
Infrared Propagators and Confinement: a Perspective from Lattice Simulations

Tereza Mendes

University of São Paulo & DESY-Zeuthen

Work in collaboration with Attilio Cucchieri

Summary

- Lattice studies of the **infrared behavior** of **gluon and ghost propagators** may offer a crucial test of **confinement scenarios** in Yang-Mills theories.
- However, **finite-volume effects** become an important issue as the infrared limit is approached.
- We study the case of **Landau gauge** and SU(2) gauge group, using data from the **largest lattices to date**.
- We propose **rigorous constraints** to gain control over the extrapolation to the infinite-volume limit. At the same time, we gain a **better understanding of the propagators** in terms of more **general quantities**.

Pathways to Confinement

- How does **linearly rising potential** (seen in **lattice QCD**) come about?
- Theories of quark confinement include:
dual superconductivity (electric flux tube connecting magnetic monopoles), **condensation of center vortices**, but also merons, calorons
- Proposal by Mandelstam (1979) linking linear potential to **infrared behavior of gluon propagator** as $1/p^4$
- **Gribov-Zwanziger** (similarly Kugo-Ojima) confinement scenario based on suppressed gluon propagator and **enhanced ghost propagator** in the infrared

Ghost-enhanced scenario natural in **Coulomb gauge**, where **classical** (non-Abelian) **Gauss's law** is written for **color-coulomb potential**. In momentum space, **IR divergence** of ghost propagator as $1/k^4$ leads to **linearly rising potential**.

Gribov's restriction beyond **quantization** using **Faddeev-Popov (FP) method**: take **minimal** gauge, i.e. FP operator has non-negative eigenvalues. **First Gribov horizon** approached in infinite-volume limit, implying ghost enhancement.

Note: in principle no obvious connection with Kugo-Ojima scenario; see **Kondo, arXiv:09044897**.

IR gluon propagator and confinement

- **Green's functions** carry all information of a QFT's physical and mathematical structure.
- **Gluon propagator** (two-point function) as **the most basic quantity of QCD**.
- Confinement given by behavior at large distances (small momenta) \Rightarrow **nonperturbative** study of **IR** gluon propagator.

Landau gluon propagator

$$\begin{aligned} D_{\mu\nu}^{ab}(p) &= \sum_x e^{-2i\pi k \cdot x} \langle A_\mu^a(x) A_\nu^b(0) \rangle \\ &= \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) \end{aligned}$$

IR ghost propagator and confinement

Ghost fields are introduced as one evaluates functional integrals by the **Faddeev-Popov method**, which restricts the space of configurations through a **gauge-fixing condition**. The ghosts are **unphysical particles**, since they correspond to anti-commuting fields with spin zero.

On the lattice, the (minimal) **Landau gauge** is imposed as a **minimization problem** and the ghost propagator is given by

$$G(p) = \frac{1}{N_c^2 - 1} \sum_{x, y, a} \frac{e^{-2\pi i k \cdot (x-y)}}{V} \langle \mathcal{M}^{-1}(a, x; a, y) \rangle ,$$

where the Faddeev-Popov (FP) matrix \mathcal{M} is obtained from the **second variation of the minimizing functional**.

Gribov-Zwanziger Confinement Scenario

- The **Gribov-Zwanziger** confinement scenario in **Landau gauge** predicts a gluon propagator $D(p^2)$ suppressed in the IR limit.
- In particular, $D(0) = 0$ implying that **reflection positivity** is maximally violated.
- This result may be viewed as an indication of **gluon confinement**.
- Infinite volume favors configurations on the **first Gribov horizon**, where λ_{min} of \mathcal{M} goes to zero.
- In turn, $G(p)$ should be **IR enhanced**, introducing long-range effects, related to the color-confinement mechanism.

Tests of Gribov-Zwanziger Scenario

Above results are also obtained by **functional methods** (e.g. **solution of DSEs** by Alkofer et al.)

$$D(p^2) \sim (p^2)^{2\kappa-1}, \quad G(p^2) \sim (p^2)^{-\kappa-1}$$

Note: other solutions by Aguilar et al., Boucaud et al.

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- Gluon propagator is **suppressed** in the limit $p \rightarrow 0$
- On “small” lattices could fit to $D(0) \rightarrow 0$
- Studies (with “small” lattices) showed **enhancement of $G(p)$** .

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After 2007:

- From data on very large lattices one sees that $D(0) > 0$
- On very large lattices $G(p)$ shows **no enhancement** in the IR

References

Studies on very large lattices presented by three groups at the **Lattice 2007 Conference** (PoS Lat2007)

- Bogolubsky et al. (Berlin): 80^4 lattices (**13 fm**), SU(3)
- Sternbeck et al. (Adelaide): 112^4 lattices (**19 fm**), SU(2)
- Cucchieri, T.M.: 128^4 lattices (**27 fm**), SU(2) plus **3d** SU(2) case with 320^3 (**85 fm**)

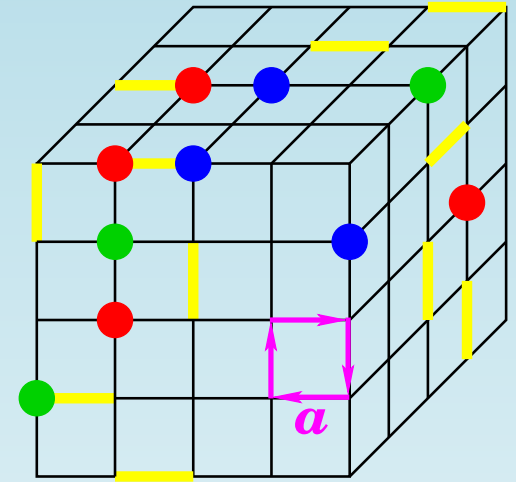
(possibly triggered by Fischer et al., Annals Phys. 2007)

Just before

- **Scaling** behavior seen on **2d lattice** (A. Maas, Phys. Rev. D 2007)
- SU(2) & SU(3) are **equivalent** in the **IR** (Cucchieri et al., Phys. Rev. D 2007)

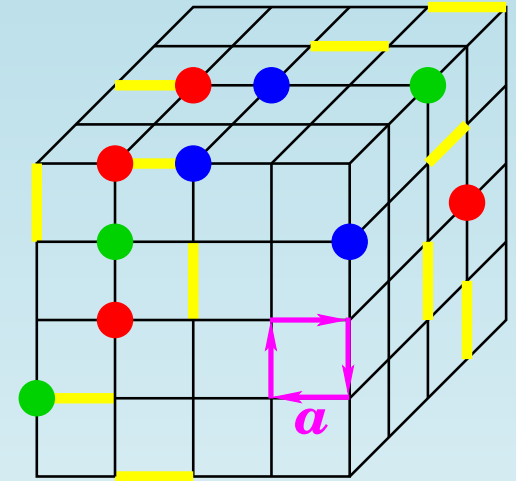
Lattice QCD

1. Quantization by **path integrals** \Rightarrow sum over configurations with “weights” $e^{iS/\hbar}$
2. **Euclidean formulation** (analytic continuation to **imaginary time**) \Rightarrow weight becomes $e^{-S/\hbar}$
3. **Discrete** space-time \Rightarrow UV cut at **momenta** $p \lesssim 1/a \Rightarrow$ **regularization**



Lattice QCD

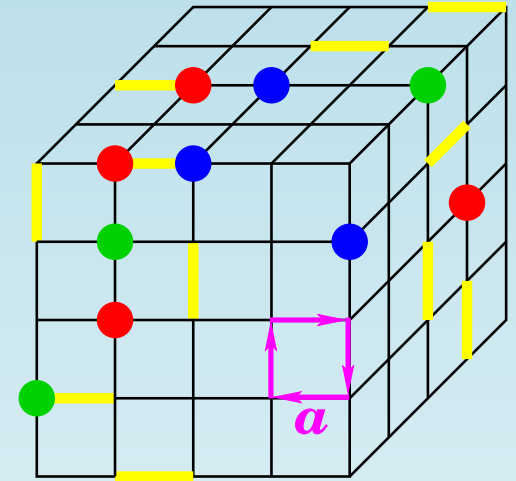
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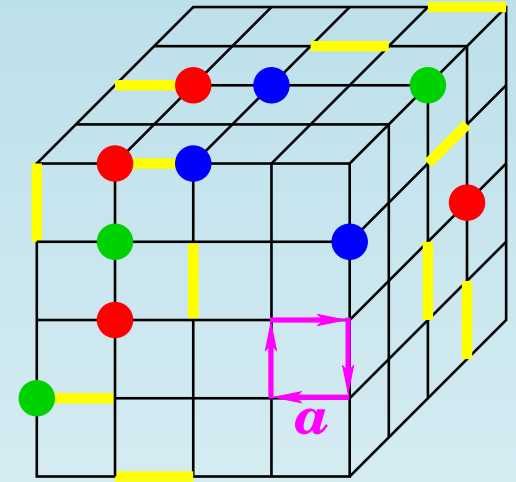
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The Wilson action

$$S = -\frac{\beta}{3} \sum_{\square} \text{ReTr} U_{\square}, \quad U_{x,\mu} \equiv e^{ig_0 a A_{\mu}^b(x) T_b}, \quad \beta = 6/g_0^2$$

