Wave function collapse

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Under "collapse of the wave function" (or "state vector reduction") one understands the 'sudden' change of the system's state in a measurement. This change is not reducible to classical "information gain", but is a genuine quantum mechanical concept, directly related to the concept of quantum state. It is especially relevant if we consider that quantum mechanics describes the behaviour of individual systems. In the following we shall first describe the role of the collapse as a formal concept in this context, then we shall discuss some variants of physical approaches to collapse. We shall comment on the notion of "individual systems" in quantum mechanics at the end of this article.

Collapse in the formalism of quantum theory.

The notion of state of a system is a fundamental concept in physics. In classical physics all quantities which can be measured upon the system ($\rightarrow$"observables": e.g., positions and momenta of a point particle) can, in principle, be simultaneously assigned precise values and this uniquely defines the state. There is therefore a one to one relation between states and observations. In quantum theory, however, only a subset of observables can be fixed at any given moment. A maximally determined state obtains by fixing a maximal set of simultaneously measurable ("compatible") observables, e.g. the position components. But there will be other observables, here the momenta, which do not posses definite values in this state. Relating states to observations is therefore a more special and not trivial procedure.

This also implies that the concept of $\rightarrow$measurement becomes essential. Here we shall only refer to an ideal measurement, which is understood as any physical arrangement by which a particular observable concerning the system of interest is fixed to some well defined value. But if the initial state of the system was such that it did not determine this particular observable beforehand, this indeterminacy will show up as irreproducibility of the result when repeating the experiment under the same conditions (same apparatus and identically "prepared" systems). Only the relative frequency of these results can be associated to a probability distribution determined by the initial state (quantum effects show up here as interference terms and non-trivial correlations when performing correlated measurements, which cannot be understood classically). After the measurement, however, the state of the system must be such that the measured observable is no longer undetermined but has now been fixed to the measured value, hence the state has changed abruptly and randomly with the given probability distribution. We speak of *collapse of the state* anterior to the measurement onto the state in which the measurement leaves the system.

The formalism of quantum theory allows to write any given state as a → superposition of other states, in particular of such states where the observable of interest has well defined values. Collapse, or state reduction means then the survival after measurement of only that state out of the superposition for which the value of the observable matches the result of the measurement.

In as much, therefore, that we can speak of individual systems and measurements, collapse is a logically necessary ingredient in the formalism. The representation of states as vectors in a →Hilbert space makes the above considerations transparent and well defined: linear combinations of vectors realize the superposition of states, with the coefficients giving the weights and their square modulus the corresponding probabilities. Here collapse appears as a sudden and generically random change in the state vector, as opposed to the continuous, deterministic transformations of the latter due to the various physical interactions the system may be subjected to. Accordingly, in this setting the axioms of quantum mechanics include a measurement and collapse postulate (von Neumann's "first intervention"), besides the definition of states as vectors in a Hilbert space (which incorporates the superposition principle), the definition of observables and expectation values and the dynamical evolution equations (von Neumann's "second intervention").

In the following we shall be slightly more formal. The reader who does not want to be bothered with technical detail may go directly to the *Physical approaches.*

The quantum mechanical Hilbert space is a generically infinitely-dimensional linear space over the complex field, with an inner scalar product and the associated norm and distance and which is complete under this norm. The states of a physical system are represented as vectors in this space and physical interventions upon the system as operators acting on these vectors. In particular observables are represented as hermitean operators, in accordance with the reality of measurements. We can use ortho-normalized bases and any vector can be decomposed in such a basis as

$$|\psi\rangle = \sum_n c_n \, |\varphi_n\rangle \, , \quad \langle \varphi_m | \varphi_n \rangle = \delta_{mn} \, , \tag{1}$$

where we used the Dirac braket notation for the vectors and scalar products (for all these concepts see the corresponding articles). In the following we shall only consider so called →pure states and use normalized vectors $\|\psi\| = 1$ with $\|\cdot\|$: the Hilbert space norm. The expectation of any operator A in the state $|\psi\rangle$ is then $\langle\psi|A|\psi\rangle$ and all information about possible observations onto the system in this state is contained in the "density operator" ("density matrix")

$$\rho = |\psi\rangle \, \langle\psi| = \sum_{n,m} c_n \, c_m^* \, |\varphi_n\rangle \, \langle\varphi_m| \, . \tag{2}$$

with the help of which we can obtain expectation values for any observable.

