Chiral freedom and the scale of weak interactions

proposal for solution of gauge hierarchy problem

- model without fundamental scalar
- new antisymmetric tensor fields
- mass term forbidden by symmetry
- chiral couplings to quarks and leptons
- chiral couplings are asymptotically free
- weak scale by dimensional transmutation

antisymmetric tensor fields

- two irreducible representations of Lorentz – symmetry : (3,1) + (1,3)
- complex representations : $(3,1)^* = (1,3)$
- similar to left/right handed spinors

$$\beta^{\pm}_{mn} = \frac{1}{2}\beta_{mn} \pm \frac{i}{4}\epsilon_{mn}{}^{pq}\beta_{pq}$$

chiral couplings to quarks and leptons

$$
\begin{aligned}
-\mathcal{L}_{ch} = {} & \bar{u}_R \bar{F}_U \tilde{\beta}_+ q_L - \bar{q}_L \bar{F}_U^\dagger \,\bar{\tilde{\beta}}_+\, u_R \\
& + \bar{d}_R \bar{F}_D \bar{\beta}_- q_L - \bar{q}_L \bar{F}_D^\dagger \beta_- d_R \\
& + \bar{e}_R \bar{F}_L \bar{\beta}_- l_L - \bar{l}_L \bar{F}_L^\dagger \beta_- e_R
\end{aligned}
$$

$$
\beta_\pm = \frac{1}{2} \beta_{mn}^{\pm} \sigma^{mn}
$$

- most general interaction consistent with Lorentz and gauge symmetry : ß are weak doublets with hypercharge
- consistent with chiral parity :

  d_R , e_R , $ß^-$ have odd chiral parity

no mass term allowed for chiral tensors

- Lorentz symmetry forbids (ß$^{+}$)* ß$^{+}$
- Gauge symmetry forbids ß$^{+}$ ß$^{+}$
- Chiral parity forbids (ß$^{-}$)* ß$^{+}$

kinetic term

$$-\mathcal{L}^{ch}_{\beta,kin} = \frac{1}{4}\int d^4x\{(\partial^\rho \beta^{\mu\nu})^*\partial_\rho\beta_{\mu\nu} - 4(\partial_\mu\beta^{\mu\nu})^*\partial_\rho\beta^\rho{}_\nu\}$$

- does not mix ß $^+$ and ß $^-$
- consistent with all symmetries, including chiral parity

quartic couplings

$$
\begin{aligned}
-\mathcal{L}_{\beta,4} = & \frac{\tau_+}{16}[(\beta^+_{\mu\rho})^\dagger \beta^{+\rho\nu}][(\beta^{+\mu\sigma})^\dagger \beta^+_{\sigma\nu}] + (+ \to -) \\
& + \frac{\tau_1}{16}[(\beta^+_{\mu\nu})^\dagger \beta^{-\mu\nu}][(\beta^-_{\rho\sigma})^\dagger \beta^{+\rho\sigma}] \\
& + \frac{\tau_2}{16}[(\beta^+_{\mu\nu})^\dagger \vec{\tau} \beta^{-\mu\nu}][(\beta^-_{\rho\sigma})^\dagger \vec{\tau} \beta^{+\rho\sigma}] \\
& + \frac{\tau_3}{64}[(\beta^+_{\mu\nu})^\dagger \beta^{-\mu\nu}][(\beta^+_{\rho\sigma})^\dagger \beta^{-\rho\sigma}] + c.c. \\
& + \frac{\tau_4}{64}[(\beta^+_{\mu\nu})^\dagger \beta^{-\rho\sigma}][(\beta^{+\mu\nu})^\dagger \beta^-_{\rho\sigma}] + c.c. \\
& + \frac{\tau_5}{64}[(\beta^+_{\mu\nu})^\dagger \beta^{-\rho\sigma}][(\beta^+_{\rho\sigma})^\dagger \beta^{-\mu\nu}] + c.c.
\end{aligned}
$$

add gauge interactions and
gauge invariant kinetic term for fermions …

classical dilatation symmetry

- action has no parameter with dimension mass
- all couplings are dimensionless

B - basis

$$\beta^{+}_{jk} = \epsilon_{jkl} B^{+}_{l} \; , \; \beta^{+}_{0k} = i B^{+}_{k}$$
$$\beta^{-}_{jk} = \epsilon_{jkl} B^{-}_{l} \; , \; \beta^{-}_{0k} = -i B^{-}_{k}$$

