Exploring the chiral and deconfinement phase structure of QCD

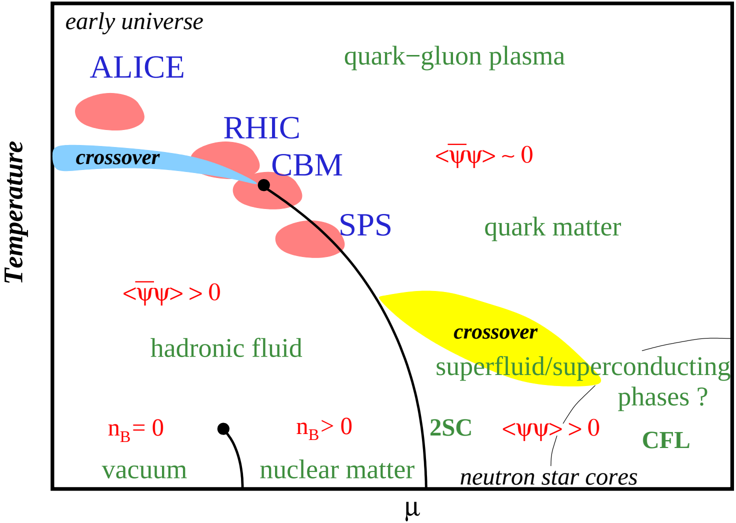
Bernd-Jochen Schaefer

Karl-Franzens-Universität Graz, Austria

30$^{\text{th}}$ January, 2009

DELTA Workshop

Heidelberg, Germany


early universe
ALICE
quark−gluon plasma
RHIC
crossover
CBM
$\langle\bar{\psi}\psi\rangle \sim 0$
SPS
quark matter
$\langle\bar{\psi}\psi\rangle > 0$
crossover
hadronic fluid
superfluid/superconducting
phases ?
$n_B = 0$
$n_B > 0$
2SC
$\langle\psi\psi\rangle > 0$
CFL
vacuum
nuclear matter
neutron star cores
Temperature
μ

QCD Phase Transitions

QCD: two phase transitions:

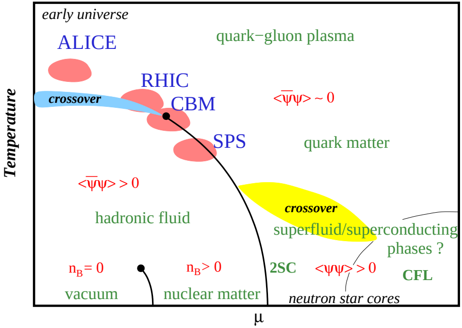
1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \to SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

   associate limit: $m_q \to 0$



chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high T

QCD: two phase transitions:

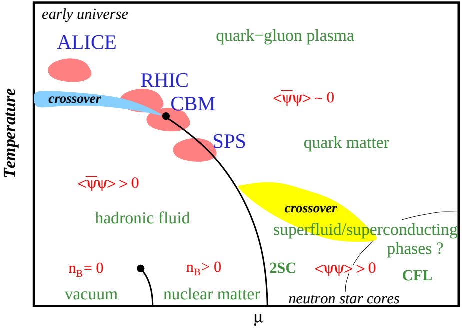
1. restoration of chiral symmetry
2. de/confinement (center symmetry)

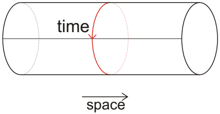
order parameter:

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$$

$$\Phi = \frac{1}{N_c} \langle \mathrm{tr}_c \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau, \vec{x})} \rangle$$

associate limit: $m_q \to \infty$





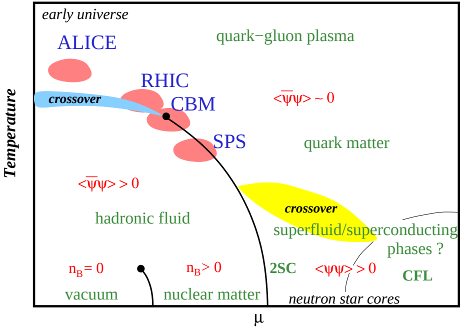
➜ related to free energy of a static quark state: $\Phi = e^{-F_q}$

QCD: two phase transitions:

1. restoration of chiral symmetry
2. de/confinement

At densities/temperatures of interest

only model calculations available



effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

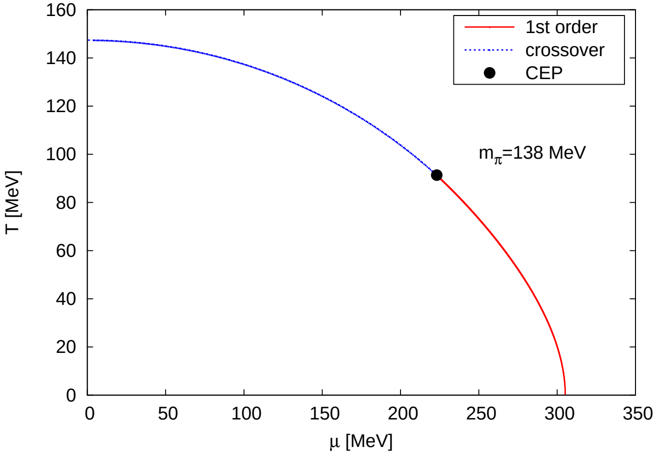
Outline

- Two-Flavor Quark-Meson Model
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics

- Lagrangian:

