The Chiral Phase Transition in QCD: on the quark mass dependence of Goldstone fluctuations

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Spontaneous Chiral Symmetry Breaking in QCD

Chiral symmetry in the chiral limit ($m_q \to 0$):

- QCD-Lagrangian exhibits chiral symmetry
- however, at low temperatures, the chiral symmetry is spontaneously broken: $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$
- order parameter is the chiral condensate: $\langle \bar{q}q \rangle = -\frac{T}{V}\frac{\partial}{\partial m_q} \log \mathcal{Z}, \qquad \langle \bar{q}q \rangle \neq 0$ for $T < T_c$
- at T_c the chiral symmetry is restored
- small m_q breaks chiral symmetry explicitly, but SχSB still provides good approximation to low energy QCD (pions as pseudo-Goldstone bosons)

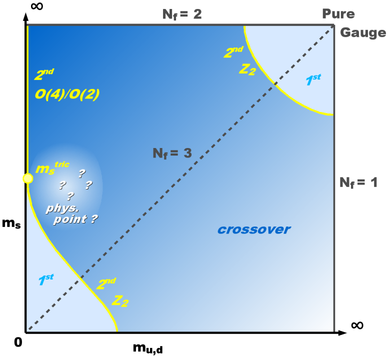
Axial symmetry:

- below T_c: nonzero chiral condensate breaks $\mathrm{U}(1)_A$ explicitly
- at some $T_{\mathrm{U}(1)_A}$, the $\mathrm{U}(1)_A$ symmetry is restored effectively

The QCD phase transitions at zero density

Columbia Plot: quark mass dependence of the order of the transition

- at physical quark masses, a crossover is expected
- for sufficiently small quark masses (both $m_{u,d}$ and m_s) the transition is first order.
- critical lines of second order transition - limiting cases:
  $N_f = 2$: O(4) universality class
  $N_f = 3$: Ising universality class



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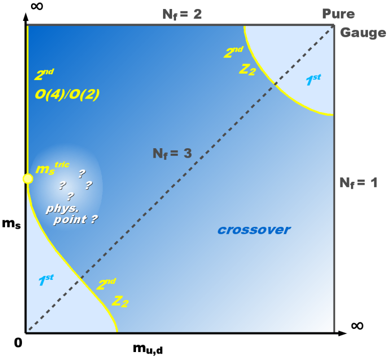

In this talk: interested in the chiral limit

$\lim m_q \to 0$ and m_s fixed at physical value.

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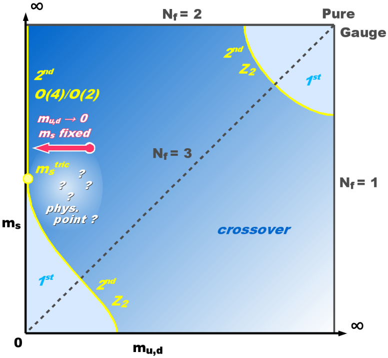

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O(N) models and $N_f = 2$ QCD

Physics of QCD at low energies can be described effective by O(N) symmetric spin models:

- $SU(2)_L \times SU(2)_R \simeq O(4) \to O(3) \simeq SU(2)_V$
- O(4) fields: $\pi^i = \bar{q}\gamma_5 t^i q$, $\sigma = -\bar{q}q$ and $\delta^i = \bar{q}t^i q$, $\eta' = \bar{q}\gamma_5 q$
- $(\vec{\pi}, \sigma)$ and $(\vec{\delta}, \eta')$ form O(4) rotation invariant vectors
- external field H corresponds to quark mass m_q
- order parameter: "magnetization" $\Sigma = \langle\sigma\rangle$

This description is valid below and in the vicinity of the chiral phase transition.

O(N) Spin Model in 3 and 4 dimensions

$N-1$ transverse Goldstone modes give corrections to Σ in case of a non-zero external field H (from calculation of expectation value of transversal two-point function $\langle \pi^i \pi^j \rangle$):

- $d=3$:

$$\Sigma_H = \Sigma_0 \left(1 - \frac{N-1}{8\pi^2} \frac{(\Sigma_0 H)^{1/2}}{F_0^3} + \mathcal{O}(H) \right)$$

- $d=4$:

$$\Sigma_H = \Sigma_0 \left(1 - \frac{N-1}{32\pi^2} \frac{\Sigma_0 H}{F_0^2} \ln\left(\frac{\Sigma_0 H}{F_0^2 \Lambda_\Sigma} \right) + \mathcal{O}(H^2) \right)$$

