Studying finite-size effects in spin-polarized Fermi gases

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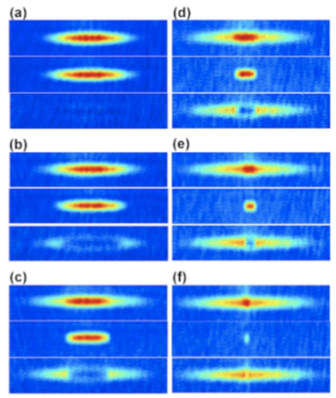
M. Ku, JB, A. Schwenk, arXiv:0812.3430
JB, J. Polonyi, A. Schwenk, work in progress

Strongly-Interacting Fermions in Nature

ultracold fermionic atoms



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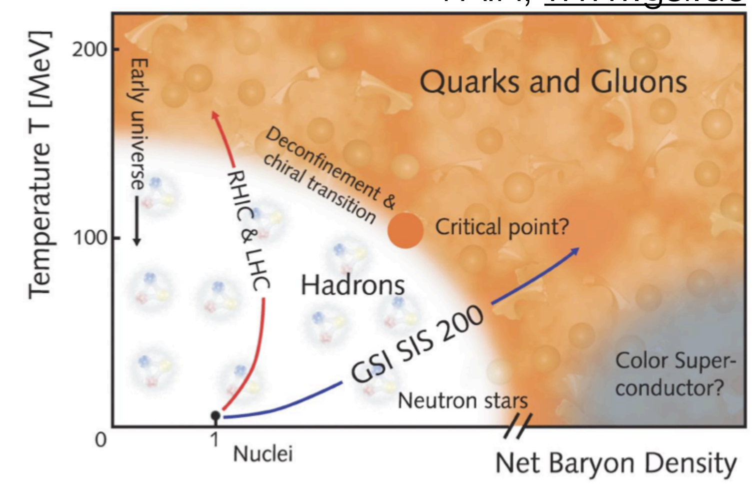
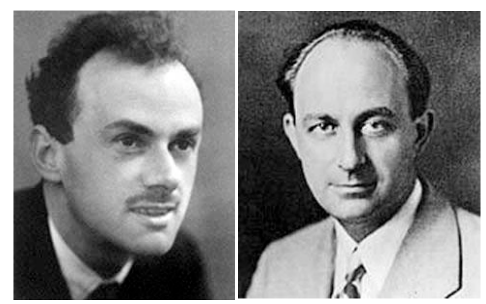
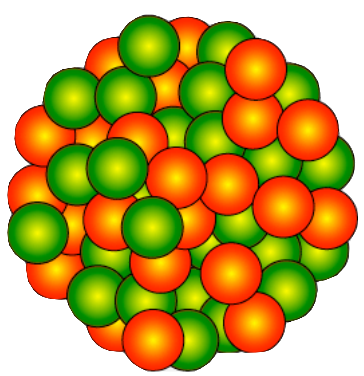




Nuclear Physics



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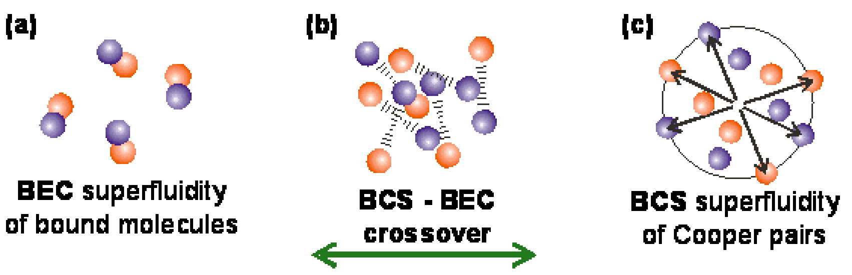
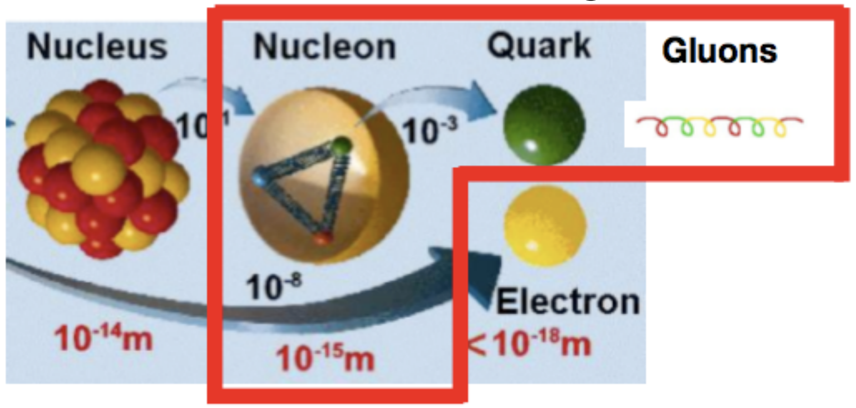
Problem: Microscopic and macroscopic DoFs

ultracold fermionic atoms



QCD (Phase Diagram)



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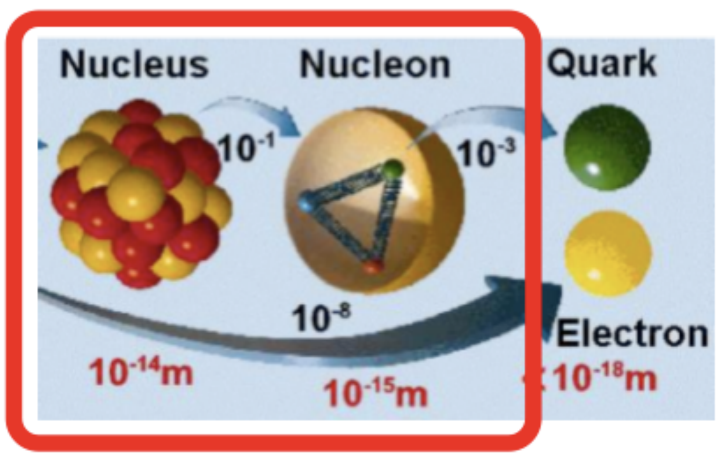

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How to tackle such strongly-interacting systems?

Monte-Carlo methods
(Lattice QCD, Quantum MC, ...)

Functional approaches
(Dyson-Schwinger Eqs.,
Functional RG methods, ...)

Hamiltonian approaches
(coupled-cluster theory, ...)

How to tackle such strongly-interacting systems?

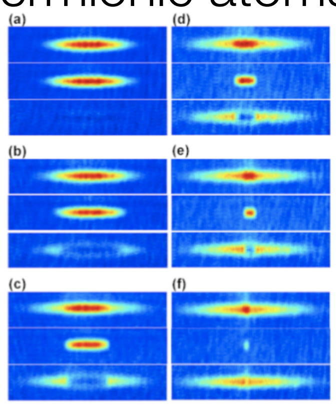
Monte-Carlo methods
(Lattice QCD, Quantum MC, ...)

Functional approaches
(Dyson-Schwinger Eqs., Functional RG methods, ...)

pros	**cons**
• allows for interpolation between finite system and continuum • no sign-problem • computationally efficient	• truncated action

Outline

1) trapped ultracold fermionic atoms



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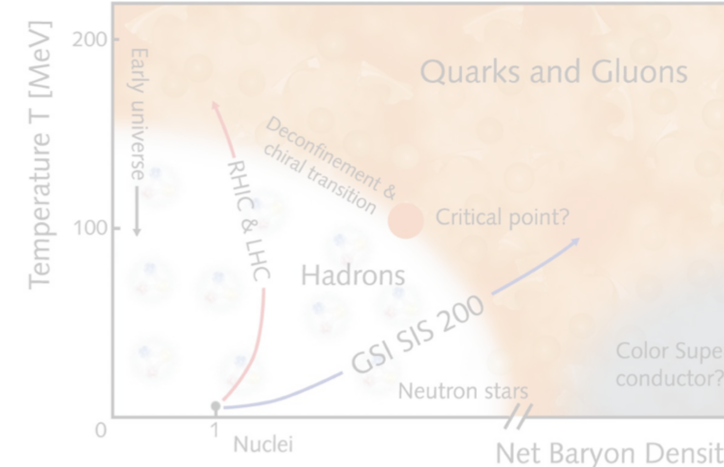
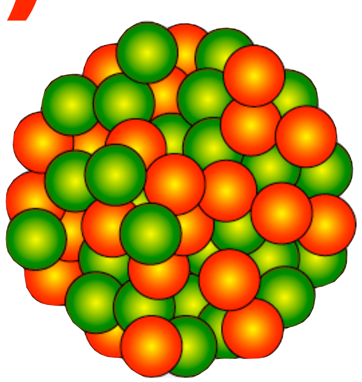
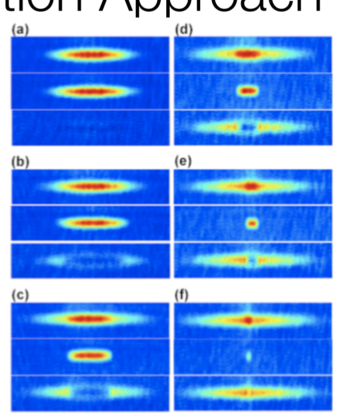


effective action:

$$\Gamma[\bar{\psi}, \psi, \phi, ...]$$

2) Density Functional Theory & RG Flow Equation Approach





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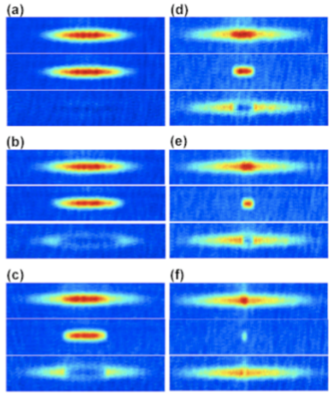
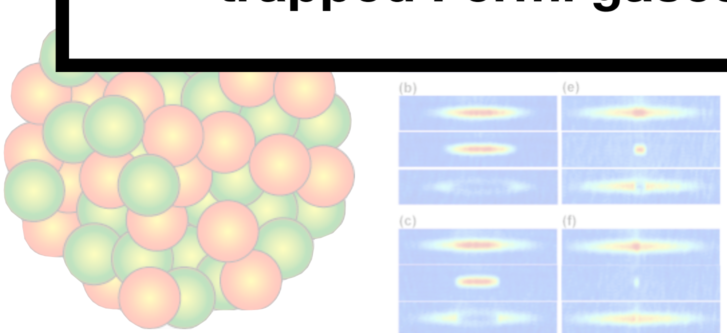
Outline

trapped ultracold
fermionic atoms



1. **Motivation**
2. **Experimental status**
3. **Theoretical study of trapped Fermi gases**



effective action:

$$\Gamma[\bar{\psi}, \psi, \phi, \ldots]$$

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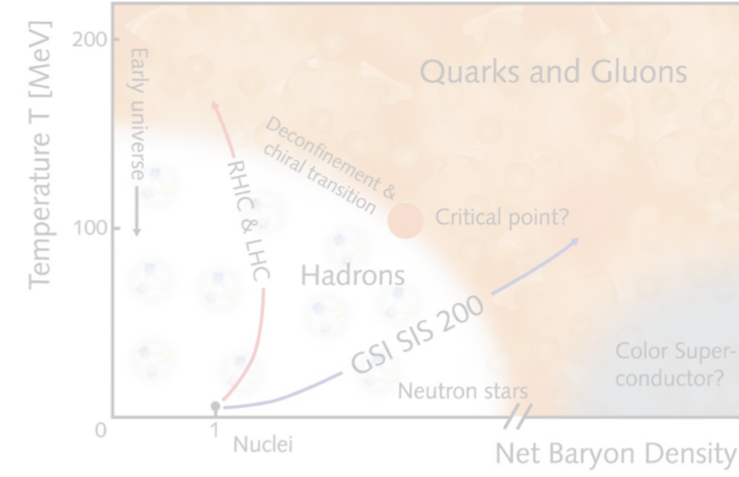

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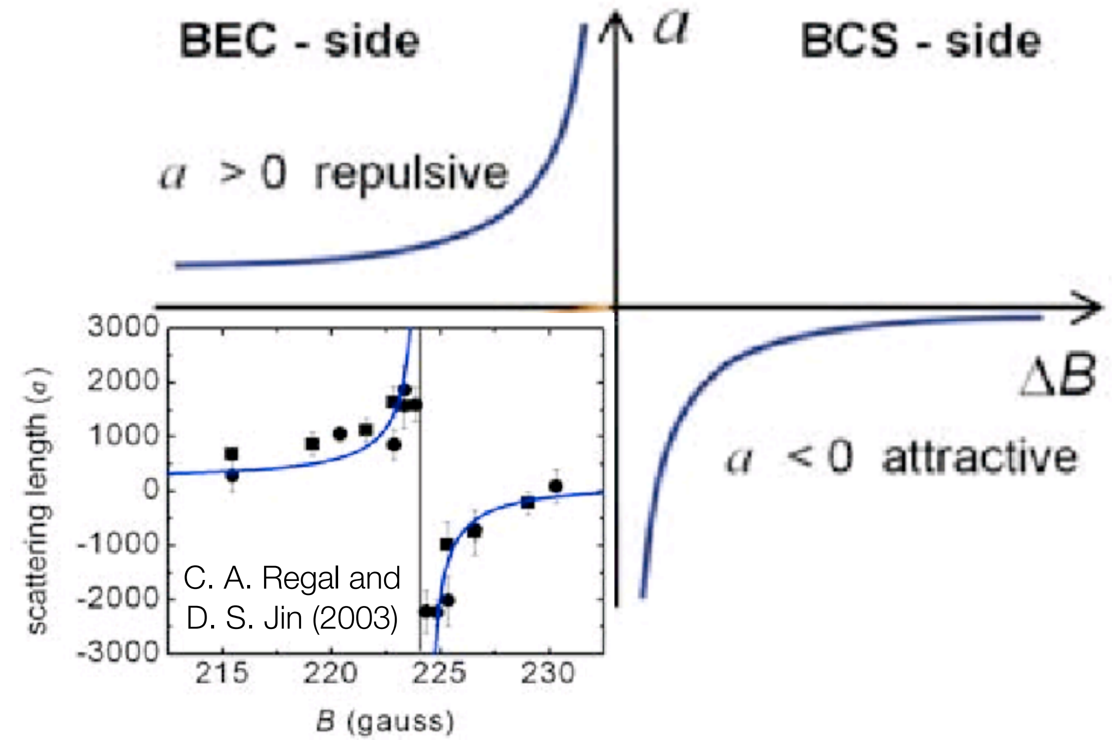
Unitary Regime