$$\mathcal{L}_{\rm qm} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

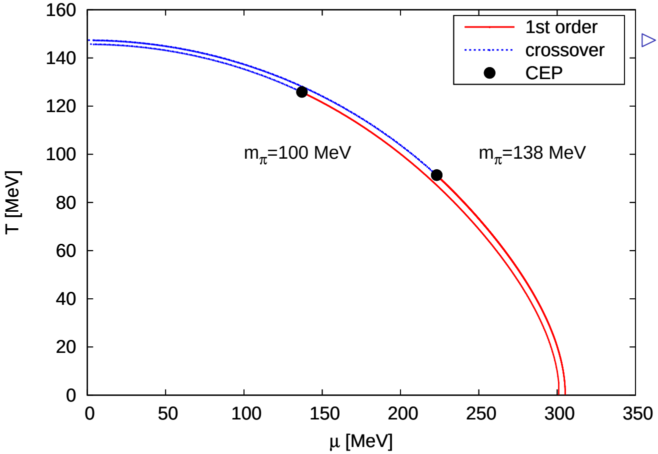
- Mean field analysis



- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis

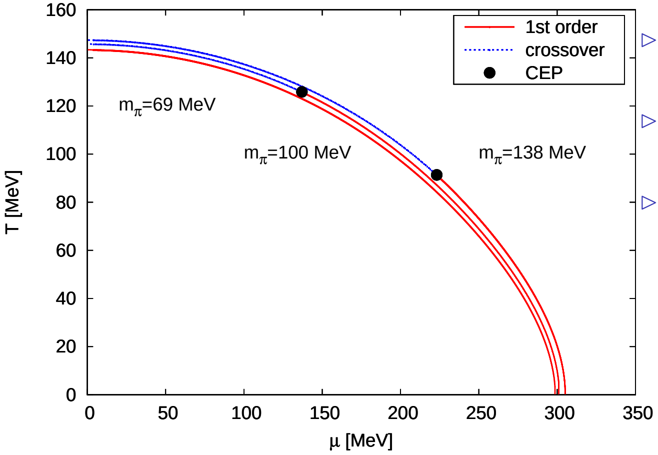


▷ as m_q decreased CEP → T-axis

- Lagrangian:

$$\mathcal{L}_{\rm qm} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 - c\sigma$$

- Mean field analysis



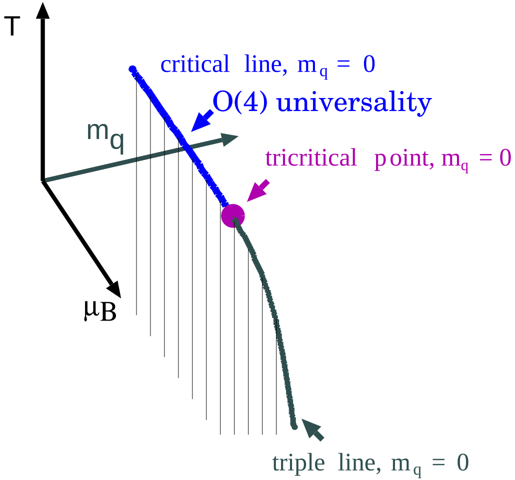
▷ as m_q decreased
CEP $\rightarrow$ T-axis

▷ chiral limit
no CEP

▷ at $\mu = 0$
$O(4)$ scaling expected
i.e. 2nd-order PT

$\rightarrow$ truncation effect

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$

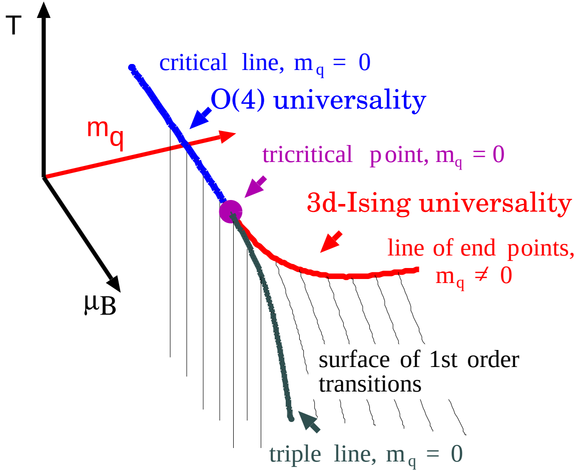


General properties

- chiral limit
  tricritical point (Gaussian fixed point)

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$: no symmetry remains $\longrightarrow$ only one critical mode σ (Ising) ($\vec{\pi}$ massive)



General properties

- chiral limit
  tricritical point
  (Gaussian fixed point)
- finite m_q
  critical endpoints
  (3D-Ising class)

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) [Wetterich]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

- Ansatz for Γ_k: (LO derivative expansion $\rightarrow$ arbitrary potential V_k)

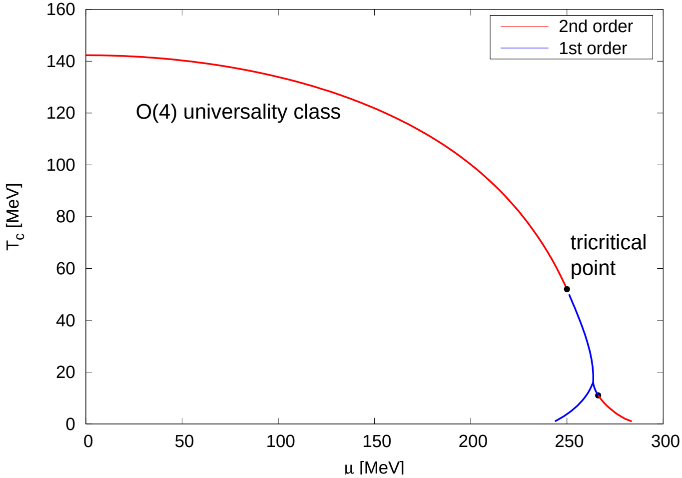
$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

[BJS, J. Wambach, '05 & '06]

RG analysis:

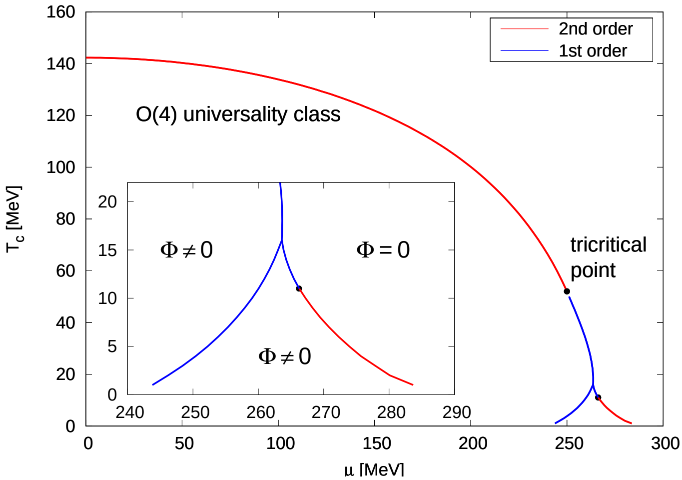
$O(4) \sim SU(2) \times SU(2)$ chiral limit

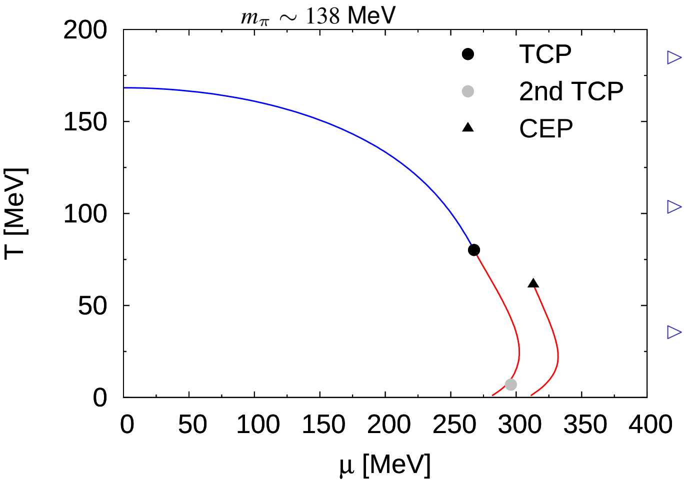


[BJS, J. Wambach, '05 & '06]

RG analysis:

$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!





