Warm Dark Matter and the LHC

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- Dark matter absolutely crucial
  for structure formation

CMB anisotropies: baryon density perturbations at recombination,
$T = 3000$ K

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{rec} \simeq \left(\frac{\delta T}{T}\right)_{CMB} = (\text{a few}) \cdot 10^{-5}$$

Matter perturbations grow as $\frac{\delta\rho}{\rho}(t) \propto T^{-1}$

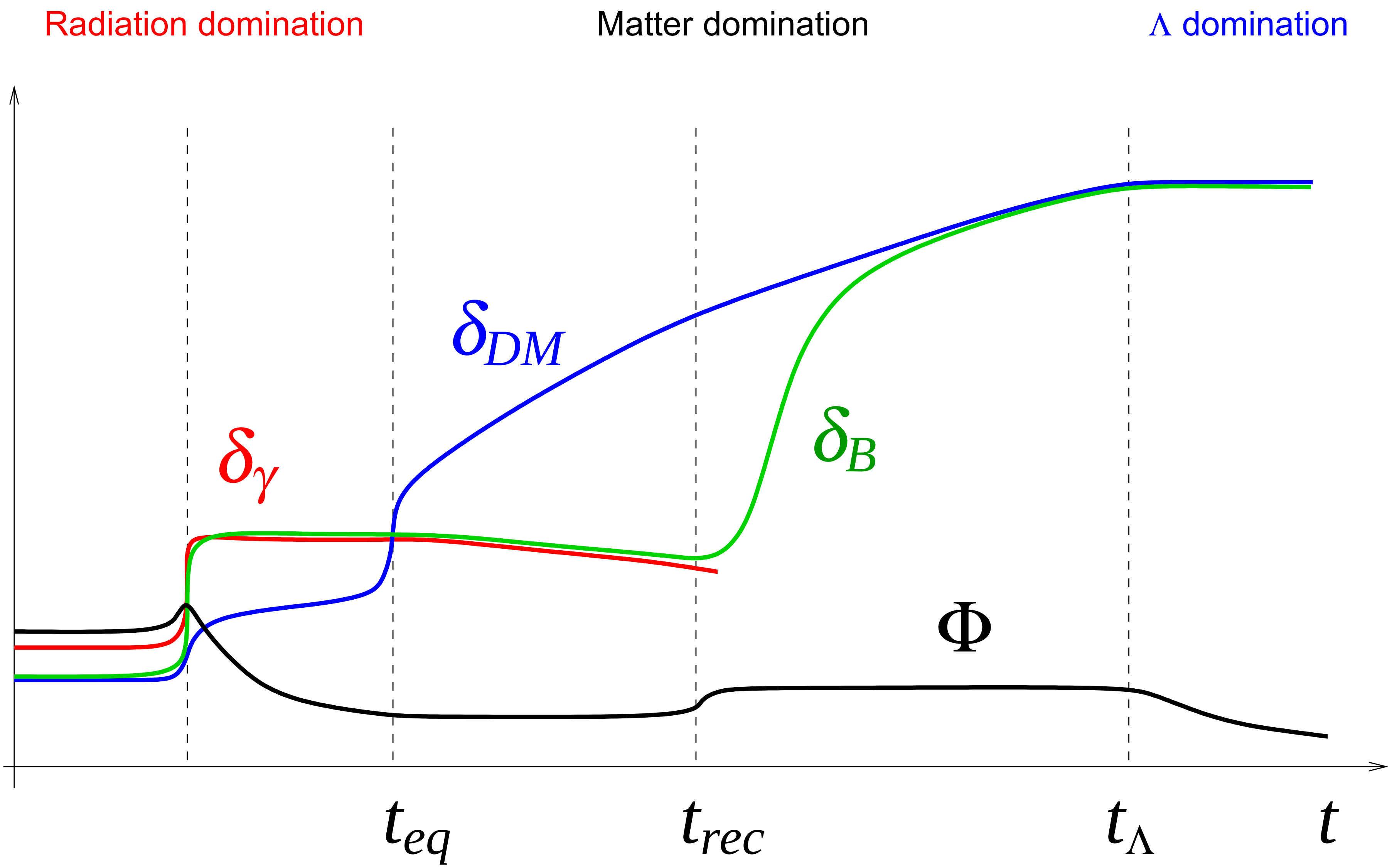
Perturbations in baryonic matter grow after recombination only.
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times (\text{a few}) \cdot 10^{-5} = (\text{a few}) \cdot 10^{-2}$$

No galaxies, no stars...

