Calorons and dyons in $SU(2)$ Yang-Mills theory at $T > 0$

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Outline of the talk:

1. Introduction: Instantons and Calorons
2. Calorons with non-trivial holonomy
3. Lattice tools for the instanton or caloron search
4. Results of the lattice search
5. Simulating a caloron gas
6. Simulating a dyon gas
7. Conclusions, outlook

1. Introduction: Instantons and Calorons

Reinvestigation of an old issue: semiclassical approach to QCD path integral

't Hooft, '76; Callan, Dashen, Gross, '78;

Ilgenfritz, M.-P., '81; Shuryak, '82; Diakonov, Petrov, '84.

Solution of $S_{\mathrm{YM}}[A] = \mathrm{Min.} \quad \Leftrightarrow \quad$ (anti-) selfduality $G_{\mu\nu} = \pm\tilde{G}_{\mu\nu}$

Belavin-Polyakov-Shvarts-Tyupkin (BPST) instanton with $Q_{top} = \pm 1$

$$A^{\mathrm{inst}}_{a,\mu}(x-z,\rho,R) = R_{a\alpha}\bar{\eta}_{\alpha\mu\nu}\frac{2\,\rho^2\,(x-z)_\nu}{(x-z)^2\,((x-z)^2+\rho^2)}, \qquad A^{\mathrm{antiinst}} : \bar{\eta} \leftrightarrow \eta$$

Take (semi-) classical fields as (sufficiently dilute) superpositions

$$A^{\mathrm{class}} = \sum_{i=1}^{N_+ + N_-} A^{(i)}_{a,\mu}(x-z^{(i)},\rho^{(i)},R^{(i)}), \qquad \rho^{(i)}\rho^{(j)} \ll (z^{(i)}-z^{(j)})^2$$

in order to approximate the functional integral by $A = A^{\mathrm{class}} + \varphi$

$$\int DA \exp(-S_{\mathrm{YM}}[A]) \simeq \sum_{\mathrm{class}} \exp(-S[A^{\mathrm{class}}]) \int D\varphi \exp\left(-\frac{1}{2!}\int \varphi \frac{\delta^2 S}{\delta A^2}|_{A^{\mathrm{class}}}\varphi\right) + \cdots$$

- Quite powerful phenomenological approach for non-perturbative quantities and phenomena related to chiral symmetry breaking and $U_A(1)$ problem $\Longleftrightarrow$ confinement hard to explain

  See reviews by Schäfer, Shuryak, '98; Dyakonov, '03;...

- Instantons found on lattice by cooling, smoothing,...

  Teper, '86; Ilgenfritz, Laursen, M.-P., Schierholz, Schiller, '86; Polikarpov, Veselov, '88;...

- Relation to models of confinement as proven on the lattice: monopole and vortex condensation?

- Are BPST instantons really the relevant semiclassical building blocks?

- Can the semiclassical approach be improved?

  Recent attempts:

  – interacting instanton (-meron) liquid model Lenz, Negele, Thies, '03-'04

  – pseudoparticle approximation of path integral M. Wagner, '06-'08

Possible answer from studies at $T > 0$?

Partition function

$$Z_{\mathrm{YM}}(T,V) \equiv \mathrm{Tr}\, e^{-\frac{\hat{H}}{T}} \propto \int DA\, e^{-S_{\mathrm{YM}}[A]} \;\; \text{with } A(\vec{x}, x_4{+}b) = A(\vec{x}, x_4), \;\; b = 1/T\,.$$

Old semiclassical treatment with Harrington-Shepard (HS) caloron solutions
$\equiv$ x_4-periodic instanton chains Gross, Pisarski, Yaffe, '81

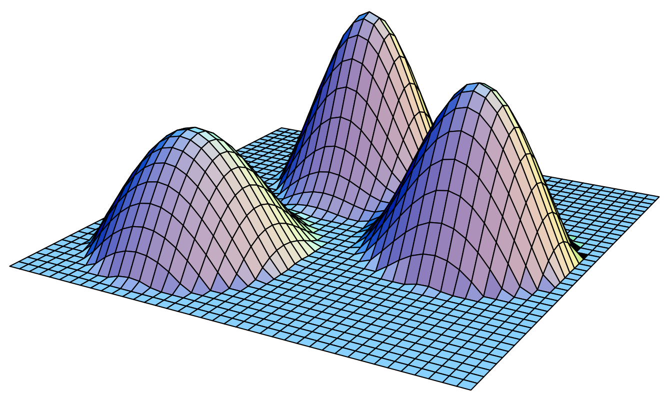
$$A^{\mathrm{HS}}_{a\mu} = \bar{\eta}_{a\mu\nu}\partial_\nu \log(\Phi(x))$$

$$\begin{aligned}\Phi(x) &= 1 + \sum_{k\in\mathbf{Z}} \frac{\rho^2}{(\vec{x}-\vec{z})^2 + (x_4 - z_4 - kb)^2} \\ &= 1 + \frac{\pi\rho^2}{b|\vec{x}-\vec{z}|}\, \frac{\sinh\left(\frac{2\pi}{b}|\vec{x}-\vec{z}|\right)}{\cosh\left(\frac{2\pi}{b}|\vec{x}-\vec{z}|\right) - \cos\left(\frac{2\pi}{b}(x_4 - z_4)\right)}\end{aligned}$$

Kraan - van Baal - Lee - Lu solutions (KvBLL)
= (multi-) calorons with non-trivial asymptotic holonomy

$$P(\vec{x}) = \mathbf{P}\exp\left(i\int_0^{b=1/T} A_4(\vec{x},t)\ dt\right) \overset{|\vec{x}|\to\infty}{\Longrightarrow} \quad \mathcal{P}_\infty \ = e^{2\pi i\omega\tau_3} \notin \ \mathbf{Z}$$

Kraan, van Baal, '98 - '99, Lee, Lu '98



Action density of an $SU(3)$ caloron (van Baal, '99)

$\Longrightarrow$ not a simple $SU(2)$ embedding into $SU(3)$!!

2. Calorons with non-trivial holonomy

K. Lee, Lu, '98, Kraan, van Baal, '98 - '99, Garcia-Perez et al. '99

- x_4-periodic, (anti)selfdual solutions from ADHM formalism,
- generalize Harrington-Shepard calorons (i.e. x_4 periodic BPST instantons).

For $SU(2)$: holonomy parameter $\bar{\omega} = 1/2 - \omega$, $0 \leq \omega \leq 1/2$.

$$
\begin{aligned}
A_\mu^C &= \frac{1}{2}\bar{\eta}^3_{\mu\nu}\tau_3\partial_\nu \log\phi + \frac{1}{2}\,\phi\,\mathrm{Re}\left((\bar{\eta}^1_{\mu\nu} - i\bar{\eta}^2_{\mu\nu})(\tau_1 + i\tau_2)(\partial_\nu + 4\pi i\omega\delta_{\nu,4})\tilde{\chi}\right) \\
&\qquad + \delta_{\mu,4}\;2\pi\omega\tau_3\,, \\
\phi(x) &= \frac{\psi(x)}{\hat{\psi}(x)}\,, \qquad x = (\vec{x}, x_4 \equiv t)\,, \qquad r = |\vec{x} - \vec{x}_1|, \quad s = |\vec{x} - \vec{x}_2|\,, \\
\psi(x) &= -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega) + \frac{r^2 + s^2 + \pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega) \\
&\quad + \frac{\pi\rho^2}{s}\sinh(4\pi s\omega)\cosh(4\pi r\bar{\omega}) + \frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\cosh(4\pi s\omega), \\
\hat{\psi}(x) &= -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega) + \frac{r^2 + s^2 - \pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega)\,, \\
\tilde{\chi}(x) &= \frac{1}{\psi}\left\{e^{-2\pi i t}\frac{\pi\rho^2}{s}\sinh(4\pi s\omega) + \frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\right\}.
\end{aligned}
$$

Properties:

- periodicity with $b = 1/T$,

- (anti)selfdual with topological charge $Q_t = \pm 1$,

- has two centers at $\vec{x}_1, \vec{x}_2$ $\rightarrow$ "instanton quarks",

- scale-size versus distance: $\pi \rho^2 T = |\vec{x}_1 - \vec{x}_2| = d$,

- limiting cases:

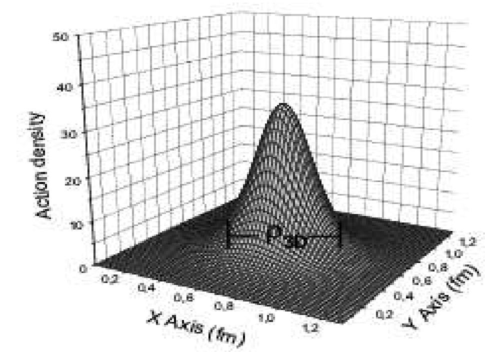
  - $\omega \rightarrow 0$ $\Longrightarrow$ 'old' HS caloron,
  - $|\vec{x}_1 - \vec{x}_2|$ large $\Longrightarrow$ two static BPS monopoles or dyons (DD) with mass ratio $\sim \bar{\omega}/\omega$,
  - $|\vec{x}_1 - \vec{x}_2|$ small $\Longrightarrow$ non-static single caloron (CAL).

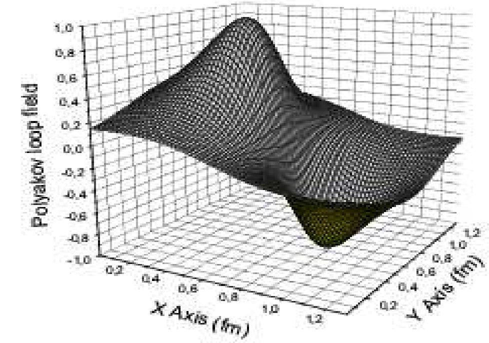
- $L(\vec{x}) = \frac{1}{2} \mathrm{tr} P(\vec{x}) \rightarrow \pm 1$ close to $\vec{x} \simeq \vec{x}_{1,2}$ $\Longrightarrow$ "dipole structure"

KvBLL $SU(2)$ caloron:

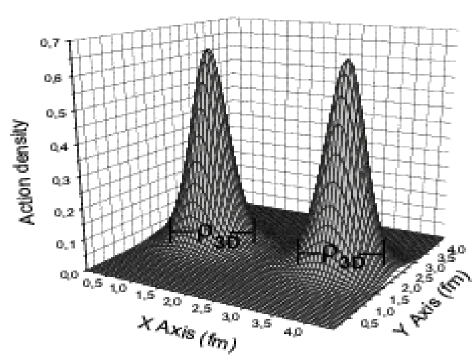
Action density

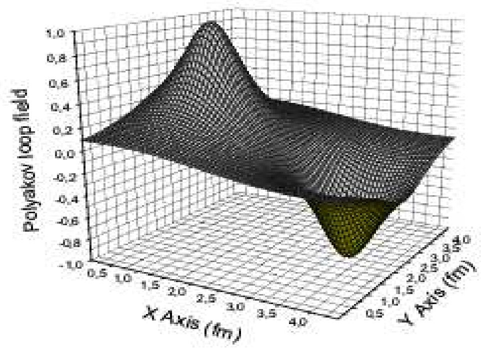
Polyakov loop





CAL





DD

- Localization of the zero-mode of the Dirac operator:

- time-antiperiodic b.c.:
  around the center with $L(\vec{x}_1) = -1$,

$$|\psi^{-}(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi r\bar{\omega})/r\right] \quad \text{for large} \quad d,$$

- time-periodic b.c.:
  around the center with $L(\vec{x}_2) = +1$,

$$|\psi^{+}(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi s\omega)/s\right] \quad \text{for large} \quad d.$$

- $SU(N_c)$ KvBLL calorons

- - consist of N_c monopole constituents becoming well-separated static BPS monopoles (dyons) in the limit of large distances or scale sizes,
  - resemble single-localized HS calorons (BPST instantons) at small distances, but are genuine $SU(N_c)$ objects - not embedded $SU(2)$.

- Eigenvalues of the (asymptotic) holonomy

$$\mathcal{P}_\infty = g \ \exp(2\pi i \, \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_N)) \ g^\dagger$$

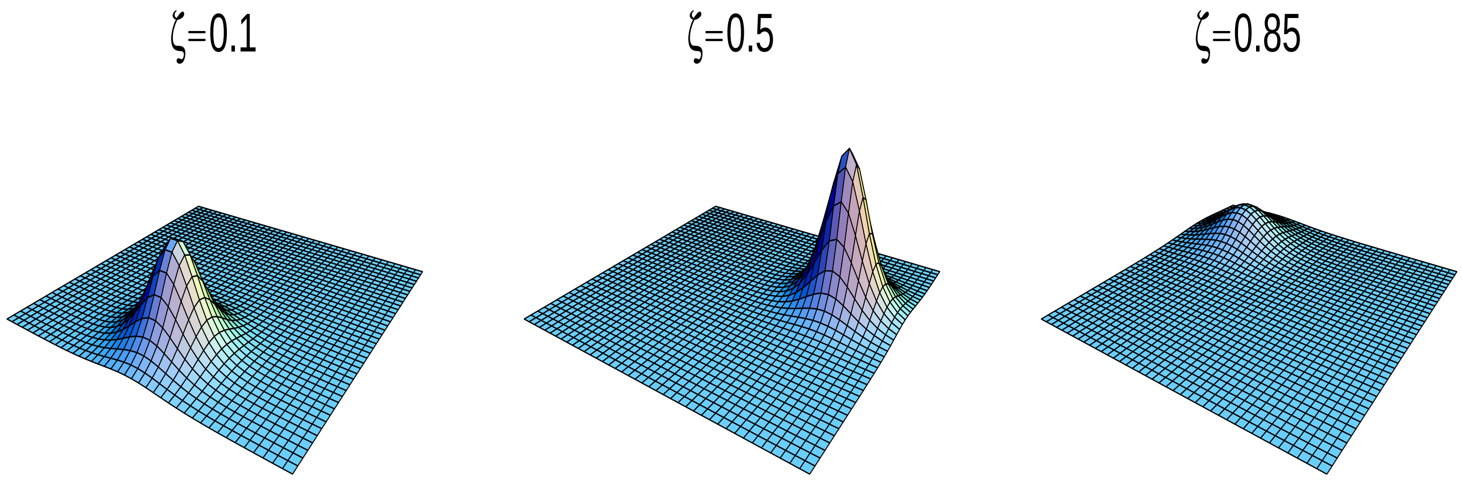
  with ordering $\mu_1 < \mu_2 < \cdots < \mu_{N+1} \equiv 1 + \mu_1 \ , \ \ \mu_1 + \mu_2 + \cdots + \mu_N = 0$ determine the masses of the dyons: $M_i = 8\pi^2(\mu_{i+1} - \mu_i), \ i = 1, \cdots, N.$

- Monopole constituents are localized at positions $\vec{x}_m$, where eigenvalues of the Polyakov loop $P(\vec{x})$ degenerate.