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$V(R) \sim \sigma R$ from **small β** expansion

In general: **Monte Carlo simulations** + ∞ -vol limit + $a \rightarrow 0$

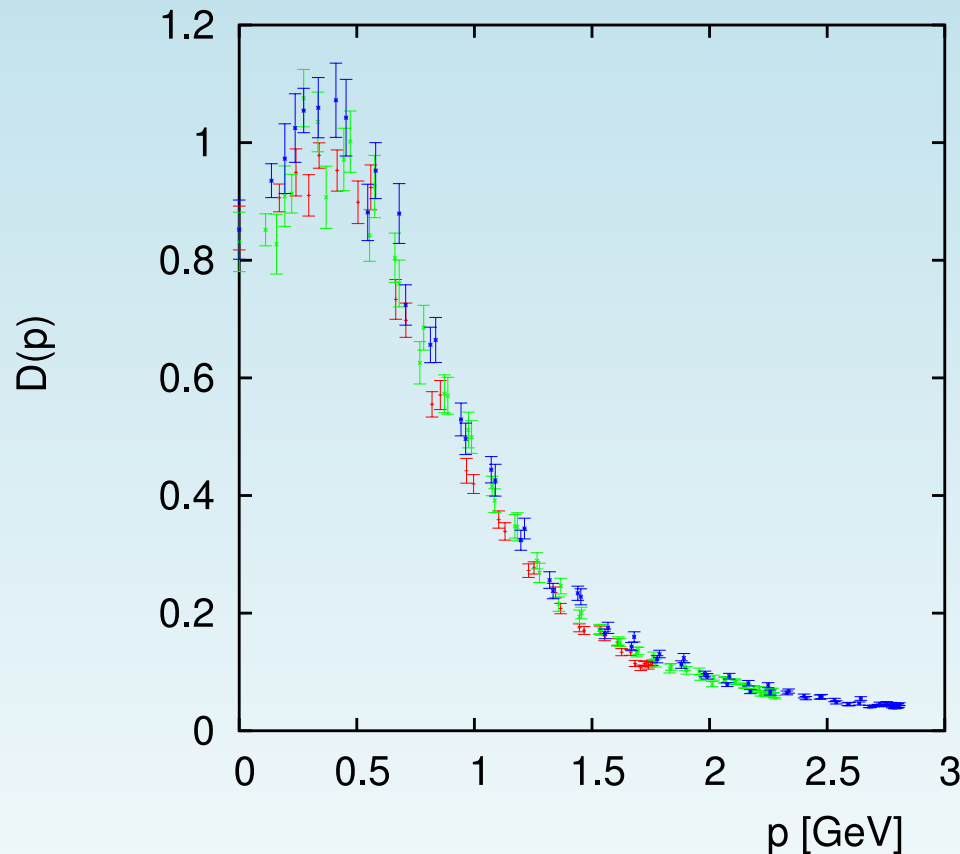
Lattice Features

- Gauge action written in terms of **oriented plaquettes** formed by the **link variables** $U_{x,\mu}$, which are group elements
- under gauge transformations: $U_{x,\mu} \rightarrow g(x) U_{x,\mu} g^\dagger(x + \mu)$, where $g \in SU(3) \Rightarrow$ closed loops are gauge-invariant quantities
- integration volume is finite: **no need for gauge-fixing**
- when gauge fixing, procedure is incorporated in the simulation, **no need to consider FP matrix**
- get FP matrix without considering **ghost fields** explicitly
- **Lattice momenta** given by $\hat{p}_\mu = 2 \sin(\pi n_\mu/N)$ with $n_\mu = 0, 1, \dots, N/2 \Leftrightarrow p_{min} \sim 2\pi/(aN) = 2\pi/L$,
 $p_{max} = 4/a$ in physical units

Bounds and Results for the Gluon Propagator

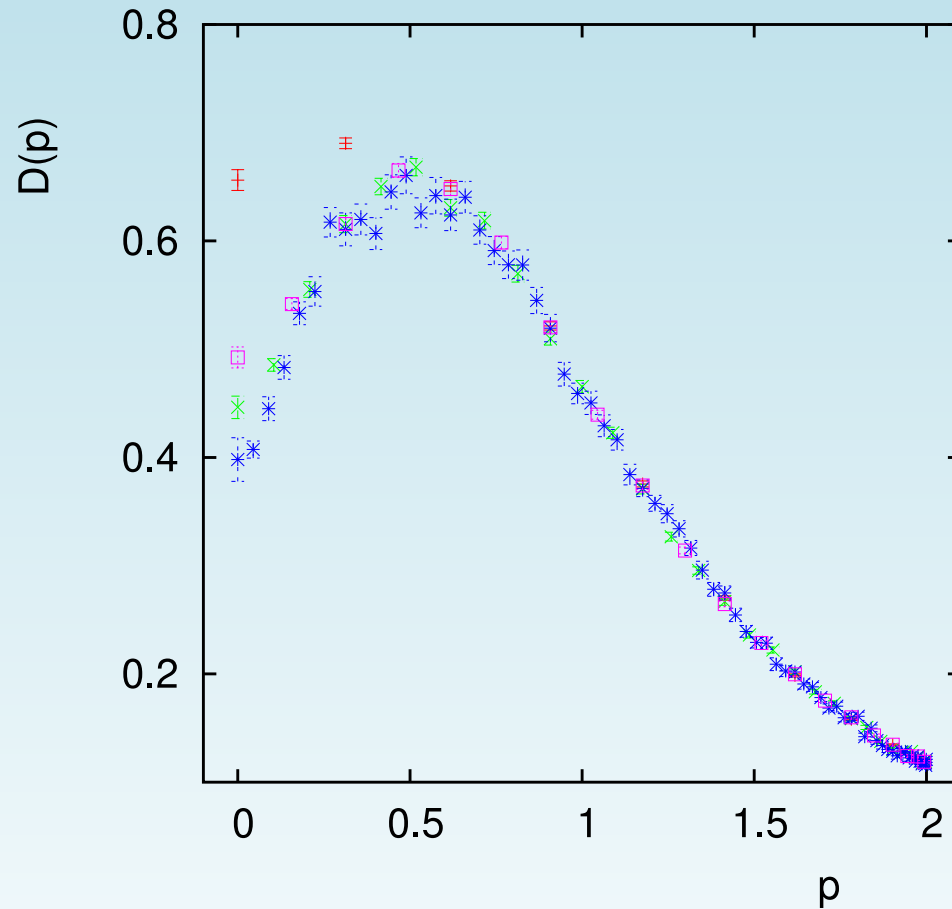
(A. Cucchieri, T.M., PoS LATTICE2007 and Phys. Rev. Lett. 2008)

Infinite-volume limit in 3d (I)