Perturbations in dark matter start to grow much earlier

Growth of perturbations (linear regime)



Clouds over CDM

Numerical simulations of structure formation with CDM show

- Too many dwarf galaxies

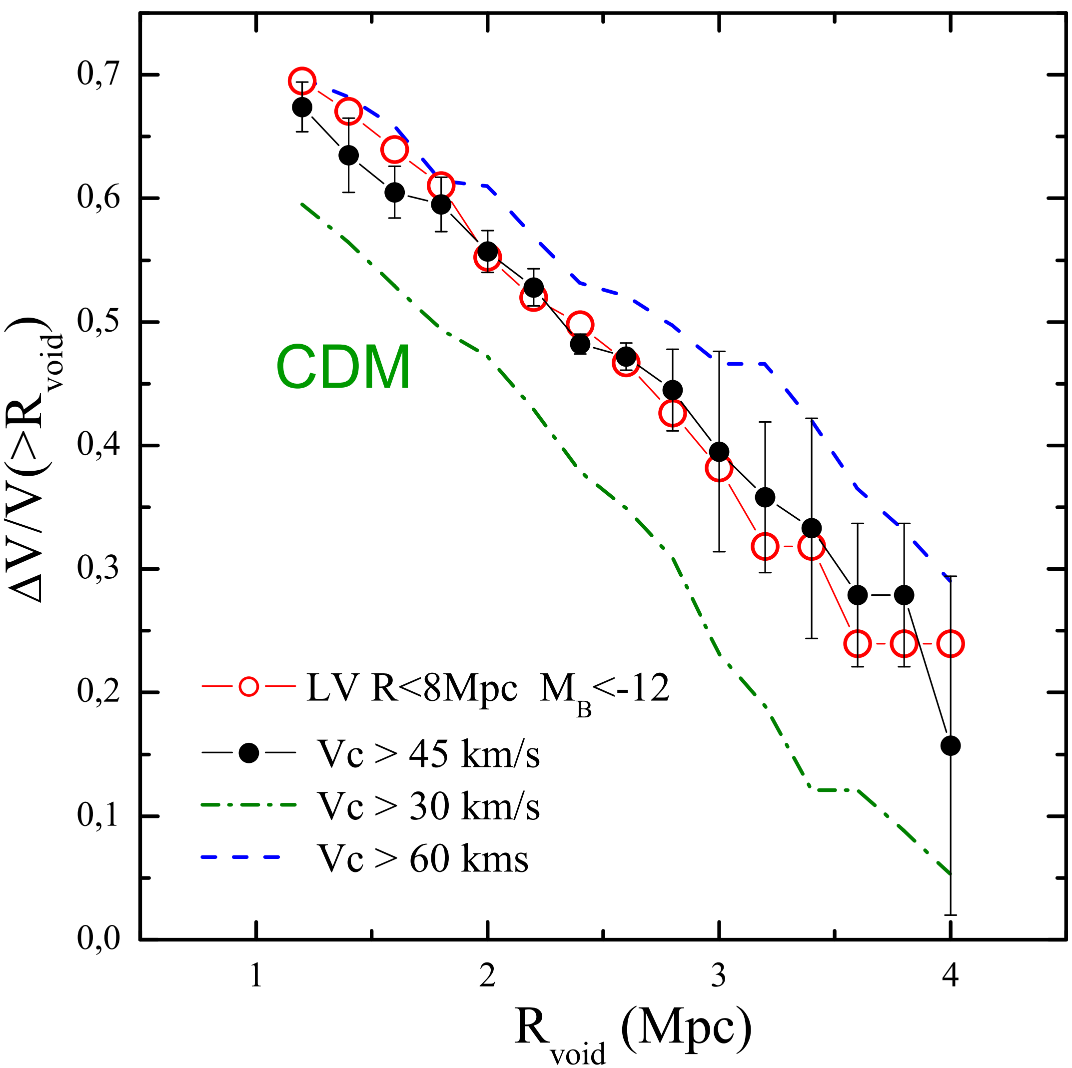
  A few hundred satellites of a galaxy like ours —

  Much less observed so far

  Kauffmann et.al.'93; Klypin et.al.'99;
  Moore et.al.'99;...; Madau et.al.'08

  - In particular: CDM predicts too few and too small empty regions (voids) in our Local Volume ($r \simeq 8$ Mpc), as compared to observations

    Tikhonov and Klypin'08



0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
1
2
3
4
$\Delta V/V(>R_{void})$
R_{void} (Mpc)
CDM
LV R<8Mpc M_B<-12
Vc > 45 km/s
Vc > 30 km/s
Vc > 60 kms

- More about CDM:
  - Too low angular momenta of spiral galaxies
  - Too high density in galactic centers (“cusps”)
- Not crisis yet

Dwarfs may simply be too dark.
Simulations may overestimate number of dwarfs
...
Clouds may go away

But what if one really needs to suppress small structures?