If we choose the basis vectors $|\varphi_n\rangle$ above to be *eigenstates* of some observable A

$$A\,|\varphi_n\rangle \;= a_n\,|\varphi_n\rangle\,, \tag{3}$$

then a measurement of A upon the system in state $|\psi\rangle$ will produce some value, say a_{n_0}, with probability $\langle\varphi_{n_0}|\,\rho\,|\varphi_{n_0}\rangle = |c_{n_0}|^2$ and leave the system in the state φ_{n_0}. This means an abrupt change of the state vector which can be seen as a sudden "rotation" of the latter aligning it with one of its components, chosen *randomly* with the mentioned probability:

$$|\psi\rangle = \sum_n c_n\,|\varphi_n\rangle \;\longrightarrow\; |\psi'\rangle = |\varphi_{n_0}\rangle\,. \tag{4}$$

This "reduction of the state vector" (collapse, or von Neumann's "first intervention") is to be contrasted with the *deterministic* dynamical evolution of the state vector due to physical interactions (von Neumann's "second intervention"), realized by a unitary operator acting continuously in time, (written in differential form this is the Schrödinger equation):

$$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle\,. \tag{5}$$

Physical approaches to collapse

The conceptual differences between von Neumann's first and second interventions have led to many interpretational problems. In standard quantum theory the collapse of the wave function is associated with the measurement but the moment of its occurrence (the "Heisenberg cut") can be anywhere between the actual interaction of the system with the apparatus and the conscious registration of the result. If the observer is considered external this appears to introduce a subjective element in the theory, with corresponding ambiguities (→"Wigner's friend"). These problems have prompted many attempts to give the collapse a more physical ground. These attempts can be divided in three classes: "no collapse" (in deterministic extensions which reproduce quantitatively quantum theory), "apparent collapse" (in quantum theory itself within a certain interpretation) and "dynamical collapse" (in the frame of theories which *approximate* quantum theory).

The first class essentially corresponds to the hidden variables theories. In this case there is no collapse at all, the state precisely determines every observable and the spread of results in a repeated experiment is due to the different values taken by the "hidden variables" which make that we in fact deal with different initial states each time, the difference escaping however our control (is hidden). An elaborated theory hereto has been set up by D. Bohm 1952 and has been further developed thereafter. It is a celebrated theorem established by J. S. Bell 1964 that demanding agreement with quantum theory requires *non-local* hidden variables. This is brought to a quantitative test in the so called →"Bell's inequalities" for correlated measurements which should be fulfilled for *local* hidden variable theories. Experiments up to date appear to violate these inequalities and show agreement with the quantum mechanical predictions. Non-local hidden variables, though allowed by this test, contradict a basic principle of physics – locality. This, and difficulties in pursuing this program for realistic physical theories diminishes the attractiveness of hidden variable theories.

In the second case the accent is on illuminating the physics of the measurement process. We shall here discuss the so called "→environmental decoherence" argument as raised by H. D. Zeh 1970 and W. H. Zurek 1981. The measurement is realized by some physical interaction with an "apparatus" understood as a quantum system. The discussion uses the observation that quantum systems which in some way form a compound have to be considered as "entangled", which means that in a generic state of the compound system the component systems do not possess a separate state. This is a generic feature of quantum theory and means among others that, in principle, the notion of isolated system is only an approximation whose goodness depends on the physical situation. Now, a measurement implies an →entanglement between the system and the apparatus. Moreover, since the latter essentially is a macroscopic system, it unavoidably will be entangled with an environment which is not accessible to our observations (e.g., light scattered from the pointers and leaving the experimental arrangement). Observations upon the system imply therefore an averaging over the states of the environment which are associated with different "pointer" states of the apparatus and are macroscopically different. This leads to the loss of observable interference between the different states of the apparatus. This simulates therefore a classical statistics.