- ▷ bending usual for RG

  Clausius-Clapeyron relation ok

- ▷ 2nd tricritical point

  in chiral limit

- ▷ features

  parameter independent

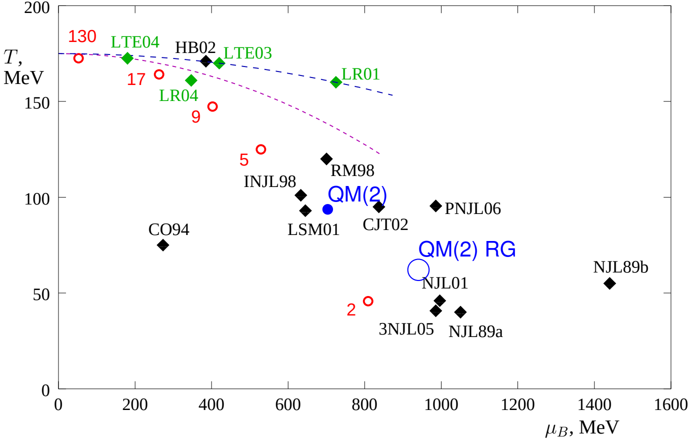
  but locations TCP/CEP

  parameter dependent

[BJS,Wambach '05 & '06]

model studies vs. lattice simulations

Black points: models — Lines & green points: lattice — Red points: Freezeout points for HIC



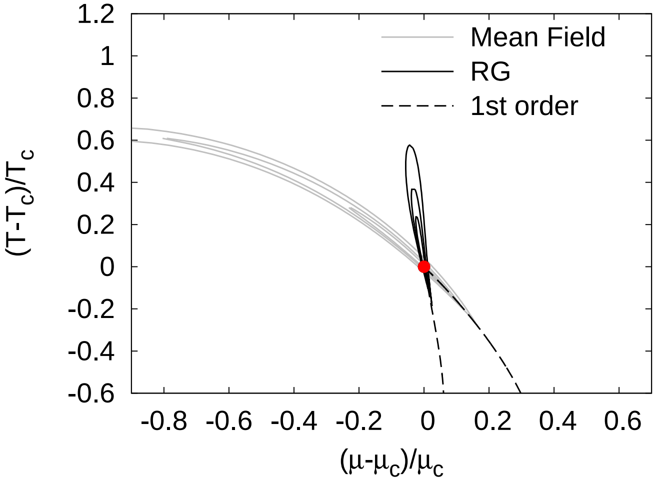
Stephanov '05 & '07

lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

Comparison with scalar χ_σ : MF↔RG

[BJS, J. Wambach '06]



Outline

- Two-Flavor Quark-Meson Model
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

2 logarithmic potential:

Rössner et al. 2007

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

3 Fukushima

Fukushima 2008

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

with

a controls deconfinement $\quad b$ strength of mixing chiral & deconfinement

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

in presence of dynamical quarks: $T_0 = T_0(N_f)$

BJS, Pawlowski, Wambach, 2007

N_f	0	1	2	2 + 1	3
T_0 [MeV]	270	240	208	187	178

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

in presence of dynamical quarks: $T_0 = T_0(N_f)$ BJS, Pawlowski, Wambach, 2007

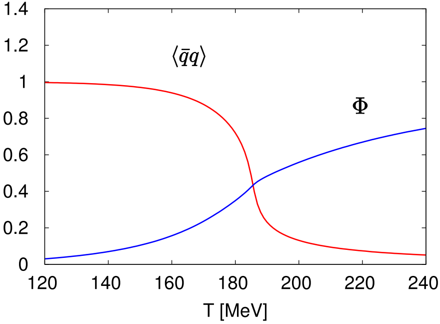
N_f	0	1	2	2+1	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0:$ $T_0 = T_0(N_f, \mu)$ $\bar{\phi} \neq \phi^*$

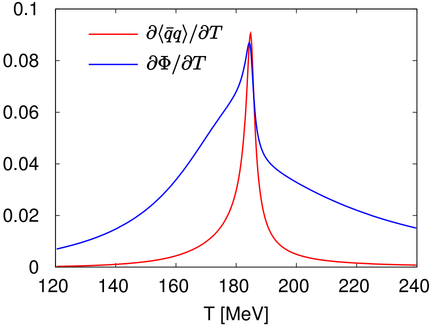
[BJS, Pawlowski, Wambach '07]

Numerical results:

order parameters



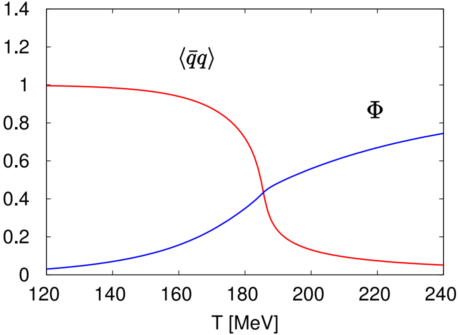
T-derivatives of order parameters



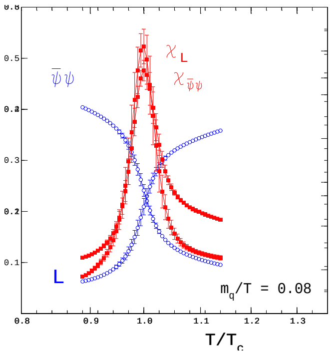
[Laermann et al.]

