Confinement and chiral symmetry breaking from functional methods
Results & challenges

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Overview

1. Motivation & chiral symmetry breaking
2. Landau gauge QCD
   - Signatures of confinement
   - Finite volume effects
3. Confinement-deconfinement phase transition
   - Polyakov loop potential
   - Results for the order parameter
4. Summary & Outlook

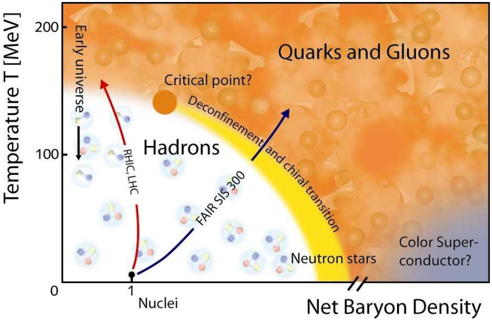
QCD phase diagram

- chiral phase transition:

  $SU_L(N_f) \times SU_R(N_f)$

  order parameter:

  $$\langle \bar{q}\, q \rangle = \begin{cases} 0, & T > T_{c,\chi} \\ > 0, & T < T_{c,\chi} \end{cases}$$



- confinement-deconfinement: Z_3

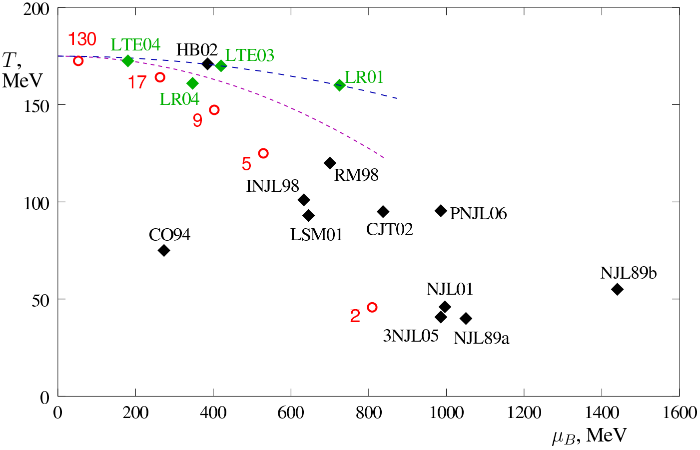
  order parameter: $\beta = 1/T$

  $$\Phi = \langle \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i \int_0^\beta dt\, A_0} \rangle = \begin{cases} > 0, & T > T_{c,\mathrm{conf}} \\ 0, & T < T_{c,\mathrm{conf}} \end{cases}$$

  Polyakov loop $\Phi = e^{-\beta F_q}$ relates to a static quark state.

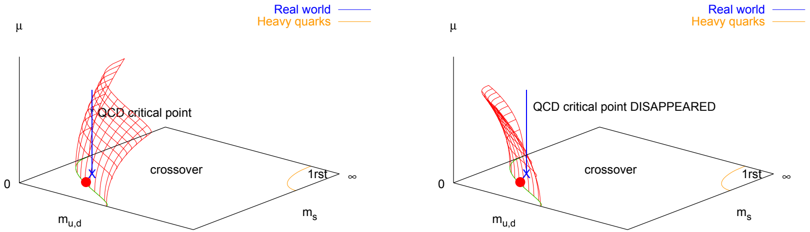
Critical point

Black: models   green, lines: lattice   Red: Freeze-out points for HIC



Stephanov '07

Critical point



Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on infinitesimal μ
Does the transition become 1rst-order (left) or crossover (right)?

Answer: little change ($\rightarrow$ surface almost vertical)

2007: measure δB_4 under $\delta\mu^2 \rightarrow$ crossover: $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{\pi T}\right)^2$

Critical point

QCD with one quark flavor: phase boundary

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)}\right)^2 + \dots$$

- large N_c expansion: $t_2 \sim \frac{N_f}{N_c}$ (D. Toublan '05, JB '08)
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97* / 0.4**	---	---	(JB '08)
Lattice: imag. μ	---	0.500(54)	0.667(6)	(de Forcrand et al. '02, '06)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

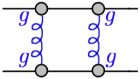
*with global $U_A(1)$ symmetry
** anomalously broken $U_A(1)$ symmetry (lower bound)

- only one single input parameter: $\alpha_s(M_Z)$

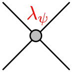
Braun, arXiv:0810.1727 [hep-ph]

$**$ also Braun, Haas, Marhauser, JMP, work in progress

Dynamical Hadronisation

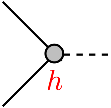


$\Rightarrow$



$\overset{\int_{p-\delta p}^{p}}{\Longleftarrow}$ $\quad g\, \bar\psi \not{A}\, \psi$