- $SU(3)$: moving localization of the fermionic zero mode from constituent to constituent when changing the boundary condition with phase $\zeta \in [0, 1]$:

$$\Psi_z(x_0 + b, \vec{x}) = e^{-2\pi i \zeta}\ \Psi_z(x_0, \vec{x})$$

(with $b = 1/T$)



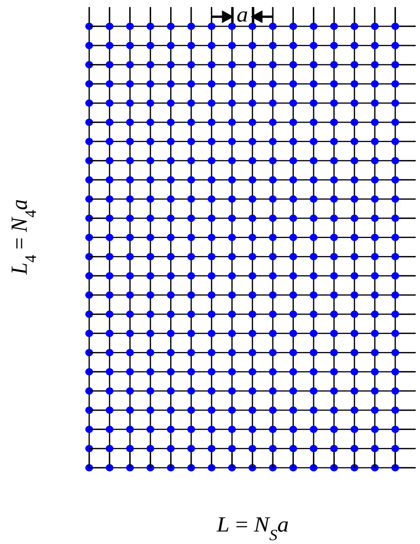
Garcia Perez, et al., '99; Chernodub, Kraan, van Baal, '00

- Multi-calorons known only in very special cases

  van Baal, Bruckmann, Nogradi, '04

- Treatment of the path integral in the background of KvBLL calorons in terms of monopole constituents: free energy favours non-trivial holonomy at $T \simeq T_c$ Diakonov, '03; Diakonov, Gromov, Petrov, Slizovskiy, '04

3. Lattice tools for the instanton and caloron search



$$\begin{aligned} x &\longrightarrow x_n \\ \phi(x) &\longrightarrow \phi_n \equiv \phi(x_n) \\ \partial_\mu \phi(x) &\longrightarrow \frac{1}{a}(\phi_{n+\hat{\mu}} - \phi_n) \end{aligned}$$

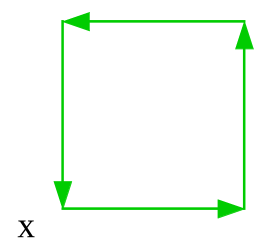
Gauge fields:

$$A_\mu(x_n) \Longrightarrow U_{n,\mu} \equiv P \exp i \int_{x_n}^{x_n+\hat{\mu}a} A_\mu dx_\mu \in SU(N_c)$$

Gauge action (Wilson '74):

$$S_W = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N_c} \mathrm{Re\ Tr\ U_{x,\mu\nu}}\right) \sim \sum_{x,\mu<\nu} a^4\, \mathrm{Tr\ G_{\mu\nu} G_{\mu\nu}(x)},$$

$$\beta = \frac{2N_c}{g_0^2}$$



Plaquette: $\quad U_{x,\mu\nu} = U_x\; U_{x+\hat{\mu},\nu}\; U^\dagger_{x+\hat{\nu},\mu}\; U^\dagger_{x,\nu}$

Path integral quantization:

$$< W >= Z^{-1} \int \prod_{n,\mu} dU_{n,\mu} W(U) \exp(-S_W(U))$$

$$Z = \int \prod_{n,\mu} dU_{n,\mu} \exp(-S_W(U))$$

Monte Carlo method: Generates ensemble of lattice fields in a Markov chain

$$\{U\}_1, \{U\}_2, \cdots, \{U\}_N$$

with resp. to probability distribution ('Importance sampling')

$$W(\{U\}) = Z^{-1} \exp(-S_W(U)) \, .$$

Take x_4-periodic quantum lattice fields as "snapshots" at $T \neq 0$ in order to search for semi-classical objects

$\Longrightarrow$ calorons with non-trivial holonomy ??

Lattice filter strategies:

(A) Lowest action plateaux, i.e. extract classical solutions with various minimization or "cooling" methods:

$S \approx n\ S_0, \quad n = 1, \cdots, 6, \qquad (S_0 \equiv 8\pi^2/g^2)$

$\Longrightarrow$ KvBLL-like topological clusters seen for $SU(2)$ (and $SU(3)$)

– "dipole (triangle)" constituent structure for the Polyakov loop,

– MAG Abelian monopoles correlated with dyon constituents,

– and fermionic mode "hopping" from constituent to constituent.

(B) Clusters of top. charge by 4d smearing $\quad S \approx n\ S_0, \quad n = O(30-40),$ string tension reduced but non-zero.

(C) Equilibrium lattice gauge fields:

low-lying modes of chirally improved or exact (overlap) Dirac operator in equilibrium without and in combination with smearing

Lattice observables and tools:

- **Cooling and smearing:**

  Successive minimization of the (Wilson plaquette) action $S(U)$ by replacing $U_{x,\mu} \to \bar{U}_{x,\mu}$

$$
\begin{aligned}
\bar{U}_{x,\mu} \quad &= \quad \mathbf{P}_{SU(N_c)} \Bigg((1-\alpha)\, U_{x,\mu} + \\
&\qquad \frac{\alpha}{6} \sum_{\nu(\neq\mu)} \Big[U_{x,\nu} U_{x+\hat{\nu},\mu} U^{\dagger}_{x+\hat{\mu},\nu} + U^{\dagger}_{x-\hat{\nu},\nu} U_{x-\hat{\nu},\mu} U_{x+\hat{\mu}-\hat{\nu},\nu} \Big] \Bigg)
\end{aligned}
$$

  with

  – $\alpha = 1.0 \quad \to \quad$ cooling

    - iteration down to action plateaus in order to search for (approximate) solutions of the classical (lattice) equations of motion $\delta S/\delta U_{x,\mu} = 0\,.$

  – $\alpha = 0.45 \quad \to \quad$ 4d APE smearing

    - iteration in order to remove short-range fluctuations

    $\to$ clusters of top. charge far from being class. solutions.

- Gluonic observables
  - *action density* $\varsigma(\vec{x}) = \frac{1}{N_t}\sum_t s(\vec{x},t)$;
  - *topological density*

$$q_t(\vec{x}) = -\frac{1}{2^9\pi^2 N_t}\sum_t \left(\sum_{\mu,\nu,\rho,\sigma=\pm 1}^{\pm 4} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left[U_{x,\mu\nu} U_{x,\rho\sigma}\right] \right);$$

  - *spatial Polyakov loop distribution*

$$L(\vec{x}) = \frac{1}{N_c}\mathrm{tr}\ \mathcal{P}(\vec{x}), \qquad P(\vec{x}) = \prod_{t=1}^{N_t} U_{\vec{x},t,4};$$

    in particular asymptotic holonomy

$$L_\infty = \frac{1}{N_c}\mathrm{tr}\left(\frac{1}{V_\alpha}\sum_{\vec{x}\in V_\alpha}\left[\mathcal{P}(\vec{x})\right]_{\mathrm{diagonal}} \right),$$

    where V_α region of minimal action (topological) density;
  - *Abelian magnetic fluxes and monopole charges* within maximally Abelian gauge (MAG).

- Fermionic modes:

  eigenvalues and eigenmode densities of lattice Dirac operator

$$\sum_y D[U]_{x,y}\ \psi(y) = \lambda\ \psi(x)$$

(with varying x_4-boundary conditions) determined numerically by applying Arnoldi method (ARPACK code package).

Standard Wilson - badly breaking chiral invariance:

$$D_W[U]_{x,y} = \delta_{xy} - \kappa \sum_\mu \left\{ \delta_{x+\hat{\mu},y}\,(\mathbf{1} - \gamma^\mu) U_{x,\mu} + \delta_{y+\hat{\mu},x}\,(\mathbf{1} + \gamma^\mu) U^\dagger_{y,\mu} \right\}$$

Chiral improvement - overlap operator:

$$D_{\mathrm{ov}} = \frac{\rho}{a}\ \left(1 + D_{\mathrm{W}} / \sqrt{D^\dagger_{\mathrm{W}}\ D_{\mathrm{W}}} \right)\ , \qquad D_{\mathrm{W}} = M - \frac{\rho}{a}\ ,$$

satisfies Ginsparg-Wilson relation $\Longrightarrow$ chiral symmetry at $a \neq 0$

$$D\gamma_5 + \gamma_5 D = \frac{a}{\rho} D\gamma_5 D\ ,$$

D_{ov} guarantees index theorem $\quad Q_{\mathrm{index}} = N_- - N_+$.

Topological charge density filtered by truncated mode expansion:

$$q_{\lambda_{\mathrm{cut}}}(x) = - \sum_{|\lambda| \le \lambda_{\mathrm{cut}}} \left(1 - \frac{\lambda}{2} \right) \psi_\lambda^\dagger \gamma_5 \psi_\lambda(x) \; ,$$

Numerical evidence for equivalence of filters:

Chirally improved fermionic filter applied to equilibrium (quantum) fields reveals similar cluster structures as 4D smearing, if mode truncation is tuned to appropriate number of smearing steps:

$$\text{Small } N_{\mathrm{smear}} \quad \Longleftrightarrow \quad \text{large } N_{\mathrm{modes}}.$$

$\Longrightarrow \quad$ moderate smearing of MC lattice fields seems justified.