J. Gasser, H. Leutwyler - Ann. Phys. 158 (1984)

P. Hasenfratz, H. Leutwyler - Nucl. Phys Proc. B343 (1990)

O(N) Scaling in 3 dimensions

Consistency of presence of Goldstone modes below T_c with critical behaviour at T_c:

- below T_c: $\Sigma_H = c_0(T) + c_1(T)H^{1/2}$
- $T \simeq T_c$: scaling functions determine dependence on external field

$$\Sigma_H = H^{1/\delta} f_s\left(tH^{-\beta\delta}\right), \qquad t = \frac{T - T_c}{T_c},$$

  with critical exponents

$$\Sigma_{H=0} \sim t^{\beta}, \quad \beta = 0.38 \qquad \text{and} \qquad \Sigma_H(t=0) \sim H^{1/\delta}, \quad \delta = 4.82$$

- magnetic equation of state: $\frac{1}{\beta}\Sigma^{\delta-1}\chi_H = \tilde{c}_0 + \tilde{c}_1(H/\Sigma^{\delta})^{-1/2}$
- Goldstone effect in O(N) and consistency with critical scaling numerically well established

D. J. Wallace, R, K. P. Zia - Phys. Rev. B12 (1975)
J. Engels, T. Mendes - Nucl. Phys Proc. Suppl. 83 (2000)

Goldstone Modes in QCD

QCD with $N_f = 2 + 1$ flavors:

- chiral condensate now in principle depends on two masses, $m_{u,d}$ and m_s
- chiral limit in $m_{u,d}$ and m_s fixed: limiting case is $N_f = 2$ QCD, hence $N_f = 2$ behaviour expected

To analyze quark mass dependence of chiral condensate and chiral susceptibilities:

- study effective Lagrangians for low energy physics, e. g.

$$\mathcal{L}_{\text{eff}}^{(1)} = \frac{1}{4} F_\pi^2 \,\text{Tr}(\partial\mu U^\dagger)(\partial\mu U) + \Sigma Re Tr \mathcal{M} U, \quad U = e^{2i\phi^a t^a / F_\pi}$$

Pressure of the Pion Gas

Pressure of the pion gas: each pion contributes with

$$P(T, M_\pi) = \frac{\pi^2 T^4}{45} - \frac{T^2 M_\pi^2}{12} + \frac{T M_\pi^3}{6\pi} - \frac{M_\pi^4}{16\pi^2} \log \frac{\Lambda}{M_\pi} + \dots$$

Gellman-Oakes-Renner: $M_\pi^2 = 2 m_q B \qquad \rightarrow \qquad \frac{\partial M_\pi}{\partial m_q} = \frac{B}{M_\pi}$

$$\begin{aligned} \langle \bar{q} q \rangle &= -\frac{T}{V} \frac{\partial}{\partial m_q} \ln \mathcal{Z} = \left\langle \frac{\partial P}{\partial m_q} \right\rangle \\ &= B \left\{ \frac{T^2}{6} + \frac{T M_\pi}{2\pi} + \frac{M \pi^2}{4\pi^2} \log \frac{\Lambda}{M_\pi} \right\} \end{aligned}$$

[H. E. Haber, H. A. Weldon - Phys. Rev. Lett. 46 (1981)]

Quark Mass dependence of Chiral Condensate

Preliminary Data of the RBC-Bielefeld Collaboration on Chiral Condensate:

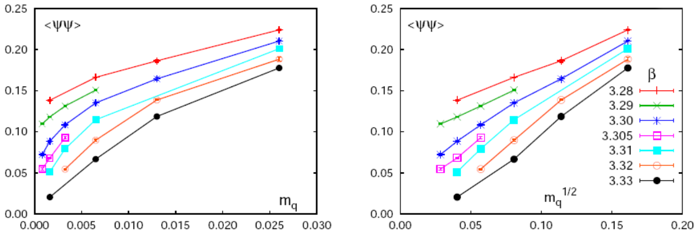


Figure: Chiral Condensate over quark mass

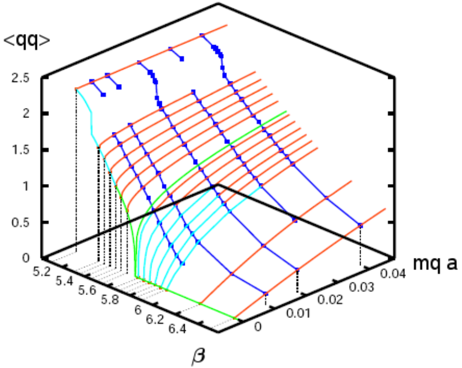
- data indicate Goldstone Effect in $N_f = 2 + 1$ QCD below T_c (orientation: $\beta = 3.30 \quad \rightarrow \quad T \simeq 196$ MeV)
- fitansatz: $\bar{q}q = a + b\sqrt{m_q} + cm_q$ describes data remarkably well