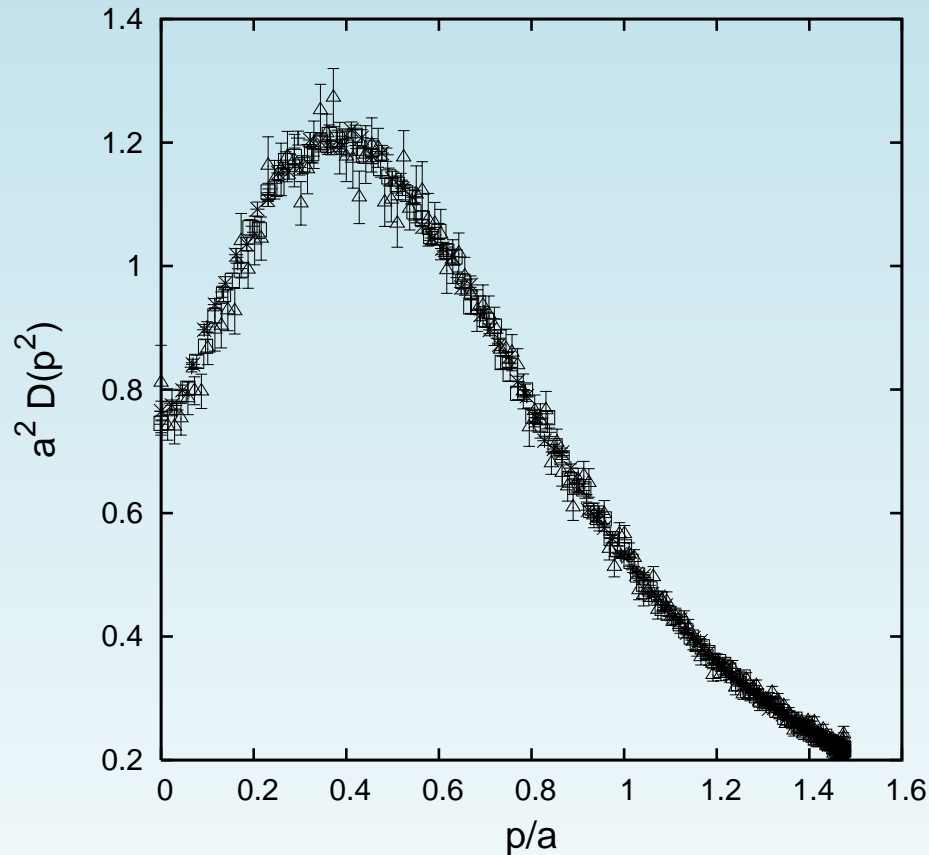
Gluon propagator as a function of the lattice momentum p for $\beta = 3.4$ and 32^3 (+), $\beta = 4.2$ and 64^3 (\times), $\beta = 5.0$ and 64^3 (*) (A. Cucchieri, Phys. Rev. D60 034508, 1999).

Infinite-volume limit in 3d (II)



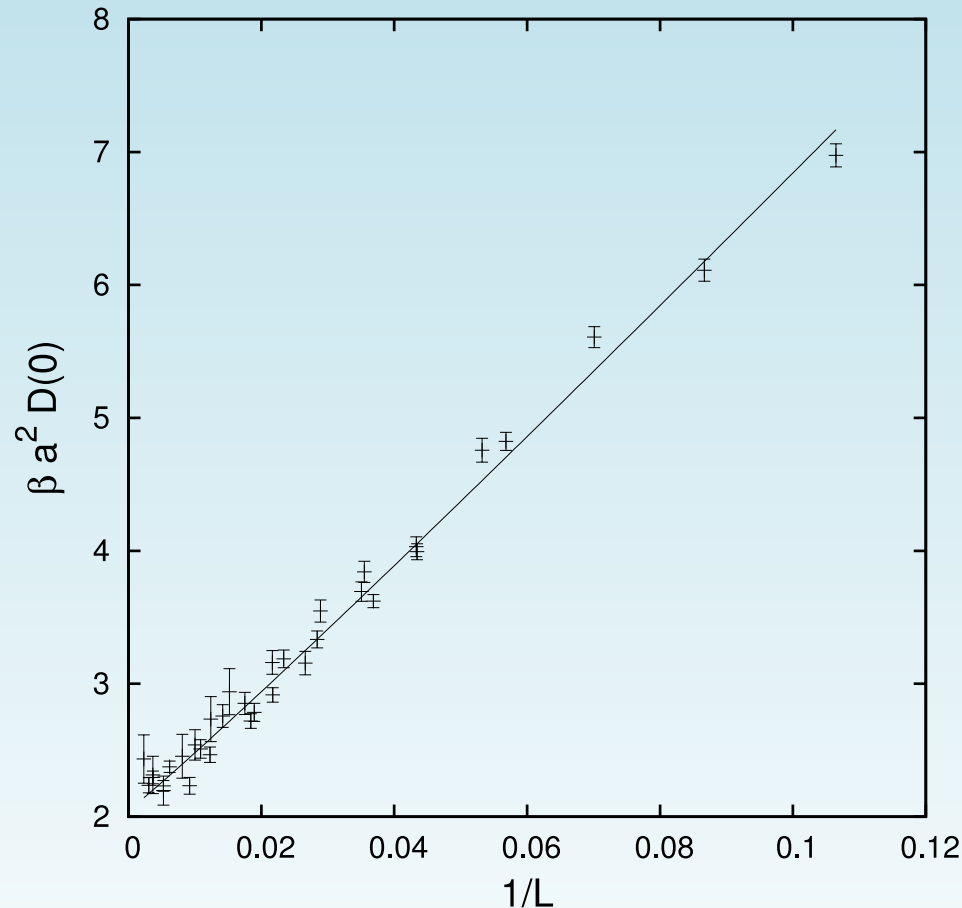
Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3 , 60^3 and 140^3 at $\beta = 3.0$ (A. Cucchieri, T. M. and A. Taurines, Phys. Rev. D67 091502, 2003).

Infinite-volume limit in 3d (III)



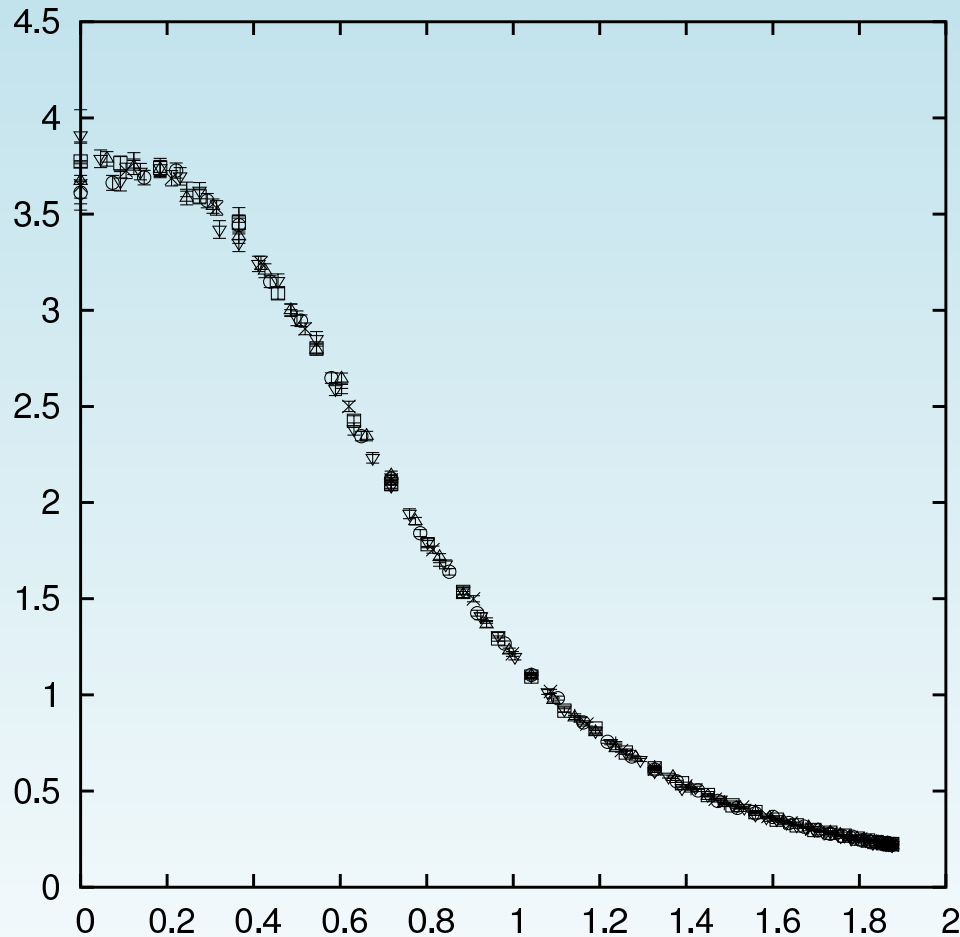
Gluon propagator as a function of the lattice momentum p including lattices up to 320^3 in the scaling region.