Numerical results:

order parameters



lattice

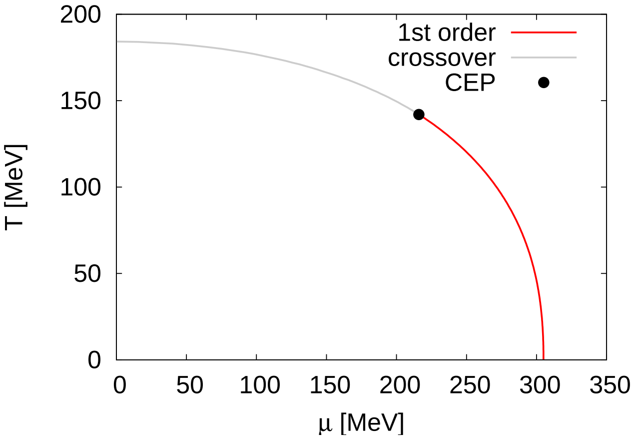


[BJS, Pawlowski, Wambach '07]

in mean field approximation

- for PQM model

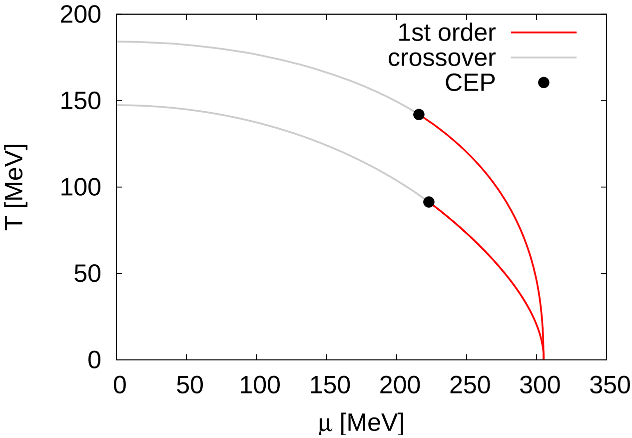
chiral transition and 'deconfinement' coincide



[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide

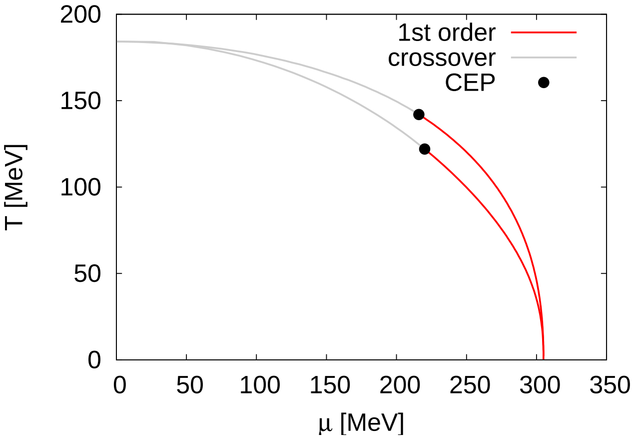


- for PQM model
- for QM model
  (lower lines)

[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide

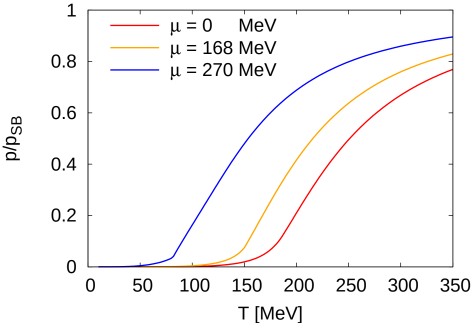


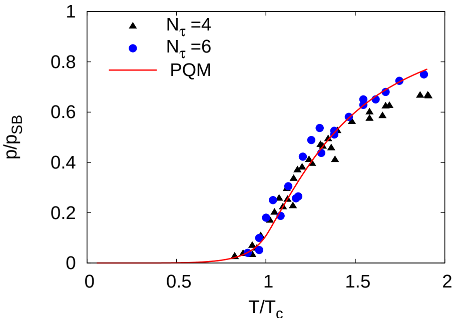
- for PQM model
- for PQM model with μ-modification in Polyakov loop potential
  (lower lines)

- perturbative pressure of QCD with N_f massless quarks

$$\frac{p}{T^4} = (N_c^2 - 1)\frac{\pi^2}{45} + N_f\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu}{T}\right)^4\right] .$$

- $N_f = 2$: lattice: $N_\tau = 4, 6$; $\mu = 0$





[Ali Khan et al. '01]

Outline

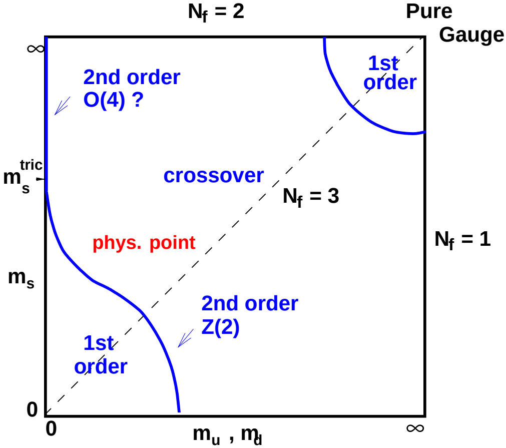
- Two-Flavor Quark-Meson Model
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics

Columbia plot: [Brown et al. '90]

$T_\chi^{N_f=2} \sim 175$ MeV

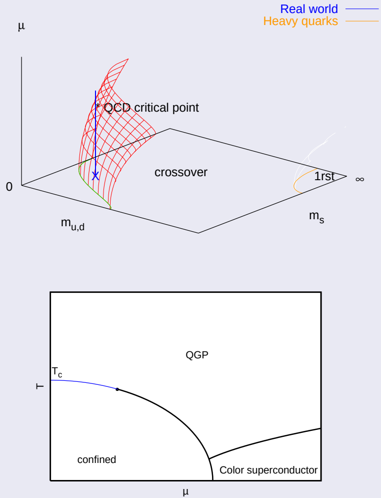
$T_d \sim 270$ MeV

$T_\chi^{N_f=3} \sim 155$ MeV

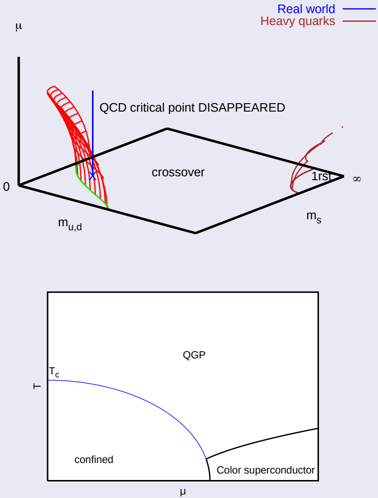


Mass Sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

Standard scenario: $m_c(\mu)$ increasing



Nonstandard scenario: $m_c(\mu)$ decr.