Hubbard-Stratonovitch



$\Longleftarrow$

$$\lambda_\psi\, (\bar\psi\psi)^2 = h\, \bar\psi\psi\, \sigma - \tfrac{1}{2} m^2 \sigma^2$$

with $m^2 = -\dfrac{h^2}{2\lambda_\psi}$ and EoM(σ)

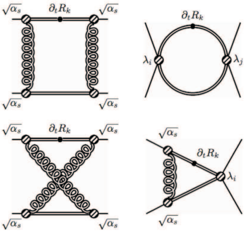
+Baryons and Glueballs

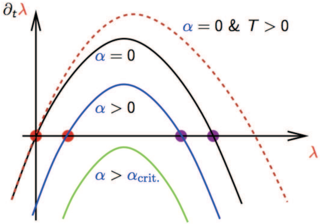
+Baryonisation

Dynamical degrees of freedom

$\psi,\ A$ + Mesons, Baryons $\phi \sim \bar\psi\psi,\ b \sim \psi^3 \Longleftarrow$ Quarks, Gluons $\psi,\ A$

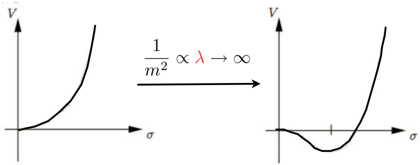
A glimpse at chiral symmetry breaking





- flow of four-fermion couplings:

$$\partial_t\lambda = 2\lambda - \lambda A(\tfrac{T}{k})\lambda - b(\tfrac{T}{k})\lambda\alpha_s - c(\tfrac{T}{k}){\alpha_s}^2$$



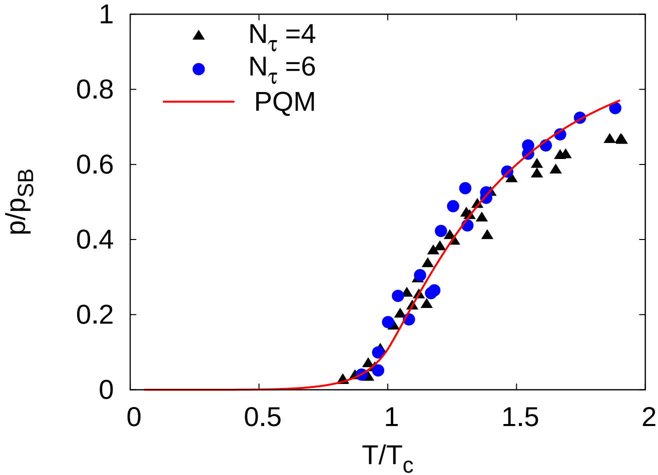
thanks to J. Braun

Polyakov–Quark-Meson Model

Schaefer, JMP, Wambach, Phys. Rev. D **76** (2007) 074023.

EoM of: $U[\Phi,\bar{\Phi}]$ $+V[\sigma,\vec{\pi}]$ $+\Omega_{\bar{q}q}(\Phi,\bar{\Phi},\sigma)$

Polyakov loop — meson — fermionic determinant

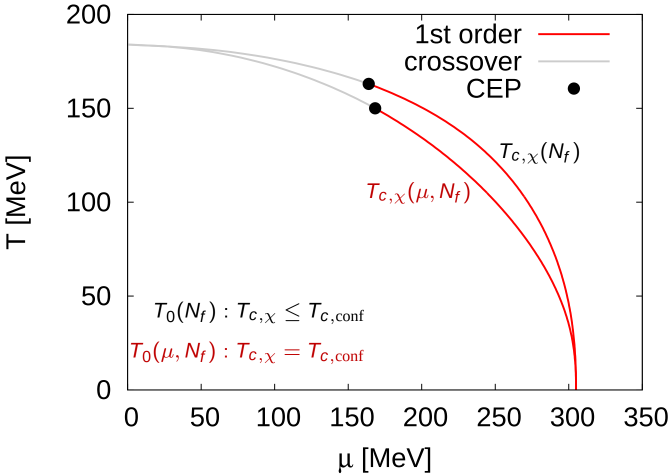


lattice data taken from Ali Khan et al. (CP-PACS), Phys. Rev. D **64** (2001)

Polyakov–Quark-Meson Model

Schaefer, JMP, Wambach, Phys. Rev. D **76** (2007) 074023.