High initial velocities of DM particles $\Longrightarrow$ Warm dark matter

Free streaming

At time t free streaming length

$$l_{fs}(t) \sim v(t) \cdot t$$

Particle velocity

$$v = \frac{p}{m} = \frac{p}{T}\frac{T}{m}$$

At radiation-matter equality (beginning of rapid growth of perturbations),

$$l_{fs}(t_{eq}) \sim \frac{p}{T}\frac{T_{eq}t_{eq}}{m}$$

Perturbations at smaller scales are suppressed.

Present size

$$l_0 \sim (1+z_{eq}) \cdot \frac{p}{T} \frac{T_{eq} t_{eq}}{m}$$

- $\frac{p}{T} \simeq 3$ (if relativistic thremal-like distribution at decoupling)
- $z_{eq} \simeq 3000$, $T_{eq} \simeq 1$ eV, $t_{eq} \simeq 60$ kyr $\simeq 20$ kpc $\Longrightarrow$

$$l_0 \sim 200 \text{ kpc} \cdot \frac{1 \text{ keV}}{m}$$

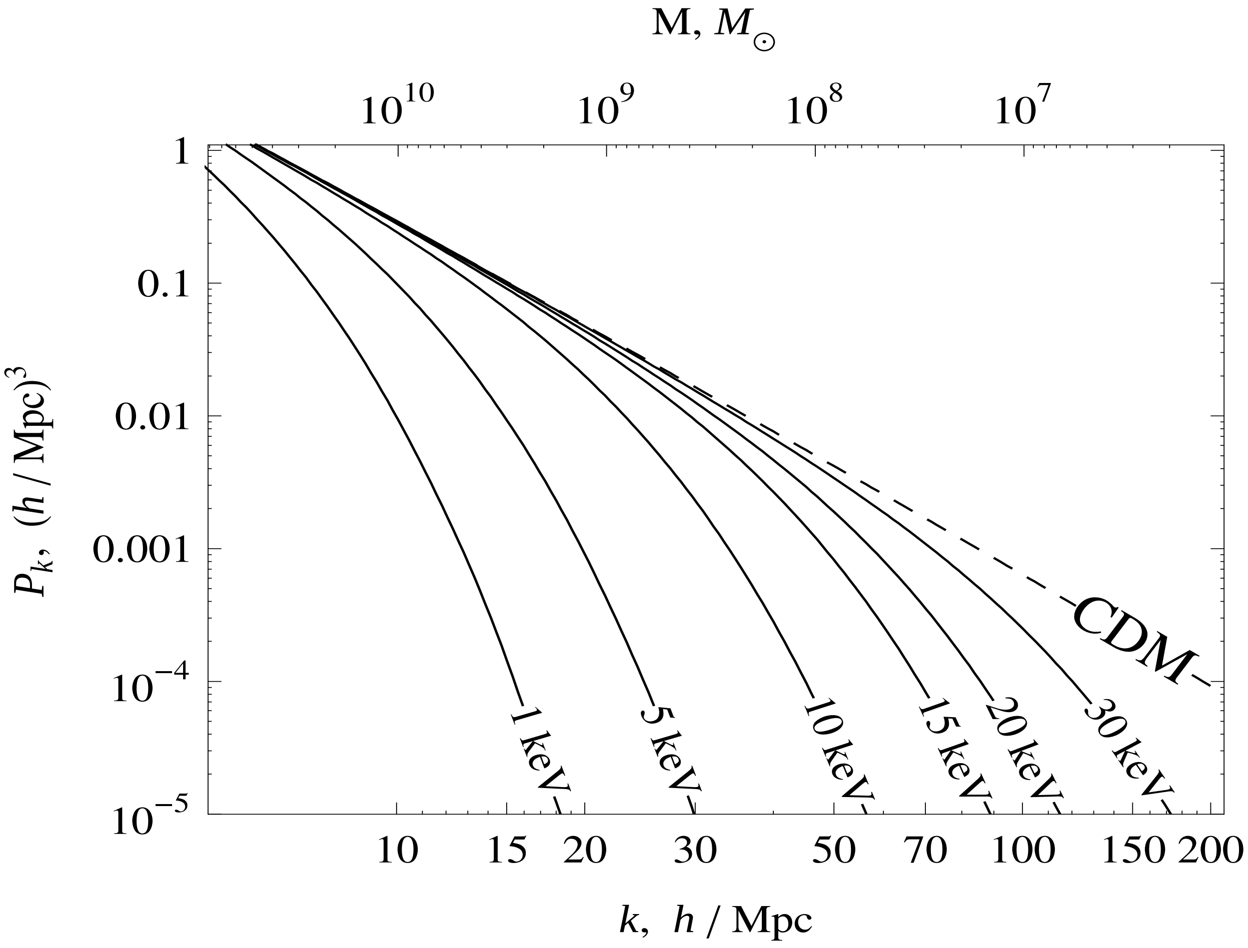
Mass of less abundant objects

$$M \lesssim \rho_{DM} \cdot \frac{4}{3} \pi l_0^3 \sim 10^9 M_\odot \cdot \left(\frac{1 \text{ keV}}{m} \right)^3$$

Cf. dwarf galaxies, $M_{dwarf} \sim 10^8 \div 10^9 M_\odot$.

NB: In fact, perturbations get suppressed already at radiation domination, so this is underestimate of M.

Power spectrum of perturbations



Assuming thermal primordial distribution normalized to $\Omega_{DM} \simeq 0.2$.

Warm dark matter: additional argument

Tremaine, Gunn
Hogan, Dalcanton;
Boyanovsky et.al., ...

- Initial phase space density of dark matter particles: $f(\vec{p})$, independent of $\vec{x}$.

  Fermions:

  $$f(\vec{p}) \leq \frac{1}{(2\pi)^3} \quad \text{by Pauli principle}$$

  Not more than one particle in quantum unit of phase space volume $\Delta\vec{x}\Delta\vec{p} = (2\pi\hbar)^3$.

  NB: Thermal distribution: $f_{max} = \frac{1}{2(2\pi)^3}$

  Expect maximum initial phase space density somewhat below $(2\pi)^{-3}$

