9. Homework sheet for quantum mechanics

To be handed in on the tutorial on the 09.06.

Q 23: *More about a Gaussian wave packet* **(7 points)**

Consider again the 1D Gaussian wave packet of Q 20 (sheet 7):

$$\psi(x,t) \;=\; \int_{-\infty}^{+\infty} \frac{dp}{2\pi\hbar}\,\phi(p)\exp\left\{\frac{i}{\hbar}\left(px-\frac{p^2}{2m}t\right)\right\} \tag{1}$$

with the amplitude function

$$\phi(p) \;=\; A\exp\left\{-\frac{(p-p_0)^2\;d^{\,2}}{\hbar^2}\right\} \tag{2}$$

and with the constants $A, d, p_0 \in \mathbb{R}$.

a) From the lectures, you know that the average momentum at time t is given by

$$< p(t) >= \; -i\hbar\int_{-\infty}^{\infty} dx\,\psi^*(x,t)\frac{\partial}{\partial x}\psi(x,t) \tag{3}$$

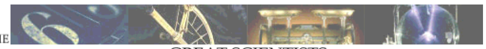
Substitute (1) into (3) in order to show that

$$< p(t) >= \int_{\infty}^{\infty}\frac{dp}{2\pi\hbar}\,|\phi(p)|^2\,p$$

Calculate $< p(t) >$ for the amplitude function $\phi(p)$ given above of a Gaussian wave packet. **(4 points)**

g) Calculate the momentum standard deviation $\Delta p = \sqrt{< (p- < p >)^2 >}$ for the Gaussian wave packet. **(2 points)**

h) What is given by the product of the uncertainty $\Delta x \cdot \Delta p$ (Δx was already calculated in Q 20 e)) ? **(1 point)**



Richard Feynman

1918 - 1988

Richard P. Feynman was born in New York City on the 11th May 1918. He studied at the Massachusetts Institute of Technology where he obtained his B. Sc. in 1939 and at Princeton University where he obtained his Ph.D. in 1942. He was Research Assistant at Princeton (1940-1941), Professor of Theoretical Physics at Cornell University 1945-1950), Visiting Professor and thereafter appointed Professor of Theoretical Physics at the California Institute of Technology (1950-1959).

Professor Feynman was a member of the American Physical Society, the American Association for the Advancement of Science; the National Academy of Science; in 1965 he was elected a foreign member of the Royal Society, London (Great Britain).

One important result of the work Sinitiro Tomonaga, Julian Schwinger, and Richard Feynman was the explanation of the Lamb-shift. Their work is, however, much more general and of deep general significance to physics. It has explained and also predicted several important phenomena. It is the continuation of some investigations performed in the late 1920's in order to find the general quantum mechanical laws according to which the atoms and in particular the electrons give rise to electromagnetic fields, e.g. emit light, and are influencedbysuchfields. By applying quantum mechanics not only to matter but also to the electromagnetic field Dirac, Heisenberg, and Pauli managed in those years to formulate a theory, called quantum electrodynamics, which contains the quantum mechanical laws for the interaction of charged particles, in particular electrons, and the electromagnetic field. It satisfies the important condition of being in agreement with the theory or relativity.

It was soon realized, however, that this theory had serious defects. When one tried to calculate a quantity of such importance as the contribution to the mass of an electron originating in its interaction with the electromagnetic field an infinite and therefore useless result was obtained. A similar difficulty occurred for the charge of the electron.

Because of the fundamental importance of having a more useful quantum electrodynamics many theoretical physicists tried during the 1930's to over come those difficulties. Some indications were forthcoming how this should be accomplished. It lasted, however, until the 1940's for decisive progress to be made.

A new area was then initiated by investigations first performed by Tomonaga. His work was primarily related to the demands imposed by the theory of relativity. In a paper published in 1943 and in later work published together with his collaborators, Tomonaga managed to give a new formulation of quantum electrodynamics and other similar theories, which marked an important progress.

Definite progress was only made as a consequence of the discovery of the Lamb-shift mentioned earlier. When this discovery was discussed at a conference the idea was accepted that the new effect could be explained by quantum electrodynamics provided the proper interpretation was given to this theory. The correctness of this idea was supported by a provisional calculation of the Lamb - shift which was published by Bethe shortly after the conference.

As soon as Tomonaga knew about the Lamb experiment and Bethe's paper he realized that an essential step to be taken was to substitute the experimental mass for the fictive mechanical mass which appeared in the equations of quantum electrodynamics and to perform a similar renormalization of the electric charge. The compensating terms which had then to be introduced in the equations should cancel the infinities. Tomonaga managed to carry out this difficult program on the basis of his earlier investigations mentioned above. He deduced further a correct formula for the Lamb-shift which was found to give results in good agreement with the measurements.

Almost simultaneously with the discovery of the Lamb-shift another pecularity was found by Kusch and his collaborator Foley, which made it clear that the magnetic moment of the electron is somewhat larger than had been assumed before. Using the method of renormalization which he also developed Schwinger was able to prove that a small anomalous contribution should be added to the value of the magnetic moment accepted until then. His calculation agreed with the experiments. Schwinger's calculation was indeed earlier than and very important for the proper interpretation of these measurements.

Schwinger had developed the formalism of the new quantum electrodynamics in several fundamental papers using partially methods similar to those of Tomonaga. He has also made this formalism more useful for practical calculations.

Feynman used more radical methods for solving the problems of quantum electrodynamics. He created a new formalism which he made very useful for practical calculations by introducing a graphical interpretation called Feynman diagrams, which have become an important feature of modern physics. In the description used by Feynman the electromagnetic field did not any more appear explicitly. His description has been very valuable in elementary particle physics where it is necessary to consider besides the electromagnetic also other interactions.

When considering the truth of quantum electrodynamics in its new form one has first of all to realize the extraordinary success of this theory in giving results in agreement with the experiments. For the Lamb-shift and for the anomalous part of the magnetic moment of the electron the agreement is within some parts in one hundred thousand respectively in a million and no disagreement has yet been found. Quantum electrodynamics is indeed one of the most accurate of all the theories of physics. Further evidence in this respect is given by the applications of the theory to the positronium atom and to the mu-particle. The new formalism has also been very important for other parts of physics in particular elementary- particle physics, but also solid-state physics, nuclear physics and statistical mechanics.

Professor Schwinger and Professor Feynman. By introducing new ideas and methods into an old theory you have, together with Professor Tomonaga, created a new and most successful quantum electrodynamics, which occupies a central position in physics. This theory has been unique in stimulating modern research. You have yourself contributed to the extension of its methods to other