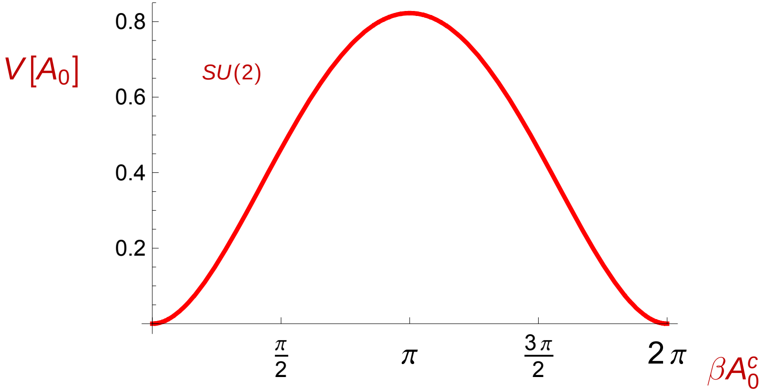
T_0 is the critical temperature from the Polyakov loop potential $U[\Phi, \bar{\Phi}]$



$N_f = 2$

Weiss Potential

$V[A_0]$: one-loop effective potential

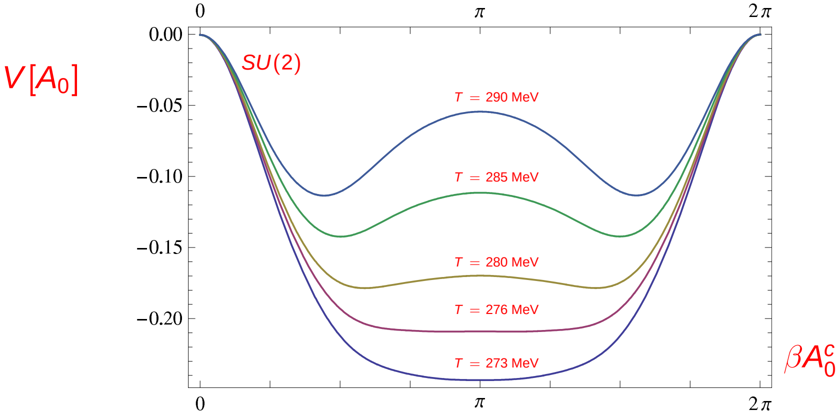


$SU(2)$: $\Phi[A_0] = \cos \frac{1}{2}\beta A_0^c$ with $A_0 = A_0^c \sigma_3$

Preview: full potential from Green functions

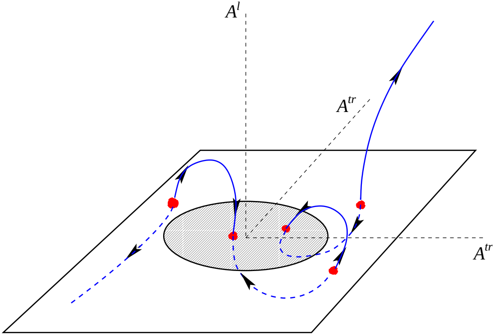
Braun, Gies, JMP, arXiv:0708.2413 [hep-th]

$T_c \simeq 276 \pm 10\mathrm{MeV}$ $\quad$ $T_c/\sqrt{\sigma} = 0.627 \pm 0.023$ $\quad$ lattice: $T_c/\sqrt{\sigma} = .709$



$$\Phi[A_0] = \cos\, \tfrac{1}{2}\beta A_0^c$$

Overview



thanks to L. von Smekal

Gribov problem

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}_0} \int dA \, \mathcal{O}[A] \, e^{-S_{YM}[A]}$$

with $S_{YM} = \frac{1}{2} \int \mathrm{tr}\, F^2$ and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA \, \delta[\partial_\mu A_\mu] \, |\det(-\partial_\mu D_\mu)| \, \mathcal{O}[A] \, e^{-S_{YM}[A]}$$

Faddeev-Popov

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Faddeev-Popov

- BRST: $|\det(-\partial_\mu D_\mu)| \to \det(-\partial_\mu D_\mu)$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA\, dC d\bar{C}\, \delta[\partial_\mu A_\mu]\, \mathcal{O}[A]\, e^{-S_{YM}[A] - \int \bar{C} \partial_\mu D_\mu C}$$

physical Hilbert space $\leftrightarrow$ nilpontency of BRST transformation s.

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but

$$\int dA \, \delta[\partial_\mu A_\mu] \, \frac{\det(-\partial_\mu D_\mu)}{|\det(-\partial_\mu D_\mu)|} = 0$$

Neuberger problem

- Observables in Yang-Mills theory: $\mathcal{O}[A^g] = \mathcal{O}[A]$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA\, \delta[\partial_\mu A_\mu]\, |\det(-\partial_\mu D_\mu)|\, \mathcal{O}[A]\, e^{-S_{YM}[A]}$$

Faddeev-Popov

- BRST: $|\det(-\partial_\mu D_\mu)| \to \det(-\partial_\mu D_\mu)$

$$\langle \mathcal{O}[A] \rangle = \frac{1}{\mathcal{N}} \int dA\, dC\, d\bar{C}\, \delta[\partial_\mu A_\mu]\, \mathcal{O}[A]\, e^{-S_{YM}[A] - \int \bar{C} \partial_\mu D_\mu C}$$