- Non-dissipative motion of particles, gravitatonal interactions only: particles tend to penetrate into empty parts of phase space $\Longrightarrow$ coarse grained distribution decreases in time; maximum phase space density also decreases in time.

  But not by many orders of magnitude

- Simulations of violent relaxation:
  (NB: not directly applicable to warm dark matter)

  phase space density indeed decreases,

$$\frac{\text{initial phase space density}}{\text{present phase space density}} = \frac{f}{f_0} = \Delta$$

with

$$\Delta \simeq 10 \div 1000$$

- Observable:

$$Q(\vec{x}) = \frac{\rho_{DM}(\vec{x})}{\langle v_{||}^2 \rangle^{3/2}}$$

$\rho_{DM}(\vec{x}) \Longleftrightarrow$ gravitational potential
$\langle v_{||}^2 \rangle \Longleftrightarrow$ velocities of stars along line of sight.

Assume dark matter particles have same velocities as stars (e.g., virialized)

$$Q \simeq m^4 \frac{n(\vec{x})}{\langle \frac{1}{3} p^2 \rangle^{3/2}} \simeq 3^{3/2} m^4 f_0(\vec{x}, \vec{p})$$

Mass bounds from primordial phase space distribution:

$$m^4 f_{max} > 3^{-3/2} Q_{max}$$

- Estimator of primordial phase space density:

$$f \simeq \Delta \frac{Q}{3^{3/2} m^4}$$

- Largest observed: dwarf galaxies (Coma Berencies, Leo IV, Canes Venaciti II)

$$Q_{max} = \left(3 \cdot 10^{-3} \div 2 \cdot 10^{-2}\right) \frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3}$$

With $M_\odot \simeq 1 \cdot 10^{63}$ keV, 1 pc$= 1.5 \cdot 10^{26}$ keV^{-1}, km/s$= 3 \cdot 10^{-6}$

$$\begin{aligned} Q_{max} &= 0.03 \div 0.2 \ \mathrm{keV}^4 \\ &\simeq 3^{3/2} \Delta^{-1} \cdot m^4 f_{max} \simeq 3^{3/2} \Delta^{-1} \cdot m^4 \frac{\#}{(2\pi)^3} \end{aligned}$$

If maximum observed Q indeed estimates the largest phase space density of DM particles in the present Universe, then

$$m \sim (1 \div 10) \cdot \text{keV}$$

NB: Independent argument,
works for bosons in somewhat different way.

