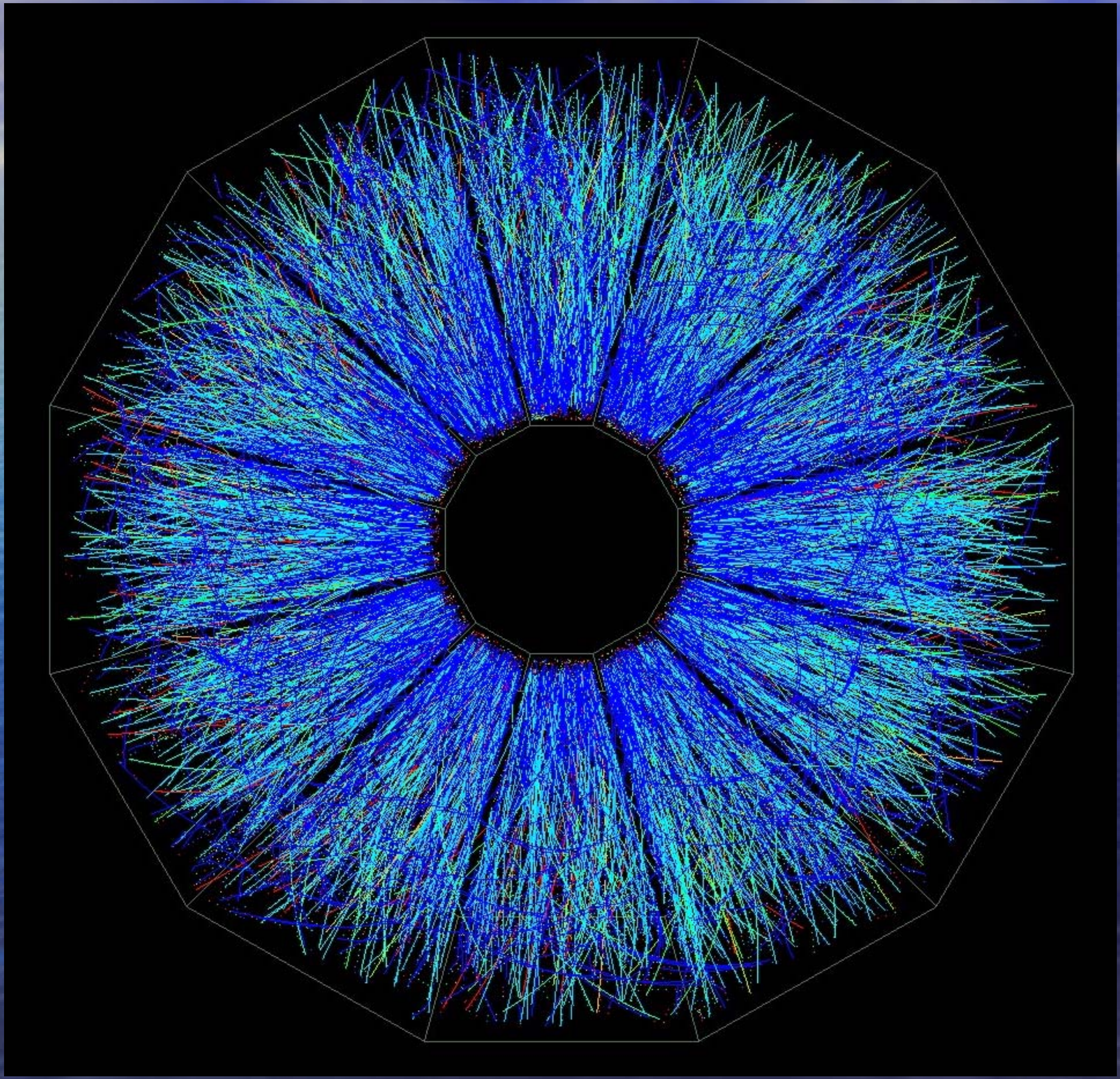
Has the critical temperature of the QCD phase transition been measured ?