[F. Karsch, presented on Lattice 2008]

Goldstone Effect in Adjoint QCD

QCD with (staggered) fermions in the adjoint representation:

- chiral symmetry breaking pattern is $SU(2N_f) \to SO(N_f)$
- exhibits Z_{N_c} symmetry as in pure gauge theory, but only asymptotically free for $N_f \leq 2$
- two distinct phase transitions in $N_f = 2$ adjQCD with $T_{\rm deconf} < T_{\rm chiral}$
- intermediate deconfined phase: chiral behaviour as expected from 3d O(N) models



[J. Engels, S. Holtmann, T. Schulze, Nucl. Phys. B724 (2005)]

Susceptibilities in Spin models and in QCD

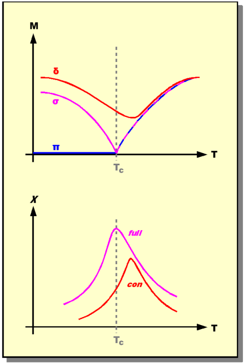
Susceptibilities allow determination of critcal temperature, as it is a measure for the fluctuations of the order parameter:

- in O(N) Spin Models: $\chi = \frac{\partial \Sigma}{\partial H} = \left\langle \Sigma^2 \right\rangle - \left\langle \Sigma \right\rangle^2 = \left\langle (\delta \Sigma^2)^2 \right\rangle$
- in QCD: also (quark-line) connected diagrams contribute to chiral susceptiblity due to possibility of additional Wick contraction:

$$
\begin{aligned}
\chi_{\text{full}} &= \frac{\partial \left\langle \bar{q} q \right\rangle}{\partial m_q} = -\frac{1}{\mathcal{Z}^2} \left(\frac{\partial}{\partial m_q} \mathcal{Z} \right)^2 + \frac{}{\mathcal{Z}} \frac{\partial^2 1}{\partial m_q^2} \mathcal{Z} \equiv N_f \chi_{\text{con}} + N_f^2 \chi_{\text{d}} \\
\chi_{\text{dis}} &= \left\langle (\bar{q} q)^2 \right\rangle - \left\langle \bar{q} q \right\rangle^2 \\
\chi_{\text{con}} &= \left\langle \bar{q}(x) q(x) \bar{q}(0) q(0) \right\rangle
\end{aligned}
$$

Connected Susceptibility Revisited

- Connected part gives only a minor contribution to finite size scaling
- nevertheless interesting to also investigate the connected susceptibility:



- connected susceptibility is related to the $U(1)_A$ anomaly
- if $U(1)_A$ is effectively restored: the isosinglet σ-meson (f_0) and the isovector δ-meson (a_0) are mass degenerated
- to lowest order:

$$\chi_{\text{con}} \sim \frac{1}{M_\delta^2} \quad \text{and} \quad \chi_{\text{full}} \sim \frac{1}{M_\sigma^2}$$

[K. Rajagopal, F. Wilczek - Nucl. Phys. B399 (1993)]
[M. Marci, E. Meggiolaro - Nucl. Phys. B665 (2003)]

Calculation of Connected Susceptiblity in Chiral Perturbation Theory

- Calculation of χ_{con} for N_f degnerated flavors yields for infrared part:

$$\chi_{\text{con}}^{\text{IR}} = \frac{N_f^2 - 4}{16\pi^2 N_f} \left(\frac{\Sigma}{F_\pi^2} \right) \log \frac{\Lambda^2}{M_\pi^2}$$