●s-wave scattering length is tunable by Feshbach resonance (ext. magnetic field)
●interaction strength is proportional to s-wave scattering length a



Unitary Regime

- s-wave scattering length is tunable by Feshbach resonance
- interaction strength is proportional to s-wave scattering length a
- limit of infinite scattering length a defines a universal regime:

$$0 \approx \frac{1}{|a|} \ll k_F \sim \frac{1}{r} \ll \frac{1}{R} \approx \infty$$

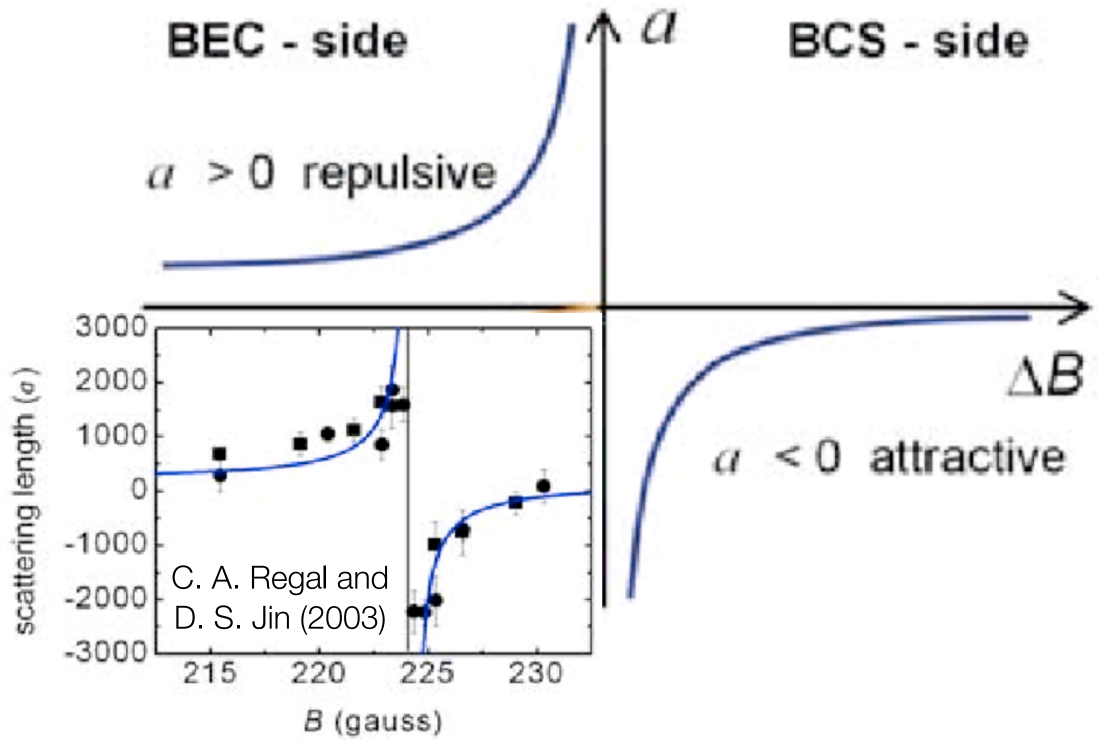
density (~Fermi momentum) is the only scale (unitarity limit)

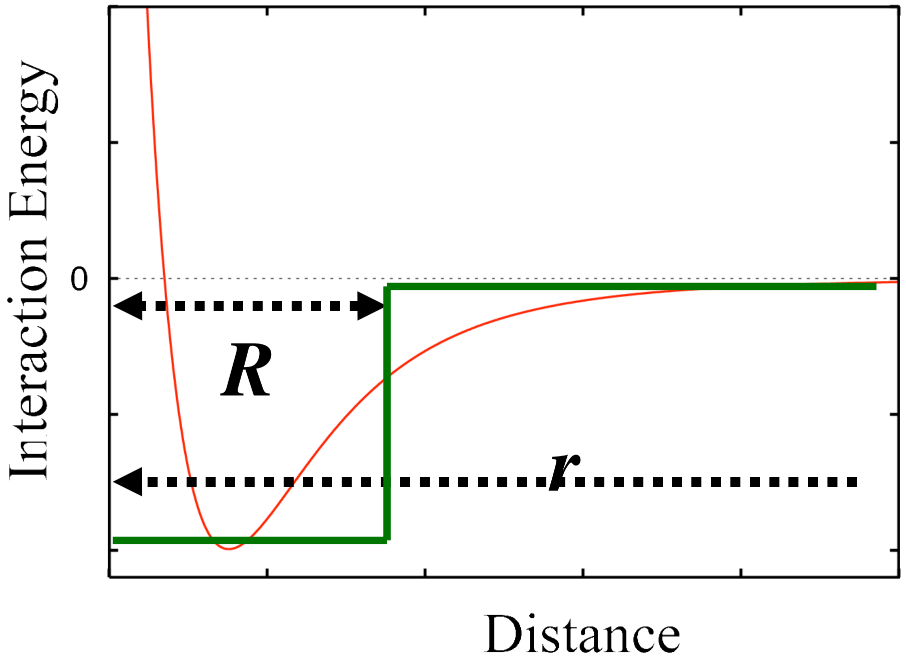
- Universal properties:

$$E/N, T_c, \cdots \propto \text{universal const(s).} \times E_F$$

- Example: dilute neutron matter

$$|a_{\rm nn}| \sim 18.5\text{fm} \gg R \sim 1.4\text{fm}$$





Symmetric Fermi Gases

•Experiment: Fermions in different hyperfine states

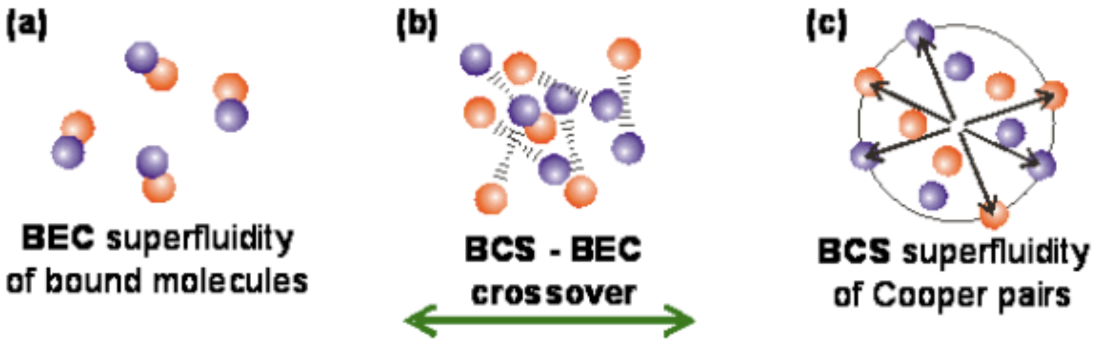
•provides an experimentally accessible environment for a study of quantum phenomena:

(a) BEC regime: tightly bound molecule $(a_s > 0)$

(b) Unitary regime: crossover - delocalized molecule with $E_B = 0$

(c) BCS regime: delocalized Cooper pairs $(a_s < 0)$

•**symmetric regime at T=0:** smooth crossover, superfluidity persists



Symmetric Fermi Gases at finite T

- Experiment: Fermions in different hyperfine states
- provides an experimentally accessible environment for a study of quantum phenomena:
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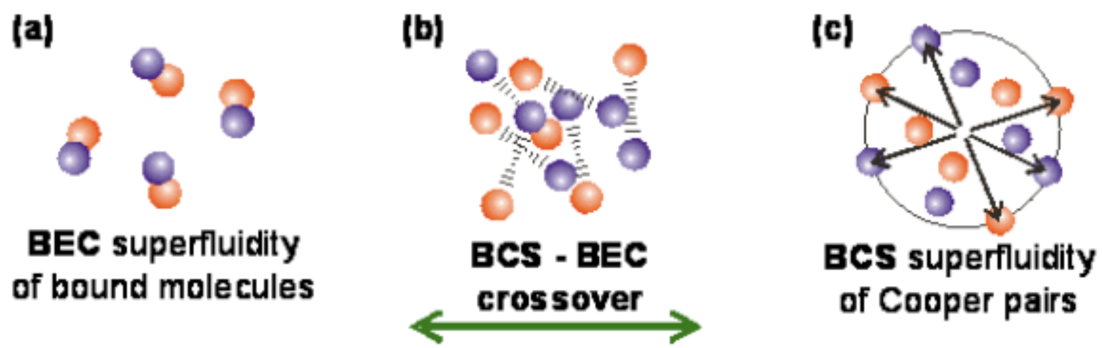
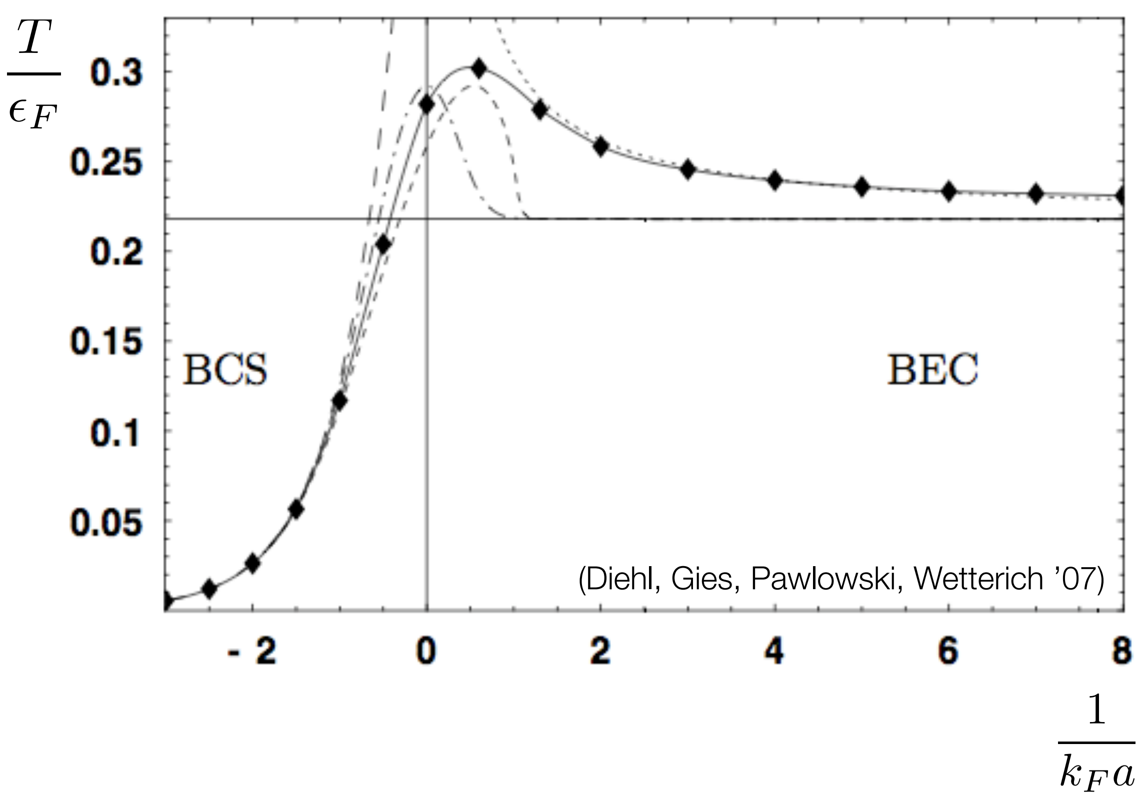
  - **(b) Unitary regime:** crossover - delocalized molecule with $E_B = 0$
  - **(c) BCS regime:** delocalized Cooper pairs $(a_s < 0)$
- **symmetric regime at T=0:** smooth crossover, superfluidity persists
- **symmetric regime at finite T:** phase transition, "melting condensate"