Infinite-volume limit in 3d: $D(0)$



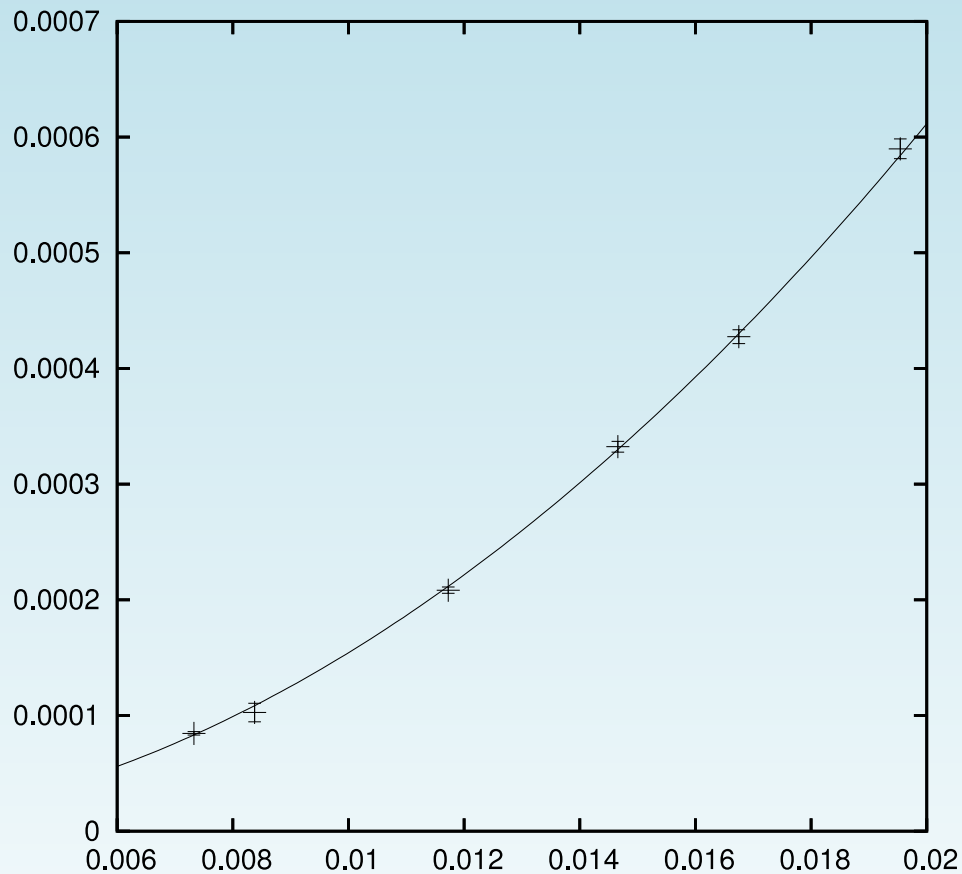
Gluon propagator at zero momentum as a function of the inverse lattice side $1/L$ (in fm^{-1}) and **extrapolation** to infinite volume. Data for lattice volumes up to 320^3 for $\beta = 3.0$.

Infinite-volume limit in 4d



Gluon propagator as a function of the lattice momentum p for lattice volume up to $V = 128^4$ at $\beta = 2.2$.

Extrapolation to infinite volume: a hint



Average absolute value of the gluon field at zero momentum $|\tilde{A}_\mu^b(0)|$ (for $\beta = 2.2$) as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. Recall that $D(0) \propto V \sum_{\mu,b} |\tilde{A}_\mu^b(0)|^2$. We also show the fit of the data using the Ansatz b/L^c (with $c = 1.99 \pm 0.02$).

Zwanziger proved that in Landau gauge this quantity should go to zero at least as fast as $1/L$.

Lower bound for $D(0)$

We can obtain a **lower bound** for the gluon propagator at zero momentum $D(0)$ by considering the quantity

$$\overline{M}(0) = \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} |\tilde{A}_\mu^b(0)|.$$

Consider the Schwarz inequality $|\vec{X} \cdot \vec{Y}|^2 \leq \|\vec{X}\|^2 \|\vec{Y}\|^2$, a vector \vec{Y} with all components equal to 1 and a vector \vec{X} with components X_i , we find

$$\left(\frac{1}{m} \sum_{i=1}^m X_i \right)^2 \leq \frac{1}{m} \sum_{i=1}^m X_i^2,$$

where m is the number of components of the vectors \vec{X} and \vec{Y} .

Lower bound for $D(0)$ (II)

We can now apply this inequality first to the vector with $m = d(N_c^2 - 1)$ components $\langle |\tilde{A}_\mu^b(0)| \rangle$, where

$$\tilde{A}_\mu^b(0) = \frac{1}{V} \sum_x A_\mu^b(x)$$

is the gluon field at zero momentum. This yields

$$\langle \bar{M}(0) \rangle^2 \leq \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |\tilde{A}_\mu^b(0)| \rangle^2 .$$

Then, we can apply the same inequality to the **Monte Carlo estimate** of the average value

$$\langle |\tilde{A}_\mu^b(0)| \rangle = \frac{1}{n} \sum_c |\tilde{A}_{\mu,c}^b(0)| ,$$

where n is the number of configurations. In this case we obtain

$$\langle |\tilde{A}_\mu^b(0)| \rangle^2 \leq \langle |\tilde{A}_\mu^b(0)|^2 \rangle .$$

Lower bound for $D(0)$ (III)

Thus, by recalling that

$$D(0) = \frac{V}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |\tilde{A}_\mu^b(0)|^2 \rangle ,$$

we find

$$\left[V^{1/2} \langle \overline{M}(0) \rangle \right]^2 \leq D(0) .$$

We obtain that $\langle \overline{M}(0) \rangle$ goes to zero exactly as $1/V^{1/2}$!

This gives

- $D(0) \geq 0.5(1) \text{ (GeV}^{-2}\text{)}$ in 3d
- $D(0) \geq 2.5(3) \text{ (GeV}^{-2}\text{)}$ in 4d

Upper bound for $D(0)$

We can now consider the inequality

$$\langle \sum_{\mu,b} |\tilde{A}_{\mu}^b(0)|^2 \rangle \leq \langle \left\{ \sum_{\mu,b} |\tilde{A}_{\mu}^b(0)| \right\}^2 \rangle .$$

This implies

$$D(0) \leq Vd(N_c^2 - 1) \langle \overline{M}(0)^2 \rangle .$$

Thus

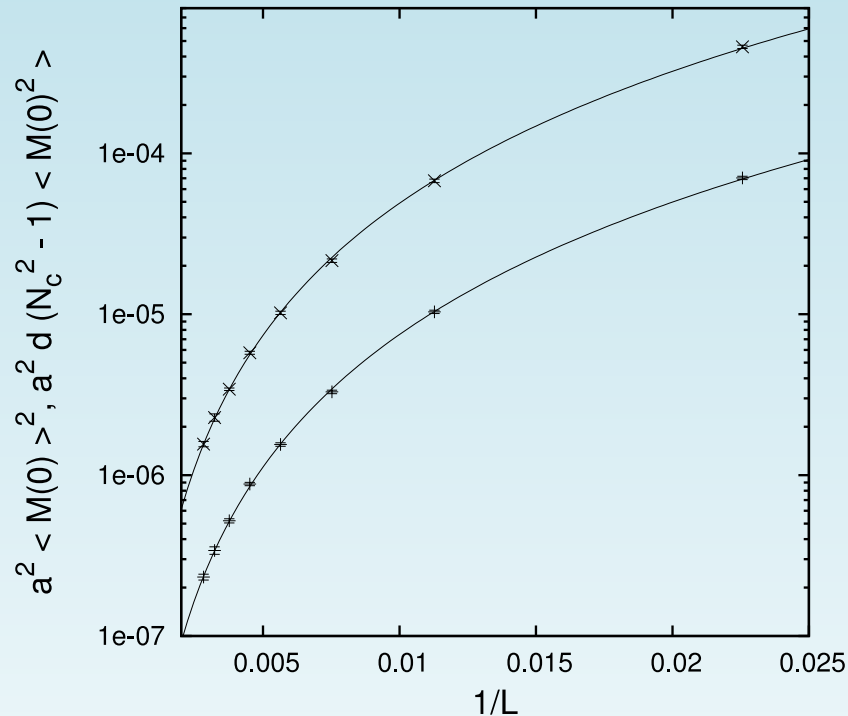
$$V \langle \overline{M}(0) \rangle^2 \leq D(0) \leq Vd(N_c^2 - 1) \langle \overline{M}(0)^2 \rangle .$$

In summary, if $\overline{M}(0)$ goes to zero as $V^{-\alpha}$ we find that

$$D(0) \rightarrow 0, \quad 0 < D(0) < +\infty \quad \text{or} \quad D(0) \rightarrow +\infty$$

respectively if α is **larger than**, **equal to** or **smaller than** $1/2$.