4. Results of the lattice search

ad (B) Topological clusters from 4d smearing - SU(2) case

Ilgenfritz, Martemyanov, M.-P., Veselov, '04 - '05

4D APE smearing:

- reduces quantum fluctuations while keeping long range physics,
- string tension becomes slowly reduced,
- lumps (clusters) of topological charge become visible.

We analyse clusters w. r. to their MAG Abelian monopole content, select
– static monopole world lines = 'distinct dyons',
– closing monopole world lines = 'distinct calorons'.

Estimate radius of top. clusters from peak values of top. density

$\Longrightarrow$ top. cluster charges.

Questions:

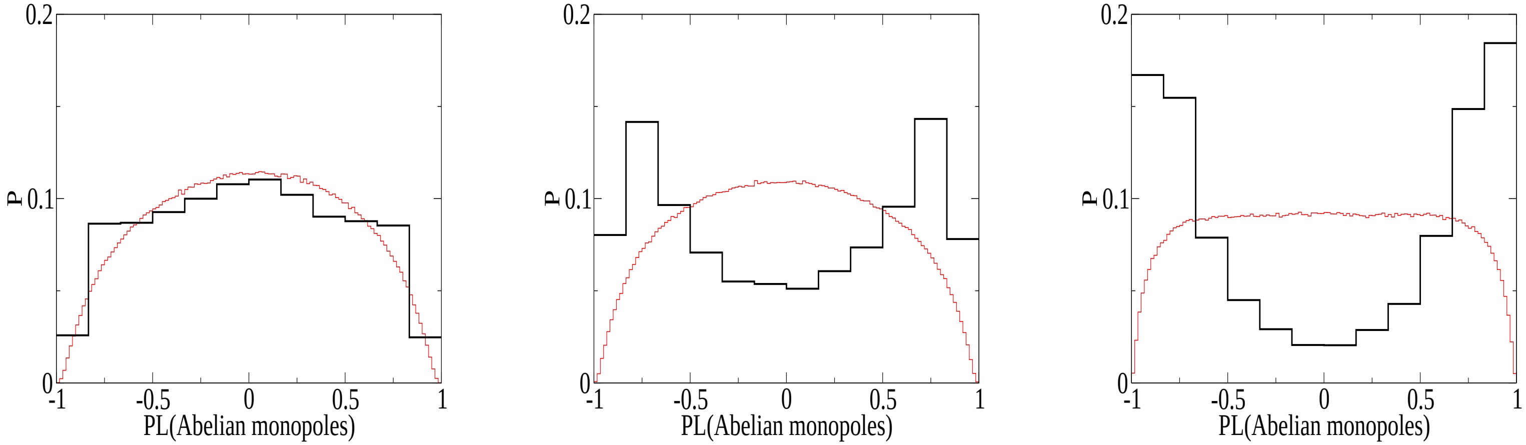
- How does the topological content changes with $T < T_c$? $N_{\mathrm{dyon}} : N_{\mathrm{caloron}}$?

- What happens for $T > T_c$?

$T < T_c$: lattice size $24^3 \times 6$, 50 4d smearing steps

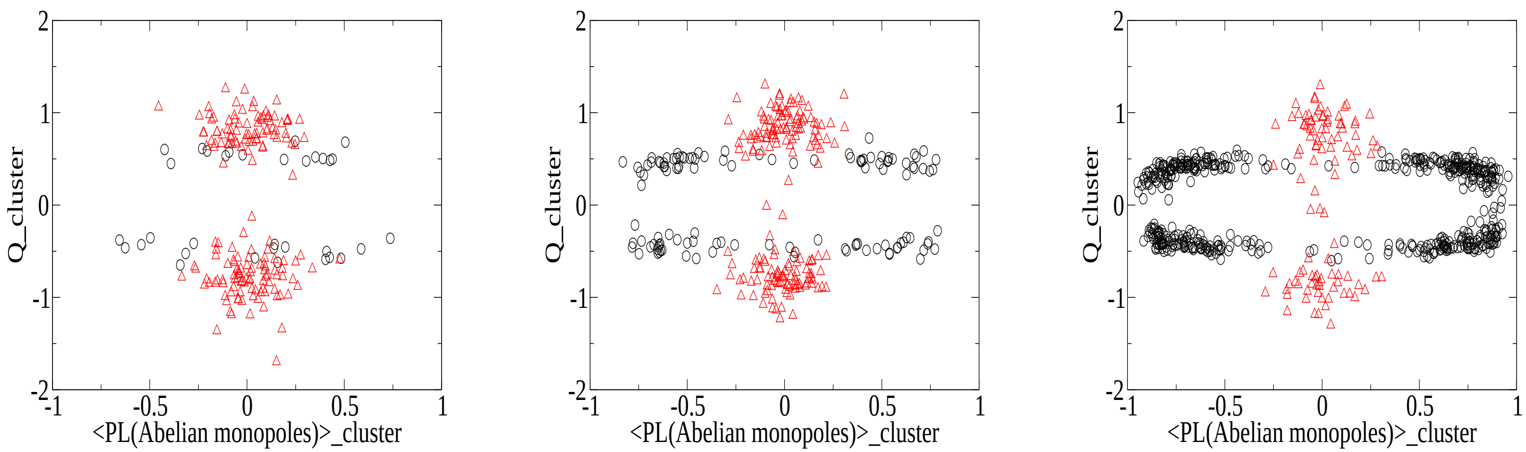
Polyakov loop distributions in lattice sites with time-like MAG Abelian monopoles.

For comparison: unbiased distribution of Polyakov loops in all sites.



$\beta = 2.2$ $\beta = 2.3$ $\beta = 2.4$

Q_{cluster} versus Pol. loop averaged over positions of time-like Abelian monopoles



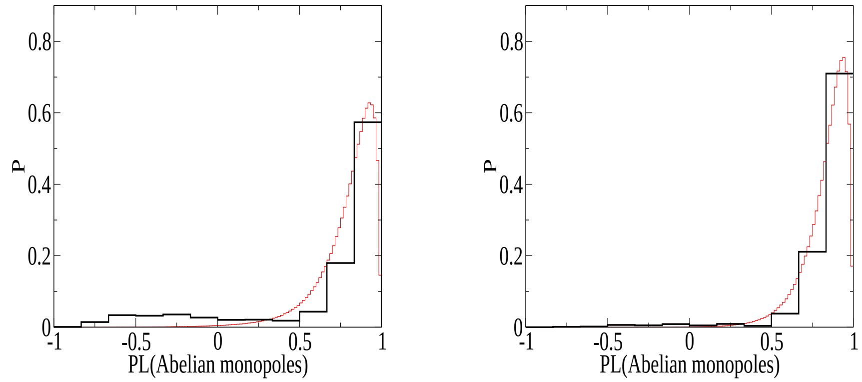
$\beta = 2.2$ $\qquad$ $\beta = 2.3$ $\qquad$ $\beta = 2.4$

$\Rightarrow$ Topological clusters with $Q_t \simeq \pm\frac{1}{2}$ identified.

$\Rightarrow$ $N_{\text{dyon}} : N_{\text{caloron}}$ of identifiable single dyons and non-dissociated calorons rises with $T \to T_c$.