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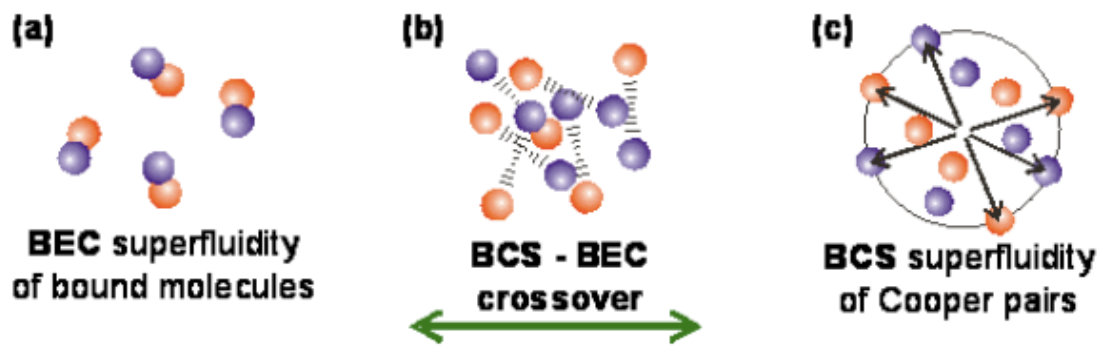
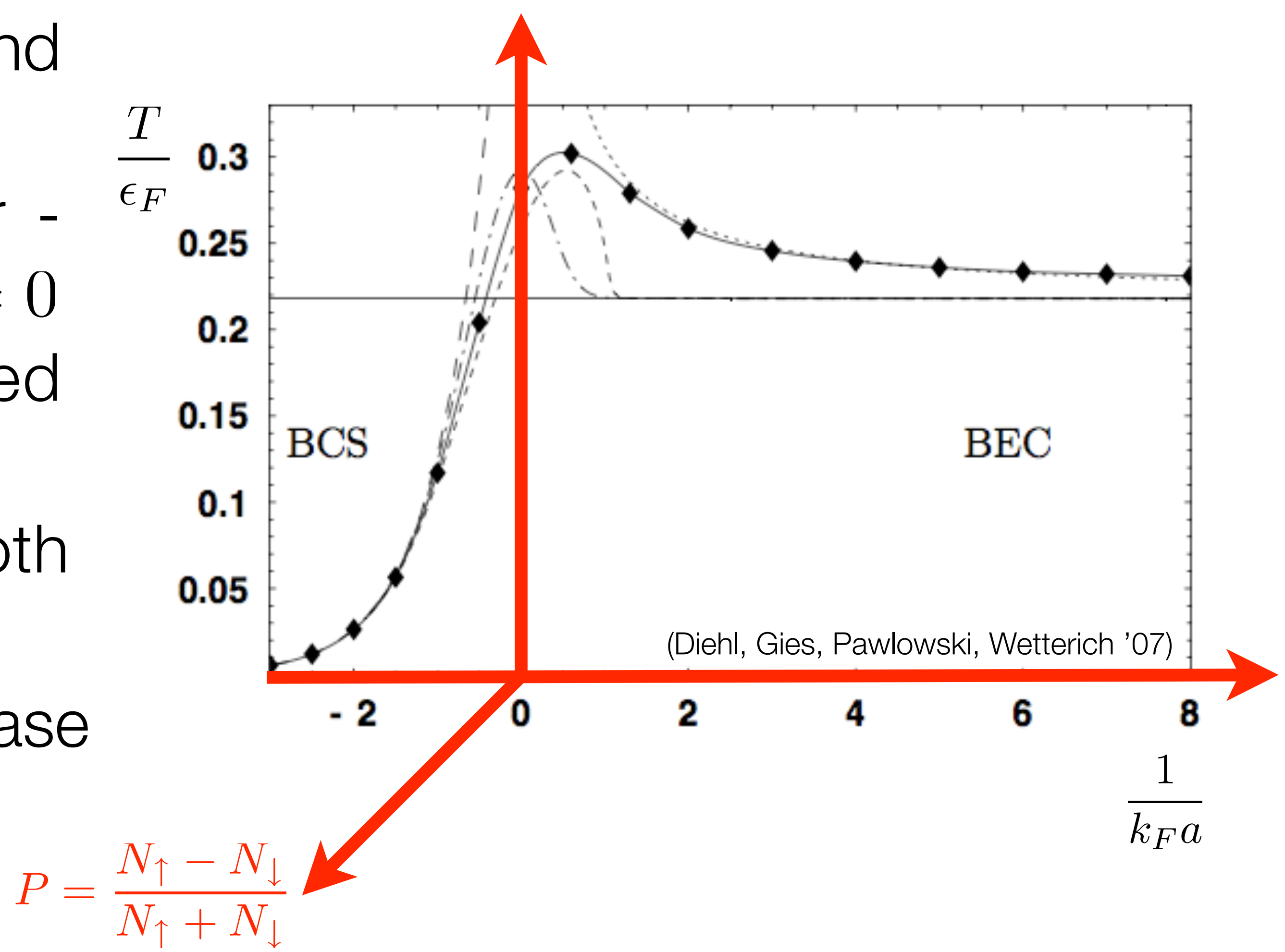
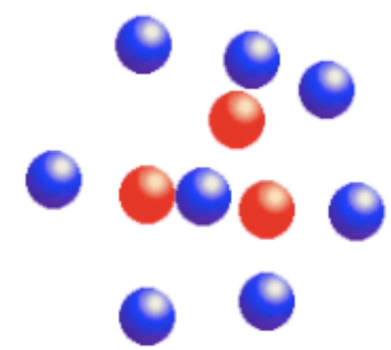
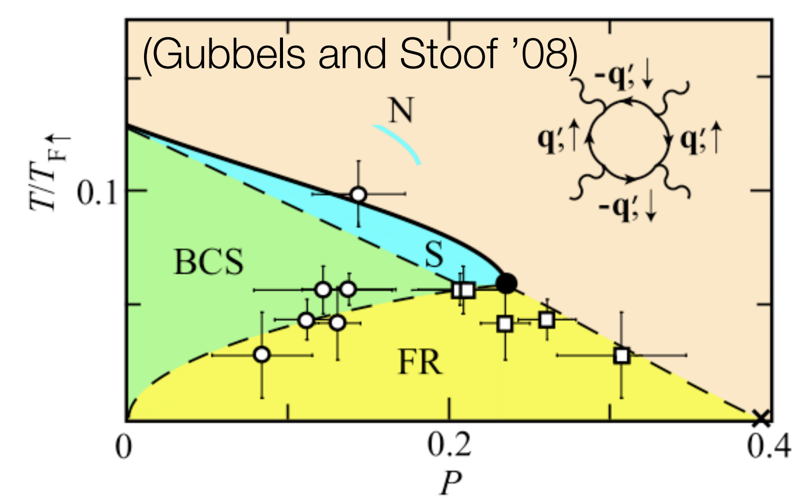


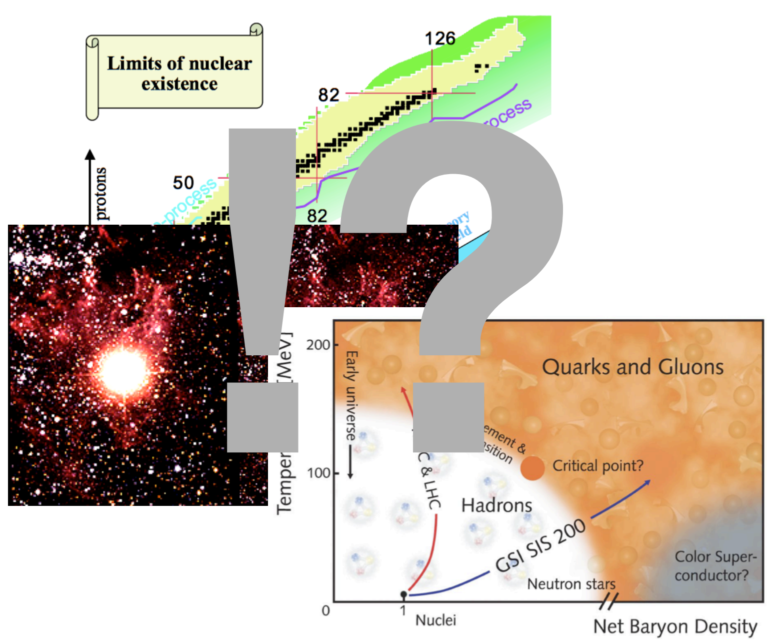


Asymmetric Fermi Gases



- Spin-polarized Fermi gases, e. g. $N_\uparrow > N_\downarrow$
  - Majority fermions $N_\uparrow$, minority fermions $N_\downarrow$
  - Polarization $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$
- **What happens when we have a population imbalance?**
- Relevance for various research fields, e. g.: Clogston limit in superconductivity, nuclear physics, astrophysics, QCD at finite T(?), ...
- Experiments with spin-polarized Fermi gases are very useful to explore asymmetric strongly-interacting Fermi systems

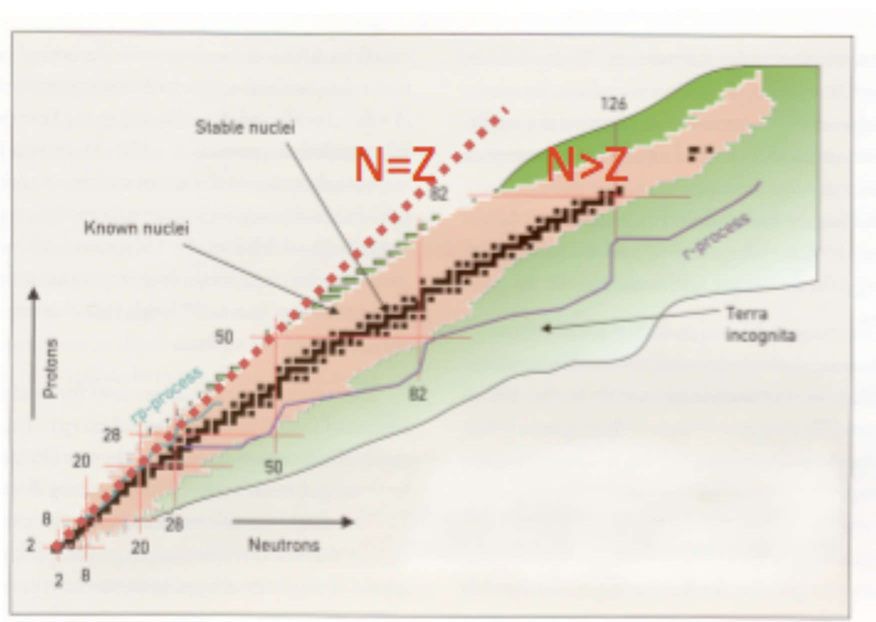


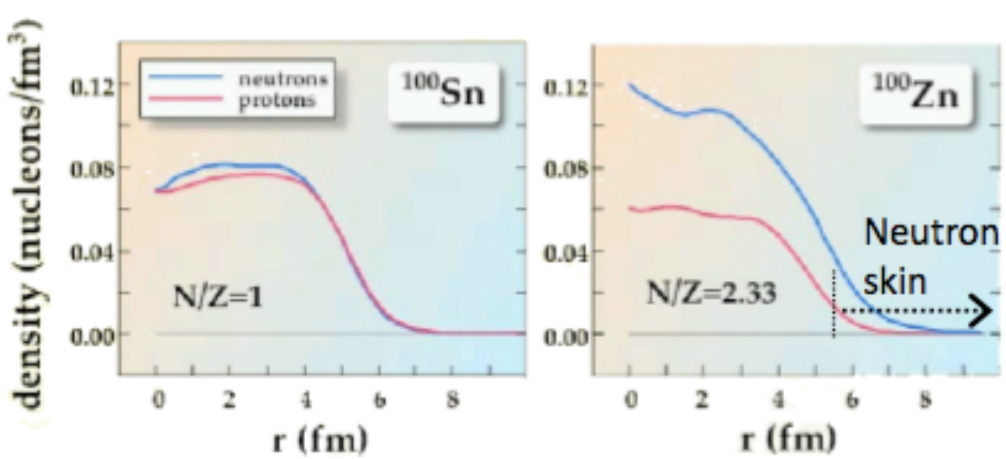


Beyond one's own nose: Asymmetric systems in nature

- **Nuclear physics:**

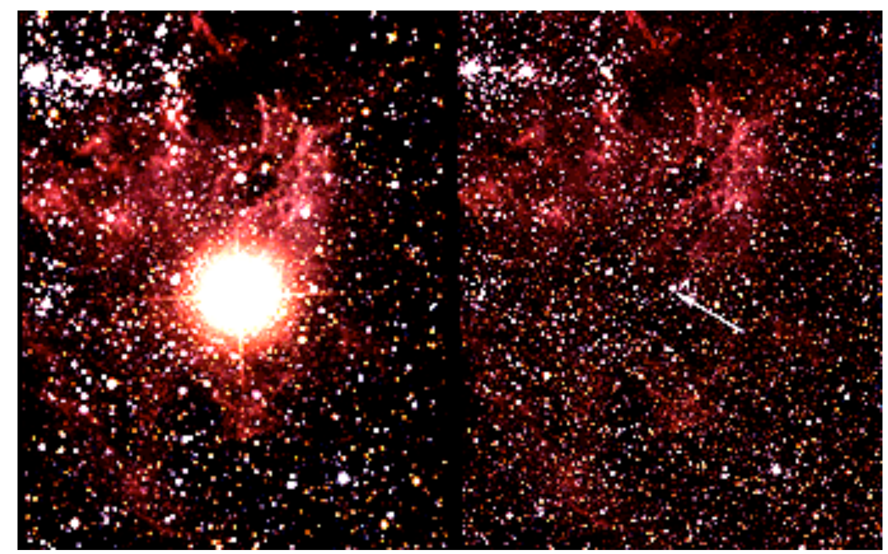
  Most nuclei N > Z (neutron skin)