- B –fields are unconstrained
- six complex doublets
- vectors under space – rotations
- irreducible under Lorentz -transformations

free propagator

$$-\mathcal{L}^{ch}_{\beta,kin} = \Omega^{-1} \int \frac{d^4q}{(2\pi)^4} \{ B_k^{+*}(q) P_{kl}(q) B_l^{+}(q) + B_k^{-*}(q) P_{kl}^{*}(q) B_l^{-}(q) \}$$

inverse propagator has unusual form :

$$P_{kl} = -(q_0^2 + q_j q_j)\delta_{kl} + 2q_k q_l - 2i\epsilon_{klj} q_0 q_j$$

$$P^\dagger = P \ , \ \ PP^* = q^4 \ , \ \ P^{-1} = \frac{1}{q^4} P^*$$

propagator is invertible ! except for pole at $q^2 = 0$

energy density

$$\rho = -T^0_0 = Z_+\{\partial_0 B_k^{+*}\partial_0 B_k^+ + 2\partial_k B_k^{+*}\partial_l B_l^+ \\ -\partial_l B_k^{+*}\partial_l B_k^+\} + (+ \rightarrow -)$$

$$\rho = 2Z_+\partial_k b_3^{+*}\partial_k b_3^+ + (+ \rightarrow -)$$

positive for longitudinal mode

vanishes for transversal modes (borderline to stability)

unstable secular classical solutions in free theory

asymptotic freedom

evolution equations for chiral couplings

$$k\frac{\partial}{\partial k}F_U = -\frac{9}{8\pi^2}F_U F_U^\dagger F_U - \frac{3}{8\pi^2}F_U F_D^\dagger F_D + \frac{1}{4\pi^2}F_U\, tr(F_U^\dagger F_U) - \frac{1}{2\pi^2}g_s^2 F_U$$

$$k\frac{\partial}{\partial k}F_D = -\frac{9}{8\pi^2}F_D F_D^\dagger F_D - \frac{3}{8\pi^2}F_D F_U^\dagger F_U + \frac{1}{4\pi^2}F_D tr(F_D^\dagger F_D + \frac{1}{3}F_L^\dagger F_L) - \frac{1}{2\pi^2}g_s^2 F_D$$

$$k\frac{\partial}{\partial k}F_L = -\frac{9}{8\pi^2}F_L F_L^\dagger F_L + \frac{1}{4\pi^2}F_L\, tr(F_D^\dagger F_D + \frac{1}{3}F_L^\dagger F_L)$$

$$F_U = Z_u^{-1/2}\bar{F}_U Z_q^{-1/2} Z_+^{-1/2}$$

evolution equations for top coupling

$$k\frac{\partial}{\partial k}F_U = -\frac{9}{8\pi^2}F_U F_U^\dagger F_U$$

$$+\frac{1}{4\pi^2}F_U\, tr(F_U^\dagger F_U)$$

fermion anomalous dimension

tensor anomalous dimension

no vertex correction

asymptotic freedom !