- two crucial relation for $\text{SU}(N_f)$ were used here: completeness relation and relation between symmetric structure constants:

$$\sum_{a=1}^{N_f^2-1} \text{tr}\left(\lambda^a A \lambda^a B\right) = -\frac{2}{N_f}\,\text{tr}\,(AB) + 2\,\text{tr}\,A\,\text{tr}\,B \qquad \sum_{a,b,c,d=1}^{N_f^2-1} d_{abc} d_{abd} = \sum_{a,b=1}^{N_f^2-1} \frac{N_f^2-4}{N_f} \delta_{cd}$$

[J. Gasser, H. Leutwyler - Nucl. Phys. B250 (1985)]
[A. V. Smilga, J. Stern - Phys. Lett. B318 (1993)]
[A. Smilga, J. J. M. Verbaarschot - Phys. Rev. D54 (1996)]

IR Divergences in χ at Zero and Nonzero Temperature

remarkable: $\chi_{\text{con}}^{\text{IR}} = 0$ for $N_f = 2$

in the chiral limit: limiting case is $N_f = 2$ QCD, hence $N_f = 2$ behaviour expected $\rightarrow$ no Goldstone Effect expected for connected susceptibility!

From $\chi_{\text{con}}^{\text{IR}}$ and $\chi_{\text{full}}^{\text{IR}}$ one also gets $\chi_{\text{disc}}^{\text{IR}}$:

- IR-Part of Full Susceptibilities at $T = 0$ and at $M_\pi \ll T \ll T_c$:

$$\chi^{\text{IR}} = \frac{N_f^2 - 1}{8\pi^2} \left(\frac{\Sigma}{F_\pi^2} \right) \log \frac{\Lambda^2}{M_\pi^2} \qquad \chi_T^{\text{IR}} = \frac{N_f^2 - 4}{8\pi^2 N_f} \frac{T}{\sqrt{m}} \left(\frac{\Lambda^2}{F_\pi^2} \right)^{3/2}$$

IR-Part of Disconnected Susceptibility at $T = 0$ and $M_\pi \ll T \ll T_c$

$$d = 3: \quad \chi_{\text{dis}}^{\text{IR}} = \frac{N_f^2 + 2}{8\pi^2 N_f} \frac{T}{\sqrt{m}} \left(\frac{\Lambda^2}{F_\pi^2} \right)^{3/2} \quad \rightarrow 1/\sqrt{m_q}\text{-behaviour}$$

$$d = 4: \quad \chi_{\text{dis}}^{\text{IR}} = \frac{N_f^2 + 2}{16\pi^2 N_f} \left(\frac{\Sigma}{F_\pi^2} \right) \log \frac{\Lambda^2}{M_\pi^2} \quad \rightarrow \log m_q\text{-behaviour}$$

Setup of Staggered Chiral Perturbation Theory

Now: turn to staggered chiral perturbation theory in order to discuss taste violations

- calculation of so-called Bubble terms $B_{a_0}(p)$, $B_{f_0}(p)$[1], which are integrated correlations functions
- fluctuation-dissipation theorem: $\chi_M = \beta \int d^3x G_M(x) = B_M(p=0)$
- relation between bubble terms and susceptibilities: $\chi^{\mathrm{IR}} = N_f \chi^{\mathrm{IR,con}} + N_f^2 \chi^{\mathrm{IR,dis}}$ with

$$\chi^{\mathrm{IR,con}} = B_{a_0}(0), \qquad \chi^{\mathrm{IR,dis}} = (B_{f_0}(0) - B_{a_0}(0))/N_f$$

- vacuum disconnected diagrams do not contribute to bubble term by definition, but they also do not contribute to IR divergences.

[1] switch of notation: $\sigma \rightarrow f_0$, $\delta \rightarrow a_0$ because now $N_f = 2+1$ is considered

Setup of Staggered Chiral Perturbation Theory

Solution to the fermion doubling problem:

- distribute 16 degrees of freedom over 2^4 hypercube in a way that reflects the Dirac structure of fermions.
- every flavor still comes in four tastes $\rightarrow$ eliminated by fourth root trick on fermion determinant
- replacement of $N_f \rightarrow N_f/4$ in observables
- in SχPT: introduce replica index n_r, which at the end is set to $n_r = 1/4$
  $N_f = 2+1 \quad \rightarrow \quad 8+4 \quad \rightarrow \quad 8n_r + 4n_r$
- $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{\text{cont}} + a^2 \sum_i C_i \mathcal{O}_i(U)$,