To be more specific (again, these technical aspects can be skipped), if $\varphi_n^{\{1\}}$, $\varphi_n^{\{2\}}$ are bases for the two component systems in a binary compound (say, two atoms in a molecule) a generic state of the latter is

$$\begin{aligned} |\Psi\rangle &= \sum_{m,n} c_{mn}\, |\varphi_m^{\{1\}}\rangle\, |\varphi_n^{\{2\}}\rangle \\ &= \sum_n c_n\, |\psi_n^{\{1\}}\rangle\, |\varphi_n^{\{2\}}\rangle\,, \end{aligned} \tag{6}$$

where for the second equation we used a certain redefinition of the states. This total wave function generally does not factorize, hence it does not allow any of the two systems to be in a definite state. With '1' designating an apparatus and '2' a system to be measured (6) is also a model for the physical interactions during a measurement process:

$$\begin{aligned} |\Psi\rangle &= \sum_{m,n} c_{mn}\, |\varphi_m^{\{app\}}\rangle\, |\varphi_n^{\{sys\}}\rangle \\ &= \sum_n c_n\, |\psi_n^{\{app\}}\rangle\, |\varphi_n^{\{sys\}}\rangle\,. \end{aligned} \tag{7}$$

The apparatus is entangled both with our system and with the environment. Let us consider the apparatus as being such that the total wave function can be written as

$$|\Psi\rangle = \sum_n c_n\, |\phi_n^{\{env\}}\rangle\, |\psi_n^{\{app\}}\rangle\, |\varphi_n^{\{sys\}}\rangle\,, \tag{8}$$

where the environmental states $|\phi_n^{\{env\}}\rangle$ differ macroscopically and are therefore orthogonal. Since we have no access to the situation of the environment (we cannot make correlated experiments involving the states of the environment), according to the quantum mechanical formalism any information we can obtain about the system is contained in the "reduced density matrix" where the environmental situation has been "traced out":

$$\begin{aligned} \rho_{red} &= \sum_k \langle\phi_k^{\{env\}}|\,|\Psi\rangle\,\langle\Psi|\,|\phi_k^{\{env\}}\rangle \\ &= \sum_n |c_n|^2\,|\psi_n^{\{app\}}\rangle\,|\varphi_n^{\{sys\}}\rangle\,\langle\psi_n^{\{app\}}|\,\langle\varphi_n^{\{sys\}}|\,. \end{aligned} \tag{9}$$

At variance to the general case (2), ρ_{red} is diagonal, which implies that we cannot observe the typical quantum mechanical interference between the different possible issues of the measurement.

This consequence – the *simulation of a classical statistics* – of the "unavoidable entanglement" with an uncontrollable environment stays at the basis of the effect called →"decoherence" which is a *specific quantum mechanical effect implying no further hypothesis*. It is always present, independently of interpretations, of measurement models, etc and is well defined in each physical situation. Its relevance for the measurement is to "de-correlate" the various possible results, as shown above, which therefore *appear* as distributed according to a classical ensemble. This does not replace collapse (which requires the choice of just one of these possible results, accompanied by the corresponding acquirement by the system of the corresponding wave function, after the interaction with the apparatus has ceased). However, it makes possible an alternative point of view, that of an "apparent collapse". The basis for this point of view is the so called "relative state interpretation" of quantum mechanics proposed by H. Everett III 1957, according to which all possible outcomes of each measurement coexist but that due to the local nature of the observations their histories form different branches of the evolution of the total system (in end effect, the world). The role of decoherence effects at measurement is now to ensure that no local observations can put into evidence correlations between the different branches, which are thus completely "unaware" of each other. From the point of view of one given branch the other components of the wave function appear therefore as irretrievably lost. Although the system is still entangled with the rest of the universe and therefore does not possess in principle a wave function for itself, any observations upon the system within one branch give the same results *as if* formal collapse had occurred (the observer is viewed as part of the quantum world and thus his consciousness follows the same branching pattern). This perspective calls for cosmological arguments. A picture of these steadily branching histories is however difficult to realize and, for instance in the so called "many-worlds" representation, somewhat unintuitive. Related interpretations are provided, e.g., in the "consistent histories" approach of R.B. Griffith 1984 and M. Gell-Mann and J. B. Hartle 1990.