Avdeev,Chizhov ;93

dimensional transmutation

$$f_t^2(k) = \frac{4\pi^2}{7\ln(k/\Lambda_{ch}^{(t)})}$$

Chiral coupling for top grows large
at chiral scale Λ_{ch}

This sets physical scale : dimensional transmutation -
similar to Λ_{QCD} in strong QCD- gauge interaction

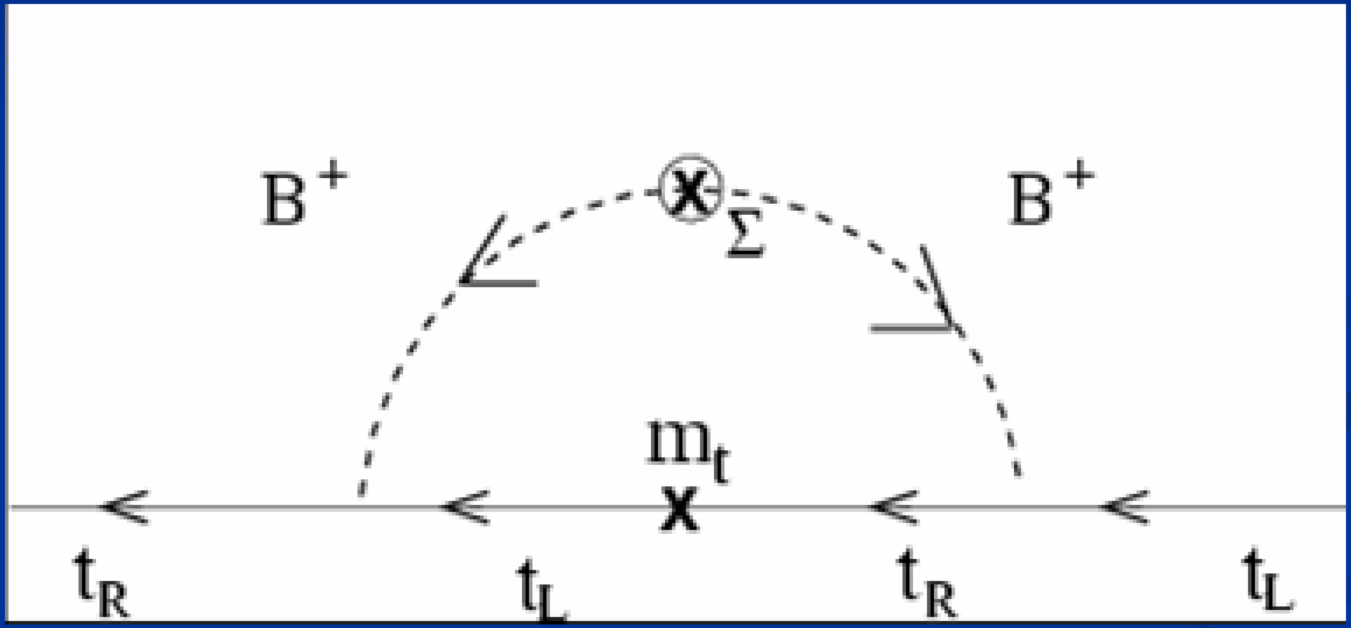
spontaneous electroweak symmetry breaking

top – anti-top condensate

- large chiral coupling for top leads to large effective attractive interaction for top quark
- this triggers condensation of top – anti-top pairs
- electroweak symmetry breaking : effective Higgs mechanism provides mass for weak bosons
- effective Yukawa couplings of Higgs give mass to quarks and leptons