Taste Violations and $U(1)_A$ Anomaly Term

- taste-index: $b \in \{P, A, T, V, I\}$ corresponds to taste channels defined via Euclidean gamma matrices. $t_b \in \{\gamma_5, i\gamma_\mu\gamma_5, i\gamma_\mu\gamma_\nu, \gamma_\mu, 1\}$
- impact of taste violations on meson masses:

$$M^2_{f,f',b} = \mu(m_f + m_f') + a^2\Delta_b$$

with taste-violations: $\Delta_P = 0$ and $\Delta_b \neq 0$ for $b \neq P$

- moreover: isosinglet states (η, η') are both modified by the taste-singlet anomaly and by two-trace (quark-line hairpin) taste-vector and taste-axial operators

[C. Aubin, C. Bernard - Phys. Rev. D68 (2003)]
[C. Bernard, C. DeTar, Z. Fu, S. Prelvsek - arXiv:0707.2402v1 (2007)]

Scalar Correlator in SχPT: f_0 and a_0 Bubble term

- Propagators for mesons made of light valence quarks f, g and f', g' resp.

$$\left\langle \phi^b_{gs,fr}(-k)\phi^b_{f'r',g's'}(k)\right\rangle = -\delta_{fg}\delta_{f'g'}\delta_{rs}\delta_{r's'}\frac{\delta_b\prod_L(q^2+M^2_{Lb})}{(q^2+M^2_{f,g,b})(q^2+M^2_{f',g',b})\prod_F(q^2+M^2_{Fb})}$$

with $b \in \{V, A, I\}$ and $\delta_V = a^2\delta'_V$, $\delta_A = a^2\delta'_A$, $\delta_I = 4m_0^2 n_r/N_f$

- L: unmixed flavor neutral mesons
- F: eigenvalues of full mass matrix

f_0 and a_0 bubble term

- general bubble term with valence flavors f, f' and e, e':

$$B_{f,f';e,e'}(p) = \tilde{\mu} \sum_k \sum_{g,s,r,b} \sum_{g',s',r',b'} \Big\{ \Big\langle \phi^b_{fr,gs}(-k) \phi^{b'}_{er',g's'}(k) \Big\rangle \Big\langle \phi^b_{gs,f'r}(k-p) \phi^{b'}_{g's',e'r'}(p-k) \Big\rangle$$
$$\Big\langle \phi^b_{fr,gs}(-k) \phi^{b'}_{g's',e'r'}(k) \Big\rangle \Big\langle \phi^b_{gs,f'r}(k-p) \phi^{b'}_{er',g's'}(p-k) \Big\rangle \Big\}$$

- IR relevant terms in $B_{a_0}(0)$:

$$B_{a_0}(0) \simeq \sum_b n_r^2 \sum_{f,k} \left(\frac{1}{k^2 + M_{fu,b}} \right)^2 - 2 \sum_k \left(\frac{1}{k^2 + M^2_{U,I}} \right)^2 + \quad \text{IR -irrev. terms}$$

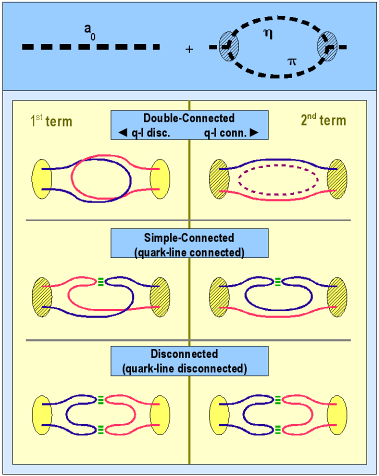
- continuum limit: $M^2_{U,b}$ degenerate

$$B_{a_0}(0) \simeq \sum_{f,k} \left(\frac{1}{k^2 + M^2_{K,b}} \right)^2 + 2 \sum_k \left(\frac{1}{k^2 + M^2_U} \right)^2 \left(\frac{1}{k^2 + M^2_\eta} \right)^2$$

lattice artefact: IR divergence for χ_{con} at $a \neq 0$ in pseudo-taste channel!