- How many particles should have so high phase space density?

$$\frac{\text{Dark matter in dwarfs}}{\text{Total dark matter}} \sim 10^{-5}$$

$\Longrightarrow$ Statistical estimates/bounds for strongly peaked initial distribution functions $f(\mathbf{p})$. Look into most populated corners of primordial phase space

Madsen

Gravitinos

- Mass $m_{3/2} \simeq F/M_{Pl}$
  $\sqrt{F} =$ SUSY breaking scale.
  $\Longrightarrow$ Gravitinos light for low SUSY breaking scale.
  E.g. gauge mediation
- Light gravitino = LSP $\Longrightarrow$ Stable
- Decay width of superpartners into gravitino + SM particles

$$\Gamma_{\tilde{S}} \simeq \frac{M_{\tilde{S}}^5}{F^2}$$

$$\Longrightarrow \qquad \Gamma_{\tilde{S}} = \frac{M_{\tilde{S}}^5}{6\, m_{3/2}^2 M_{Pl}^2}$$

$M_{\tilde{S}}$ = mass of superpartner $\tilde{S}$

Gravitino production in decays of superpartners

Moroi, Murayama; ...

$$\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s}\Gamma_{\tilde{S}}$$

$s =$ entropy density

$n_{\tilde{S}}/s = \text{const} \sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, while $n_{\tilde{S}} \propto \mathrm{e}^{-M_{\tilde{S}}/T}$ for $T \ll M_{\tilde{S}}$
$\Longrightarrow$ production most efficient at $T \sim M_{\tilde{S}}$ (slow cosmological expansion with unsuppressed $n_{\tilde{S}}$)

$$\frac{n_{3/2}}{s} \simeq \frac{\Gamma_{\tilde{S}}}{g_* H(T \sim M_{\tilde{S}})} \simeq \frac{M_{Pl}^*}{g_* M_{\tilde{S}}^2} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}$$

Mass-to-entropy ratio

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \sum_{\tilde{S}} \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

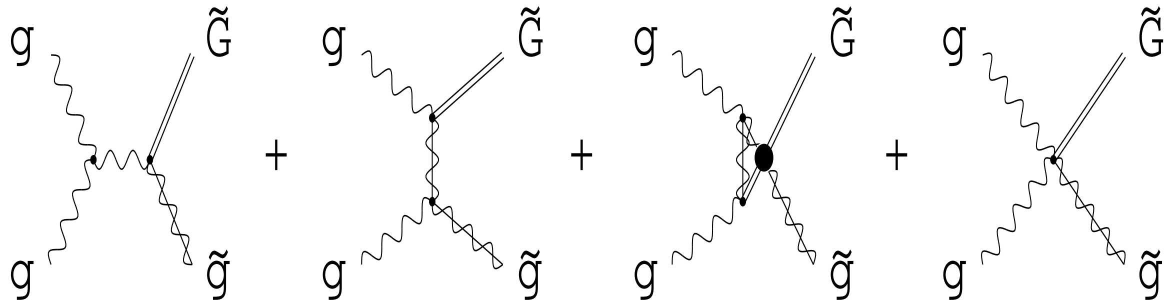
For $m_{3/2} =$ a few keV, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV

$$M_{\tilde{S}} \simeq 100 \div 300 \text{ GeV}$$

Need light superpartners

Production in scattering

Moroi et.al.; Boltz et. al.; Pradler; Rychkov, Strumia;...



Gravitino (goldstino) coupling to gauginos, dominant at high temperatures:

$$L_{int} = \frac{M_{\tilde{g}}}{F} \tilde{G}[\gamma^\mu, \gamma^\nu]\tilde{g}F_{\mu\nu} = \frac{M_{\tilde{g}}}{M_{Pl} m_{3/2}} \tilde{G}[\gamma^\mu, \gamma^\nu]\tilde{g}F_{\mu\nu}$$

Gravitinos produced at temperature T (modulo soft logs):

$$\left(m_{3/2}\frac{n_{3/2}}{s}\right)_T \sim g^2 \frac{M_{\tilde{g}}^2}{M_{Pl}^2 m_{3/2}} \frac{n_g}{s} \cdot n_g H^{-1}(T) \propto g^2 \frac{M_{\tilde{g}}^2}{m_{3/2}} T$$

Maximum production at highest possible temperature:

$$\Omega_{\tilde{G}} = \# \cdot g^2 \left(\frac{M_{\tilde{g}}}{100 \text{ GeV}} \right)^2 \cdot \left(\frac{1 \text{ keV}}{m_{3/2}} \right) \cdot \left(\frac{T_R}{1 \text{ TeV}} \right)$$

Need low maximum temperature in the Universe, $T_R \lesssim 1$ TeV to avoid overproduction in collisions.