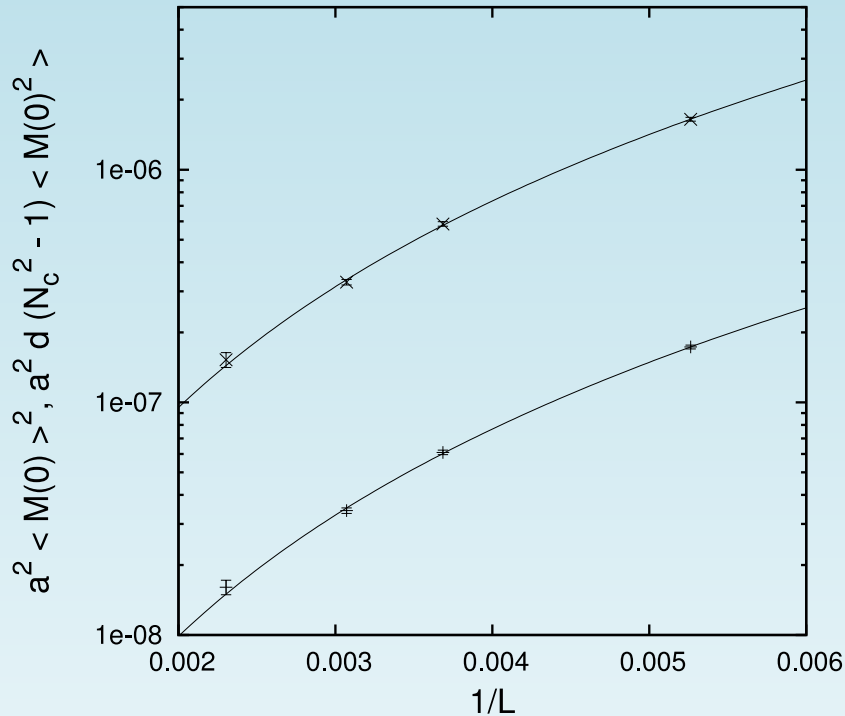
Upper and lower bounds for $D(0)$



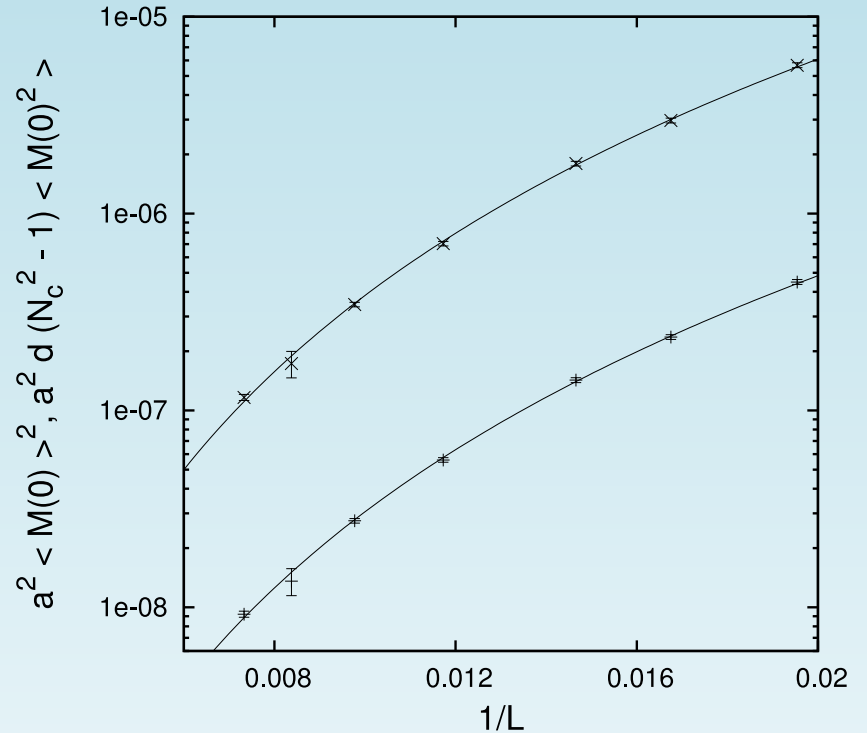
Two-dimensional case: B_l/L^l (for $a\langle\overline{M}(0)\rangle$) and the Ansatz B_u/L^u (for $a^2\langle\overline{M}(0)^2\rangle$), with $B_l = 1.48(6)$, $l = 1.367(8)$ and $\chi/d.o.f. = 1.00$ and $B_u = 2.3(2)$, $u = 2.72(1)$ and $\chi/d.o.f. = 1.02$.

Upper and lower bounds **extrapolate to zero faster than $1/V$** ,
implying $D(0) = 0$.

Upper and lower bounds for $D(0)$ (II)



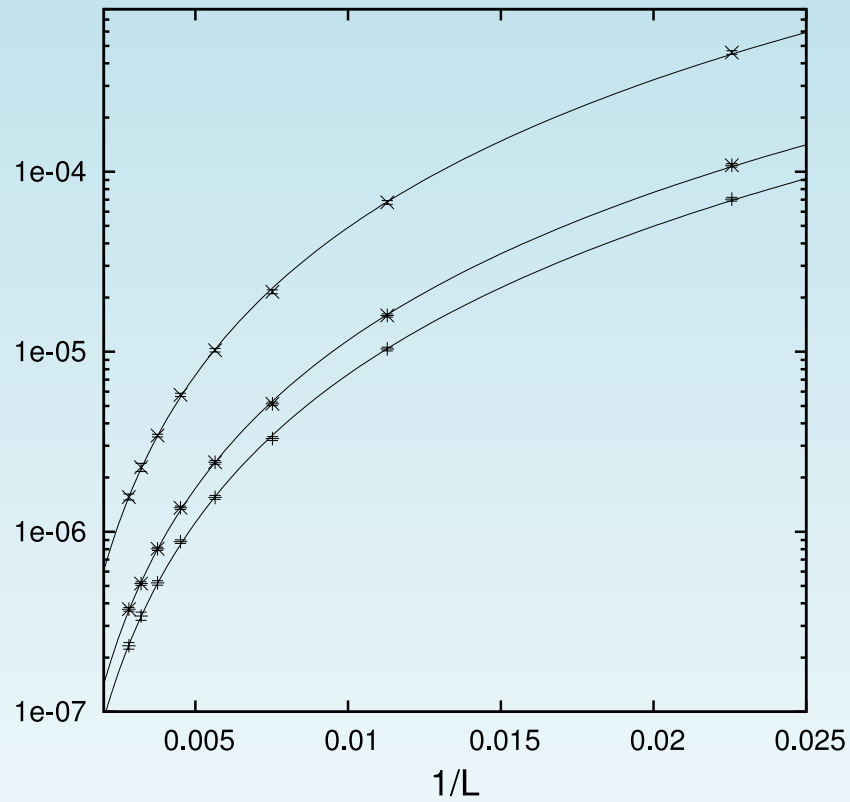
Similarly for 3d: $l = 1.48(3)$; $B_u = 1.0(3)$, $u = 2.95(5)$ and $\chi/d.o.f. = 0.95$.



Similarly for 4d: $l = 1.99(2)$; $B_u = 3.1(5)$, $u = 3.99(4)$ and $\chi/d.o.f. = 0.96$.

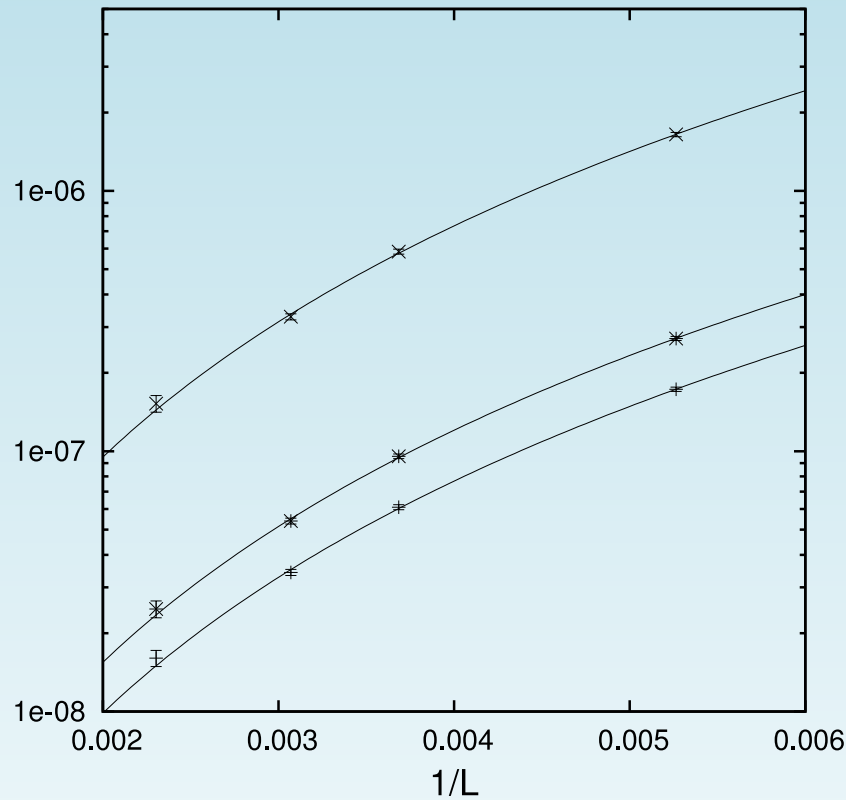
Upper / lower bounds **extrapolate to zero as $1/V$** , implying $D(0) > 0$.

