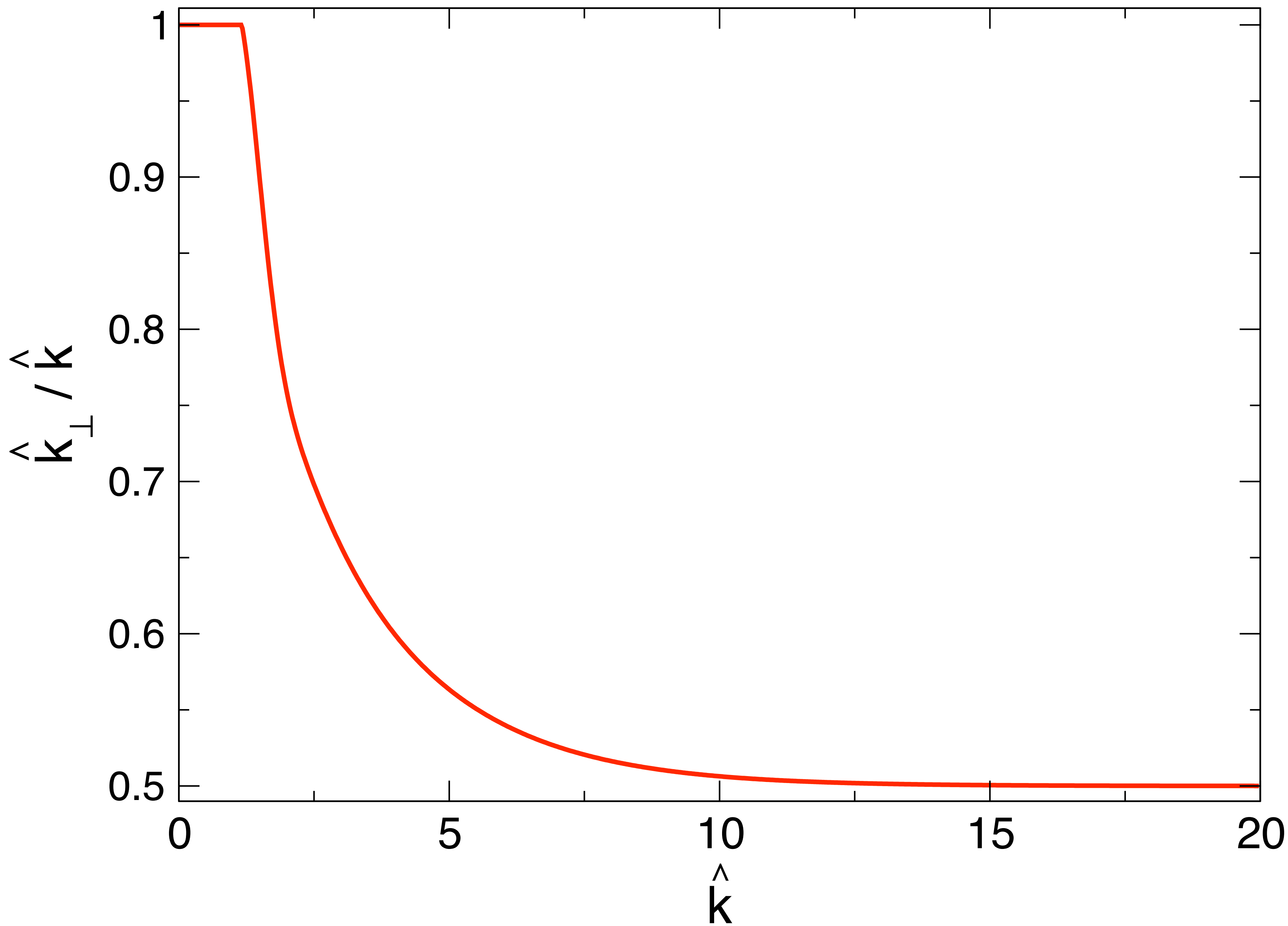
Flow schematically given by

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\Gamma_k^{(2)} + R_A}\right)_{00} \partial_t R_k + \mathrm{Tr}\,\partial_t \left[\ln(S_{YM}^{(2)} + R_A)\right]_{ii} .$$