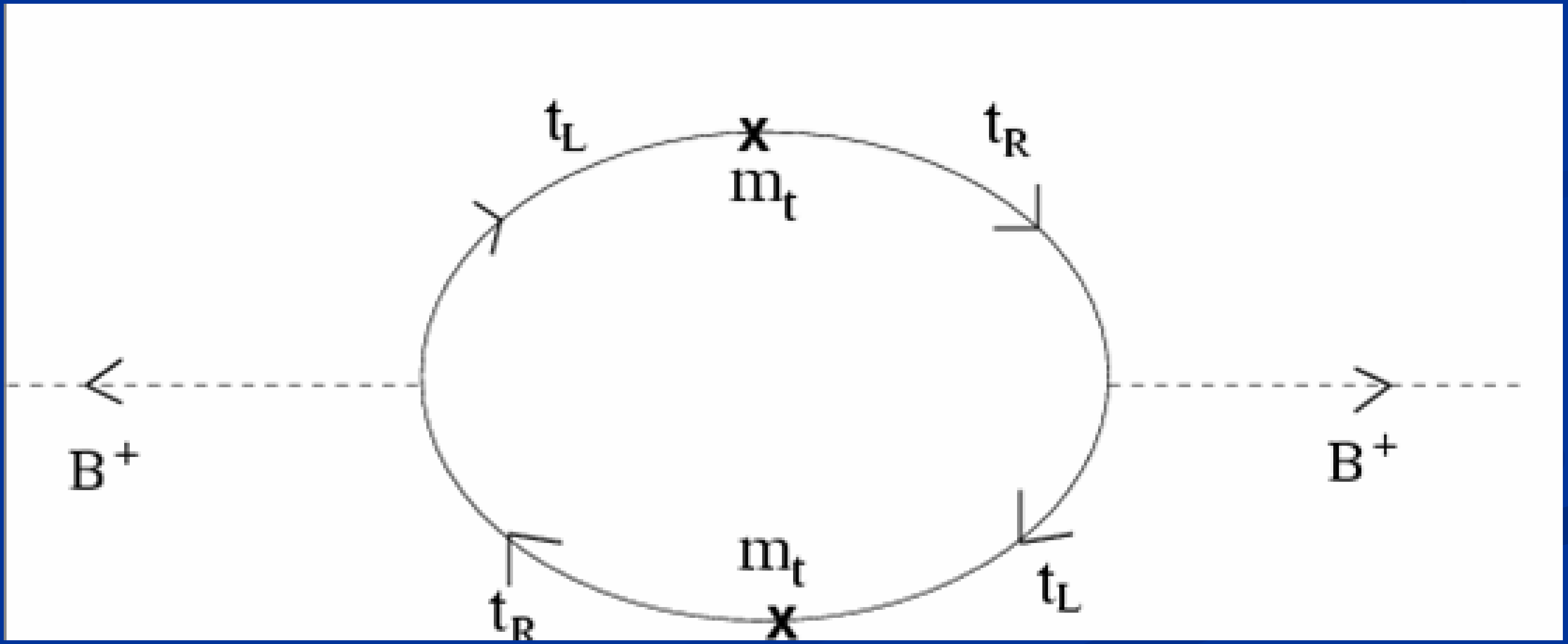
cf : Miranski ; Bardeen, Hill, Lindner

Schwinger - Dyson equation for top quark mass



solve gap equation for top quark propagator

SDE for B-B-propagator



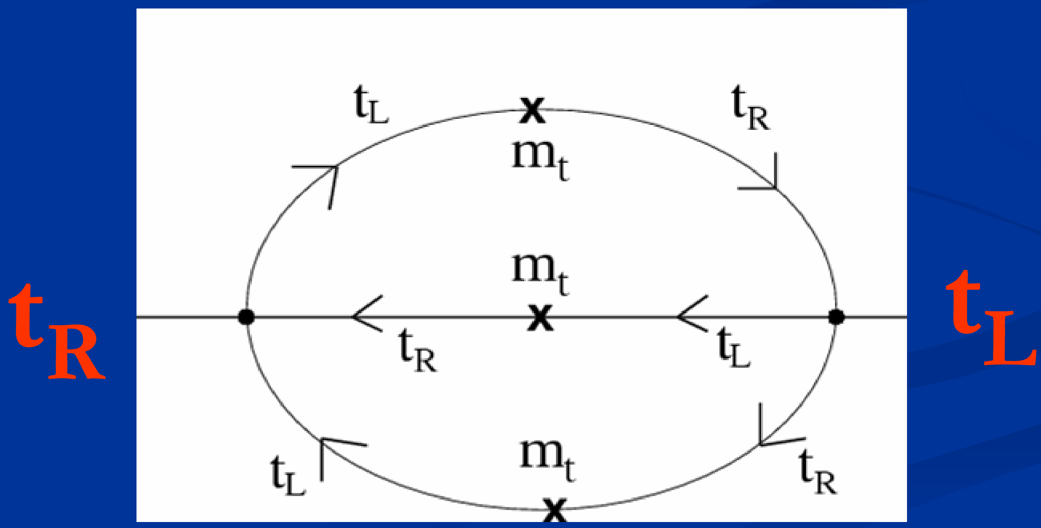
gap equation for top quark mass

$$\int_{M_\beta^2/m_t^2}^{\infty} \frac{dx}{x(x+1)} \frac{\ln\left(\frac{\Lambda_t^4}{m_t^4(1+x/4)^2}+1\right)}{\left(\ln\left(\frac{m_t^2}{\Lambda_{ch^2}^{(t)}}\right)+\ln x\right)^2} = \frac{49}{36}$$

has reasonable solutions for m_t

two loop SDE for top-quark mass

contract B- exchange to pointlike four fermion interaction



effective interactions

- introduce composite field for top- antitop bound state
- plays role of Higgs field
- new effective interactions involving the composite scalar φ

effective scalar tensor interactions

$$
\begin{aligned}
-\mathcal{L}_{M\beta} = \frac{1}{8} tr \{ & \sigma_1 [\varphi_t^\dagger \varphi_b][\bar{\beta}_- \beta_+] + \sigma_2 [\varphi_t^\dagger \beta_+][\bar{\beta}_- \varphi_b] + \\
& + \sigma_+ [\bar{\beta}_+ \varphi_t][\bar{\beta}_+ \varphi_t] + \sigma_- [\bar{\beta}_- \varphi_b][\bar{\beta}_- \varphi_b)] \\