Upper and lower bounds plus $D(0)/V$

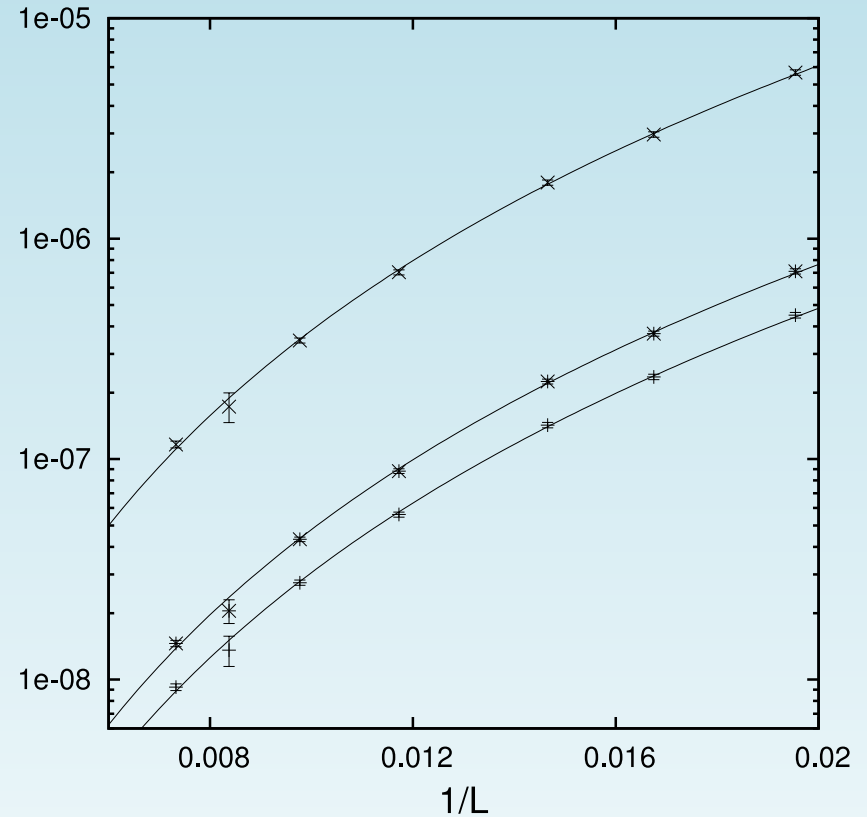


2d case

Upper and lower bounds plus $D(0)/V$ (II)



3d case



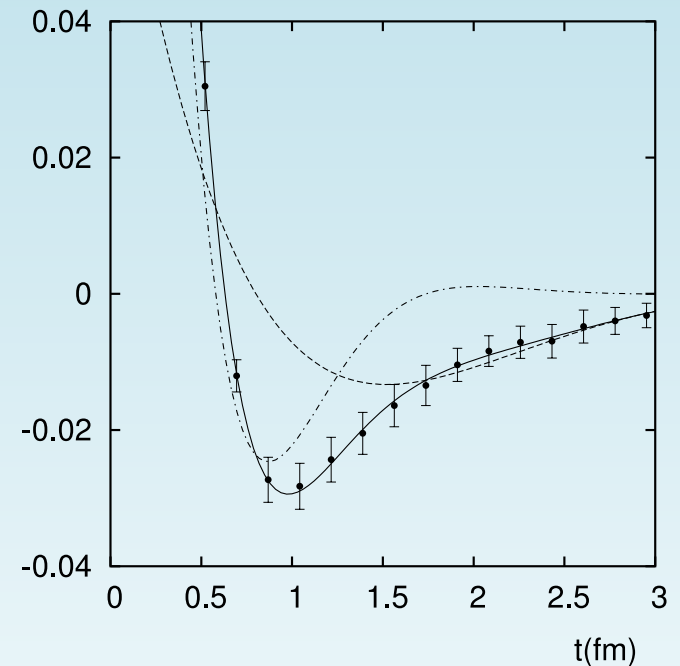
4d case

Gluon Propagator at Infinite Volume

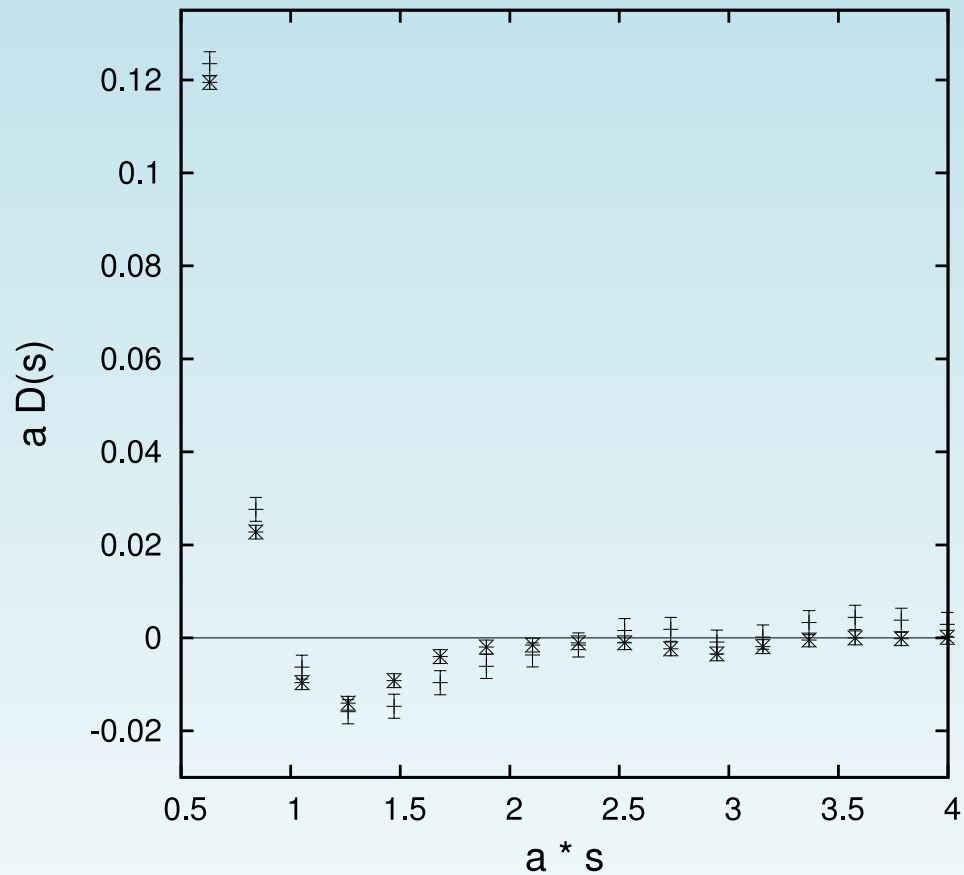
- Gluon propagator in Landau gauge **IR finite** in 3d and 4d, as a consequence of “self-averaging” of a **magnetization-like quantity** [i.e. $M(0)$, **without the absolute value**].
- May think of $D(0)$ as a **response function** (susceptibility) of this observable (“**magnetization**”). In this case it is natural to expect $D(0) \sim \text{const}$ in the infinite-volume limit.
- **2d case is different**, the magnetization is “over self-averaging”, the susceptibility is zero.
- **Question**: why is the **2d** case different? Possible solution from S. Sorella and collaborators.
- **Note**: violation of **reflection positivity** in 2d, 3d and in 4d.

Violation of reflection positivity in 3d

- The transverse gluon propagator **decreases** in the IR limit for momenta smaller than p_{dec} , which corresponds to the mass scale λ in a Gribov-like propagator $p^2/(p^4 + \lambda^4)$. We can estimate $p_{dec} = 350_{-50}^{+100}$ MeV.
- Clear violation of **reflection positivity**: this is one of the manifestations of **gluon confinement**. In the scaling region, the data are well described by a sum of Gribov-like formulas, with a light-mass scale $M_1 \approx 0.74(1)\sqrt{\sigma} = 325(6)$ MeV and a second mass scale $M_2 \approx 1.69(1)\sqrt{\sigma} = 745(5)$ MeV.



Violation of reflection positivity in 4d



Clear violation of **reflection positivity** for lattice volume $V = 128^4$ at $\beta = 2.2$.