Finally, the class 3 models define collapse as a genuine physical effect. This obtains as a supplementary postulate, which, in the formulation of G. C. Ghirardi,

A. Rimini and T. Weber 1975, (→GRW Theory) states that the wave function of any spatial degree of freedom collapses spontaneously in a random manner, thereby fixing this degree of freedom to a value randomly chosen with the distribution given by the wave function before collapse ("spontaneous collapse" or "spontaneous localization" hypothesis). There are also other possibilities to achieve a dynamical collapse, for instance turning the Schrödinger equation into a stochastic differential equation through the addition of a non-linear noise term as proposed by P. Pearle 1976. In this case the collapse is only approximate, the collapsed wave function retaining an exponentially falling tail. The main features are, however, similar, namely:

- Even if for each degree of freedom the collapse occurs extremely rarely, the apparatus being a *macroscopic* object will be steadily subject to collapses. Since the (*microscopic*) system to be measured becomes entangled with the apparatus, *see (7),* the collapse acting in the latter and retaining some term, say $|\psi_{n_0}^{\{app\}}\rangle$ of the superposition automatically selects the corresponding component vector of the system, $|\varphi_{n_0}^{\{sys\}}\rangle$, fixing in this way the corresponding observable and leaving the system in a pure state. Therefore this model *explains* measurement.

- Collapse as a physical random process is not compatible with quantum mechanics in the sense that it leads to measurable deviations from the predictions of the latter. The details (parameters) of this process can be, however, so tuned, that these effects are detectable only for macroscopic systems, where they are welcome, but not for microscopic systems, where to a good precision the standard quantum mechanical predictions should hold.

  To be more specific, in the discrete random collapse model, for instance, with a frequency of spontaneous collapses of, e.g., $10^{17}s^{-1}$ the wave function of a microscopic system will collapse about once in 10^{10} years, the age of the universe, while a macroscopic body with typically 10^{23} degrees of freedom would undergo a collapse as often as 10^6 times per second. This is compatible both with the behaviour of atoms, with the action of an apparatus and with the localized appearance of macroscopic objects, for which the successive spontaneous localizations of internal degrees of freedom soon pins down the center of mass of the body. Similar effects are obtained in the noisy dynamics models.

- The collapse is assumed to act on spatial degrees of freedom ("spontaneous localization") which is reasonable since usual interactions are local. It seems difficult, however, to obtain relativistic generalizations of the model, in particular for local quantum field theories.

Replacing the formal postulate of "collapse in the measurement" by the postulate of "general stochastic evolution" of the wave function appears somewhat arbitrary and one would like to have corroboration from further observations. This, however, appears very difficult, since the predicted new physics has similar signature with environmental decoherence and would be masked by the latter even if present. As long as we have no independent evidence for such a universal stochastic dynamics its postulate remains however *ad hoc*.

Note that none of these proposals really solves the problem, namely to provide a non-formal explanation for the collapse and the measurement process of *standard quantum mechanics*: either we modify the theory in an *in principle* measurable way (even if we may tune the parameters to ensure that the difference does not show up *in practice*), or we only provide an "as if" effect (even if the difference to true collapse might be of only cosmological relevance). This has prompted Bell to speak of "good for all practical purposes" in connection with some of these (and others) "solutions". Finally, non-local hidden variables might not be seen as a real alternative. But even if not solving the problem the various theoretical studies contributed very much to illuminate it.

As already mentioned, the problem of collapse is relevant in an interpretation of quantum theory pertaining to individual events. Many of the conceptual problems can be discarded in a statistical interpretation which states that wave function, collapse etc. are only mathematical instruments which allow us to make statistical predictions, and the latter are the only place where theory meets the real world. It may appear, however, that this "economical" point of view unnecessarily impoverishes the theory. In fact statistics is not a real "thing" or event in itself, but is a conclusion drawn from the observation of many single events. The theory does refer to the latter individually and in some special cases does this in an unambiguous way, for instance when it predicts probability 0 or 1 for a certain event. These are incentives to assume that it does account for individual events generally, even if we cannot make an intuitive picture of this reference. It would seem, in some sense, quite a miracle and in fact unintuitive to have the extraordinary explanatory power of quantum theory based on a lucky choice of theoretical "instruments" completely detached from reality. This does not mean that wave functions etc. should exist as such in reality, but that there are things and a structure in reality which support such abstractions. On the other hand it seems rather difficult to grasp this structure. Its features, as they might be suggested by the theory, do not appear unambiguous and easily understandable. The foregoing discussion of the collapse illustrates these problems.