Hairpin Diagrams and Contributions to B_{a_0}

- important to realize which diagrams are quark-line connected and which are quark-line disconnected
- only quark line connected diagrams contribute to a_0 !
- vacuum disconnected diagrams are per definition of bubble term neglected



continuum limit: f_0 and a_0 bubble term

- for $N_f = 2+1$ (further simplification: $m_0 \to \infty$):

$$B_{a_0}(p) = \tilde{\mu}^2 \sum_k \left\{ 16 n_r^2 \sum_f \frac{1}{k^2+M_{fu}^2} \frac{1}{(k+p)^2+M_{fu}^2} - \frac{4}{3} \frac{1}{(k+p)^2+M_\pi^2} \left(\frac{3/2}{k^2+M_\pi^2} - \frac{1/2}{k^2+M_\eta^2} \right) \right\}$$

$$B_{f_0}(p) = B_{a_0}(p) + \tilde{\mu}^2 \sum_k \left\{ 32 n_r^2 \frac{1}{k^2+M_\pi^2} \frac{1}{(k+p)^2+M_\pi^2} + \frac{4}{9} \left(\frac{3/2}{(k+p)^2+M_\pi^2} - \frac{1/2}{(k+p)^2+M_\eta^2} \right) \left(\frac{3/2}{k^2+M_\pi^2} - \frac{1/2}{k^2+M_\eta^2} \right) \right\}$$

- with $n_r = \frac{1}{4}$ at $p = 0$:

$$B_{a_0}(0) = \tilde{\mu}^2 \sum_k \left\{ \frac{2}{3} \frac{1}{k^2+M_\pi^2} \frac{1}{k^2+M_\eta^2} - \frac{2}{(k^2+M_K^2)^2} \right\}$$

$$B_{f_0}(0) = B_{a_0}(0) + \tilde{\mu}^2 \sum_k \left\{ 3 \frac{1}{(k^2+M_\pi^2)^2} - \frac{2}{3} \frac{1}{k^2+M_\pi^2} \frac{1}{k^2+M_\eta^2} + \frac{1}{9} \frac{1}{(k^2+M_\eta^2)^2} \right\}$$

- surviving modes in a_0: $\bar{K}K$ and $\pi\eta$, in f_0: $\pi\pi$, $\pi\eta$ and $\eta\eta$

IR divergence induced by taste violations

- taste violation contribution in 3D for N_f degenerated flavors:

$$\chi^{\mathrm{con,IR}}_{\mathrm{S}\chi\mathrm{PT}} = B^{\mathrm{IR}}_{a_0}(0) = \frac{\tilde{\mu}}{4\pi}\frac{n_r^2 N_f}{2}\frac{1}{\sqrt{2\mu m_l}}$$

$$\chi^{\mathrm{dis,IR}}_{\mathrm{S}\chi\mathrm{PT}} = \left(B^{\mathrm{IR}}_{f_0}(0) - B^{\mathrm{IR}}_{a_0}(0)\right)/N_f = \frac{\tilde{\mu}}{4\pi}\frac{n_r^2}{2}\frac{1}{\sqrt{2\mu m_l}}$$

- taste violation contribution in 3D for $N_f = 2+1$:

$$\chi^{\mathrm{con,IR}}_{\mathrm{S}\chi\mathrm{PT}} = B^{\mathrm{IR}}_{a_0}(0) = \frac{\tilde{\mu}}{4\pi} n_r^2 \frac{1}{\sqrt{2\mu m_l}}$$

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IR divergence of susceptibilities in the chiral limit (3 dimensions)

- 3D continuum limit for N_f degenerated flavors:

$$\chi^{\text{con,IR}}_{\text{S}\chi\text{PT}} = B^{\text{IR}}_{a_0}(0) = \frac{\hat{\mu}}{8\pi}\left(N_f - \frac{4}{N_f}\right)\frac{1}{\sqrt{2\mu m_l}}$$

$$\chi^{\text{dis,IR}}_{\text{S}\chi\text{PT}} = \left(B^{\text{IR}}_{f_0}(0) - B^{\text{IR}}_{a_0}(0)\right)/N_f = \frac{\hat{\mu}}{8\pi}\left(1 + \frac{2}{N_f^2}\right)\frac{1}{\sqrt{2\mu m_l}}$$

- 3D continuum limit for $N_f = 2 + 1$:

$$\chi^{\text{con,IR}}_{\text{S}\chi\text{PT}} = B^{\text{IR}}_{a_0}(0) = 0$$

$$\chi^{\text{dis,IR}}_{\text{S}\chi\text{PT}} = \left(B^{\text{IR}}_{f_0}(0) - B^{\text{IR}}_{a_0}(0)\right)/N_f = \frac{3\hat{\mu}}{16\pi}\frac{1}{\sqrt{2\mu m_l}}$$