Remedies:

- weighting of copies: loss of BRST-invariance — loss of nilpotency
- topological gauge fixing — Witten index

Confinement from Green functions

C. S. Fischer, A. Maas and J. M. Pawlowski, arXiv:0810.1987 [hep-ph].

Global BRST

Kugo-Ojima criterion

- physical Hilbert space $\mathcal{H}$
- mass gap
- global color charge

Broken global BRST

Signatures of confinement

- construction of $\mathcal{H}$?
- consequences of mass gap ?
- definition of global color charge ?

Landau gauge

- infrared enhanced ghost

$$\langle C\bar{C}\rangle(p) \sim \frac{1}{p^{2(1+\kappa)}}$$

- infrared suppressed gluon

$$\langle AA\rangle(p) \sim \frac{1}{p^{2(1-2\kappa)}}$$

- no constraint: lattice: $\kappa \approx 0$

$$\langle C\bar{C}\rangle(p) \sim \frac{1}{p^2}$$

- no constraint: lattice: $2\kappa \approx 1$

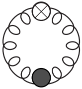
$$\langle AA\rangle(p) \sim \frac{1}{p^0}$$

A glimpse at the Functional RG

Wetterich, Phys. Lett. B **301** (1993) 90

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)}\,k\partial_k R_k(p^2)$$

- Yang-Mills theory: $\phi = (A, C, \bar{C})$

$$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$$

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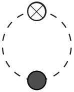
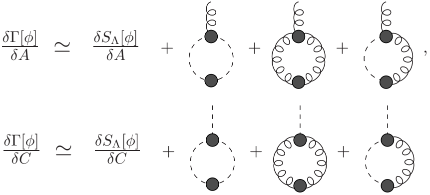


- fermions straightforward though'physically' complicated
  - no'sign problem' numerics as in scalar theories!
  - chiral fermions reminder: Ginsparg-Wilson fermions from RG argument!
  - bound states via dynamical hadronisation effective field theory techniques applicable!

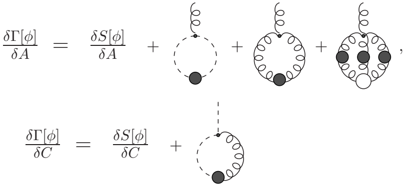
Functional IR scaling of FRG & DSE

Fischer, JMP, Phys. Rev. D **75** (2007) 025012; work in preparation

functional RG



functional DSE



- conformal scaling

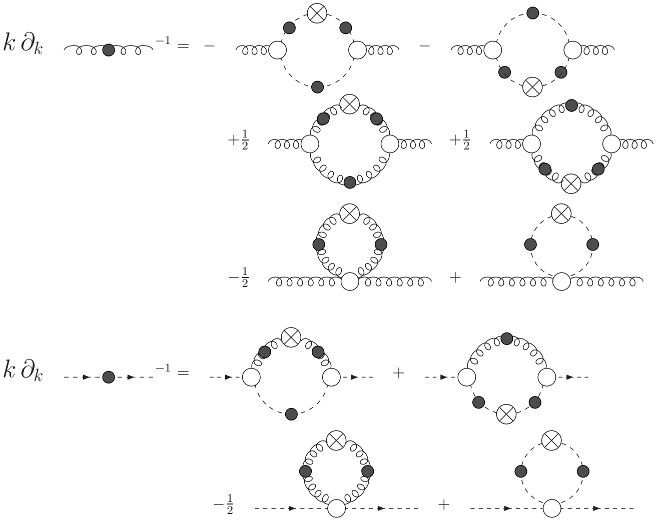
$$\Gamma^{(2n,m,\text{quarks})} \sim p^{2(n-m)\kappa+\text{quarks}}$$

$\Gamma^{(2n,m)}$: vertex with n ghost and anti-ghost lines, m gluons

confirms Alkofer, Fischer, Llanes-Estrada, Phys. Lett. **B611** (2005) 279–288
see also Alkofer, Huber, Schwenzer '08

- decoupling: $\kappa_{2n,m} = 0$ & massive gluon $\leftrightarrow$ loss of (global) BRST

UV-IR flow: Diagrams



UV-IR flow of propagators: Truncation

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- functional optimisation JMP'05
- functional relations between diagrams: Flow=Flow(DSE)