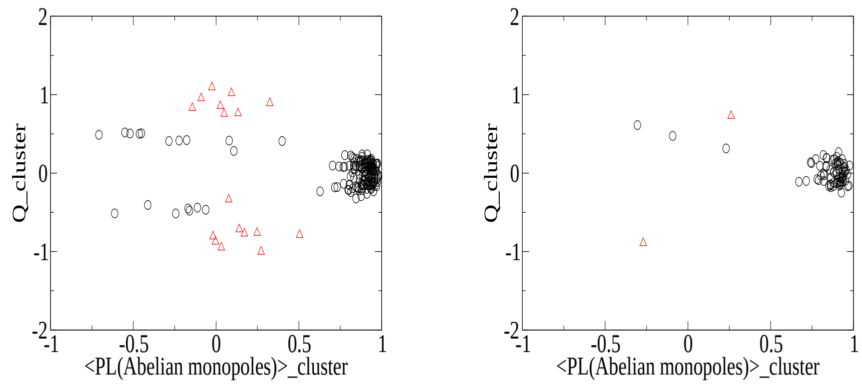
$T > T_c$: lattice size $24^3 \times 6$, 25 (20) smearing steps for $\beta = 2.5$ (2.6).

Polyakov loop distributions in lattice sites with time-like MAG Abelian monopoles.



$\beta = 2.5$ $\beta = 2.6$

Q_{cluster} versus Pol. loop averaged over positions of time-like Abelian monopoles



$\Rightarrow$ dominantly light monopoles (dyons) found, calorons suppressed for $T > T_c$.

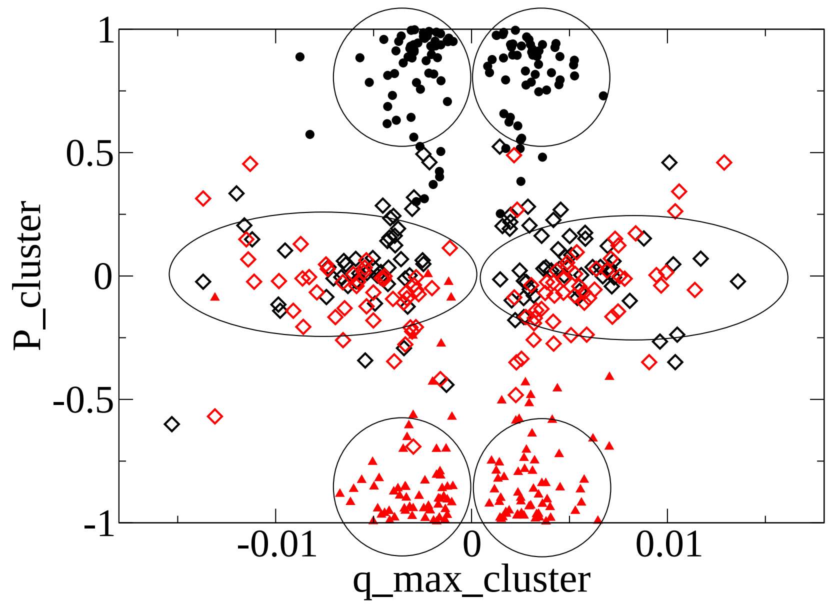
ad (C) Equilibrium fields: low-lying fermionic modes ($SU(2)$)

Bornyakov, Ilgenfritz, Martemyanov, Morozov, M.M.-P., Veselov, '07

Use tadpole-improved Lüscher-Weisz action
for good performance of the overlap operator,
lattice size $20^3 \times 6$, $\beta = 3.25 \simeq \beta_c$.

Observables:

- $q_{\lambda_{cut}}(x)$ with 20 lowest-lying modes
  for periodic and (anti-)periodic b.c.,
- identify topological clusters of both sign,
  find $q_{\mathrm{max\ cluster}}$,
- Polyakov loop $P(x)$ inside top. clusters after 10 APE smearings,
  find $P_{\mathrm{extr\ cluster}}$,
- identify clusters of type "CAL $\equiv$ DD" and "D"



D's shown at their $(q_{\rm max\ cluster}, P_{\rm extr\ cluster})$ positions. CAL's re-located according to the average $\overline{P}_{\rm cluster}$ over opposite sign values $P_{\rm max}$ and $P_{\rm min}$.

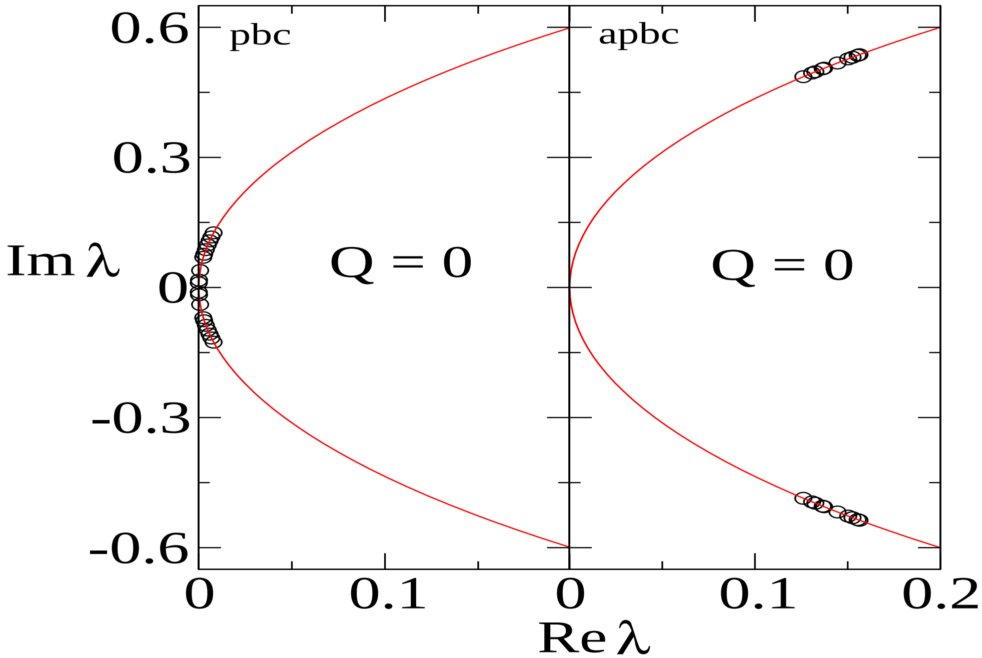
$\Longrightarrow$ KvBLL-like clusters identified at $T \simeq T_c$.

Situation at $T = 1.5\ T_c$??

Bornyakov, Ilgenfritz, Martemyanov, M.M.-P., '09

Realized with $20^3 \times 4$, $\beta = 3.25$, within $Z(2)$ sector with $\langle L \rangle > 0$.

$\Longrightarrow$ Overlap eigenvalues of a typical MC configuration:

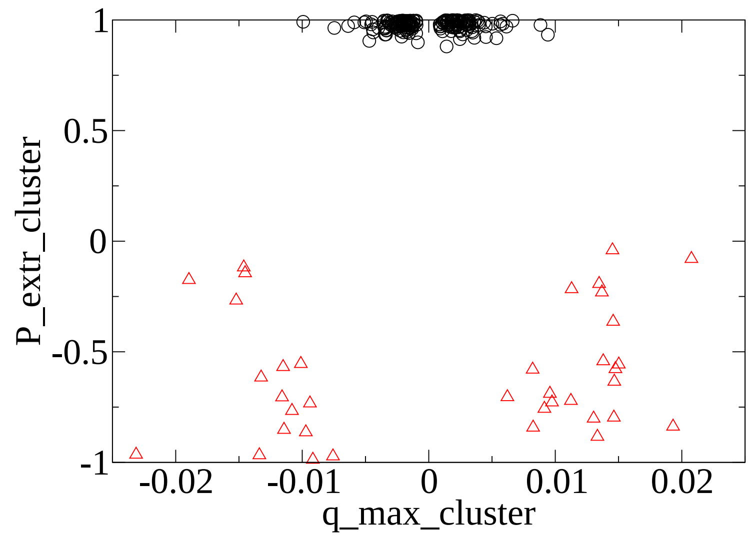


For $\langle L \rangle < 0$ Figs. for "pbc" and "apbc" would interchange!

For clusters containing static MAG Abelian monopoles show
- the extremal value of the topological charge density,
- the peak value of the local Polyakov line.