Heavy ion collision



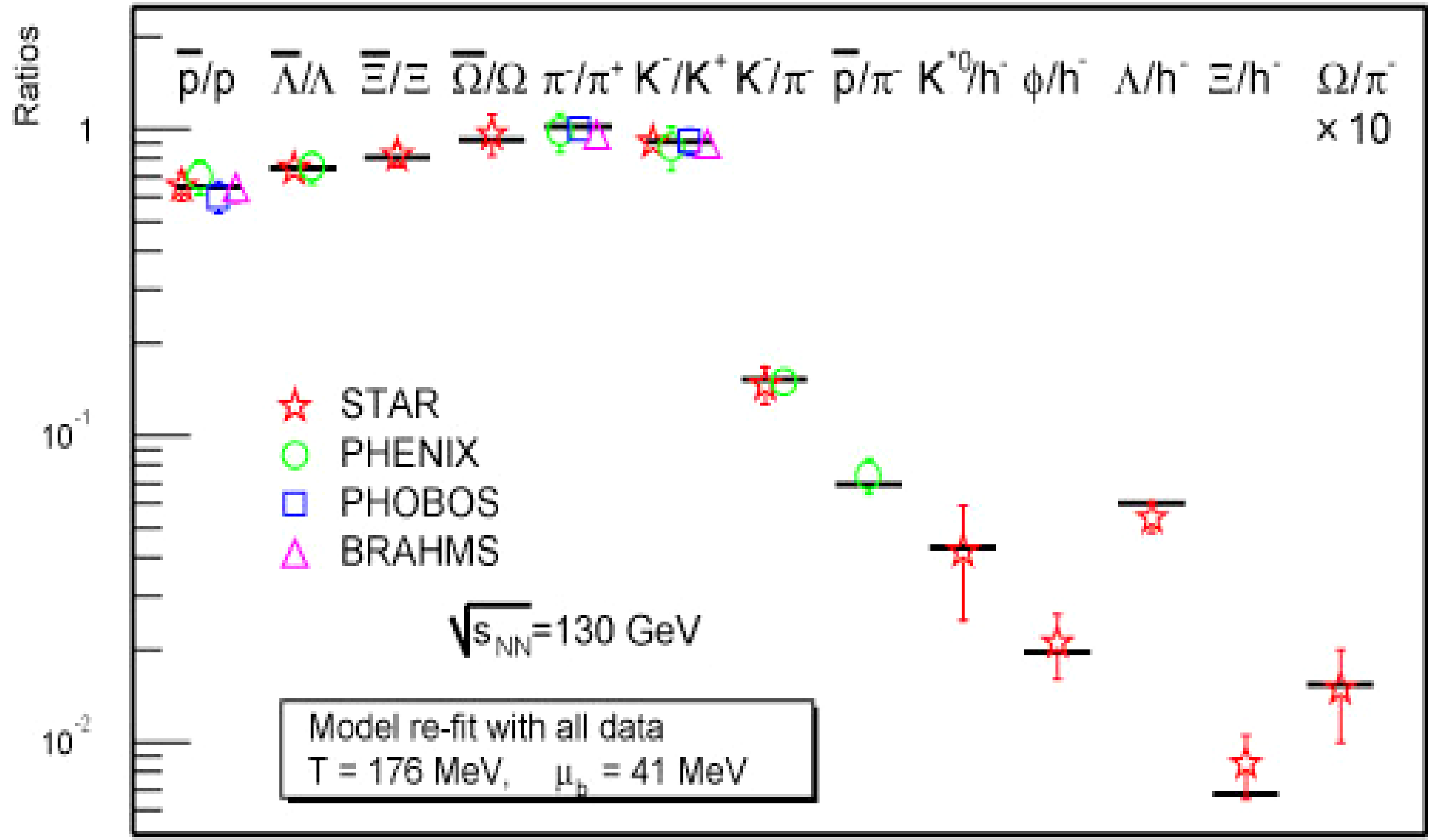
Yes !

$0.95\ T_c < T_{ch} < T_c$

- not : " I have a model where $T_c \approx T_{ch}$ "
- not : " I use T_c as a free parameter and find that in a model simulation it is close to the lattice value (or T_{ch}) "

$T_{ch} \approx 176$ MeV (?)

Hadron abundancies



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Has T_c been measured ?