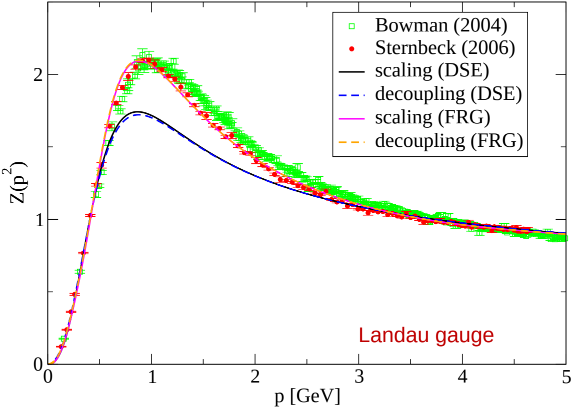
$$\Rightarrow k\partial_k \langle A(p)\,A(-p)\rangle \quad = \quad \mathrm{Flow}_A[\langle A\,A\rangle\,,\,\langle C\bar{C}\rangle]$$

$$k\partial_k \langle C(p)\,\bar{C}(-p)\rangle \quad = \quad \mathrm{Flow}_C[\langle A\,A\rangle\,,\,\langle C\bar{C}\rangle]$$

- scaling/decoupling via boundary conditions at $p^2 = 0$

UV-IR flow of propagators: Results

$p^2 \langle AA \rangle (p^2)$



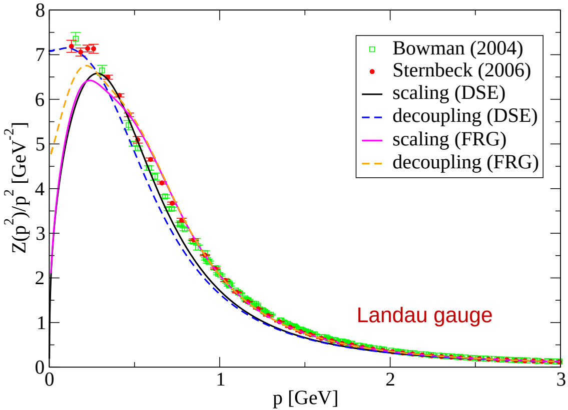
p [GEV]

C. S. Fischer, A. Maas and J. M. Pawlowski, arXiv:0810.1987 [hep-ph].

- quantitative agreement between lattice and FRG

UV-IR flow of propagators: Results

$\langle AA\rangle(p^2)$



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C. S. Fischer, A. Maas and J. M. Pawlowski, arXiv:0810.1987 [hep-ph].

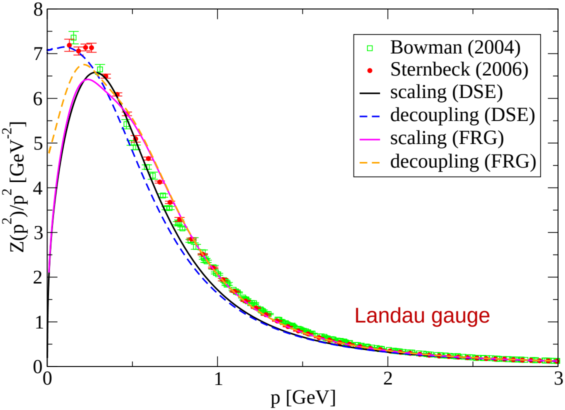
- scaling: $\kappa = 0.59535\ldots, \quad \alpha_s = 2.9717\ldots$

Pawlowski, Litim, Nedelko, von Smekal, Phys. Rev. Lett. **93** (2004) 152002

equals DS/StochQuant-result: Lerche, von Smekal, Phys. Rev. D **65** (2002) ’02
D. Zwanziger, Phys. Rev. D **65** (2002)
RG-confirmation: C. S. Fischer and H. Gies, JHEP **0410** (2004)

UV-IR flow of propagators: Results

$\langle AA \rangle (p^2)$



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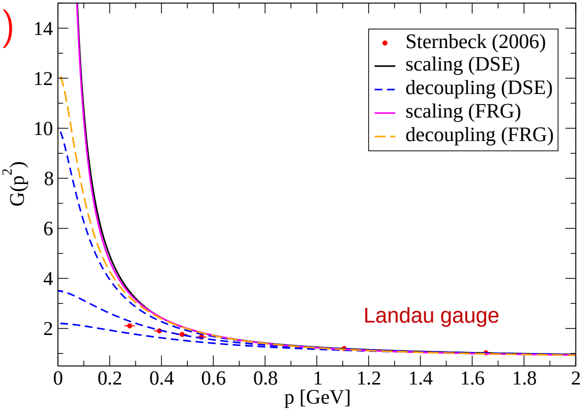
UV-IR flow of propagators: Results