- **Astrophysics:**

  Neutron star (95% n, 5% p)



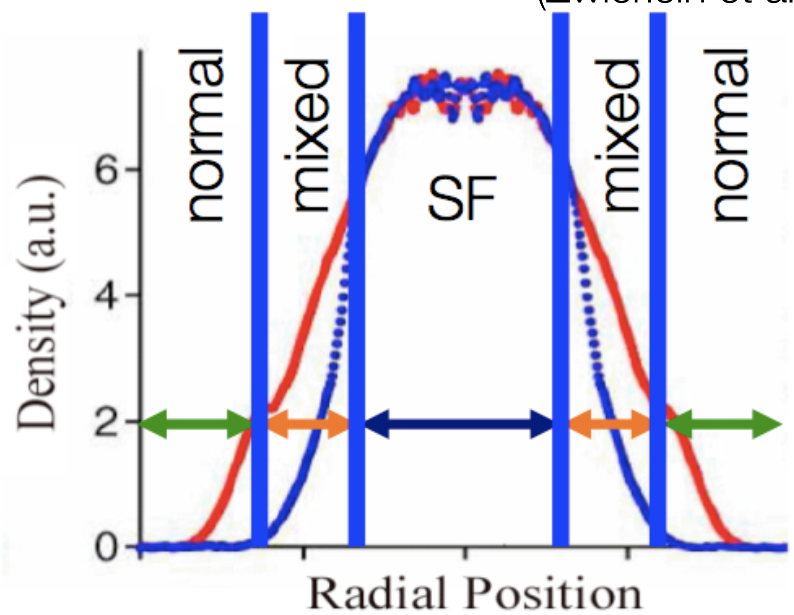
(SN 1987A from NASA image server)

Asymmetric spin-polarized systems: **MIT Experiment**

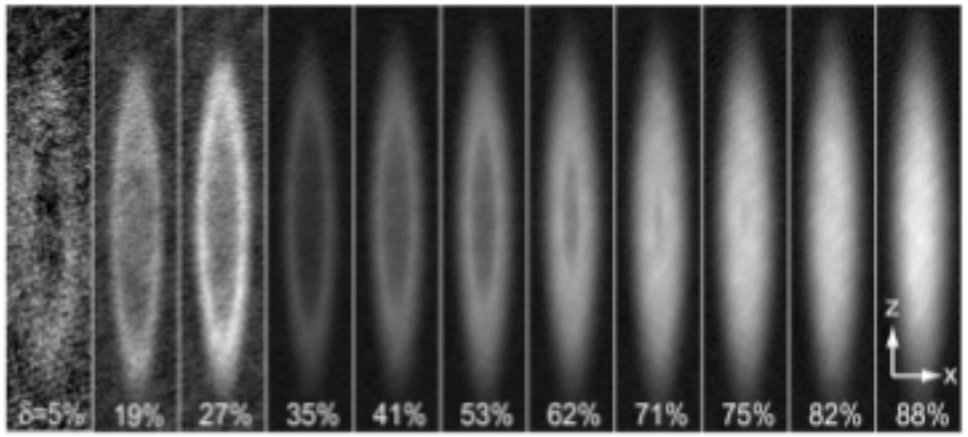
- **Experimental setup:** harmonic trap with cylindrical symmetry
  - $\omega_x = \omega_y = \alpha\omega, \omega_z = \omega; \alpha \sim 5$
  - $N_{\text{tot}} = N_\uparrow + N_\downarrow \sim 10^6 \dots 10^7$
- **Phase separation:**
  - Equal density core
  - Partially-polarized shell: diff. densities
  - Outer region of *normal* majority atoms
- **Critical polarization** above which equal density core ceases to exist:

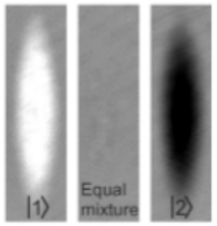
$$P_c = 0.70(3)$$

(Zwierlein et al. '06)



(Y.-i. Shin et al. '06)

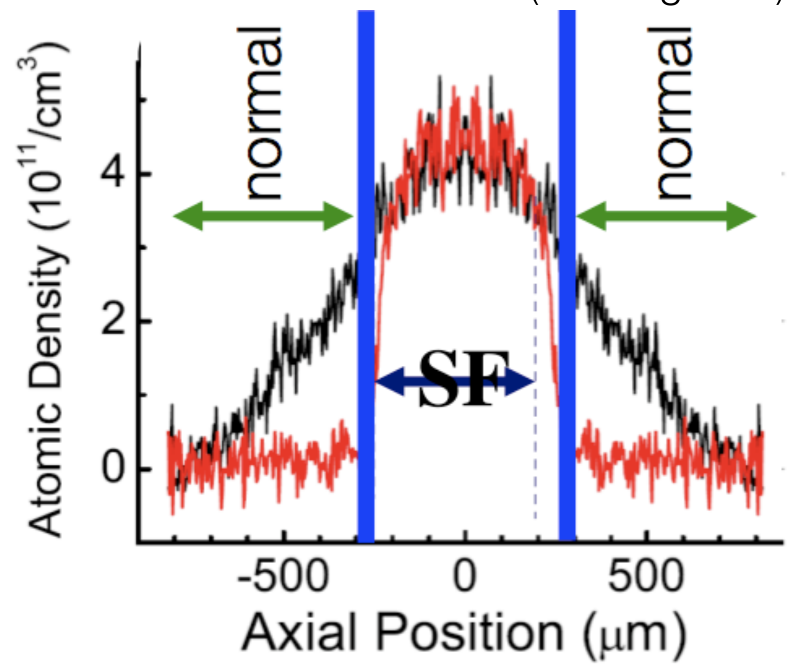


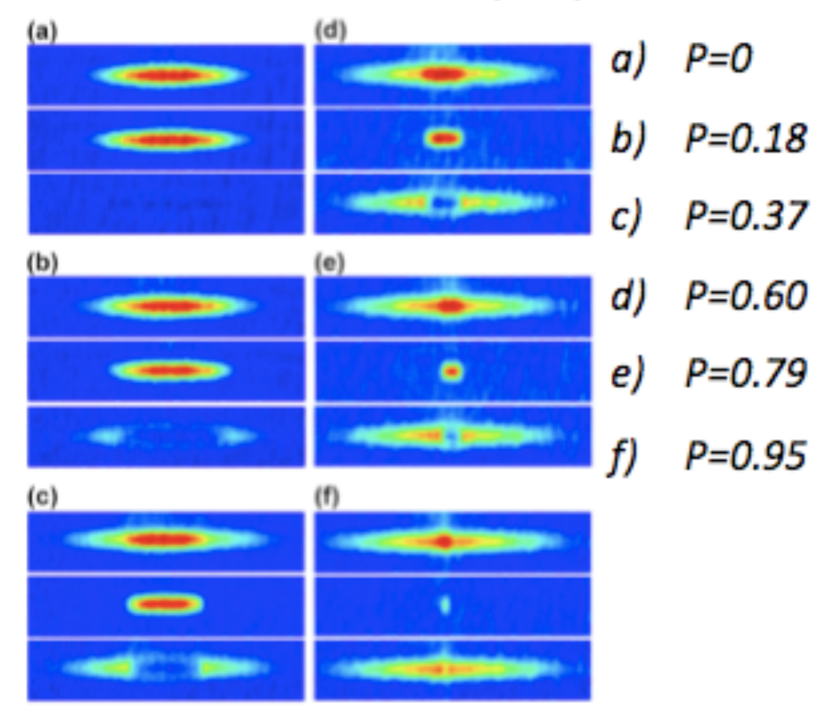


Asymmetric spin-polarized systems:
Rice Experiment

- **Experimental setup:** harmonic trap with cylindrical symmetry
  - $\omega_x = \omega_y = \alpha\omega, \omega_z = \omega;$ $\alpha \sim 35 - 45$
  - $N_{\rm tot} = N_\uparrow + N_\downarrow \lesssim 10^5$
- **Phase separation:**
  - superfluid core
  - very narrow (almost no) partially polarized region
  - Outer region of *normal* majority atoms
- **Critical polarization**: $P_c > 0.9$

(Partridge '06)



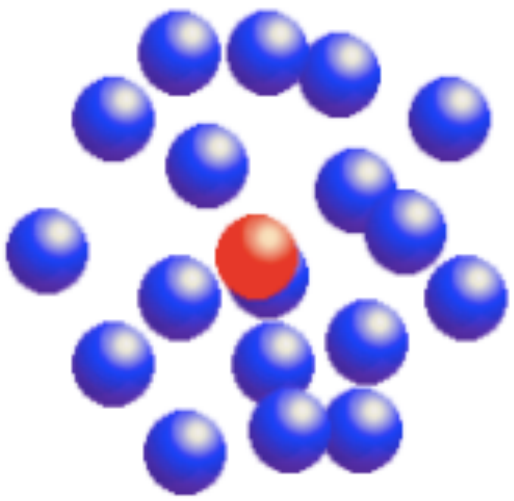


Summary of experimental differences

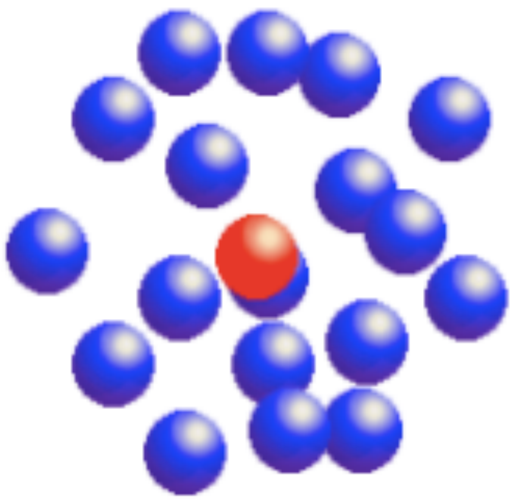
	MIT	**Rice university**
N_{tot}	$10^6 \ldots 10^7$	$\lesssim 10^5$
asymmetry α	~5	35..45
partially-polarized shell	clearly visible	extremely thin
critical polarization P_c	0.70(3)	> 0.9

Equation of state (*uniform system*)

- **single particle energy** $E_\downarrow$ **:**
  energy gain of one single minority fermion interacting resonantly with a majority Fermi sea
- systems with **large asymmetry**:
  leading contribution to EoS due to interactions, $E = E_\uparrow^{\mathrm{FS}} + E_\downarrow$



Single particle energy (*uniform system*)

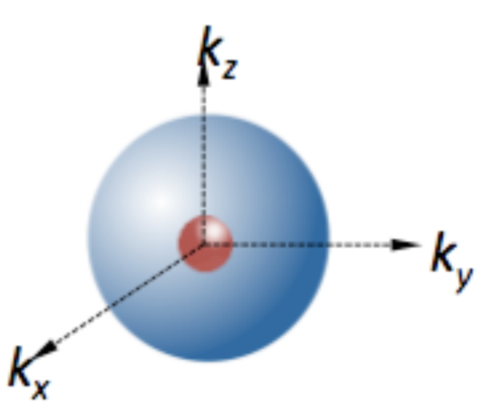


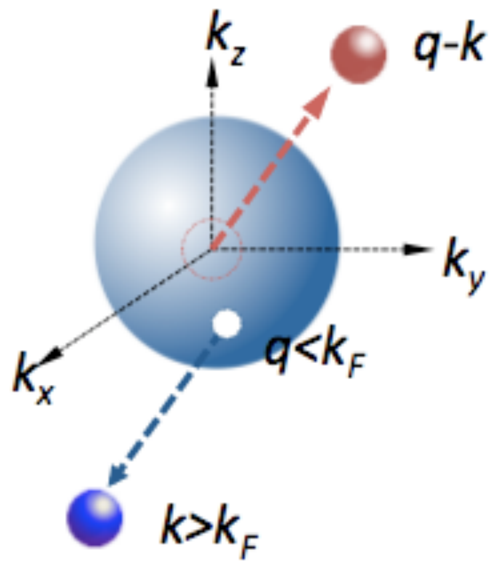
- **Hamiltonian:** (Chevy '06)

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}}\, a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',q} a^\dagger_{\mathbf{k}+\mathbf{q},\uparrow} a_{\mathbf{k}'-\mathbf{q},\downarrow} a^\dagger_{\mathbf{k}',\downarrow} a_{\mathbf{k},\uparrow}$$

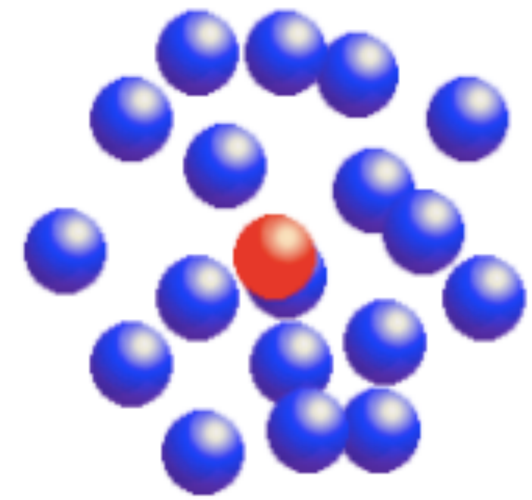
- variational ansatz (include 1p1h excitations):

$$|\psi\rangle \quad = \quad \phi_0 |\Omega\rangle \qquad + \qquad \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{k},\mathbf{q}} |\mathbf{k},\mathbf{q}\rangle$$