[de Forcrand, Philipsen: hep-lat/0611027]

- Lagrangian $\mathcal{L} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - g\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = & \, tr(\partial_\mu\phi^\dagger\partial^\mu\phi) - m^2 tr(\phi^\dagger\phi) - \lambda_1(tr(\phi^\dagger\phi))^2 \\ & -\lambda_2 tr((\phi^\dagger\phi)^2) + c\,(\det(\phi) + \det(\phi^\dagger)) \\ & + trH(\phi + \phi^\dagger)\end{aligned}$$

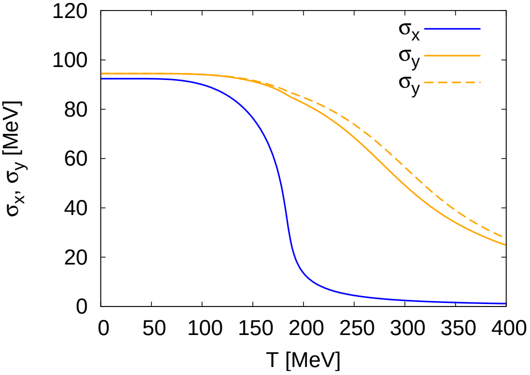
fields: $\phi = \sum_a \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$ and $H = \sum_a \frac{\lambda_a}{2} h_a$

σ_a scalar and π_a pseudoscalar nonet

[BJS, M. Wagner, arXiv:0808.1491]

➜ two condesates: nonstrange $\sigma_x(T, \mu_f)$ and strange $\sigma_y(T, \mu_f)$

with (solid) and without (dashed) $U(1)_A$ anomaly



▷ almost no effect due $U(1)_A$ anomaly

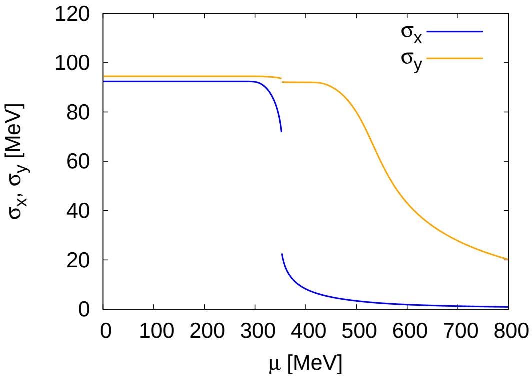
▷ T_c depends on m_σ

Chiral symmetry restoration

[BJS, M. Wagner, arXiv:0808.1491]

➜ two condesates: nonstrange $\sigma_x(T,\mu_f)$ and strange $\sigma_y(T,\mu_f)$

with (solid) and without (dashed) $U(1)_A$ anomaly



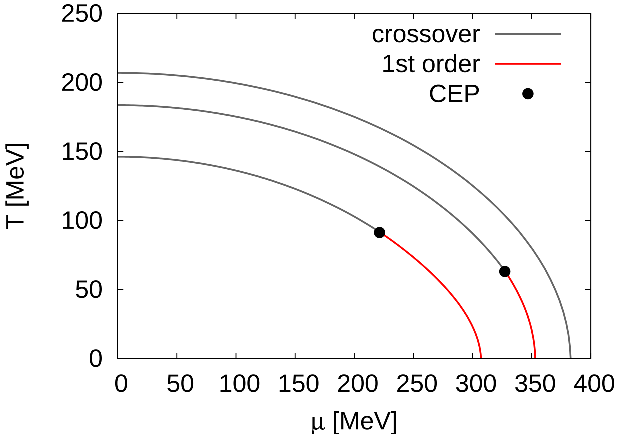
- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ T_c depends on m_σ
- ▷ $\mu \equiv \mu_q = \mu_s$
- ▷ μ_c depends on m_σ

[BJS, M. Wagner, arXiv:0808.1491]

$m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV

PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV

➜ influence of existence of CEP!



In-medium meson masses

[BJS, M. Wagner, arXiv:0808.1491]

- ▷ genuine problem of linear sigma model w/o quarks at finite T

  $\rightarrow$ negative meson masses

- ▷ but not in this approximation

  ➜Ward identities, Goldstone theorem etc. are all valid in-medium e.g.

$$h_x = f_\pi m_\pi^2$$

  similar in strange sector

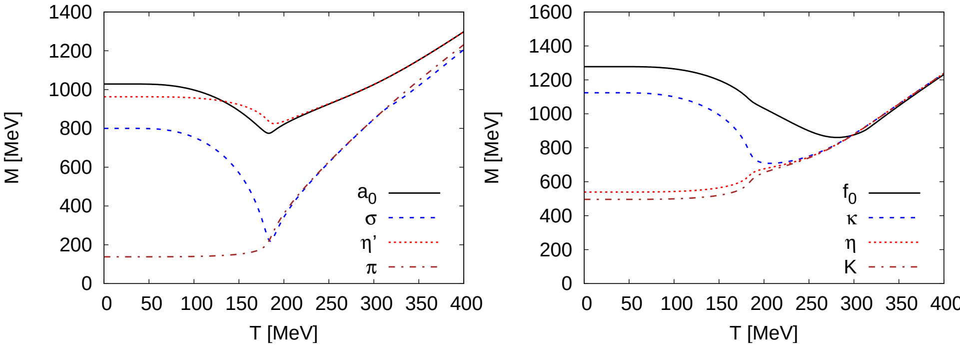
- ▷ At low temperatures: mesons dominate