Bell's inequalities

The non-classical character of the correlation in the expectations concerning correlated measurements on two entangled subsystems which do not possess states of their own, i.e., if it is not possible to rewrite (6) as a product of two factors, can be quantitatively exhibited in corresponding experiments. Assume we measure the properties A, A' on system '1' *and* B, B' on '2', that is, we use the observables (hermitean operators) $\{O\} = \{A \otimes B,\ A' \otimes B,\ \cdots\}$ and construct the quantity:

$$\Delta(A, A'; B, B') \equiv |\mathcal{E}(AB) - \mathcal{E}(AB')| + |\mathcal{E}(A'B) - \mathcal{E}(A'B')|\,, \tag{10}$$

where $\mathcal{E}$ denote the corresponding expectations in the given state of the total system:

$$\mathcal{E}(O) = \langle \Psi | O | \Psi \rangle \, . \tag{11}$$

Then we have (we choose $\|O\| \leq 1$, i.e. $\|O\psi\| \leq \|\psi\|$, $\forall \psi$):

$$\Delta(A, A'; B, B') = |\langle \Psi | A(B - B') | \Psi \rangle| + |\langle \Psi | A'(B + B') | \Psi \rangle| \tag{12}$$
$$= |\langle A\Psi | (B - B')\Psi \rangle| + |\langle A'\Psi | (B + B')\Psi \rangle|$$
$$\leq \|A\Psi\|.\|(B - B')|\Psi\| + \|A'\Psi\|.\|(B + B')\Psi\|$$
$$\leq \|(B - B')\Psi\| + \|(B + B')\Psi\|$$
$$\leq \sqrt{2[\|(B - B')\Psi\|^2 + \|(B + B')\Psi\|^2]} \tag{13}$$
$$= \sqrt{4[\|B\Psi\|^2 + \|B'\Psi\|^2]} \leq 2\sqrt{2} \, . \tag{14}$$

If we were dealing with a classical problem, that is the expectations were taken with respect to a classical ensemble:

$$\mathcal{E}_c(O) = \int O d\mu \, , \tag{15}$$

with $d\mu$ a (positive semidefinite) probability measure and $\{O\}$ *real* valued functions (assumed to be less than 1 in absolute value) we would had instead:

$$\Delta_c(A, A'; B, B') = |\mathcal{E}_c(A(B - B'))| + |\mathcal{E}_c(A'(B + B'))| \tag{16}$$
$$\leq \mathcal{E}_c(|A|.|B - B'|) + \mathcal{E}_c(|A'|.|B + B'|) \leq \mathcal{E}_c(|B - B'|)| + \mathcal{E}_c(|B + B'|)$$
$$= \mathcal{E}_c(|B - B'| + |B + B'|) \leq 2 \, , \tag{17}$$

since the general inequality:

$$\|a\| + \|b\| \leq \sqrt{2(\|a\|^2 + \|b\|^2)} \tag{18}$$

which was used in (13) could be replaced by the equality:

$$|a| + |b| = |a + b.\mathrm{sgn}(ab)| \tag{19}$$

if a, b are *real numbers*. The inequality (12,14) can be saturated if B, B' (A, A') do not commute and the subsystems are non-trivially correlated, i.e., $|\Psi\rangle$ does not factorize and the subsystems are not in pure states. Notice that (16,17) would also hold if our quantum mechanical problem were reducible to a classical one (local hidden variables). These are the well known *Bell's inequalities*, 1980, and the experimental evidence to date seems to violate the bound (16,17) and to support (12,14).

References

Discussion of these problems beyond that in the standard textbooks can be found, e.g., in

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