& + \sigma_{v1} [\varphi_b^\dagger \varphi_t][\bar{\beta}_- \beta_+] + \sigma_{v2} [\varphi_b^\dagger \beta_+][\bar{\beta}_- \varphi_t] \\
& + \sigma_{v+} [\bar{\beta}_+ \varphi_b][\bar{\beta}_+ \varphi_b] + \sigma_{v-} [\bar{\beta}_- \varphi_t][\bar{\beta}_- \varphi_t] \} + c.c.
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{8} tr \bar{\beta}_- \beta_+ &= \frac{1}{4} \beta_{\mu\nu}^{-*} \beta^{+\mu\nu} = B_k^{-*} B_k^+ \ , \\
\frac{1}{8} tr \beta_\pm \beta_\pm &= \frac{1}{4} \beta_{\mu\nu}^{\pm} \beta^{\pm\mu\nu} = B_k^{\pm} B_k^{\pm}
\end{aligned}
$$

chiral tensor – gauge boson - mixing

$$
\begin{aligned}
-\mathcal{L}_{F\beta} = {} & \nu_{y+}[\varphi_t^\dagger \beta_{\mu\nu}^+] Y^{\mu\nu} + \nu_{y+}^*[(\beta_{\mu\nu}^+)^\dagger \varphi_t] Y^{\mu\nu} \\
& + \nu_{w+}[\varphi_t^\dagger \vec{\tau} \beta_{\mu\nu}^+] \vec{W}^{\mu\nu} + \nu_{w+}^*[(\beta_{\mu\nu}^+)^\dagger \vec{\tau} \varphi_t] \vec{W}^{\mu\nu} \\
& + \nu_{y-}[\varphi_b^\dagger \beta_{\mu\nu}^-] Y^{\mu\nu} + \nu_{y-}^*[(\beta_{\mu\nu}^-)^\dagger \varphi_b] Y^{\mu\nu} \\
& + \nu_{w-}[\varphi_b^\dagger \vec{\tau} \beta_{\mu\nu}^-] \vec{W}^{\mu\nu} + \nu_{w-}^*[(\beta_{\mu\nu}^-)^\dagger \vec{\tau} \varphi_b] \vec{W}^{\mu\nu}
\end{aligned}
$$