  At high temperatures: quarks dominate

In-medium meson masses

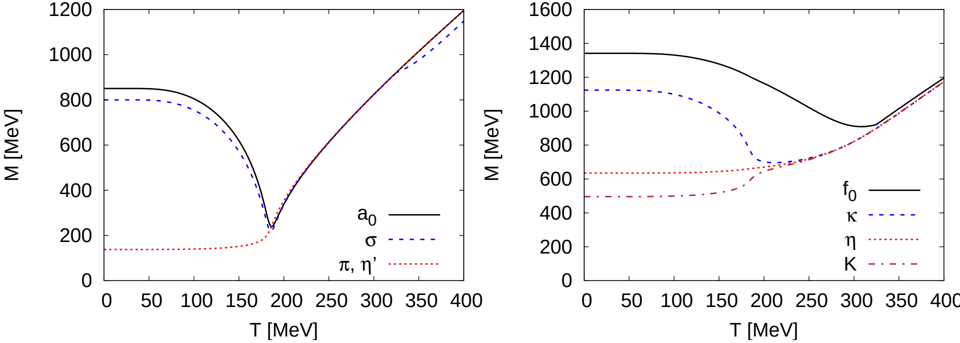
[BJS, M. Wagner, arXiv:0808.1491]

masses with $U(1)_A$ anomaly



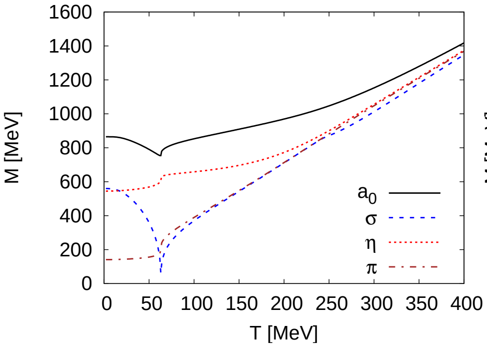
[BJS, M. Wagner, arXiv:0808.1491]

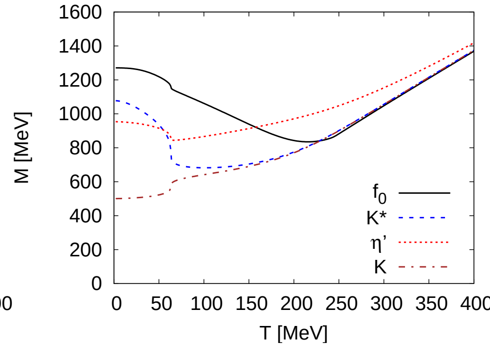
masses without $U(1)_A$ anomaly



[BJS, M. Wagner, arXiv:0808.1491]

masses with $U(1)_A$ anomaly

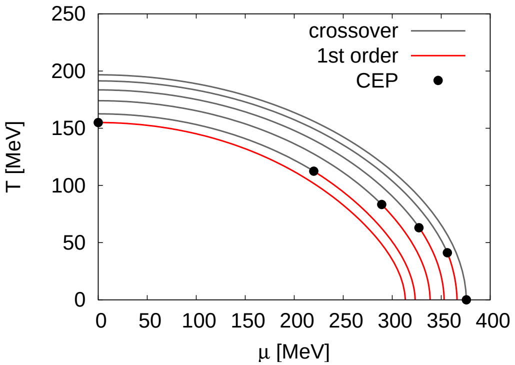




RG arguments predict for $N_f = 3$ 1st-order in chiral limit

▷ $m_\pi / m_\pi^* = 0.49$ (lower line), $0.6, 0.8 \ldots, 1.36$ (upper line) $m_\pi^* = 138$ MeV

with $U(1)_A$, $m_\sigma = 800$ MeV

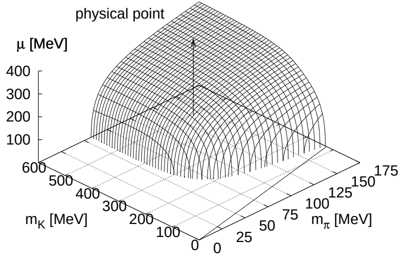


Chiral critical surface $(m_\sigma = 800$ MeV)

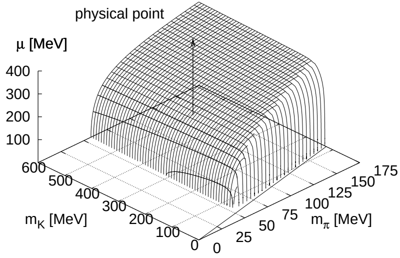
[BJS, M. Wagner, arXiv:0808.1491]

- chiral critical surface in (m_π, m_K) plane

with $U(1)_A$



without $U(1)_A$



Chiral critical surface for different m_σ

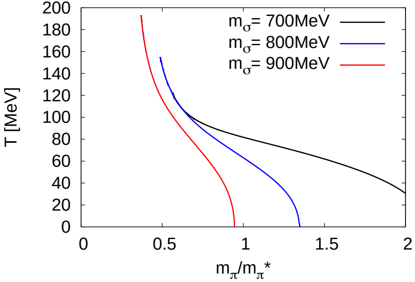
[BJS, M. Wagner, arXiv:0808.1491]

- chiral critical surface in (m_π, m_K) plane

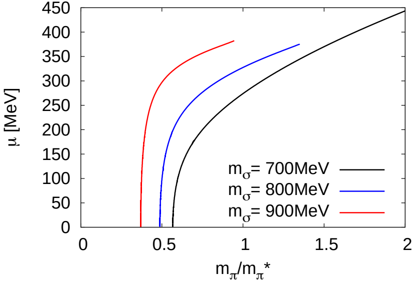
  ➜ cuts along fixed m_π / m_K ratio through physical point

$m_\pi^* = 138$ MeV, $m_K^* = 496$MeV (physical point)

critical T_c



critical μ



Outline

◦ Two-Flavor Quark-Meson Model

▷ Mean field approximation

▷ Renormalization Group study

◦ Polyakov–Quark-Meson Model

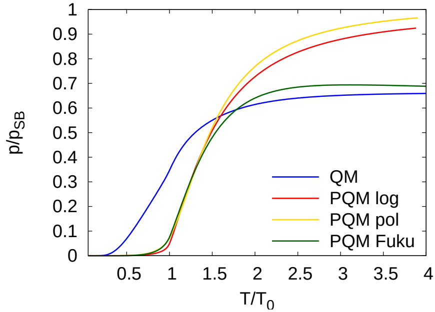
◦ Three-Flavor Quark-Meson Model

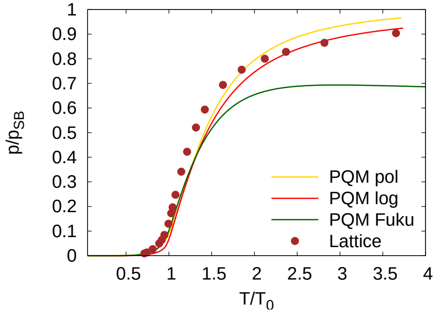
◦ ...with Polyakov loop dynamics

Thermodynamics $N_f = 2+1$

[BJS, M. Wagner; to be published ’08]