(Anti)selfduality with field strength from low-lying modes is well satisfied.

Circles $\longleftrightarrow$ clusters found with pbc (light dyons),
triangles $\longleftrightarrow$ clusters found with apbc (heavy dyons).



$\Longrightarrow$ KvBLL-like constituents again visible.
$\Longrightarrow$ But D's (not CAL's) are statistically dominant.

5. Simulating a caloron gas

[HU Berlin master thesis by P. Gerhold, '06; Gerhold, Ilgenfritz, M.-P., '06]

Model based on random superpositions of KvBLL calorons.

Superpositions made in the algebraic gauge – A_4-components fall off.
Gauge rotation into periodic gauge

$$A_\mu^{per}(x) = e^{-2\pi i x_4 \vec{\omega}\vec{\tau}} \cdot \sum_i A_\mu^{(i),alg}(x) \cdot e^{+2\pi i x_4 \vec{\omega}\vec{\tau}} + 2\pi\vec{\omega}\vec{\tau} \cdot \delta_{\mu,4}.$$

First important check: study the influence of the holonomy

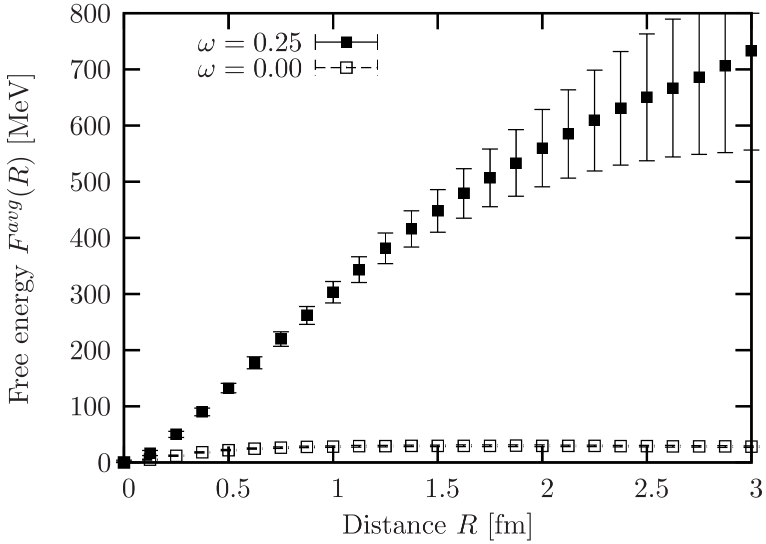
- same fixed holonomy for all (anti)calorons: $\mathcal{P}_\infty = \exp 2\pi i \omega \tau_3$
  $\omega = 0$ – trivial, $\omega = 1/4$ – maximally non-trivial,
- put equal number of calorons and anticalorons randomly but with fixed distance between monopole constituents $d = |\vec{x}_1 - \vec{x}_2| = \pi\rho^2 T$, in a 3d box with open b.c.'s,
- for measurements use a $32^3 \times 8$ lattice grid and lattice observables,
- fix parameters and lattice scale: temperature: $T = 1\ \mathrm{fm}^{-1} \simeq T_c$, density: $n = 1\ \mathrm{fm}^{-4}$, scale size: fixed $\rho = 0.33$ fm vs. distribution $D(\rho) \propto \rho^{7/3} \exp(-c\rho^2)$ such that $\overline{\rho} = 0.33$ fm.

Polyakov loop correlator $\rightarrow$ quark-antiquark free energy

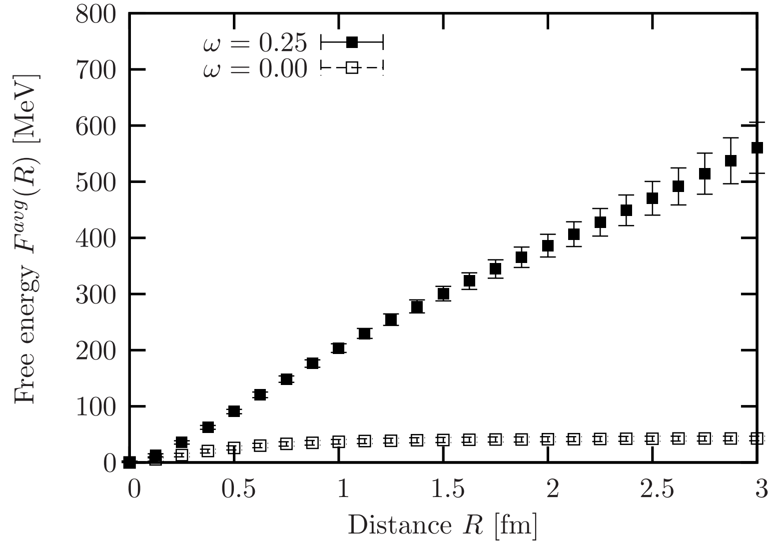
$$F(R) = -T \log\langle L(\vec{x}) L(\vec{y})\rangle, \quad R = |\vec{x} - \vec{y}|$$

with trivial ($\omega = 0$) and maximally non-trivial holonomy ($\omega = 0.25$).

ρ fixed



ρ sampled with distribution



$\Longrightarrow$ Non-trivial (trivial) holonomy (de)confines

for standard instanton or caloron liquid model parameters.

Building a more realistic model for the deconfinement transition

Main ingrediences:

- Holonomy parameter: $\omega = \omega(T)$
  lattice results for the (renormalized) average Polyakov loop.
  Digal, Fortunato, Petreczky, '03; Kaczmarek, Karsch, Zantow, Petreczky, '04
  $\omega = 1/4$ for $T \leq T_c$, ω smoothly decreasing for $T > T_c$.
- Density parameter: $n = n(T)$ for uncorrelated caloron gas to be identified with top. susceptibility $\chi(T)$ from lattice results
  Alles, D'Elia, Di Giacomo, '97
- ρ-distribution:
  $T = 0$: Ilgenfritz, M.-P., '81; Dyakonov, Petrov, '84
  $T > 0$: Gross, Pisarski, Yaffe, '81

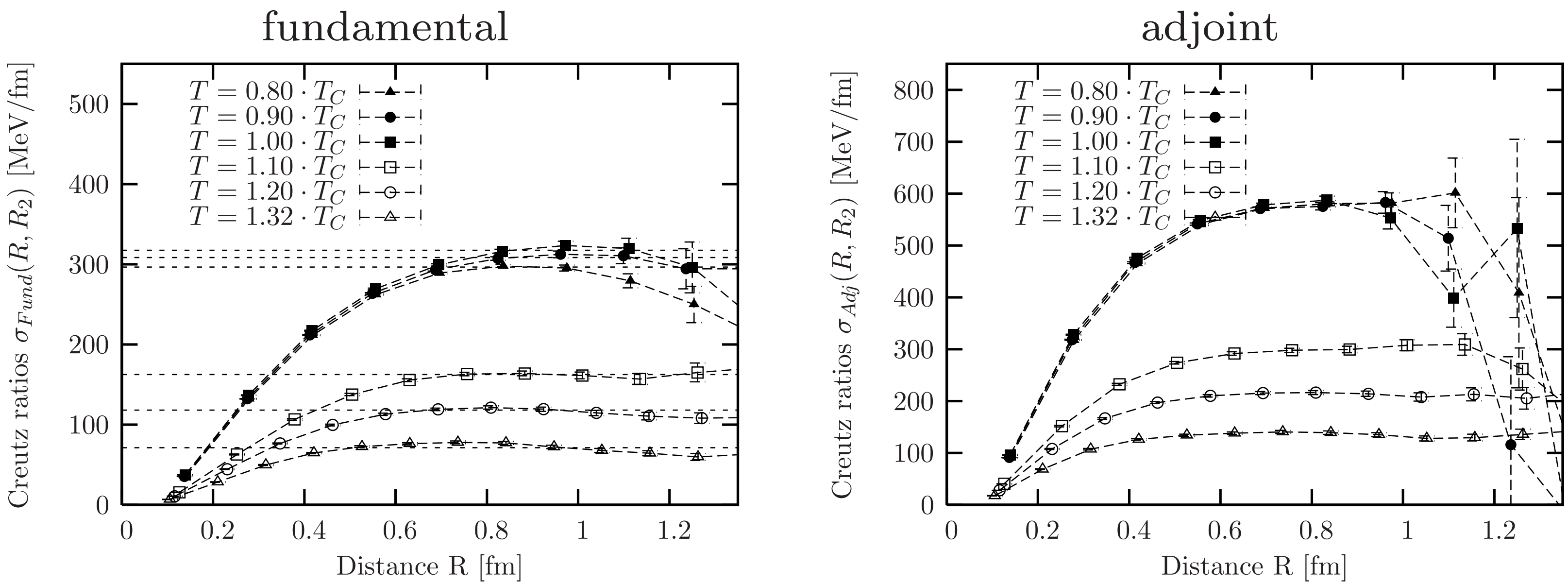
$T < T_c$	$D(\rho,T) = A \cdot \rho^{7/3} \cdot \exp(-c\rho^2)$	$\int D(\rho,T)d\rho = 1, \quad \bar{\rho}$ fixed
$T > T_c$	$D(\rho,T) = A \cdot \rho^{7/3} \cdot \exp(-\frac{4}{3}(\pi\rho T)^2)$	$\int D(\rho,T)d\rho = 1, \quad \bar{\rho}$ running