Rather contrived scenario, but generating warm dark matter is always contrived

NB: $\Gamma_{NLSP} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \Longrightarrow c\tau_{NLSP} = \text{a few} \cdot \text{mm} \div \text{a few} \cdot 100 \text{ m}$

for $m_{3/2} = 1 \div 10$ keV, $M_{\tilde{S}} = 100 \div 300$ GeV

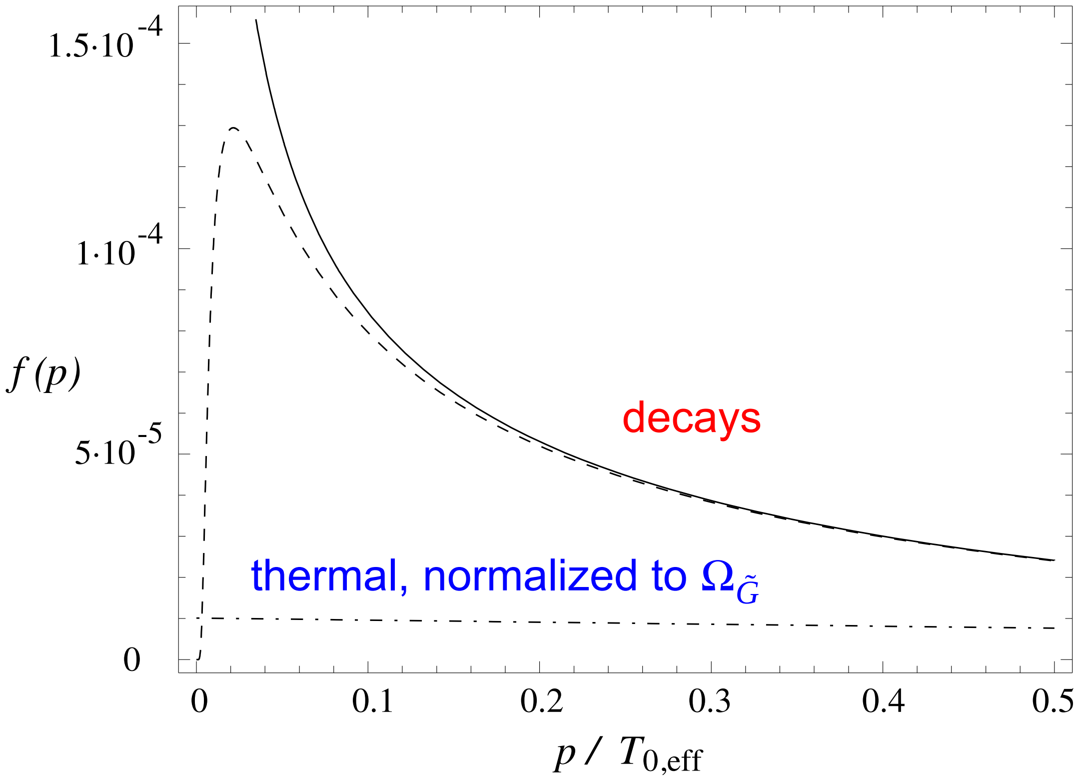
Longer lifetime for heavier gravitino (CDM candidate)

Corner in phase space

Low momentum gravitino produced in decays of heavy superpartners.