and more …

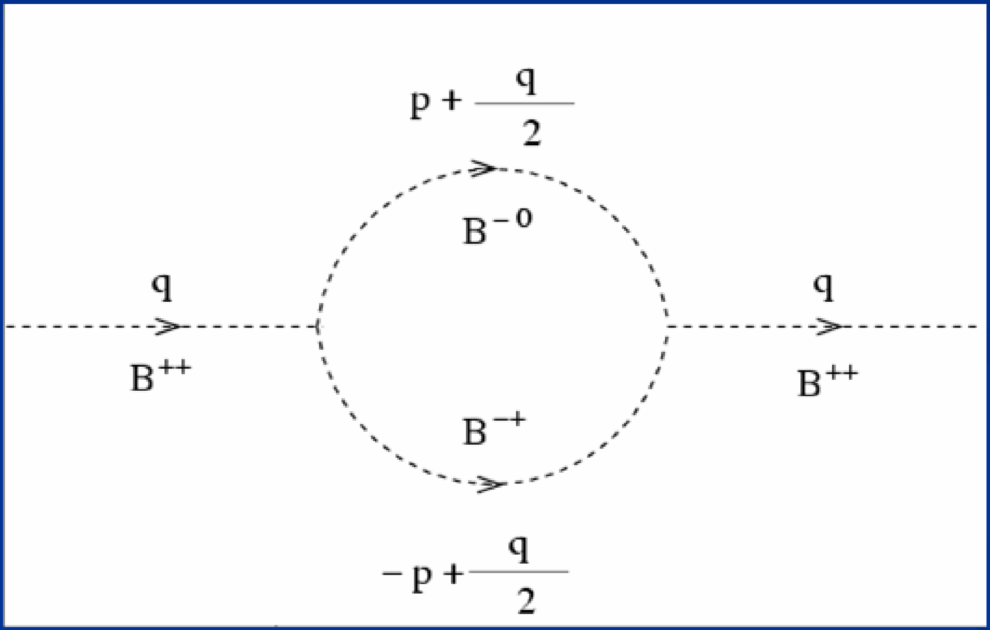
massive chiral tensor fields

effective cubic tensor interactions

$$-\mathcal{L}_{3\beta} = \gamma_t \epsilon_{klm} [\varphi_t^\dagger B_k^-][(B_l^+)^\dagger B_m^-] + \gamma_b \epsilon_{klm} [\varphi_b^\dagger B_k^+][(B_l^-)^\dagger B_m^+] + c.c.$$

generated by electroweak symmetry breaking

propagator corrections from cubic couplings



$$i\,J_{kl}(q) = \frac{1}{16\pi^2}\frac{P_{kl}(q)}{q^2}$$

non – local !

effective propagator for chiral tensors

$$\tilde{P}_{kl}(q) = P_{kl}(q) + i\left(|\gamma_t^* \varphi_t|^2 + |\gamma_b^* \varphi_b|^2\right) J_{kl}(q)$$

$$iJ_{kl}(q) = \frac{1}{16\pi^2} \frac{P_{kl}(q)}{q^2}$$

massive effective inverse propagator : pole for massive field

$$\tilde{P}_{kl}(q) = \frac{P_{kl}(q)}{q^2}(q^2 + m^2)$$

mass term :

$$m^2 = \frac{1}{16\pi^2}\left(|\gamma_t^* \varphi_t|^2 + |\gamma_b^* \varphi_b|^2\right)$$

massive spin one particles

- new basis of vector fields:

$$S^{\pm}_{\mu} = \frac{\partial_\nu}{\sqrt{\partial^2}} \beta^{\pm\nu}{}_{\mu} \ , \ \partial_\mu S^{\pm\mu} = 0$$