Lattices investigated so far

- used lattice action: p4fat3 with treelevel Symanzik improved
- configurations were generated via RHMC
- susceptibilities were measured with up to 20 random vectors
- strange quark mass fixed to $m_s a = 0.065$
- configurations (separated by 10 trajectories):
  $32^3 \times 4, \quad m_q = m_s/80, \quad \# \simeq \mathcal{O}(1500),$
  $16^3 \times 4, \quad m_q = m_s/40, \quad \# \simeq \mathcal{O}(500),$
  $16^3 \times 4, \quad m_q = m_s/40, \quad \# \simeq \mathcal{O}(200),$
  $16^3 \times 4, \quad m_q = m_s/20, \quad \# \simeq \mathcal{O}(1000),$
- Note: with m_s physical we have
  - $m_q = m_s/20: \quad \rightarrow \quad M_\pi \simeq 140$ MeV
  - $m_q = m_s/80: \quad \rightarrow \quad M_\pi \simeq 70$ MeV

Disconnected Part versus Connected Part

Peak locations: not well determined for connected part

for $T > T_c$ disconnected part becomes mass independent as expected

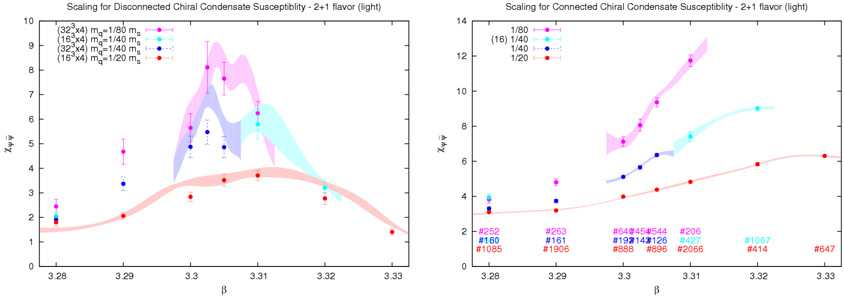
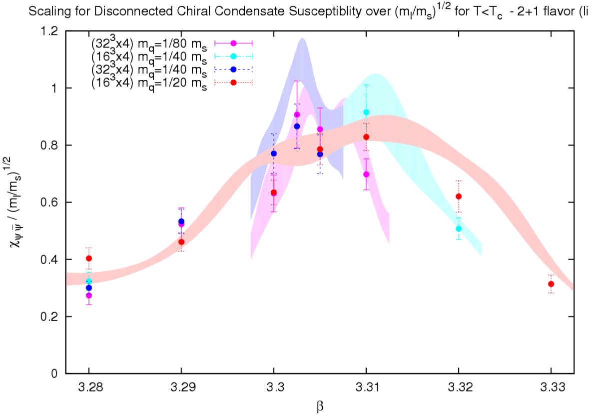


Figure: Chiral Susceptibility - left: disconected Part, right: connected Part

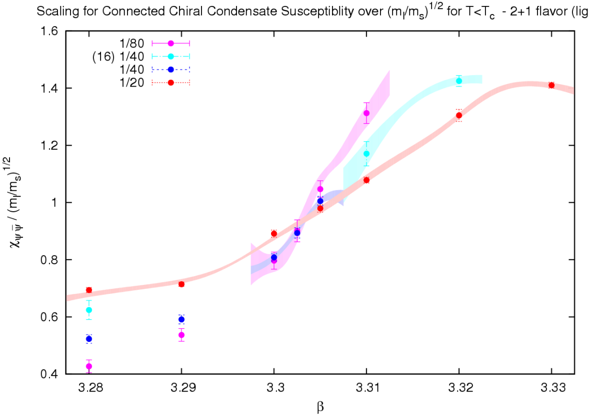
Disconnected Part versus Connected Part - $T < T_c$

Disconnected Part: Rescaling with $\sqrt{m_l/m_s}$
- Goldstone Effect extends into peak region



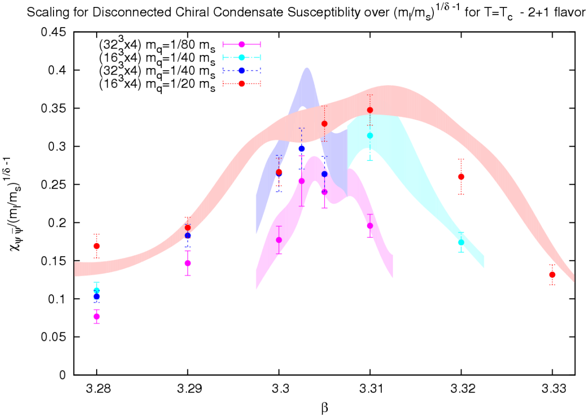
Disconnected Part versus Connected Part - $T < T_c$