SB limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$



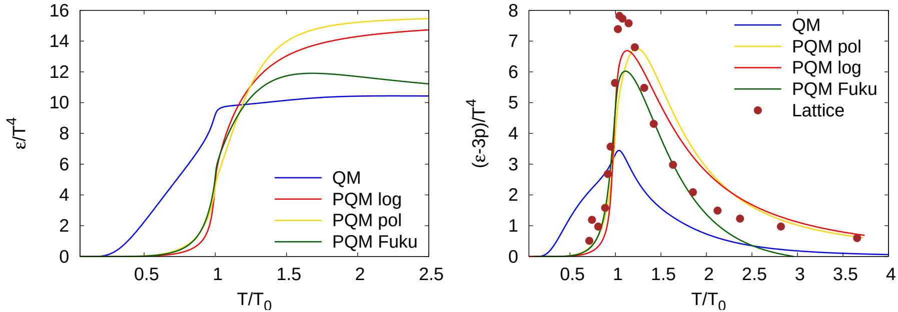


$m_\pi \sim 220$ MeV $\quad N_\tau = 8$

[Cheng et al. ’08]

[BJS, M. Wagner; to be published '08]

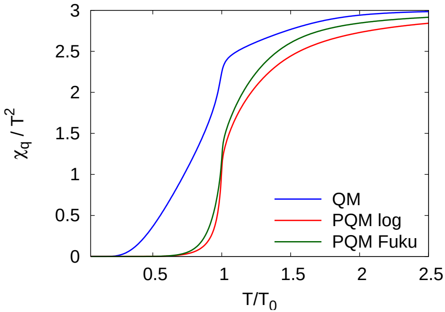
SB limit: $\frac{p_{\rm SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

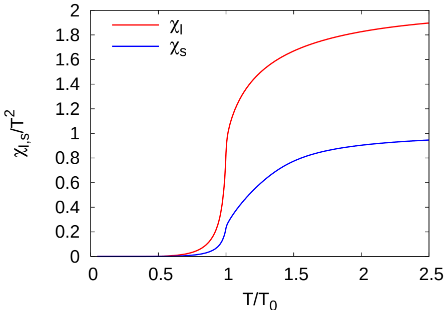


Thermodynamics $N_f = 2 + 1$

[BJS, M. Wagner; to be published '08]

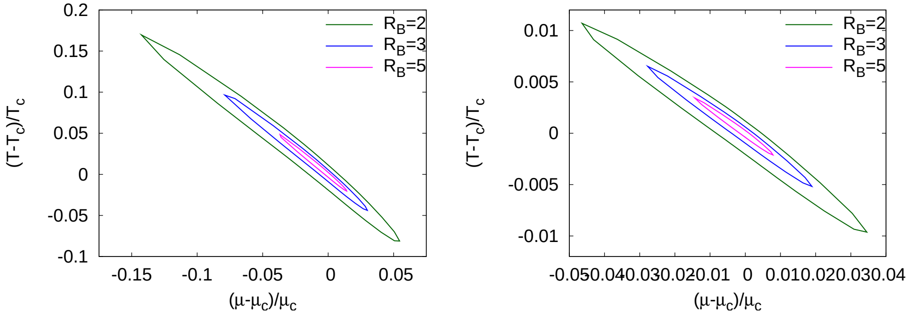
SB limit: $\frac{p_{\mathrm{SB}}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$





[BJS, M. Wagner; to be published '08]

size of the critical region:



[BJS, M. Wagner; to be published '08]

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\Bigg|_{\mu=0}$$

high temperature limits:

$$c_0(T \to \infty) = \frac{7 N_c N_f \pi^2}{180} \ ,$$

$$c_2(T \to \infty) = \frac{N_c N_f}{6} \ ,$$

$$c_4(T \to \infty) = \frac{N_c N_f}{12\pi^2}$$

$$c_n(T \to \infty) = 0 \text{ for } n > 4.$$

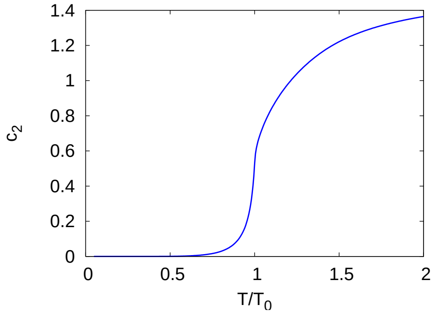
Finite density extrapolations $N_f = 2+1$

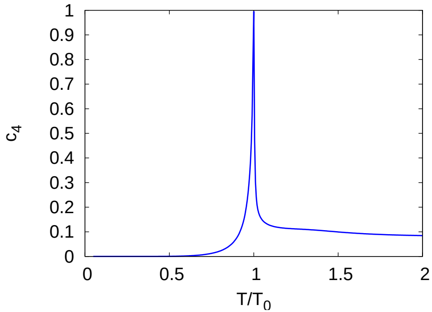
[BJS, M. Wagner; to be published '08]

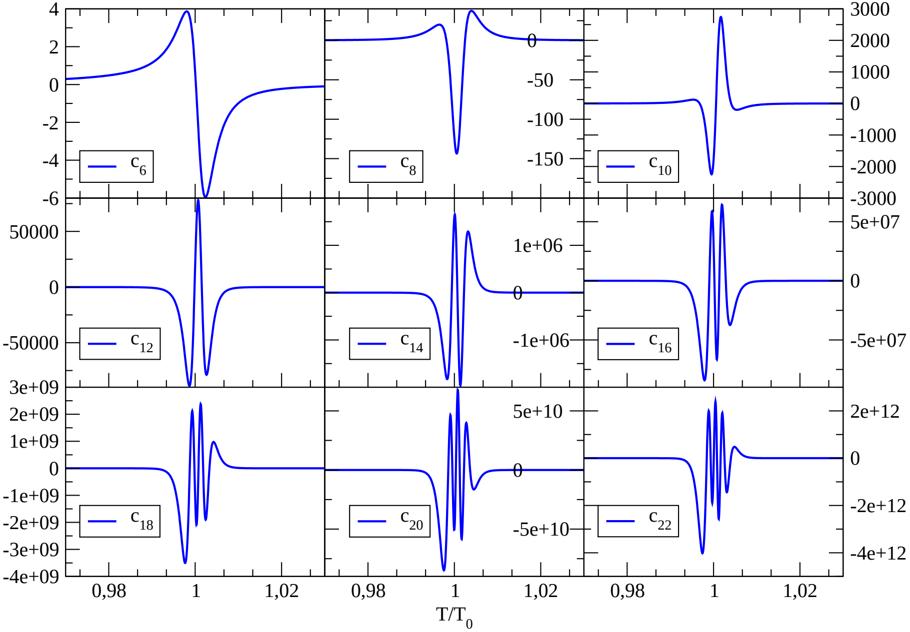
Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\bigg|_{\mu=0}$$