- standard action for massive vector fields

$$\Gamma^{ch}_{\beta,kin} = -\int_q Z(q) q_\mu q_\nu (\beta^{\mu\rho}(q))^\dagger \beta^\nu|_\rho(q)$$
$$= \int_q (q^2 + m^2) S^{\mu\dagger}(q) S_\mu(q)$$

- classical stability !

classical stability

- massive spin one fields : stable
- free theory : borderline stability/instability,
  actually unstable (secular solutions , no ghosts)
- mass term moves theory to stable region
- positive energy density

phenomenology

new resonances at LHC ?

- production of massive chiral tensors at LHC ?
- signal : massive spin one resonances
- rather broad : decay into top quarks
- relatively small production cross section : small chiral couplings to lowest generation quarks , no direct coupling to gluons

mixing of charged spin one fields

$$-\mathcal{L}_{F\beta} = -2\sqrt{2}W_d^{-\mu}\sqrt{\partial^2}\left(\frac{\nu_{w+}\varphi_t^*}{\sqrt{Z_+}}S_\mu^{+,+} + \frac{\nu_{w-}\varphi_b^*}{\sqrt{Z_-}}S_\mu^{-,+}\right)$$
$$+c.c.$$

$$\Gamma_C^{(2)} = \begin{pmatrix} q^2 + m_+^2, & \hat{m}^2, & \varepsilon_+^*\sqrt{-q^2} \\ \hat{m}^2, & q^2 + m_-^2, & \varepsilon_-^*\sqrt{-q^2} \\ \varepsilon_+\sqrt{-q^2}, & \varepsilon_-\sqrt{-q^2}, & q^2 + \bar{M}_w^2 \end{pmatrix}$$

- modification of W-boson mass
- similar for Z – boson
- **watch LEP – precision tests !**

mixing between chiral tensor and photon

$$\Gamma_c^{(2)} = \begin{pmatrix} q^2 + m_R^2 \,, & \beta\sqrt{-q^2} \\ \beta\sqrt{-q^2} \,, & q^2 \end{pmatrix} .$$

$$\det = q^2(q^2 + m_R^2 + \beta^2)$$

$$-\mathcal{L}_{ch} \to \alpha_\gamma \bar{e}_L F_L^\dagger \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c.$$

Pauli term contributes to g-2

$$-\mathcal{L}_{ch} \to \alpha_\gamma \bar{e}_L F_L^\dagger \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c.$$

suppressed by

- inverse mass of chiral tensor
- small chiral coupling of muon and electron
- small mixing between chiral tensor and photon

effective interactions from chiral tensor exchange

$$
\begin{aligned}
-\mathcal{L} = {} & (J^{+\mu})^\dagger S^+_\mu + (J^{-\mu})^\dagger S^-_\mu + h.c. \\
& + (\partial^\mu S^{+\nu})^* \partial_\mu S^+_\nu + (\partial^\mu S^{-\nu})^* \partial_\mu S^-_\nu \\
& + m_+^2 (S^\mu_+)^* S_{+\mu} + m_-^2 (S^\mu_-)^* S_{-\mu} \\
& + \hat{m}^2 ((S^\mu_+)^* S_{-\mu} + (S^\mu_-)^* S_{+\mu})
\end{aligned}
$$

$$
\begin{aligned}
(J^{+\mu})^\dagger &= \epsilon_+ \sqrt{\partial^2} W^{\mu *} + \frac{\partial_\nu}{\sqrt{\partial^2}} \bar{u}_R F_U \sigma^{\nu\mu} d_L \\
(J^{-\mu})^\dagger &= \epsilon_- \sqrt{\partial^2} W^{\mu *} \\
& \quad + \frac{\partial_\nu}{\sqrt{\partial^2}} (\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R)
\end{aligned}
$$