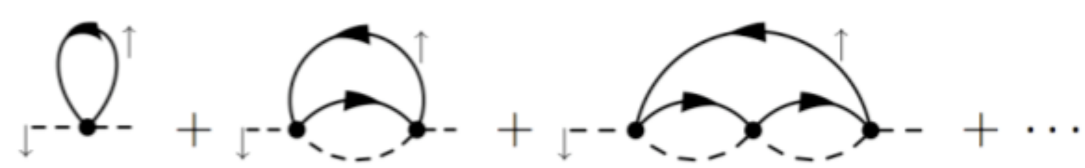


Single particle energy (*uniform system*)



- **gap equation:** (Chevy '06)

$$E_\downarrow = \frac{1}{V}\sum_{|\mathbf{q}|<k_F} \frac{1}{\frac{1}{g}+\frac{1}{V}\sum_{|\mathbf{k}|>k_F}\frac{1}{\epsilon_\mathbf{k}+\epsilon_{\mathbf{q}-\mathbf{k}}-\epsilon_\mathbf{q}-E_\downarrow}}$$

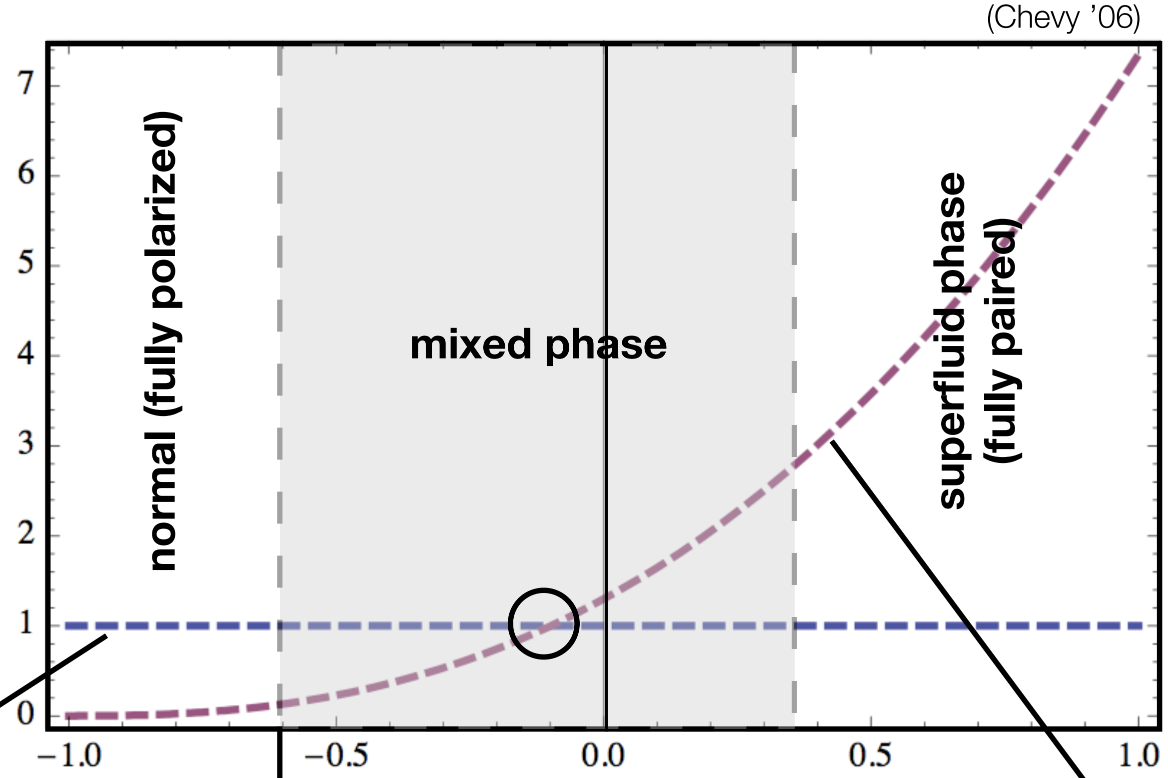
$$=$$ 

- at unitarity:
  - $\eta = E_\downarrow/E_F \approx -0.607$
  - agrees well with MC studies (see e. g. Lobo et al. '06)
  - 2p2h contributions shown to be small (Combescot & Giraud '08)

Phase diagram of an imbalanced system of spin-polarized atoms at unitarity (*uniform system)*

- consider a system of spin-up and spin-down fermion at unitarity with $N_\uparrow > N_\downarrow$
- phase diagram at T=0

pressure $\frac{P}{P_N}$



$y = \frac{\mu_\downarrow}{\mu_\uparrow}$

η

maximal stress for SF:

$\mu_\uparrow - \mu_\downarrow \leq 2\Delta$

$\Longrightarrow$ SF at unitarity:

$\frac{\mu_\downarrow}{\mu_\uparrow} \geq 0.09(3)$

(Bulgac & Forbes '07)

ideal Fermi gas

$$P_N = \frac{1}{15\pi^2}(2m)^{\frac{3}{2}}\mu_\uparrow^{\frac{5}{2}}$$

set by the energy gained when one spin-down fermion is added to the sea of spin-up fermions

superfluid phase

$$P_S = \left(\frac{P_N}{(\xi_S)^{\frac{3}{2}}}\right)(1+y)^{\frac{5}{2}}$$

Trapped system of imbalanced spin-polarized atoms at unitarity (T=0)

- energy density functional ($N_\uparrow \gg N_\downarrow$) in LDA: (see e. g. Recati et al. '08)

$$
\begin{aligned}
\Gamma[n_S, n_\uparrow, n_\downarrow] \;=\; & 2\int_{|\mathbf{r}|<R_S} d\mathbf{r}\Big\{\xi_S \frac{3}{5}\frac{(6\pi^2 n_S(\mathbf{r}))^{\frac{2}{3}}}{2m} + V(\mathbf{r}) - \mu_S\Big\} n_S(\mathbf{r}) \\
& + \int_{R_S<|\mathbf{r}|<R_\uparrow} d\mathbf{r}\left\{\frac{3}{5}\frac{(6\pi^2 n_\uparrow)^{\frac{2}{3}}}{2m}\left(1-\frac{5}{3}\eta\left(\frac{n_\downarrow}{n_\uparrow}\right)+\frac{m}{m^*}\left(\frac{n_\downarrow}{n_\uparrow}\right)^{5/3}+B\left(\frac{n_\downarrow}{n_\uparrow}\right)^2\right) n_\uparrow(\mathbf{r})\right. \\
& \left. + V(\mathbf{r})(n_\downarrow(\mathbf{r})+n_\uparrow(\mathbf{r})) - \mu_\uparrow n_\uparrow(\mathbf{r}) - \mu_\downarrow n_\downarrow(\mathbf{r})\right\}
\end{aligned}
$$

- ground state: $\frac{\delta\Gamma}{\delta n_S} = \frac{\delta\Gamma}{\delta n_\uparrow} = \frac{\delta\Gamma}{\delta n_\downarrow} = 0\,, \quad \frac{\delta\Gamma}{\delta R_S} = 0\,, \quad (\mu_\uparrow + \mu_\downarrow) = 2\mu_S$

- what we know from the *continuum*:
  - superfluid phase: $\epsilon_S = \xi_S \frac{(6\pi^2 n_S)^{\frac{2}{3}}}{2m}$
  - energy gained when a spin-down fermion is added to a Fermi sea of spin-up fermions: $E_\downarrow = \eta \frac{(6\pi^2 n_\uparrow)^{\frac{2}{3}}}{2m}$

Trapped system of imbalanced spin-polarized atoms at unitarity (T=0)

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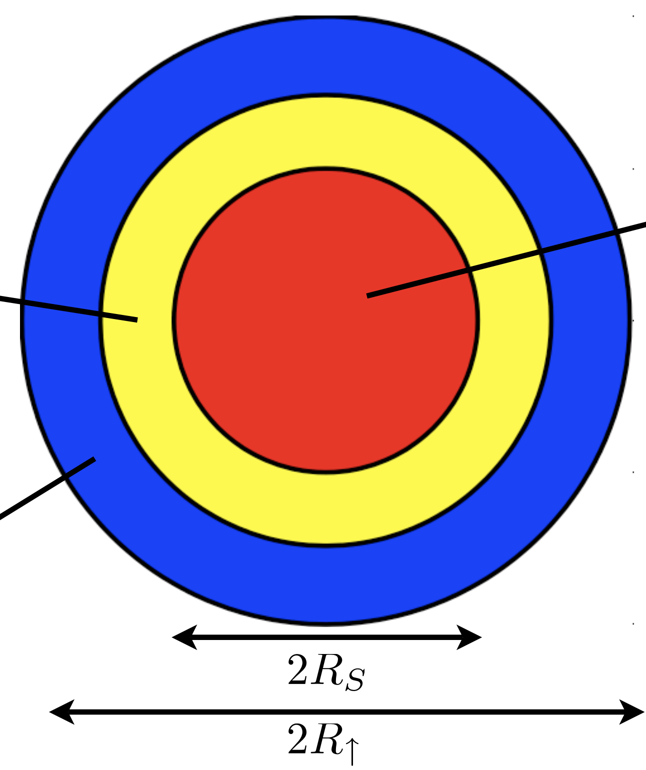
- phase diagram:

mixed phase of spin-up and spin-down fermions

$\eta \approx -0.6$ (Chevy '06)



superfluid phase: $\xi_S \approx 0.42$ (Carlson et al. '03)

normal phase: only spin-up fermions

critical polarization:

$P_c \approx 0.77$ (Recati et al. '08)

Trap effects & single particle energy

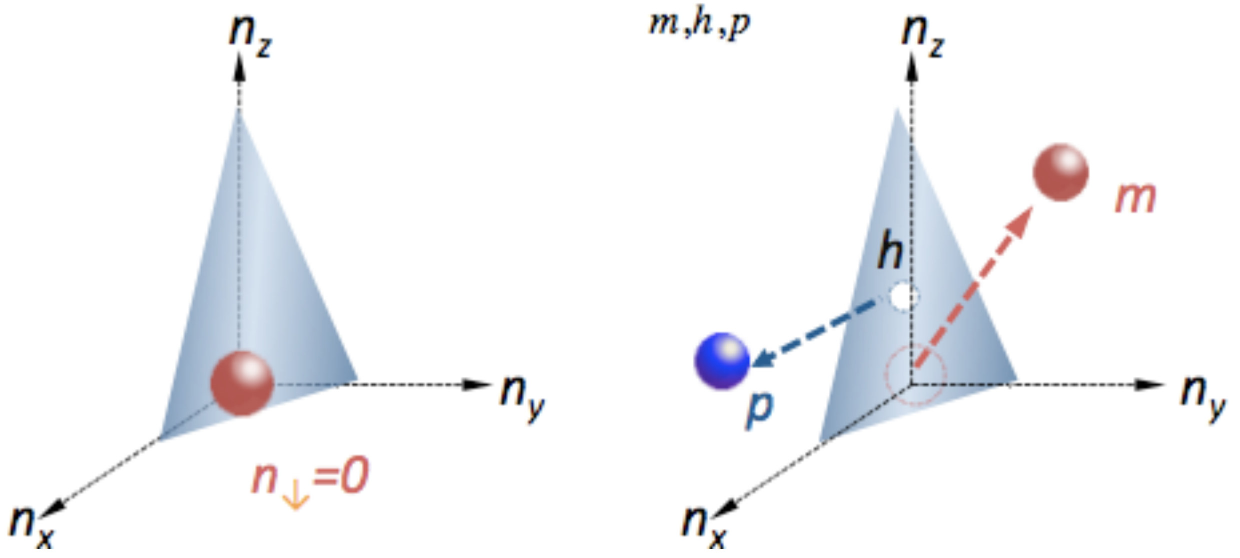
Ku, JB, Schwenk '08

- **revisited**: energy gain of a (trapped) Fermi sea of spin-up fermions when a spin-down fermion is added; Hamiltonian:

$$H = \sum_{\mathbf{n},\sigma} \varepsilon_{\mathbf{n}}\, a^\dagger_{\mathbf{n},\sigma}\, a_{\mathbf{n},\sigma} + \sum_{\mathbf{n}_\uparrow,\mathbf{n}_\downarrow,\mathbf{n}'_\uparrow,\mathbf{n}'_\downarrow} \langle \mathbf{n}'_\uparrow, \mathbf{n}'_\downarrow | V | \mathbf{n}_\uparrow, \mathbf{n}_\downarrow \rangle\, a^\dagger_{\mathbf{n}'_\uparrow,\uparrow}\, a^\dagger_{\mathbf{n}'_\downarrow,\downarrow}\, a_{\mathbf{n}_\downarrow,\downarrow}\, a_{\mathbf{n}_\uparrow,\uparrow}$$