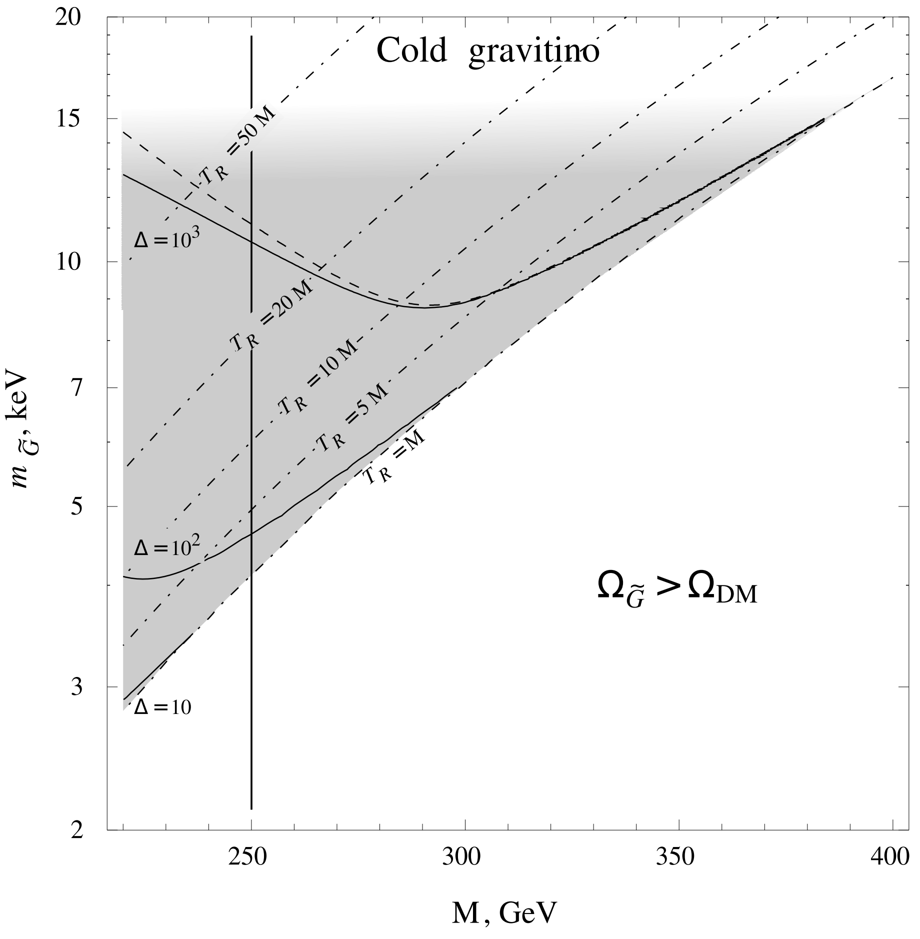
How is that possible?

Fast moving $\tilde{S}$, decaying backwards.



Scenario 1

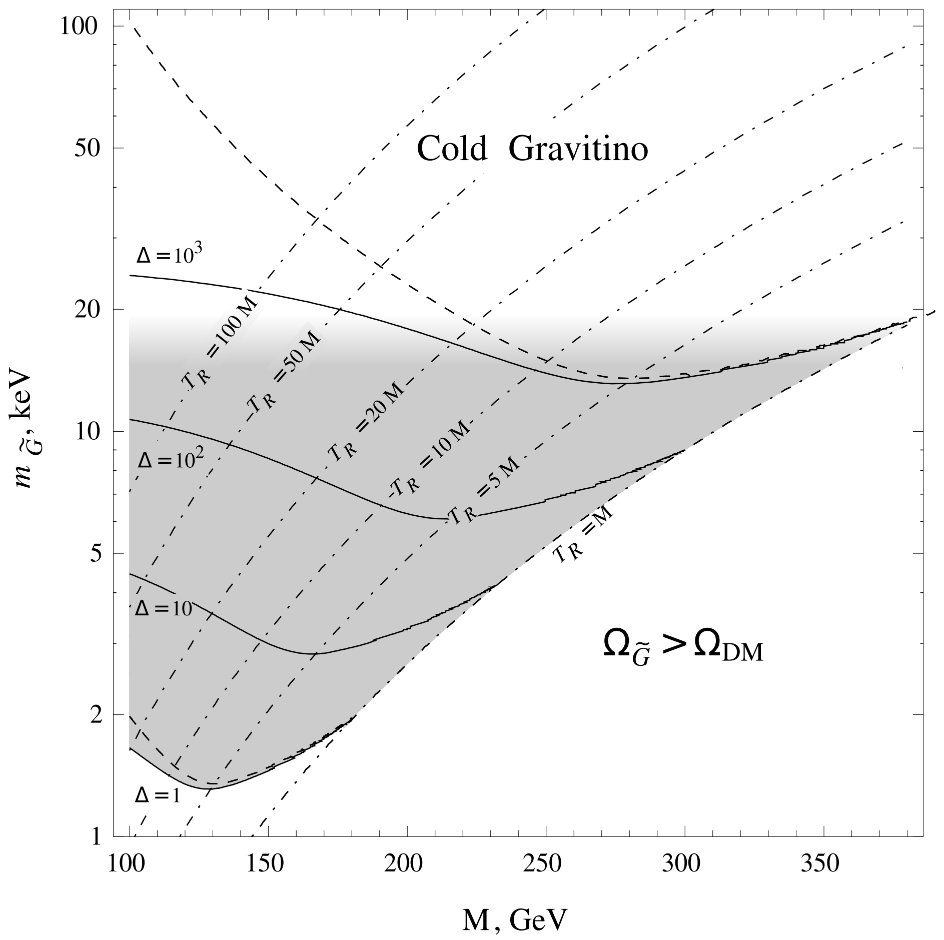
All superpartners have the same mass M.