$$V_{\perp,k} = \frac{4T}{(2\pi)^2}\int_0^{k_\perp(k)} dp p^2 \left\{\ln\Big(1 - 2\cos(\varphi)e^{-\beta k_\perp(k)} + e^{-2\beta k_\perp(k)}\Big) - \ln(1 - 2\cos(\varphi)e^{-\beta p} + e^{-2\beta p})\right\} + V_W$$

matching scales

$$\frac{T}{k^2} f(k/T) \quad = \quad T \sum_{n=-\infty}^{\infty} \frac{1}{\omega_n^2 + k_\perp^2}$$

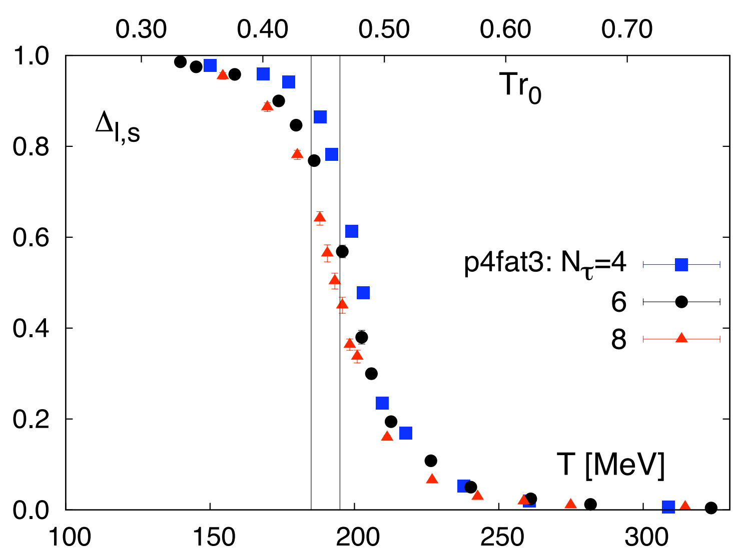


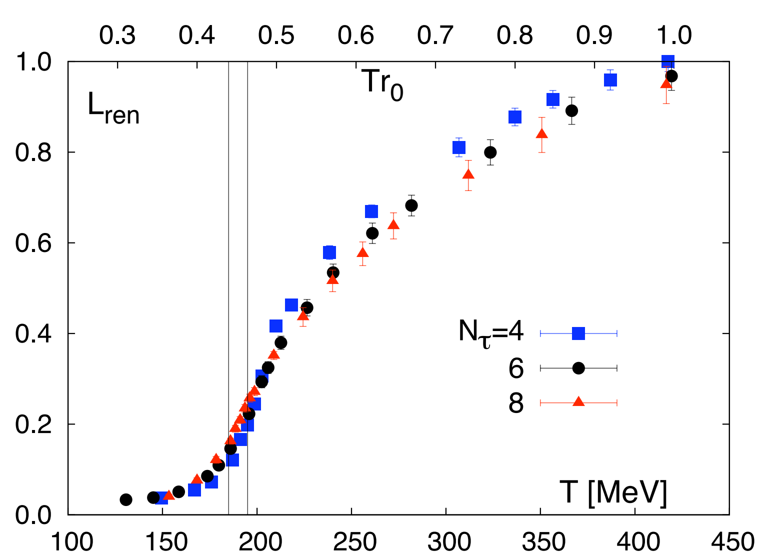
Deconfinement vs. χSB

lattice results

Karsch:

$$T_{\text{conf}} = T_{\chi\text{SB}}$$





Fodor:

$$T_{\text{conf}} > T_{\chi\text{SB}}$$