- variational ansatz (include 1p1h excitations):

$$|\psi\rangle = \phi_0 |\Omega\rangle + \sum_{m,h,p} \phi_{m,h,p} |m,h,p\rangle$$



Trap effects & single particle energy

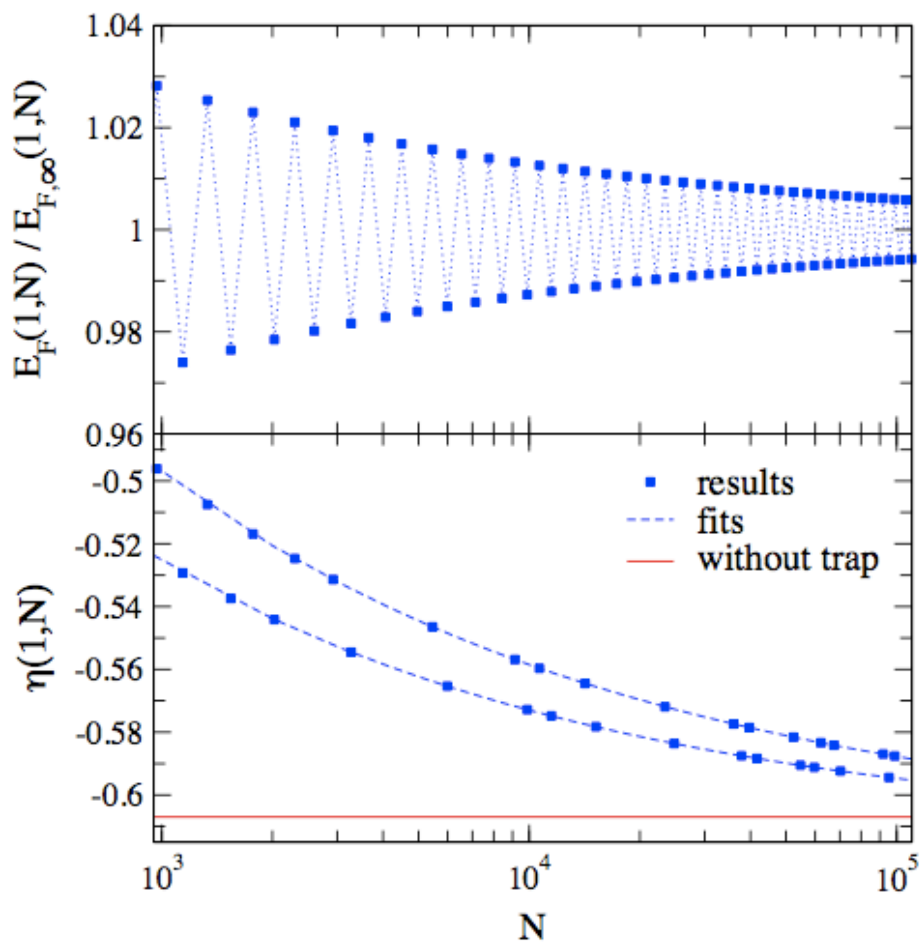
Ku, JB, Schwenk '08

- self-consistent equation for energy gain:

$$E_\downarrow - \varepsilon_{\mathbf{0}} = \sum_{\varepsilon_{\mathbf{h}} \leqslant \varepsilon_F} \sum_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{S}) \left[M^{-1}(\varepsilon_F, E_\downarrow + \varepsilon_{\mathbf{h}})\right]_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{L}) \, ,$$

- choose: $E_\downarrow = \eta(\alpha, N) E_F(\alpha, N)$ with $E_F(\alpha, N) = \dfrac{(6\pi^2 n_\uparrow(0))^{\frac{2}{3}}}{2m} \xrightarrow{N \gg 1} \omega(48N)^{\frac{1}{3}}$

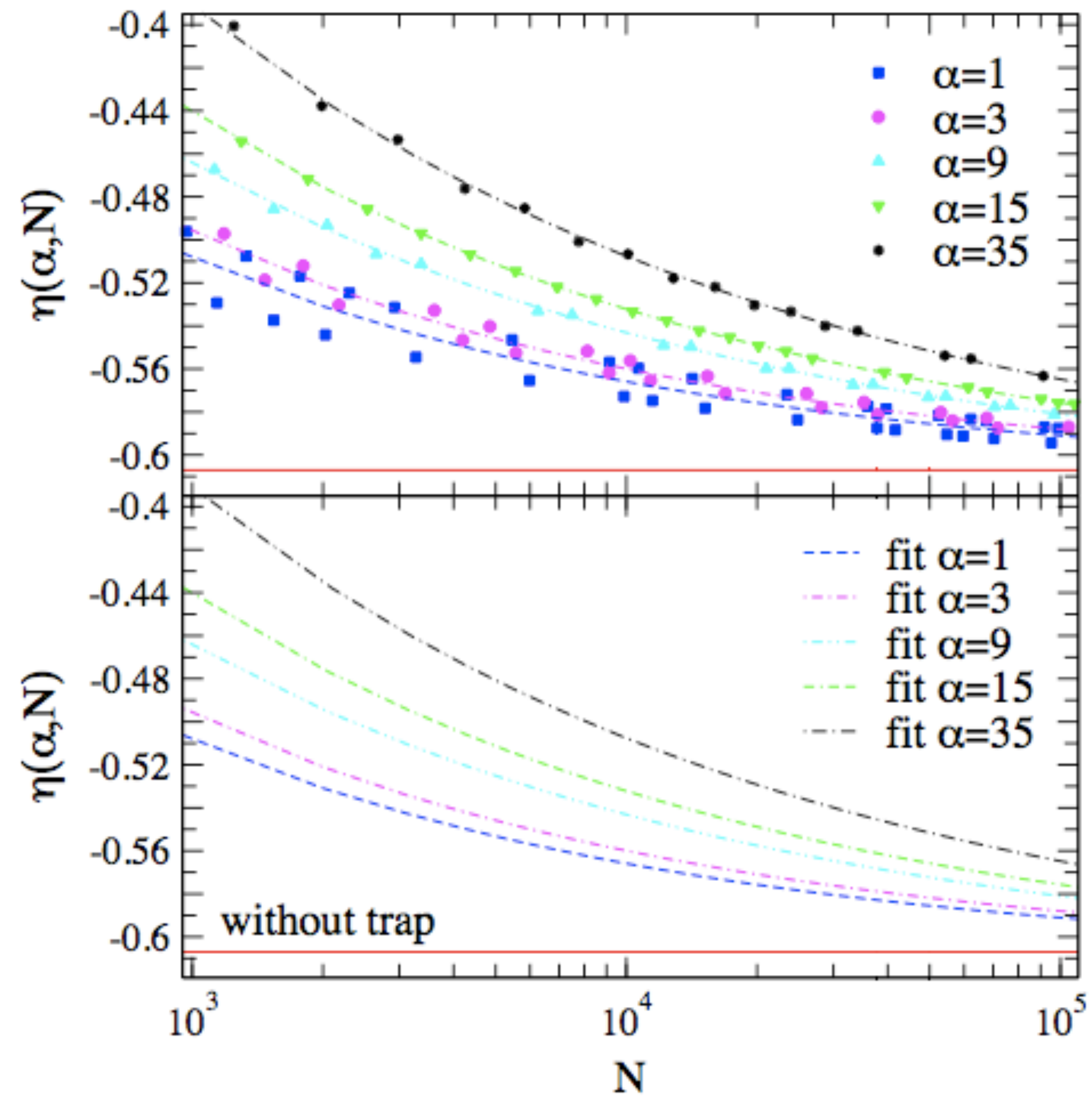
- results for the isotropic trap:



Trap effects & single particle energy

Ku, JB, Schwenk '08

- Clear finite-size and confinement effects
- fixed α, we find that the energy ...
  - ... decreases with $N_{\rm tot}$
  - ... stronger dependence on $N_{\rm tot}$ for larger α
  - ... saturates to -0.61 for large $N_{\rm tot}$
- fixed $N_{\rm tot}$, we find that the energy ...
  - ... inreases with α



Critical polarization & trap dependence

Ku, JB, Schwenk '08

- How does the critical polarization P_c depend on the trap configuration?
- energy density functional ($N_\uparrow \gg N_\downarrow$) in LDA:

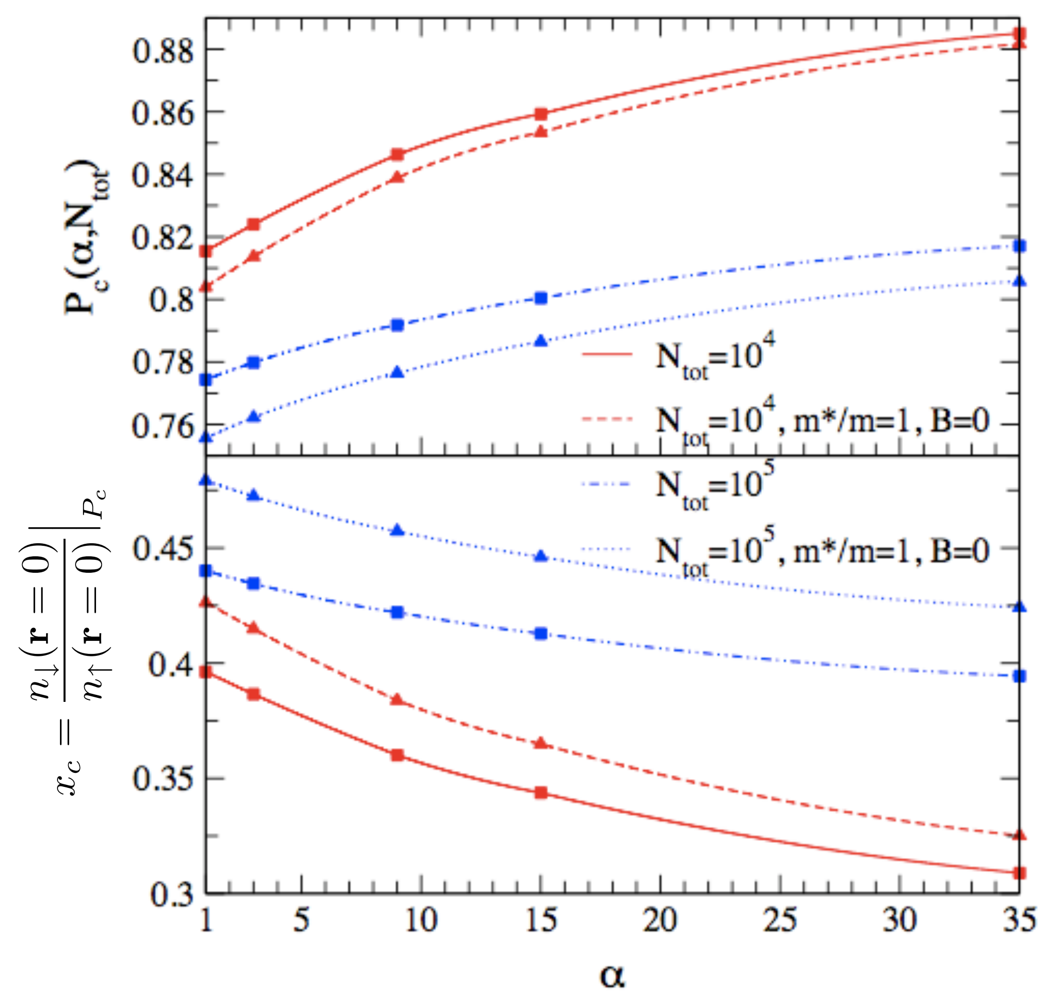
$$\begin{aligned}\Gamma[n_S, n_\uparrow, n_\downarrow] &= 2\int_{|\mathbf{r}|<R_S} d\mathbf{r}\Big\{\xi_S \frac{3}{5}\frac{(6\pi^2 n_S(\mathbf{r}))^{\frac{2}{3}}}{2m} + V(\mathbf{r}) - \mu_S\Big\} n_S(\mathbf{r}) \\ &+ \int_{R_S<|\mathbf{r}|<R_\uparrow} d\mathbf{r}\left\{\frac{3}{5}\frac{(6\pi^2 n_\uparrow)^{\frac{2}{3}}}{2m}\left(1-\frac{5}{3}\eta\left(\frac{n_\downarrow}{n_\uparrow}\right)+\frac{m}{m^*}\left(\frac{n_\downarrow}{n_\uparrow}\right)^{5/3}+B\left(\frac{n_\downarrow}{n_\uparrow}\right)^2\right)n_\uparrow(\mathbf{r})\right. \\ &\left. + V(\mathbf{r})(n_\downarrow(\mathbf{r})+n_\uparrow(\mathbf{r})) - \mu_\uparrow n_\uparrow(\mathbf{r}) - \mu_\downarrow n_\downarrow(\mathbf{r})\right\}\end{aligned}$$

- ground state: $\dfrac{\delta\Gamma}{\delta n_S} = \dfrac{\delta\Gamma}{\delta n_\uparrow} = \dfrac{\delta\Gamma}{\delta n_\downarrow} = 0\,, \quad \dfrac{\delta\Gamma}{\delta R_S} = 0\,, \quad (\mu_\uparrow + \mu_\downarrow) = 2\mu_S$
- for simplicity, calculations are done for
  (I) MC values: $\dfrac{m^*}{m} = 1.09, B = 0.14$ and $\xi_S \approx 0.4$ (see Pilati & Giorgini '08, Carlson '03)
  (II) $\dfrac{m^*}{m} = 1, B = 0$

Critical polarization & trap dependence

Ku, JB, Schwenk '08

- How does the critical polarization P_c depend on the trap configuration?