first three coefficients: c_0: pressure $\mu = 0$

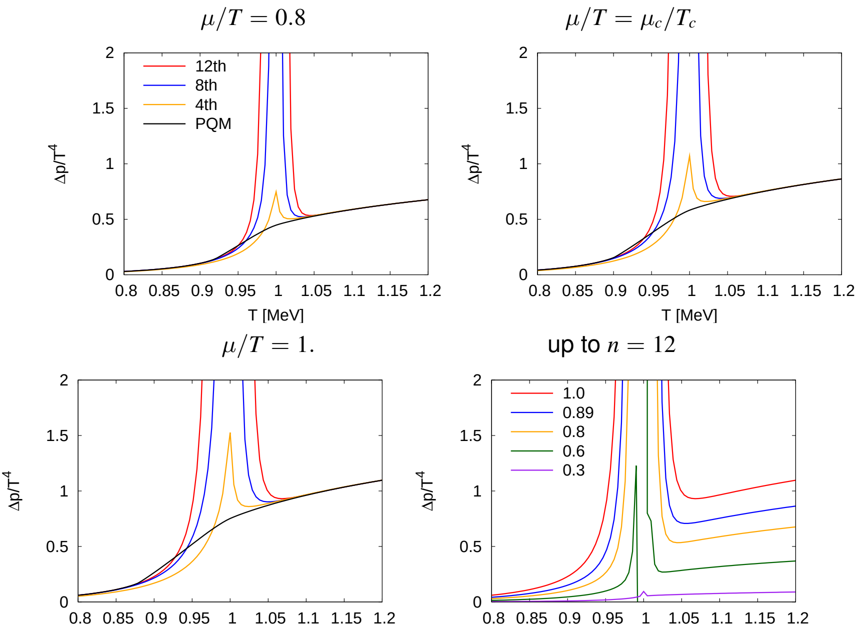




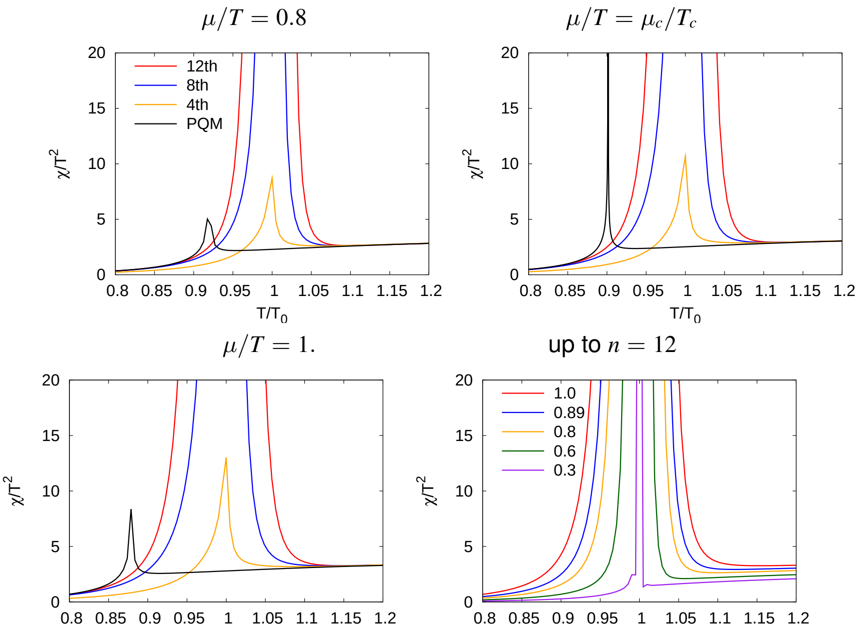


4
2
0
-2
-4
-6
c_6
0
-50
-100
-150
c_8
3000
2000
1000
0
-1000
-2000
-3000
c_{10}
50000
0
-50000
c_{12}
1e+06
0
-1e+06
c_{14}
5e+07
0
-5e+07
c_{16}
3e+09
2e+09
1e+09
0
-1e+09
-2e+09
-3e+09
-4e+09
c_{18}
5e+10
0
-5e+10
c_{20}
2e+12
0
-2e+12
-4e+12
c_{22}
0,98
1
1,02
T/T_0

Finite density extrapolations $N_f = 2 + 1$ PQM



Finite density extrapolations $N_f = 2 + 1$ PQM



Summary

Quark-meson model study for $N_F = 2$

➜ Mean field versus RG

Influence of fluctuations on phase diagram

Findings:

- ▷ MF phase diagram: no TCP (in chiral limit) found
- ▷ RG phase diagram: two TCP's (in chiral limit) & CEP found
- ▷ Size of critical region via susceptibilities: "compressed" with fluctuations

Quark-meson model study for $N_F = 3$

➜ Mean-field approximation

no need for Optimized Perturbation Theory

with and without axial anomaly

Summary

Polyakov–quark-meson model study for $N_F = 2$

➜ mean-field approximation

Findings:

- ▷ Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$

  pure gauge: $T_0 \sim 270$ MeV

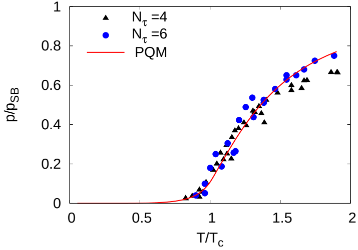
  $N_f = 2$: $T_0 \sim 210$ MeV

- ▷ Chiral & deconfinement transition coincide

- ▷ Mean-field approximation encouraging

  Quark-meson model is renormalizable

  $\rightarrow$ no UV cutoff parameter (cf. PNJL model)



- ▷ Taylorcoefficient $c_n(T) \rightarrow$ high order

- ▷ useful to develop general arguments to determine CEP location

Outlook

- include quark-meson dynamics in PQM model and for $N_f = 3$ with FRG

- include glue dynamics with FRG $\rightarrow$ full QCD
  (step by step)