- solve for S_μ in presence of other fields
- reinsert solution

general solution

$$-\mathcal{L} = -(J^{\beta\mu})^\dagger G^{\beta\alpha} J^\alpha_\mu$$

$$(J^{+\mu})^\dagger = \epsilon_+ \sqrt{\partial^2} W^{\mu *} + \frac{\partial_\nu}{\sqrt{\partial^2}} \bar{u}_R F_U \sigma^{\nu\mu} d_L$$
$$(J^{-\mu})^\dagger = \epsilon_- \sqrt{\partial^2} W^{\mu *} + \frac{\partial_\nu}{\sqrt{\partial^2}} (\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R)$$

propagator for charged chiral tensors

$$G = (-\partial^2 + m_{R1}^2)^{-1} (-\partial^2 + m_{R2}^2)^{-1} \begin{pmatrix} -\partial^2 + m_-^2 \,, & -\hat{m}^2 \\ -\hat{m}^2 & , -\partial^2 + m_+^2 \end{pmatrix}$$

momentum dependent Weinberg angle

$$\frac{g^2}{M_W^2 + q^2} \longrightarrow \frac{g^2}{\bar{M}_W^2 + q^2(1 + p_W(q^2))} = \frac{g_{eff}^2(q^2)}{M_W^2 + q^2}$$

new four fermion interactions

$$
\begin{aligned}
-\mathcal{L}_{4Fch} = & -\{\bar{u}_R F_U \sigma^{\nu\mu} d_L\} G^{++}(-\partial^2) \\
& \tilde{P}_{\nu\rho}\{\bar{d}_L F_U^\dagger \sigma^\rho{}_\mu u_R\} \\
& -\{\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R\} G^{--}(-\partial^2) \\
& \tilde{P}_{\nu\rho}\{\bar{d}_R F_D \sigma^\rho{}_\mu u_L + \bar{e}_R F_L \sigma^\rho{}_\mu \nu_L\} \\
& -\{\bar{u}_R F_U \sigma^{\nu\mu} d_L\} G^{+-}(-\partial^2) \\
& \tilde{P}_{\nu\rho}\{\bar{d}_R F_D \sigma^\rho{}_\mu u_L + \bar{e}_R F_L \sigma^\rho{}_\mu \nu_L\} \\
& -\{\bar{u}_L F_D^\dagger \sigma^{\nu\mu} d_R + \bar{\nu}_L F_L^\dagger \sigma^{\nu\mu} e_R\} G^{-+}(-\partial^2) \\
& \tilde{P}_{\nu\rho}\{\bar{d}_L F_U^\dagger \sigma^\rho{}_\mu u_R\}
\end{aligned}
$$

typically rather small effect for lower generations
more substantial for bottom , top !

mixing of chiral tensors with ρ - meson

$$\tilde{P}_{\nu\rho} = \frac{\partial_\nu \partial_\rho}{\partial^2} \; , \; \tilde{P}_{\nu\rho} \tilde{P}^{\rho}{}_{\mu} = \tilde{P}_{\nu\mu}$$

$$-\mathcal{L}^{(\rho)}_{4F2} = -\kappa^{(\rho)} \partial_\nu (\bar{\nu}_L \sigma^{\mu\nu} e_R)(\bar{d}\gamma_\mu u) + c.c.$$

$$\frac{\kappa^{(\rho)} q}{G_F} \sim \frac{\nu_\rho f_e g_\rho}{g^3} \frac{M_W^3 M_\pi}{M_{ch}^2 M_\rho^2}$$

could contribute to anomaly in radiative pion decays

conclusions

- chiral tensor model has good chances to be consistent
- mass generation needs to be understood quantitatively
- interesting solution of gauge hierarchy problem
- phenomenology needs to be explored !
- if quartic couplings play no major role:
- less couplings than in standard model → Predictivity !

end