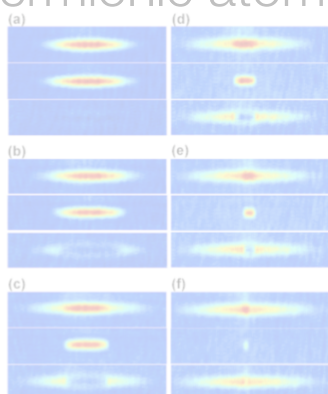


→ clear indications for strong trap dependence which helps to understand the different findings at MIT and Rice U.

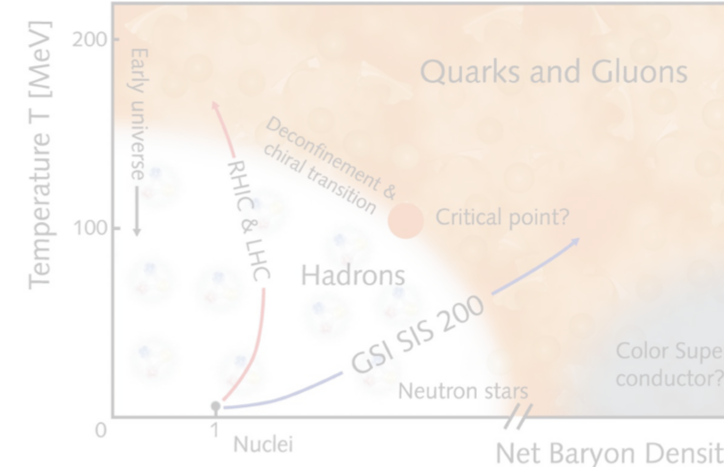
→ however, our calculations need still to be improved: dependence of effective mass on trap, terms beyond LDA, ...

Outline

trapped ultracold
fermionic atoms



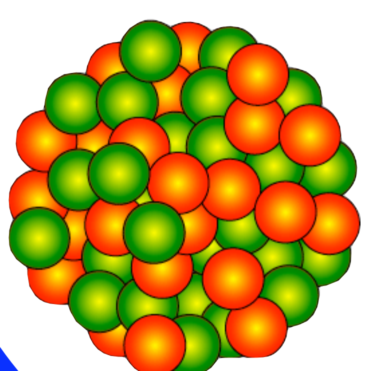
FAIR, www.gsi.de

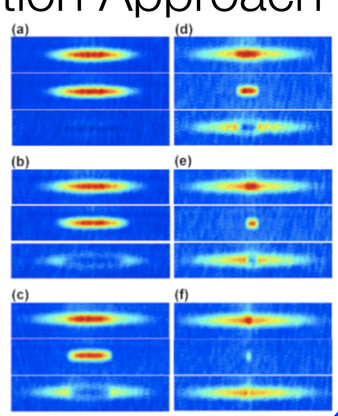


effective action:

$$\Gamma[\bar{\psi}, \psi, \phi, ...]$$

Density Functional Theory &
RG Flow Equation Approach

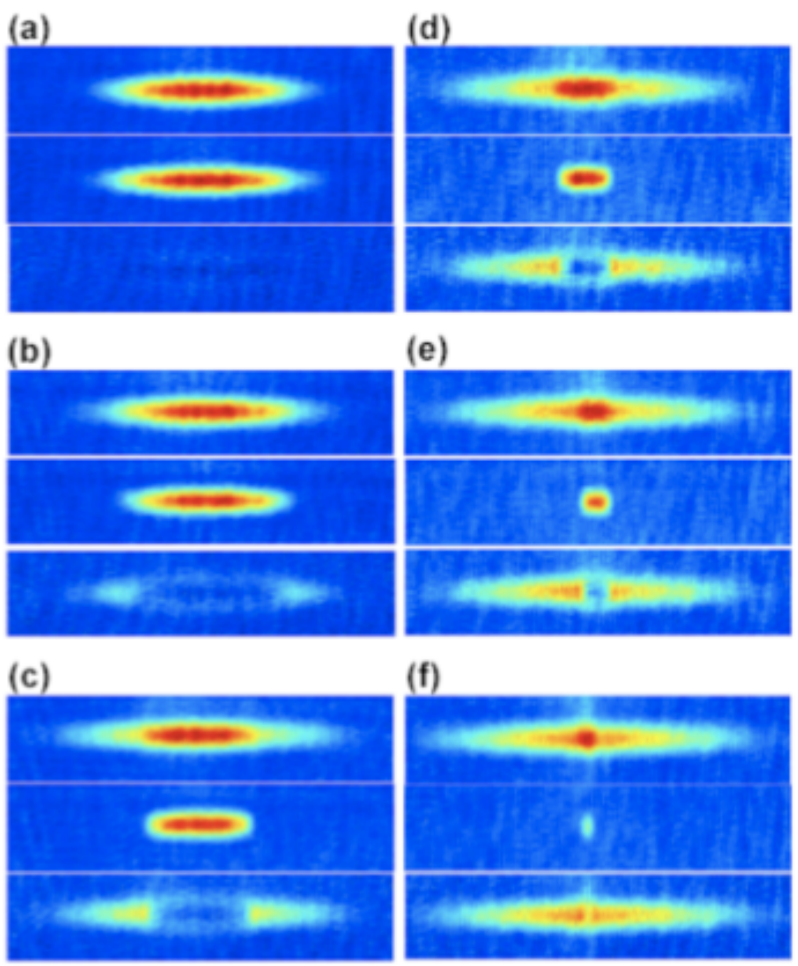




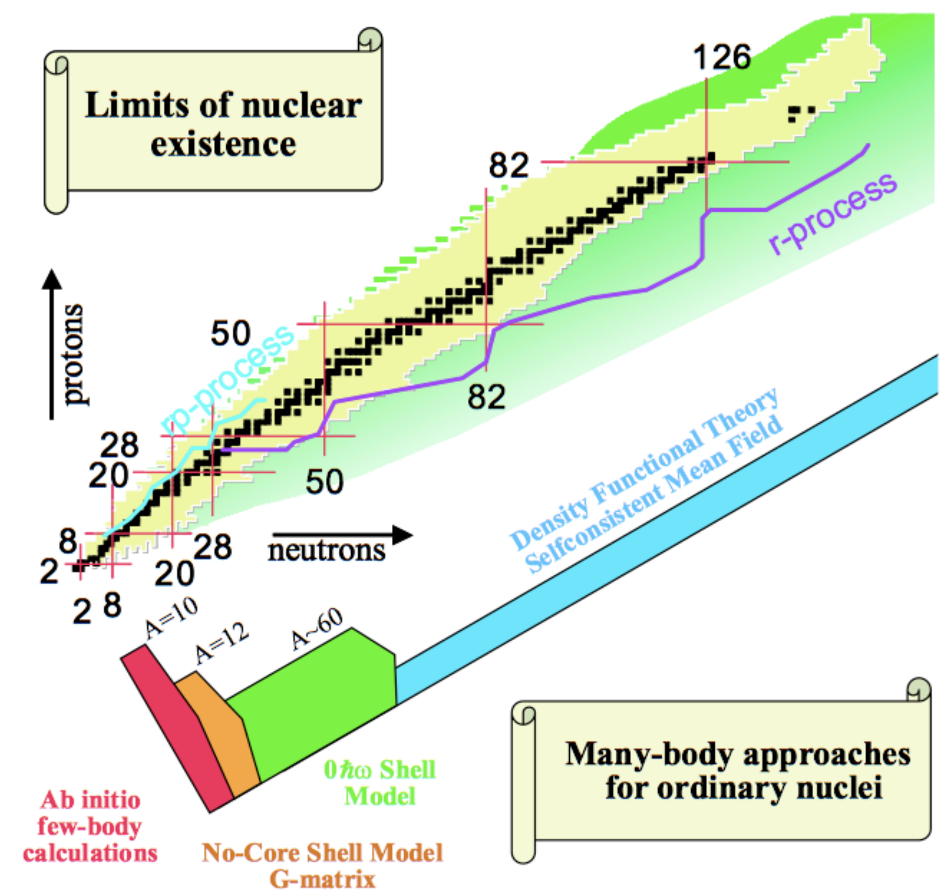
...

Ground-state properties of strongly-interacting many-body problems

Trapped Fermions



Nuclear Landscape



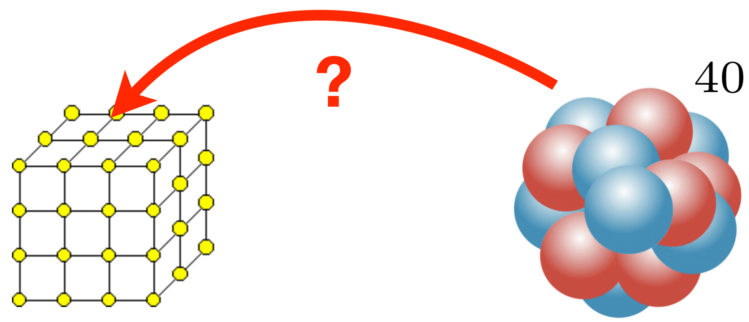
ground-state properties for heavy nuclei from microscopic interactions?

How to study ground-state properties from first principles?

- MC simulations?

How to study ground-state properties from first principles?

- MC simulations
  - “volume” problem:



$^{40}\mathrm{Ca}$

$r \sim 4.8\,\mathrm{fm}$

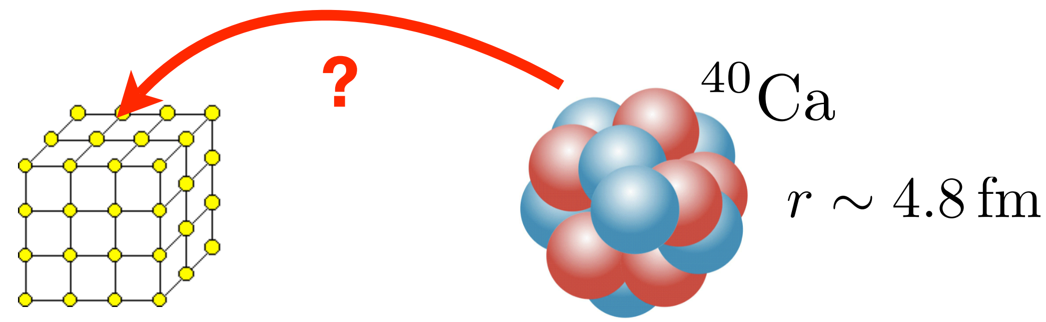
  - “construct” nucleus out of quarks: computationally (almost) impossible

$\Longrightarrow$ restricted to small nuclei (A<8?) (Savage ’06)

- Hamiltonian approaches, such as coupled-cluster theory?

How to study ground-state properties from first principles?

- MC simulations
  - "volume" problem:

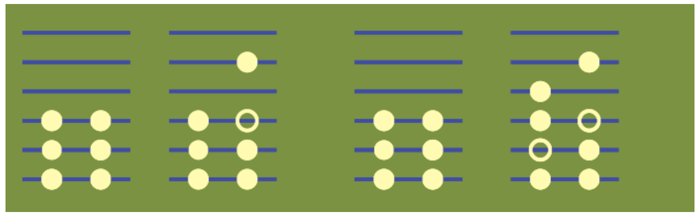


  - "construct" nucleus out of quarks: computationally (almost) impossible

  restricted to small nuclei (A<8?) (Savage '06)

- Hamiltonian approaches, such as coupled-cluster theory:
  - rewrite ground-state wave-function: $|\psi\rangle = \mathrm{e}^{(T_1+T_2+\dots)}|\phi\rangle$

  allows to include systematically n-particle-n-hole excitations of a given (reference) state



  - still computationally expensive
  - sets benchmarks for closed-shell medium-mass nuclei (Hagen et al. '07)

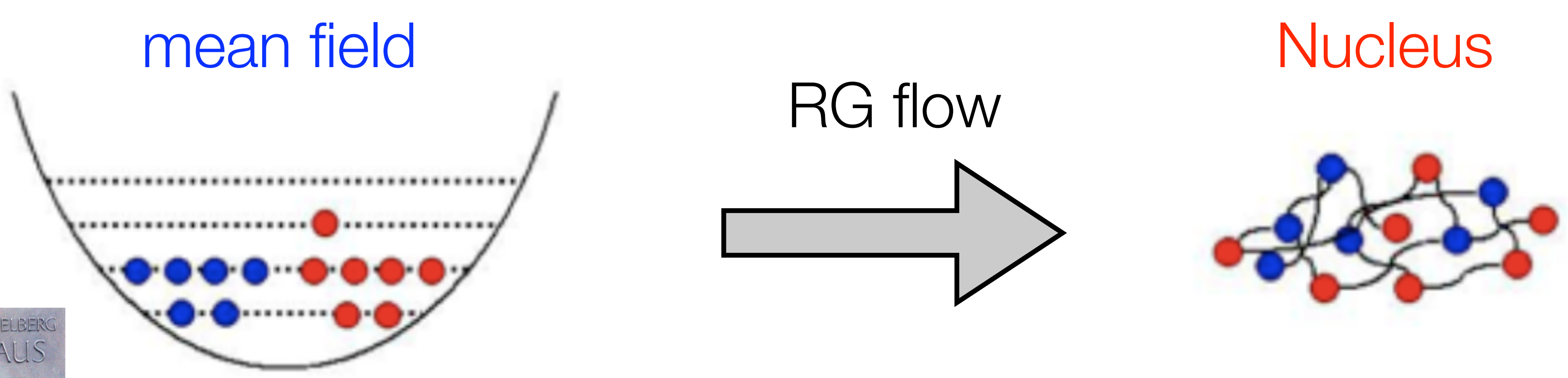
- Density Functional Theory?