$p^2 \, \langle C\bar{C}\rangle(p^2)$



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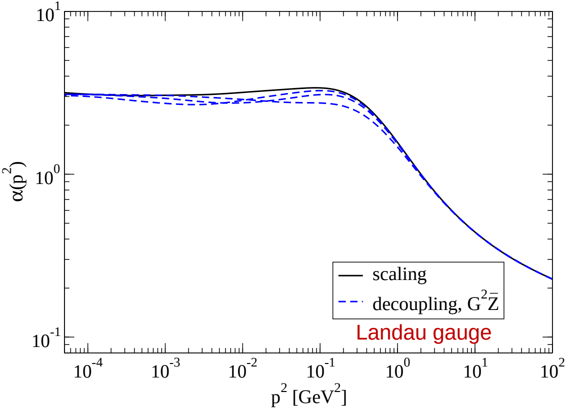
UV-IR flow of propagators: Results

$\alpha_s(p^2)$



p^2 [GEV]

C. S. Fischer, A. Maas and J. M. Pawlowski, arXiv:0810.1987 [hep-ph].

- scaling: $\kappa = 0.59535\ldots\,, \quad \alpha_s = 2.9717\ldots$
- decoupling: $\alpha_{s,\text{decoupling}}(0) := \alpha_{s,\text{scaling}}$

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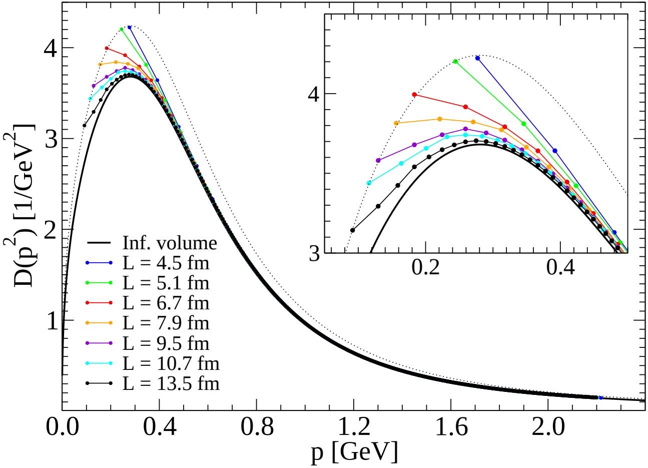
Finite volume effects

Fischer, Maas, Pawlowski, von Smekal, Annals Phys.322:2916-2944,2007

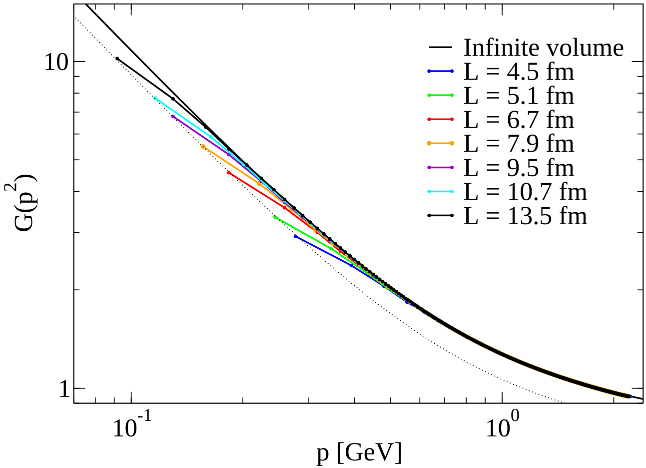
$$D(p^2) = \frac{1}{\Gamma_A^{(2)}(p)}$$



Finite volume effects

Fischer, Maas, Pawlowski, von Smekal, Annals Phys.322:2916-2944,2007

$$G(p^2) = \frac{p^2}{\Gamma_C^{(2)}(p)}$$



Functional methods–lattice puzzle

- lower dimensions
  - quantitative agreement in $d = 2$ Maas '07
  - qualitative agreement in $d = 3$ A. Maas (St Goar '08)
- large volumes on the lattice
  - in $d = 4$ up to 128^4 at $\beta = 2.2$ Cucchieri et al '07
- gauge fixings
  - improved gauge fixings Bogolubsky et al '07, von Smekal et al '07, Maas (St Goar '08)
  - stochastic quantisation with D. Spielmann, I.O. Stamatescu
- $SU(2)$ versus $SU(3)$ Cucchieri et al '07, Sternbeck et al '07
- $\beta = 0$: evidence for gauge fixing/finite size differences von Smekal (St Goar '08)

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Polyakov loop Potential

Braun, Gies, JMP, arXiv:0708.2413 [hep-th]

$$\text{RG-flow :} \quad V[\bar{A}_0] = -\frac{1}{2}\,\mathrm{Tr}\ln \langle AA\rangle[\bar{A}_0] + O(\partial_t \langle AA\rangle[\bar{A}_0]) + O(V''[A_0])$$

