## 2 Präsenzübung for the Quantum Mechanics lectures (PTP 4)

These exercises will be jointly discussed with guidance from the tutors in the tutorials of the 15th and 16th of April.

Two points are given for active participation

Q P6: Basis transformation
$\left\{\left|a_{1}\right\rangle,\left|a_{2}\right\rangle\right\}$ form an orthonormal basis for a two-dimensional complex Hilbert space (the $\{a\}$ representation basis). Show that the vectors
$\left|b_{1}\right\rangle=\frac{1}{\sqrt{2}}\left(\left|a_{1}\right\rangle+i\left|a_{2}\right\rangle\right) \quad\left|b_{2}\right\rangle=\frac{1}{\sqrt{2}}\left(\left|a_{1}\right\rangle-i\left|a_{2}\right\rangle\right)$ also form an orthonormal basis (the $\{b\}$ representation basis).

Q P7: Stern-Gerlach thought experiment

See on the other side of this page an experimental set up of two Stern-Gerlach apparatus, one behind the other. After exiting apparatus 1 (z-SG) the particles are in one of two states

$$
\left|\varphi_{1}\right\rangle=\binom{1}{0} \quad \text { or } \quad\left|\varphi_{2}\right\rangle=\binom{0}{1} .
$$

The particle which are in state $\left|\varphi_{1}\right\rangle$ then go through apparatus 2 (x-SG). After this the particles are in either state $\left|\psi_{1}\right\rangle$ or $\left|\psi_{2}\right\rangle$.
a) Of which operator are the states $\left|\psi_{1}\right\rangle$ and $\left|\psi_{2}\right\rangle$ Eigenstates?
b) Calculate the components of $\left|\psi_{1}\right\rangle$ and $\left|\psi_{2}\right\rangle$ in the basis $\left\{\left|\varphi_{1}\right\rangle,\left|\varphi_{2}\right\rangle\right\}$.
c) What \% of the original particles are in state $\left|\psi_{1}\right\rangle$ and state $\left|\psi_{2}\right\rangle$ ? Also calculate $\left|\left\langle\psi_{1} \mid \varphi_{1}\right\rangle\right|^{2}$ and $\left|\left\langle\psi_{2} \mid \varphi_{1}\right\rangle\right|^{2}$.


Abb. 1