Distributions sewed together at $T_c \quad \Longrightarrow \quad$ relates $\overline{\rho}(T=0)$ to T_c,
then $\overline{\rho}(T=0)$ to be fixed from known lattice space-like string tension
$T_c/\sqrt{\sigma_s(T=0)} \simeq 0.71$: $\quad \overline{\rho} = 0.37$ fm

Effective string tension $\sigma(R, R_2)$ from Creutz ratios of spatial Wilson loops (with $R_2 = 2 \cdot R$) versus distance R

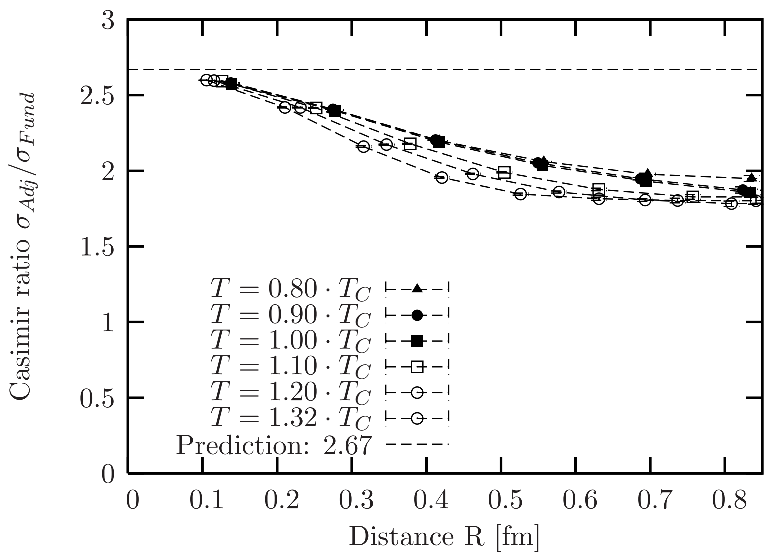
$T/T_c = 0.8,\ 0.9,\ 1.0$ for confined phase,

$T/T_c = 1.10,\ 1.20,\ 1.32$ for deconfined phase.

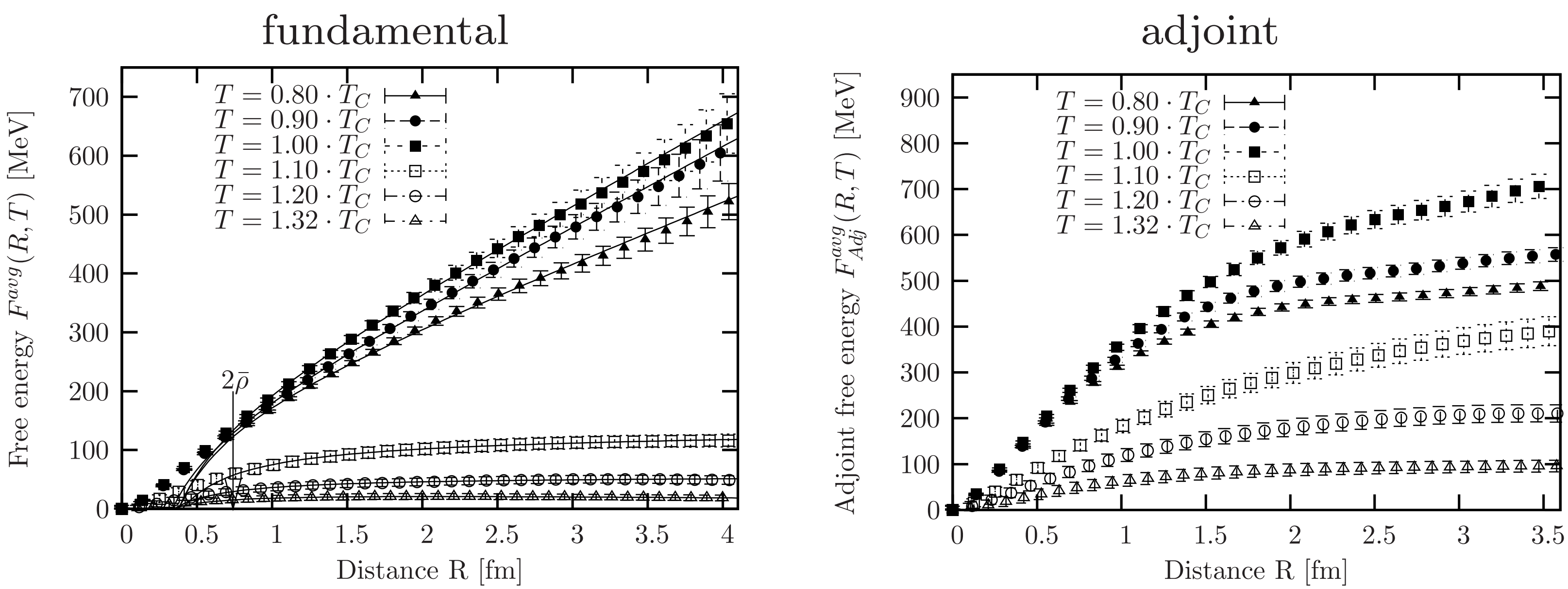


$\Longrightarrow$ Nice plateaux, but no rising $\sigma(T)$ for $T > T_c$.

Test of Casimir scaling for ratio $\sigma_{Adj}/\sigma_{Fund}$ at various T:



Color averaged free energy versus distance R at different temperatures from Polyakov loop correlators.