Confinement Criterion

$$f(\kappa_A, \kappa) = 1 + \frac{(d-1)\kappa_A - 2\kappa}{d-2} < 0$$

- scaling solution with sum rule $\kappa_A = -2\kappa - \frac{4-d}{2}$

$$\kappa > \frac{d-3}{4}$$

- decoupling solution with $\kappa_A \approx -1$ & $\kappa \approx 0$ (lattice)

$$f(-1,0) = -\frac{1}{d-2}$$

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Braun, Gies, JMP, arXiv:0708.2413 [hep-th]

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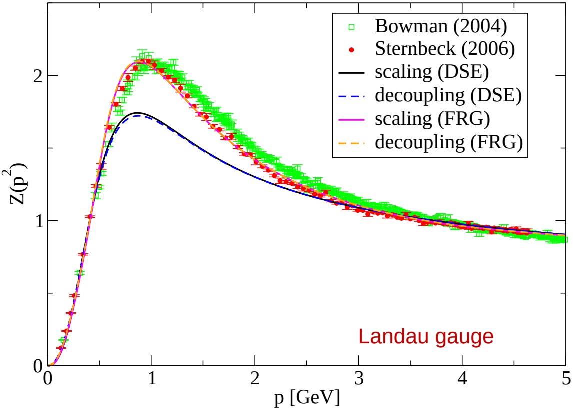
90% 10%

+ghosts

$p^2 \langle AA\rangle(p^2)$



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C. S. Fischer, A. Maas and J. M. Pawlowski, arXiv:0810.1987 [hep-ph].

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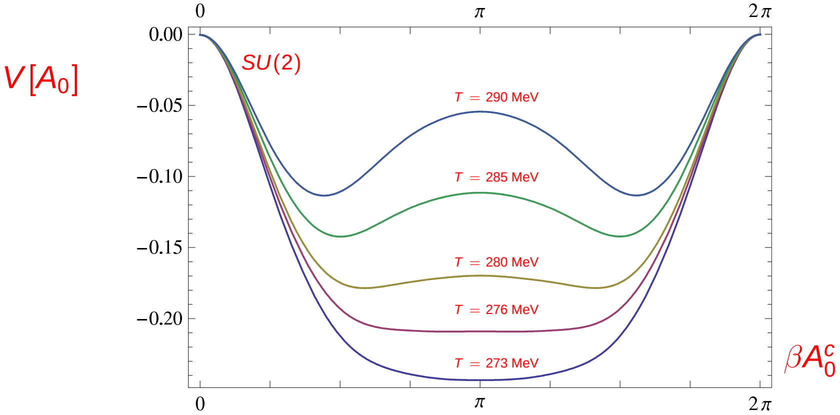

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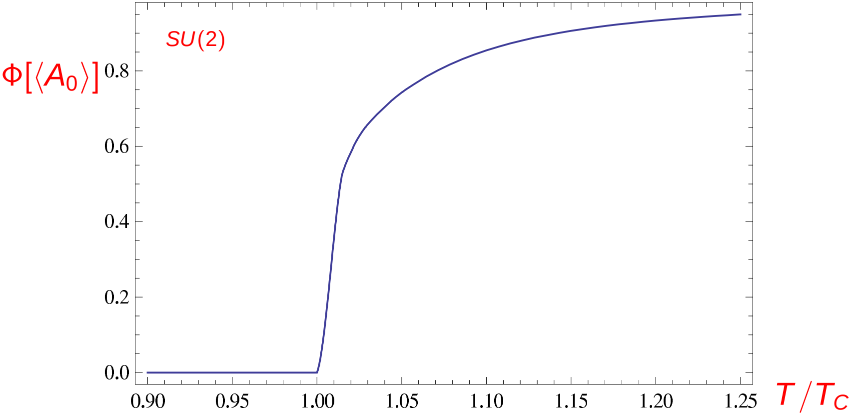
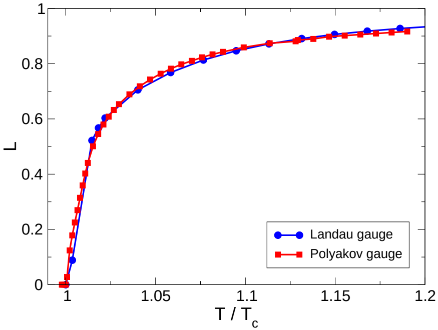


Universal properties & gauge dependence

Marhauser, JMP, arXiv:0812.1144 [hep-ph]

RG-flow in Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$



- ——: Polyakov gauge: crit. exp. $\nu = 0.65$ $\qquad \nu_{\text{Ising}} = 0.63$
- ——: Landau gauge propagators