Connected Part: Rescaling with $\sqrt{m_l/m_s}$
- no Goldstone-like behaviour



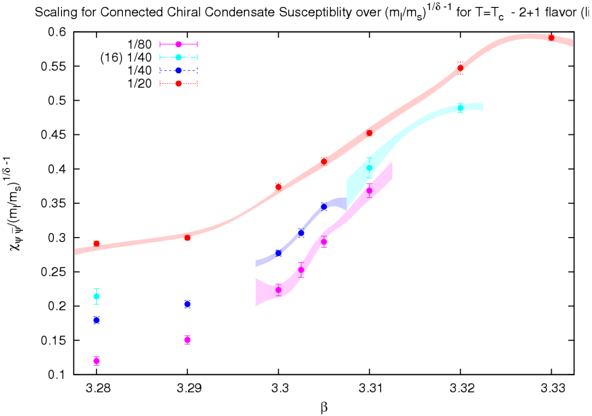
Disconnected Part versus Connected Part - $T \simeq T_c$

Disconnected Part: Rescaling with $m_l^{1/\delta-1}$
- does not match well (due to Goldstone IR divergence?)



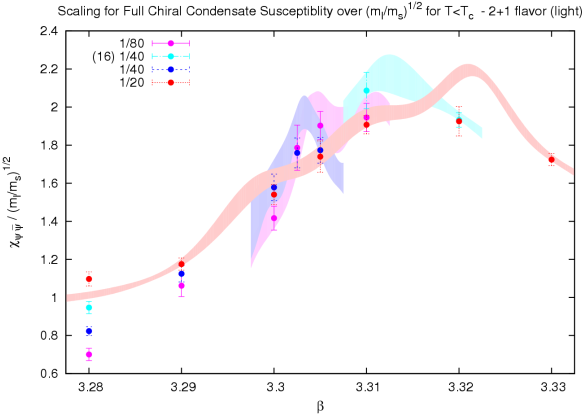
Disconnected Part versus Connected Part - $T \simeq T_c$

Connected Part: Rescaling with $m_l^{1/\delta-1}$
- does not match well (no surprise)



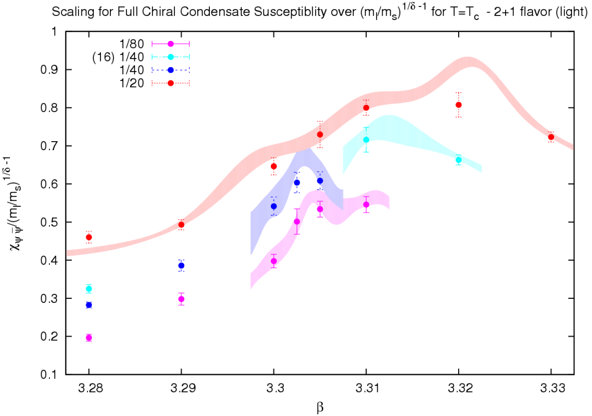
Full Susceptibility - $T < T_c$

Full Chiral Susceptibility: rescaled with $\sqrt{m_l/m_s}$
- Goldstone-behaviour spoiled?



Full Susceptibility

Full Chiral Susceptibility: rescaled with $m_l^{1/\delta-1}$



Conclusions

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- as O(N), χPT predicts Goldstone effect for an arbitrary number of flavors: $\bar{q}q \sim \sqrt{m_l} \rightarrow \chi \sim 1/\sqrt{m_l}$
- some evidence for Goldstone effect seen in $\bar{q}q$
- detailed discussion of χ: staggered fermions on the lattice induce taste violations: $\chi^{\mathrm{IR}}_{\mathrm{con}} \sim \chi^{\mathrm{IR}}_{\mathrm{dis}} \sim 1/\sqrt{m_l}$
- Goldstone effect might explain why it is difficult to see O(4)/O(2) scaling in chiral limit
- physical point not necessarily far from critical line
- analysis suggests: taste violations do not seem to play a prominent role here

Outlook:

- $N_\tau = 8$ data on the way

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Thank you for your attention!