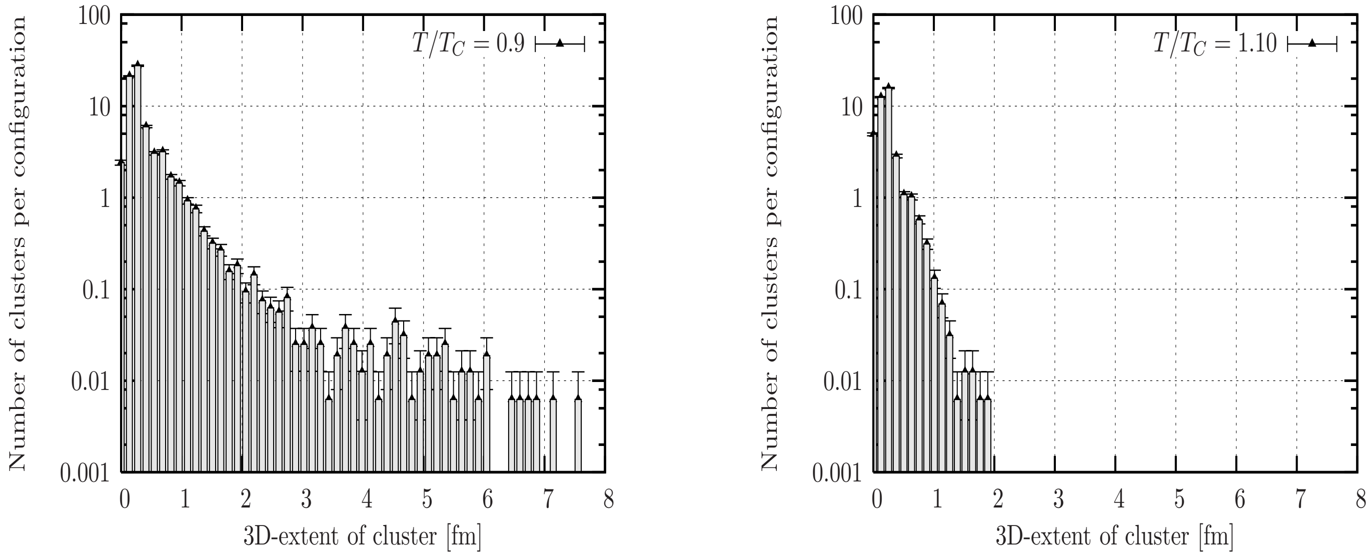


$\Longrightarrow$ successful description of the deconfinement transition,

$\Longrightarrow$ but still no realistic description of the deconf. phase.

Test of the magnetic monopole content in MAG:

histograms of 3-d extensions of dual link-connected monopole clusters



$\Longrightarrow$ Some percolation seen for $T < T_c$ as well as its disappearance for $T > T_c$

5. Simulating a dyon gas

[HU Berlin master thesis, S. Dinter, '09; Bruckmann, Dinter, Ilgenfritz, M.-P., Wagner, '09]

Working hypothesis (cf. Polyakov, '77):
Confinement evolves from magnetic monopoles effectively in 3D viewed here as superpositions of static dyon fields (as KvBLL caloron constituents).

Diakonov, Petrov, arXiv:07045.3181 [hep-th]:
Construct the integration measure over the moduli space of all kind dyons (but no antidyons, i.e. CP violated). Dyon ensemble statistics can be analytically solved. Provides confinement from Abelian fields.

Our numerical simulation:
moduli space metric satisfies positivity only for small number of dyon configurations and only for low density $\Longrightarrow$ inconsistency.

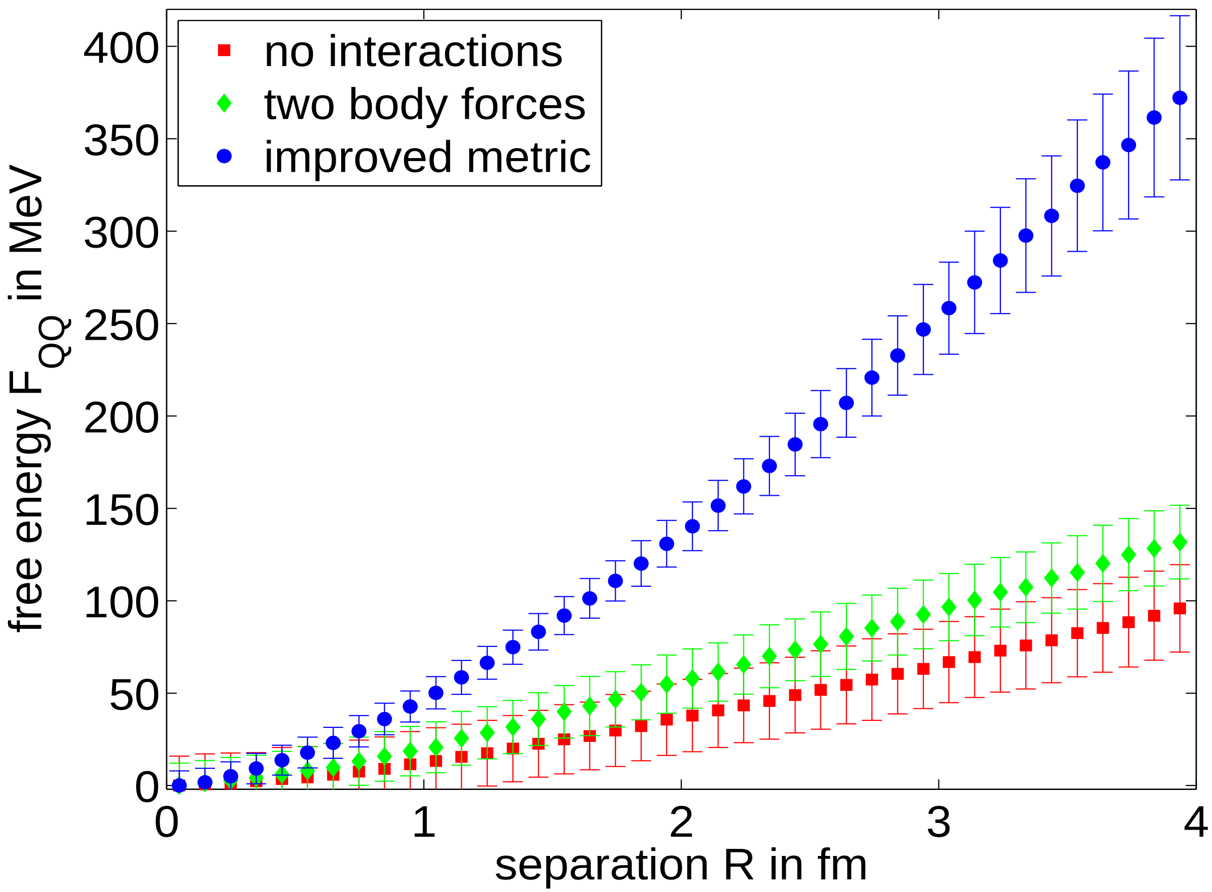
Linearly rising free energy of a static quark-antiquark pair computed from Polyakov loop correlators:

– for random dyon gas,

– for gas with Coulomb-like two-body forces taken into account,

– for gas with full moduli space metric taken into account,
constrained to be positive (realized in a HMC simulation algorithm).

Result: Free energy of a pair of static charges $F_{Q\bar{Q}}$ vs. separation R
here for dyon number $n_D = 200$ and quite low density $\rho = 1/125\,\mathrm{fm}^3$.



7. Conclusions

- KvBLL calorons with non-trivial holonomy have been clearly identified by cooling, 4d smearing and with fermionic modes in the confinement phase.
- For $T \to T_c$ calorons seem to dissociate more and more into well-separated monopoles.
- For $T > T_c$ (corresp. to trivial holonomy) light monopole pairs with opposite top. charge are dominating.
  $\Longrightarrow$ Requires more investigations.
- Semiclassical KvBLL caloron and dyon gas models very encouraging !!

Running and future tasks:

- Improving the KvBLL caloron and dyon gas models: varying holonomy, better ρ-distribution, ...
- Searching for calorons with non-trivial holonomy in full QCD at $T \neq 0$.

Collaborators and references:

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KvBLL solutions as seen from cooling - $SU(3)$ case:

- D. Peschka, diploma thesis Sept. '04.
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KvBLL caloron and dyon gas simulations at $T \neq 0$ - $SU(2)$ case:

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- P. Gerhold, E.-M. Ilgenfritz, M. M.-P., Nucl. Phys. B760 (2007) 1; hep-ph/0607315.
- P. Gerhold, E.-M. Ilgenfritz, M. M.-P., Nucl. Phys. B774 (2007) 268; hep-ph/0610426.
- S. Dinter, diploma thesis Jan. '09.
- F. Bruckmann, S. Dinter, E.-M. Ilgenfritz, M. M.-P., M. Wagner, to be submitted.

Thank you for your attention !!