Bounds and Results for the Ghost Propagator

(A. Cucchieri, T.M., PoS LATTICE2007 and Phys. Rev. D 2008)

Upper and Lower Bounds for $G(p)$

Consider eigenvectors $\psi_i(a, x)$ and associated eigenvalues λ_i of the FP matrix $\mathcal{M}(a, x; b, y)$. The ψ 's form a complete orthonormal set

$$\sum_{i=1}^{(N_c^2-1)V} \psi_i(a, x) \psi_i(b, y)^* = \delta_{ab} \delta_{xy} \quad \text{and} \quad \sum_{a,x} \psi_i(a, x) \psi_j(a, x)^* = \delta_{ij} .$$

If we now write

$$\mathcal{M}^{-1}(a, x; b, y) = \sum_{i, \lambda_i \neq 0} \frac{1}{\lambda_i} \psi_i(a, x) \psi_i(b, y)^* ,$$

we get for $G(p)$ the expression

$$G(p) = \frac{1}{N_c^2 - 1} \sum_{i, \lambda_i \neq 0} \frac{1}{\lambda_i} \sum_a |\tilde{\psi}_i(a, p)|^2 ,$$

where

$$\tilde{\psi}_i(a, p) = \frac{1}{\sqrt{V}} \sum_x \psi_i(a, x) e^{-2\pi i k \cdot x} .$$

Upper and Lower Bounds for $G(p)$ (II)

From the above expression we immediately get for $G(p)$ the **lower bound**

$$\frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_a |\tilde{\psi}_{min}(a, p)|^2 \leq G(p)$$

and the **upper bound**

$$G(p) \leq \frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_{i, \lambda_i \neq 0} \sum_a |\tilde{\psi}_i(a, p)|^2 .$$

Now by adding and subtracting the contribution from the null eigenvalue and using the completeness relation, the upper bound may be rewritten as

$$G(p) \leq \frac{1}{\lambda_{min}} \left[1 - \frac{1}{N_c^2 - 1} \sum_{j, \lambda_j = 0} \sum_a |\tilde{\psi}_j(a, p)|^2 \right] .$$

Upper and Lower Bounds for $G(p)$ (III)

In **Landau gauge** the eigenvectors corresponding to null λ are constant modes. Thus for any nonzero p we have

$$\frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_a |\tilde{\psi}_{min}(a, p)|^2 \leq G(p) \leq \frac{1}{\lambda_{min}} .$$

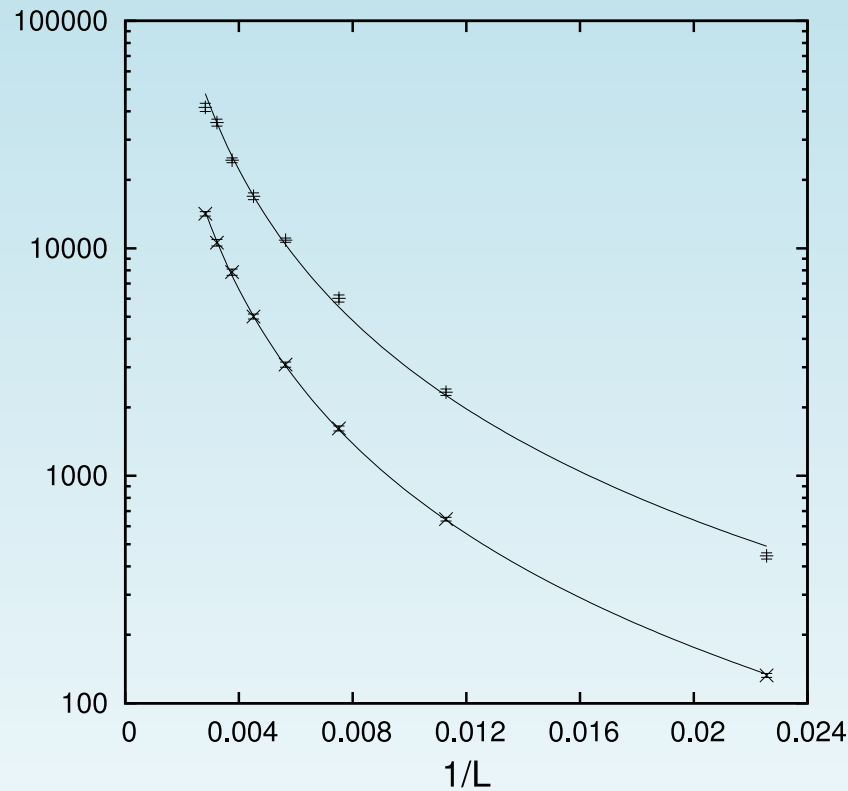
Now, assuming $\lambda_{min} \sim N^{-\alpha}$ and the power-law behavior $p^{-2-2\kappa}$ for the IR ghost propagator, we expect to have

$$2 + 2\kappa \leq \alpha$$

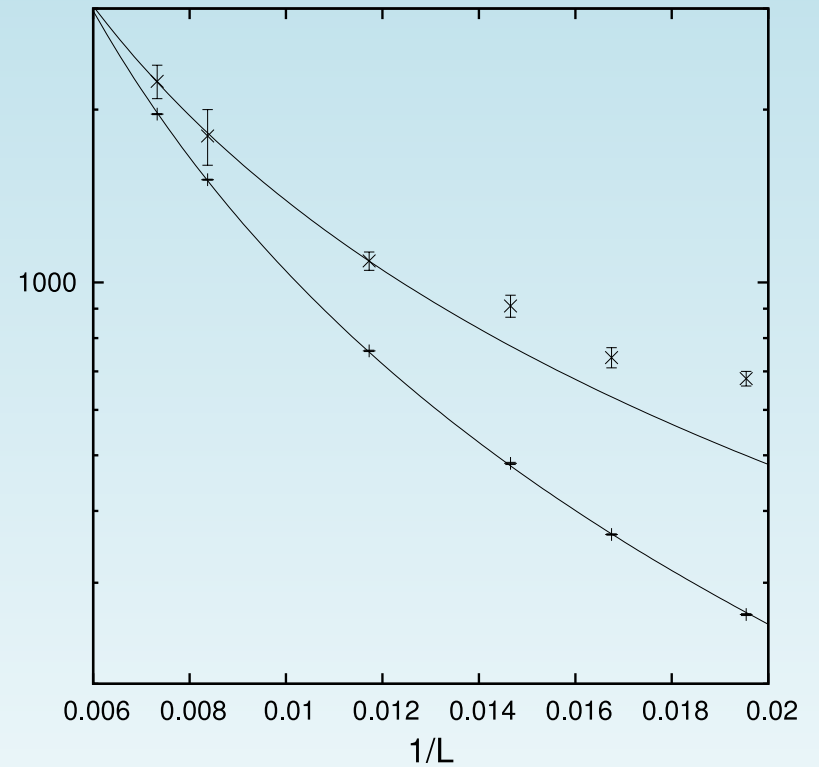
and a necessary condition for IR enhancement of $G(p)$ is

$$\alpha > 2 .$$

Upper bound for $G(p_{min})$



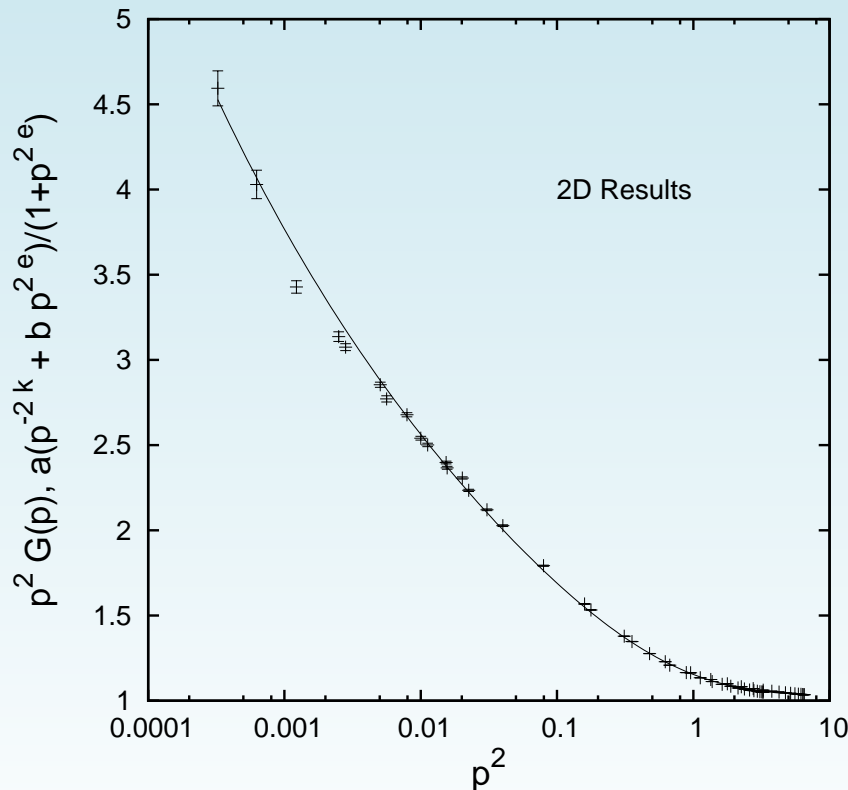
For 2d: $2\kappa = 0.251(9)$, $\alpha = 2.20(4)$.