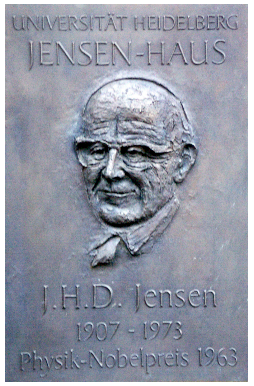
Density Functional RG for Fermionic Systems

- Density Functional: $\Gamma[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S[\psi^\dagger,\psi] + \int \frac{\delta\Gamma}{\delta\rho}\cdot(\psi^\dagger\psi)}$

  with $S[\psi^\dagger,\psi] = \int \psi^\dagger \left[\partial_t - \frac{1}{2m}\Delta\right]\psi + \frac{1}{2}\int \psi^\dagger\psi V_{2b}\psi^\dagger\psi + \mathcal{O}(V_{3b})$

- Idea:





start from mean-field (background potential)
and include many-body correlations

Density Functional RG for Fermionic Systems

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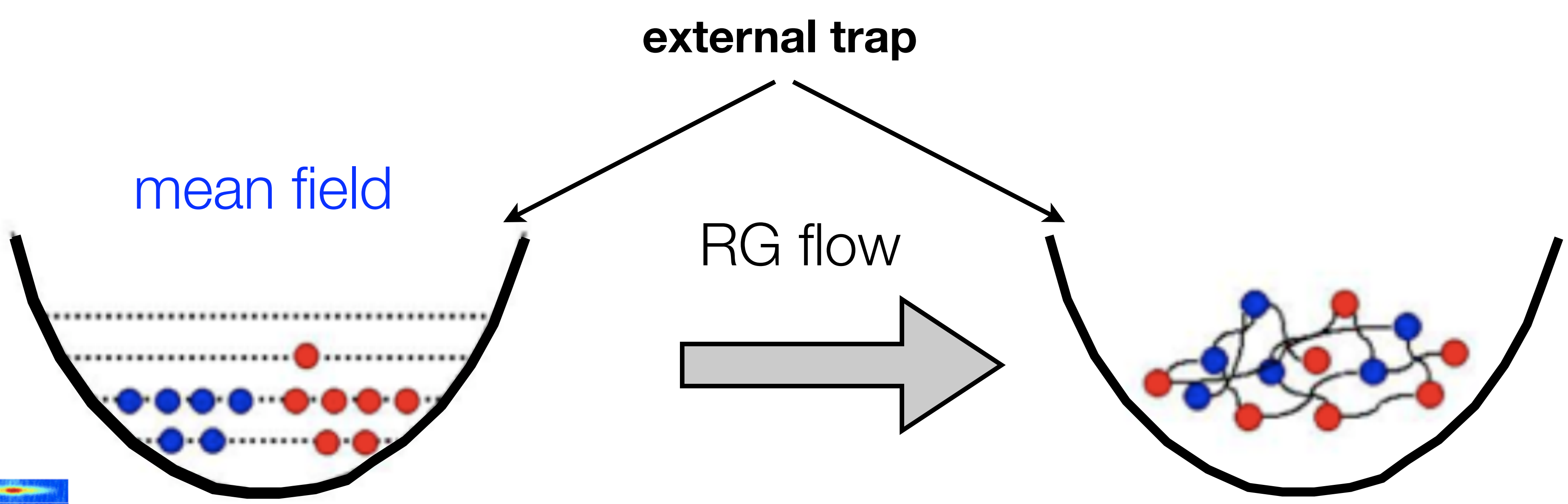
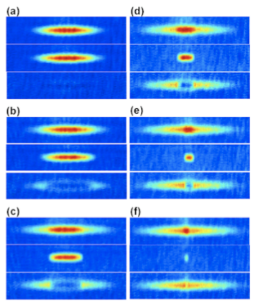
- Idea:





start from mean-field (background potential)
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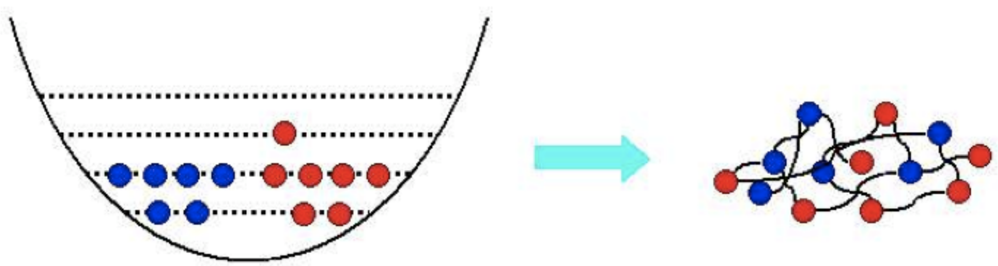
Density Functional RG for Fermionic Systems

- Density Functional: $\Gamma_\lambda[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S_\lambda[\psi^\dagger,\psi] + \int \frac{\delta \Gamma_\lambda}{\delta\rho}\cdot(\psi^\dagger\psi)}$

with $S_\lambda[\psi^\dagger,\psi] = \int \psi^\dagger \left[\partial_t - \frac{1}{2m}\Delta + (1-\lambda)U_\lambda\right]\psi + \frac{1}{2}\int \psi^\dagger\psi\lambda V_{2\mathrm{b}}\psi^\dagger\psi + \lambda\mathcal{O}(V_{3\mathrm{b}})$

external potential, e. g. trap potential

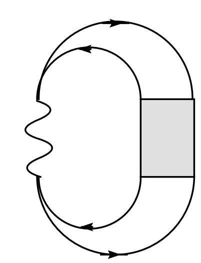
- DFT-RG flow:



$\lambda = 0$ $\lambda = 1$

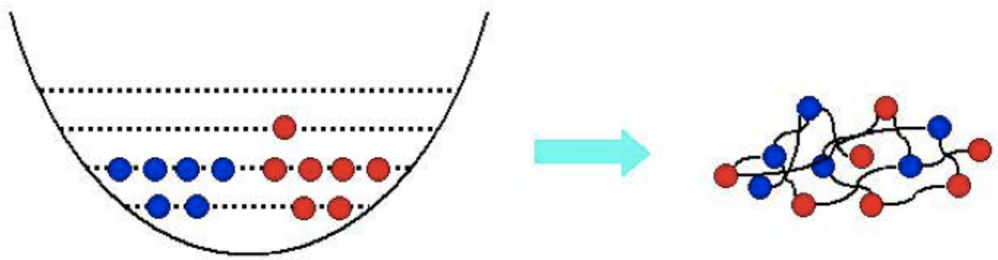
start from mean-field (background potential U_λ) and include many-body correlations

$$\partial_\lambda \Gamma_\lambda[\rho] = \frac{1}{2} V_{2\mathrm{b}}$$



Density Functional RG for Fermionic Systems

(JB, Schwenk, Polonyi, in prep.)



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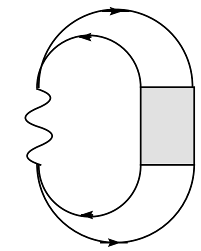
start from mean-field (background potential U_λ) and include many-body correlations

$$\partial_\lambda \Gamma_\lambda[\rho] = \frac{1}{2}\, V_{2b}$$ 

- density basis expansion scales favorably to heavy nuclei
- allows for a calculation of ground-state (g.s.) properties from microscopic interactions
- **Currently:** application to trapped systems of ultracold fermions in 1+1d, compare to Green's Function MC (Casula, Ceperley, Mueller '08)
- **For Nuclei:** validate results for medium-mass nuclei against coupled-cluster calculations

Conclusions

- energy gain in N+1 body problem depends on trap geometry and total particle number

- sensible finite-size and confinement effects in experiments with imbalanced systems of trapped spin-polarized **ultracold atoms, contributes to understand experimental discrepancies, e. g. critical polarization**

- **Density Functional Theory + FRG**: promising tool for first-principle description of many-body systems, e. g. cold atoms in traps or nuclei

Outlook

- go beyond the N+1-body problem: N+M-body problem (Ku, JB, Schwenk)
- **cold atoms**: trap effects on the finite-T phase diagram?
- computation of full energy density functional with MC?! (with M. M. Forbes)
- **DFT-RG**: large(r) 1+1d systems and comparison to GFMC

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- **finite-temperature QCD**: A. Eichhorn (Jena), H. Gies (Jena), Florian Marhauser (Heidelberg U.), J. Pawlowski (Heidelberg U.), Lisa Speyer (Heidelberg U.)

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- **(nuclear) many-body physics**: R. Furnstahl (Ohio State U.), **J. Polonyi (Louis Pasteur U.), A. Schwenk (TRIUMF)**

Appendix I

- Strongly-interacting Fermi gas in a harmonic-oscillator trap:

$$H = \sum_{\mathbf{n},\sigma} \varepsilon_{\mathbf{n}}\, a^{\dagger}_{\mathbf{n},\sigma}\, a_{\mathbf{n},\sigma} + \sum_{\mathbf{n}_\uparrow,\mathbf{n}_\downarrow,\mathbf{n}'_\uparrow,\mathbf{n}'_\downarrow} \langle \mathbf{n}'_\uparrow, \mathbf{n}'_\downarrow | V | \mathbf{n}_\uparrow, \mathbf{n}_\downarrow \rangle\, a^{\dagger}_{\mathbf{n}'_\uparrow,\uparrow}\, a^{\dagger}_{\mathbf{n}'_\downarrow,\downarrow}\, a_{\mathbf{n}_\downarrow,\downarrow}\, a_{\mathbf{n}_\uparrow,\uparrow}$$

- Contact interaction with separable cutoff in momentum space:

$$\langle \mathbf{p} | V | \mathbf{p}' \rangle = C(\Lambda)\, e^{-(p^2+p'^2)/\Lambda^2} \text{ where } C(\Lambda) = \frac{4\pi/m}{\frac{1}{a_s} - \frac{\Lambda}{\sqrt{2\pi}}}$$

- Interaction matrix elements: can be expressed as a sum over separable functions (**S**: center of mass quantum numbers.)

$$\langle \mathbf{n}_1, \mathbf{n}_2 | V | \mathbf{n}_3, \mathbf{n}_4 \rangle = C(\Lambda) \sum_{\mathbf{S}} F(\mathbf{n}_1, \mathbf{n}_2, \mathbf{S}) F(\mathbf{n}_3, \mathbf{n}_4, \mathbf{S})$$

Appendix II

- Strongly-interacting Fermi gas in a harmonic-oscillator trap:

$$H = \sum_{\mathbf{n},\sigma} \varepsilon_{\mathbf{n}}\, a^\dagger_{\mathbf{n},\sigma}\, a_{\mathbf{n},\sigma} + \sum_{\mathbf{n}_\uparrow,\mathbf{n}_\downarrow,\mathbf{n}'_\uparrow,\mathbf{n}'_\downarrow} \langle \mathbf{n}'_\uparrow, \mathbf{n}'_\downarrow | V | \mathbf{n}_\uparrow, \mathbf{n}_\downarrow \rangle\, a^\dagger_{\mathbf{n}'_\uparrow,\uparrow}\, a^\dagger_{\mathbf{n}'_\downarrow,\downarrow}\, a_{\mathbf{n}_\downarrow,\downarrow}\, a_{\mathbf{n}_\uparrow,\uparrow}$$

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Appendix III

- Self-consistent equation for E in anistropic traps

$$E - \varepsilon_{\mathbf{0}} = \sum_{\varepsilon_{\mathbf{h}} \leqslant \varepsilon_{\mathrm{F}}} \sum_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{S}) \left[M^{-1}(\varepsilon_{\mathrm{F}}, E + \varepsilon_{\mathbf{h}})\right]_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{L})$$

Matrix *M*: $$M(\varepsilon_{\mathrm{F}}, E + \varepsilon_{\mathbf{h}})_{\mathbf{S},\mathbf{L}} = \left[\frac{1}{C(\Lambda)} - D(\alpha, \Delta\widetilde{E})\right] \delta_{\mathbf{S},\mathbf{L}} + \sum_{\varepsilon_{\mathbf{p}} \leqslant \varepsilon_{\mathrm{F}}} \sum_{\mathbf{m}} \frac{F(\mathbf{m},\mathbf{p},\mathbf{S}) F(\mathbf{m},\mathbf{p},\mathbf{L})}{E + \varepsilon_{\mathbf{h}} - (\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{m}})}$$

Where $\Delta\widetilde{E} = \alpha(S_x + S_y + 2) + S_z + 1 - (E + \varepsilon_{\mathbf{h}})/\omega$

Appendix III

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Matrix M:

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General α: D has series expression

- Divergent part cancels with the cutoff term in $1/C(\Lambda)$