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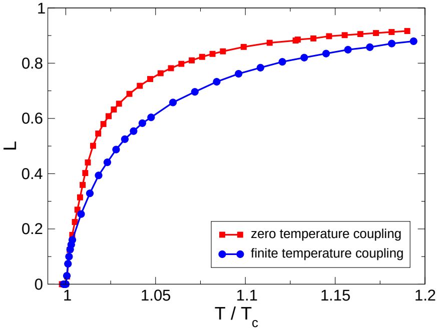
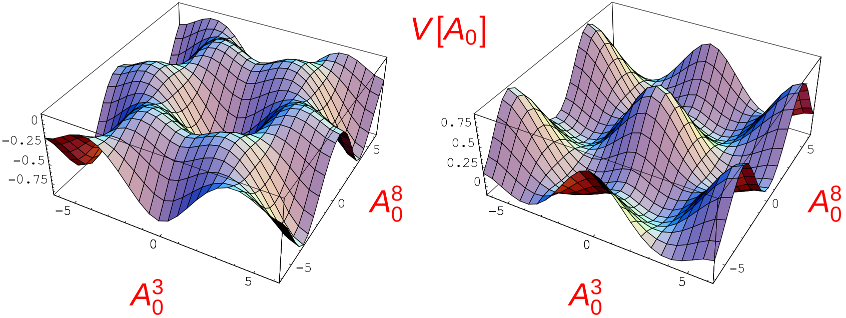


- ——: Polyakov gauge with $T = 0$ coupling
- ——: Polyakov gauge with finite T coupling

Polyakov Loop Potential: $SU(3)$

Braun, Gies, JMP, arXiv:0708.2413 [hep-th]

$T_c \simeq 284 \pm 10\mathrm{MeV}$ $\quad$ $T_c/\sqrt{\sigma} = 0.646 \pm 0.023$ $\quad$ lattice: $T_c/\sqrt{\sigma} = .646$



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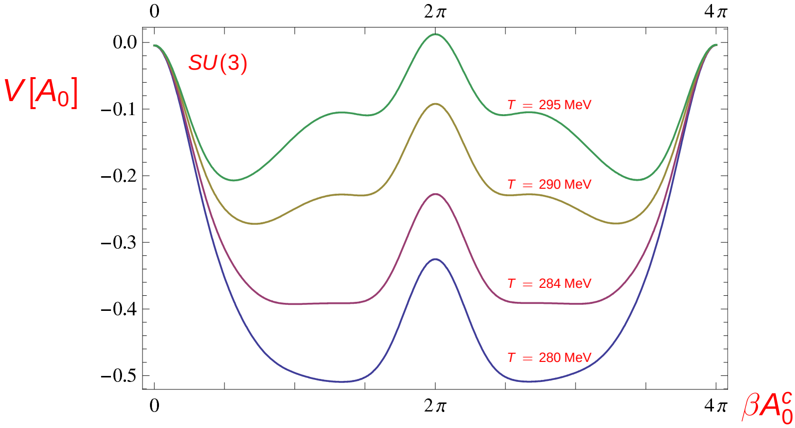

$$\Phi[\beta A_0^c = \tfrac{4}{3}\pi] = 0$$

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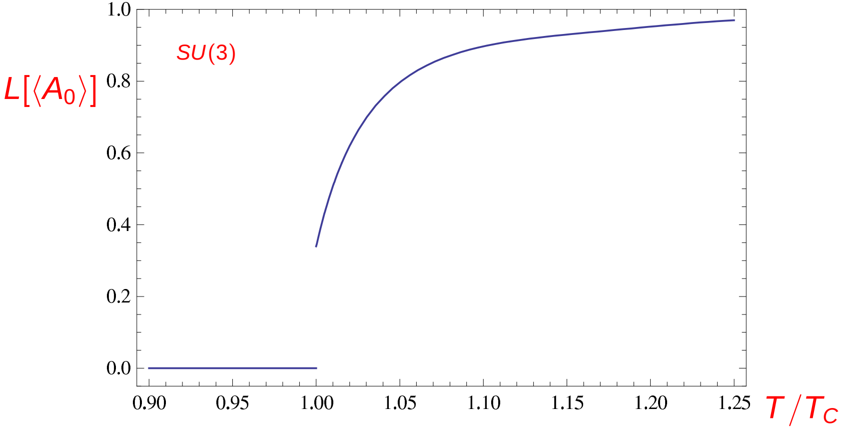

Polyakov gauge: F. Marhauser, JMP, work in progress

Summary & Outlook

- results
  - confinement-deconfinement phase transition
  - dynamical chiral symmetry breaking ⇔ ultracold atoms
  - ’QCD phase diagram’ from models
- challenges
  - full QCD
  - QCD at finite temperature & density ⇔ ultracold atoms
  - Non-equilibrium effects in QCD ⇔ ultracold atoms