For 4d: $2\kappa = 0.043(8)$, $\alpha = 1.53(2)$.

Ghost fits (I)

Fit of the ghost dressing function $p^2 G(p^2)$ as a function of p^2 (in GeV) for the 2d case ($\beta = 10$ with volume 320^2). We find that $p^2 G(p^2)$ is best fitted by the form $p^2 G(p^2) = a(p^{-2k} + bp^{2e})/(1 + p^{2e})$, with

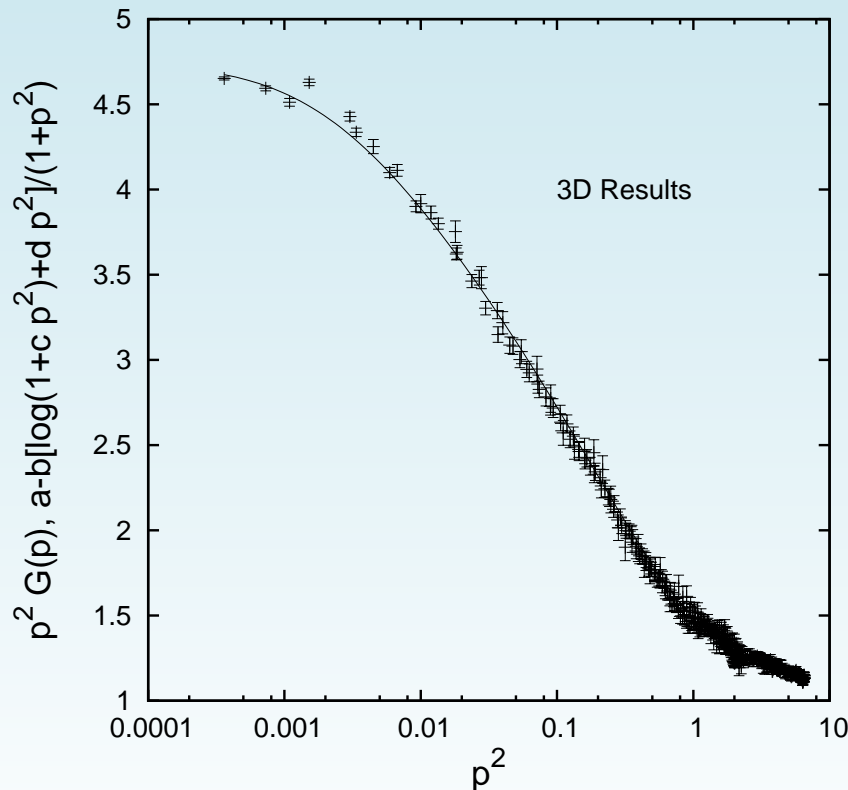


$$\begin{aligned} a &= 1.24(3) \text{ GeV}^{2(e+\kappa)}, \\ \kappa &= 0.16(2), \\ b &= 0.86(3) \text{ GeV}^{-2(e+\kappa)}, \\ e &= 0.75(15). \end{aligned}$$

In the **infrared** limit
 $p^2 G(p^2) \sim p^{-2k}$.

Ghost fits (II)

Fit of the ghost dressing function $p^2 G(p^2)$ as a function of p^2 (in GeV) for the 3d case ($\beta = 3$ with volume 240^3). We find that $p^2 G(p^2)$ is best fitted by the form $p^2 G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with

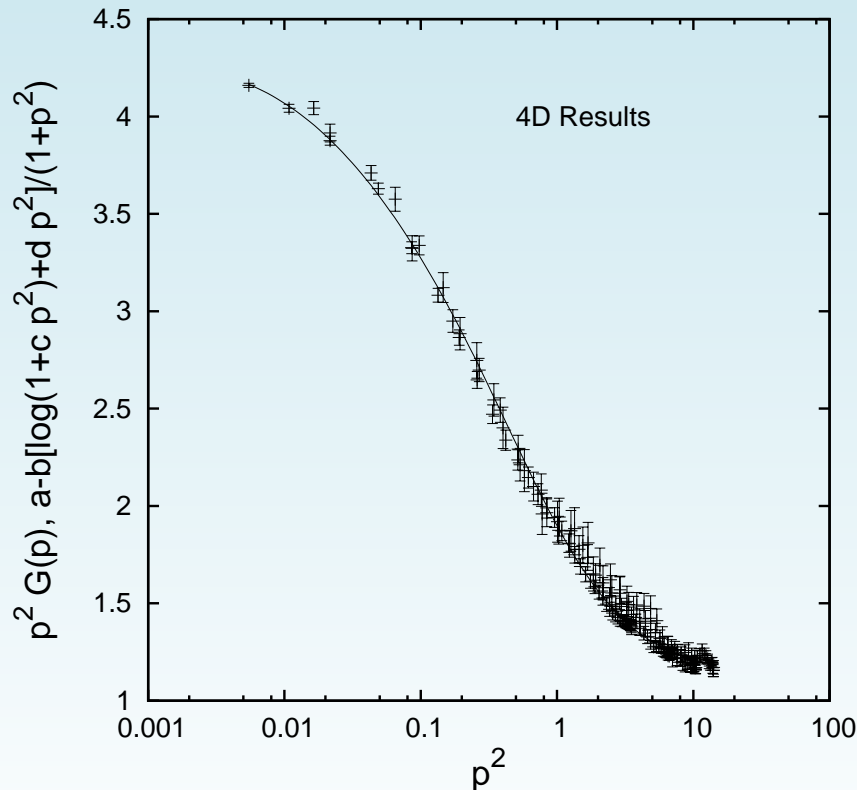


$$\begin{aligned} a &= 4.75(1), \\ b &= 0.491(5) \text{ GeV}^2, \\ c &= 450(30) \text{ GeV}^{-2}, \\ d &= 7.1(1) \text{ GeV}^{-2}. \end{aligned}$$

In the **infrared** limit
 $p^2 G(p^2) \sim a$.

Ghost fits (III)

Fit of the ghost dressing function $p^2 G(p^2)$ as a function of p^2 (in GeV) for the 4d case ($\beta = 2.2$ with volume 80^4). We find that $p^2 G(p^2)$ is best fitted by the form $p^2 G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with



$$\begin{aligned} a &= 4.32(2), \\ b &= 0.38(1) \text{ GeV}^2, \\ c &= 80(10) \text{ GeV}^{-2}, \\ d &= 8.2(3) \text{ GeV}^{-2}. \end{aligned}$$

In the **infrared** limit
 $p^2 G(p^2) \sim a$.

Ghost Propagator at Infinite Volume

- From present fits we have $\alpha > 2$ in 2d [implying IR enhancement of $G(p)$], but $\alpha < 2$ in 4d.
- On the other hand the expected relation $2 + 2\kappa \leq \alpha$ is **not** satisfied, although the upper bound is.
- Of course, we should get **better data** for λ_{min} in 2d, 3d and 4d.
- From fits of the ghost dressing function we find $p^2 G(p^2) \sim p^{-2k}$ in 2d and $p^2 G(p^2) \sim a$ in 3d and in 4d.

Propagators at $\beta = 0$

In **agreement** with the simulations at $\beta > 0$ we find (arXiv:0904.4033[hep-lat]) that

- the gluon propagator $D(p)$ violates **reflection positivity**,
- the gluon propagator at zero momentum $D(0)$ seems to be **finite and nonzero**,
- the ghost propagator $G(p)$ is **not infrared enhanced** in the deep infrared limit, but it is enhanced at larger momenta,
- a very good fit for the ghost propagator $G(p)$ is given by $f(x) = [a - b \log(p^2 + c^2)] / p^2$,
- see also recent work by A. Sternbeck and L. von Smekal.

Conclusions

- Simple properties of gluon and ghost propagators constrain (by **upper and lower bounds**) their IR behavior. For the **gluon case** we define a **magnetization-like quantity**, while for the **ghost case** we relate the propagator to λ_{min} of the FP matrix. These quantities are studied as a function of the lattice volume, to gain better control of the **infinite-volume limit** of IR propagators.
- For the gluon propagator, data & extrapolation (plus explanation as response function) support a **finite value in the IR**
- For the ghost case, enhancement seems unlikely...
- **Questions**: just considering large volumes is not enough? is the behavior of the propagators at $p = 0$ so crucial for **confinement**?
- Interesting to consider **other gauges**: MAG, λ -gauges, linear covariant gauge (collaboration with A. Maas, A. Mihara, E. Santos)