Scenario 2

Gluinos and squarks heavier than T_R,
never existed in cosmic plasma.

Electroweak sparticles relevant only



To summarize:

- Gravitinos are still warm dark matter candidates
- Possible only if superpartners are light,

$$M \lesssim 300 \text{ GeV}$$

Will soon be ruled out (or confirmed) by LHC

Competitor: strile neutrino

- Simplest production mechanism: via active-sterile mixing.

Dodelson, Widrow; Dolgov, Hansen; Asaka et.al.

Almost thermal primordial spectrum normalized to $\Omega_{DM} \simeq 0.2$

$$f(p) = \frac{g_{\nu_s}}{(2\pi)^3} \frac{\beta}{\mathrm{e}^{p/T_\nu} + 1}$$

$\Omega_\nu = \Omega_{DM} \Longrightarrow$

$$\beta = 10^{-2} \left(\frac{1 \text{ keV}}{m} \right) \propto \sin^2 2\theta$$

Phase space bound:

Also: Boyarsky et. al.

$$m^4 f_{max} > \# \cdot Q_{max} \qquad \Longrightarrow$$

$m > 5.7$ keV $\Longrightarrow$ $\sin^2 2\theta = (\text{a few}) \cdot 10^{-9}$

Similar to, and independent from Ly-α bounds.

Ly-α: Abazajan; Seljak et.al.; Viel et.al.

$m > 10 \div 28$ keV

Inconsistent with X-ray limits: $m < 4$ keV

X-ray limits: Boyarsky et. al.; Riemen-Sorensen et.al., Watson et.al.; Abazajan et.al.

- Other production mechanisms survive
  - Resonant production in the presence of lepton asymmetry
  - Decays of heavier particles
- In the latter case sterile neutrinos may be quite heavy
  - Suppose the heavy particles of mass M have decoupled and slowly decay, producing sterile neutrino of mass $m \ll M$. Momentum of sterile neutrino at the moment of the decay

    $$p_{\nu_s} \sim M/2$$

    Phase space density of sterile neutrinos

    $$f(p) \sim \frac{n_{\nu_s}(t_{dec})}{p^3} \sim \frac{n(t_{dec})}{M^3} \sim \frac{T_{dec}^3}{T_0^3} \frac{n_{\nu_s}(t_0)}{M^3}$$

Decays occur at

$$H(t_{dec}) \sim \frac{T_{dec}^2}{M_{Pl}} \sim \Gamma_M \implies T_{dec} \sim (\Gamma_M M_{Pl})^{1/2}$$

Present number density of sterile neutrinos

$$n_{\nu_s}(t_0) = \frac{\Omega_{DM}\rho_c}{m}$$

Putting all together

$$m^4 f(p) \sim m^3 \frac{(\Gamma_M M_{Pl})^{3/2}}{M^3} \frac{\Omega_{DM}\rho_c}{T_0^3}$$

Require $m^4 f(p) > Q$:

$$m \gtrsim 100\ \text{eV} \cdot \left(\frac{M^2}{\Gamma_M M_{Pl}}\right)^{1/2}$$

Heavy particles with long lifetime produce warm sterile neutrinos even if the latter are heavy (say, $m \sim$ TeV):

sterile neutrinos are fast at production (large M), and

momentum does not redshift that much (small Γ_M, late decays).

Need to fine tune freeze out of heavy particles to get Ω_{DM} right.

To conclude

- Particle physicist's viewpoint:
  - WDM is an interesting alternative to CDM.
  - Needs unconventional particle physics and cosmology
- Is it really useful for structures?
- Presently: considerable uncertainty in estimates of parameters even for known primordial phase space distributions.
  To zeroth order parametrized by dilution factor Δ: how strongly phase space density decreases during non-linear evolution of structures?
- LHC may give strong input