- Observation : statistical distribution of hadron species with "chemical freeze out temperature " T_{ch}=176 MeV
- T_{ch} cannot be much smaller than T_c : hadronic rates for $T< T_c$ are too small to produce multistrange hadrons (Ω,..)
- Only near T_c multiparticle scattering becomes important (collective excitations …) – proportional to high power of density

⇒ $T_{ch} \approx T_c$

P.Braun-Munzinger,J.Stachel,CW

Exclusion argument

Assume T is a meaningful concept - complex issue, to be discussed later

$T_{ch} < T_c$: hadrochemical equilibrium

Exclude T_{ch} much smaller than T_c :
say $T_{ch} > 0.95\ T_c$

$$0.95 < T_{ch} / T_c < 1$$

Estimate of critical temperature

For $T_{ch} \approx 176$ MeV :

$0.95 < T_{ch}/T_c$

- $176 \text{ MeV} < T_c < 185 \text{ MeV}$

$0.75 < T_{ch}/T_c$

- $176 \text{ MeV} < T_c < 235 \text{ MeV}$

Quantitative issue matters!

needed :

lower bound on

T_{ch} / T_c

Key argument

- Two particle scattering rates not sufficient to produce Ω
- “multiparticle scattering for Ω-production “ : dominant only in immediate vicinity of T_c

Mechanisms for production of multistrange hadrons

Many proposals

- Hadronization
- Quark-hadron equilibrium
- Decay of collective excitation (σ – field)
- Multi-hadron-scattering

Different pictures !

Hadronic picture of Ω - production

Should exist, at least semi-quantitatively, if $T_{ch} < T_c$

(for $T_{ch} = T_c$: $T_{ch} > 0.95\ T_c$ is fulfilled anyhow)

e.g. collective excitations ≈ multi-hadron-scattering

(not necessarily the best and simplest picture)

multihadron -> Ω + X should have sufficient rate

Check of consistency for many models

Necessary if $T_{ch} \neq T_c$ and temperature is defined

Way to give quantitative bound on T_{ch} / T_c

Rates for multiparticle scattering

2 pions + 3 kaons -> Ω + antiproton

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi.$$

Very rapid density increase

…in vicinity of critical temperature

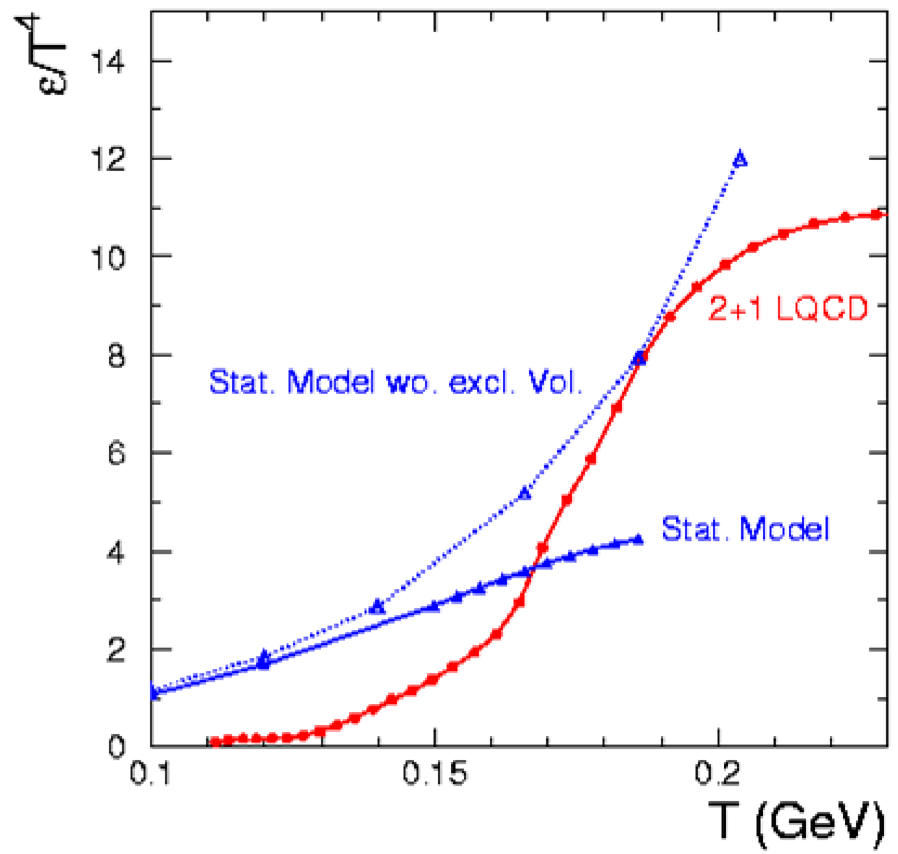
Extremely rapid increase of rate of multiparticle scattering processes

(proportional to very high power of density)

Energy density

Lattice simulations
Karsch et al

even more dramatic
for first order
transition



Phase space

- increases very rapidly with energy and therefore with temperature
- effective dependence of time needed to produce Ω

$$\tau_{\Omega} \sim T^{-60} \ !$$

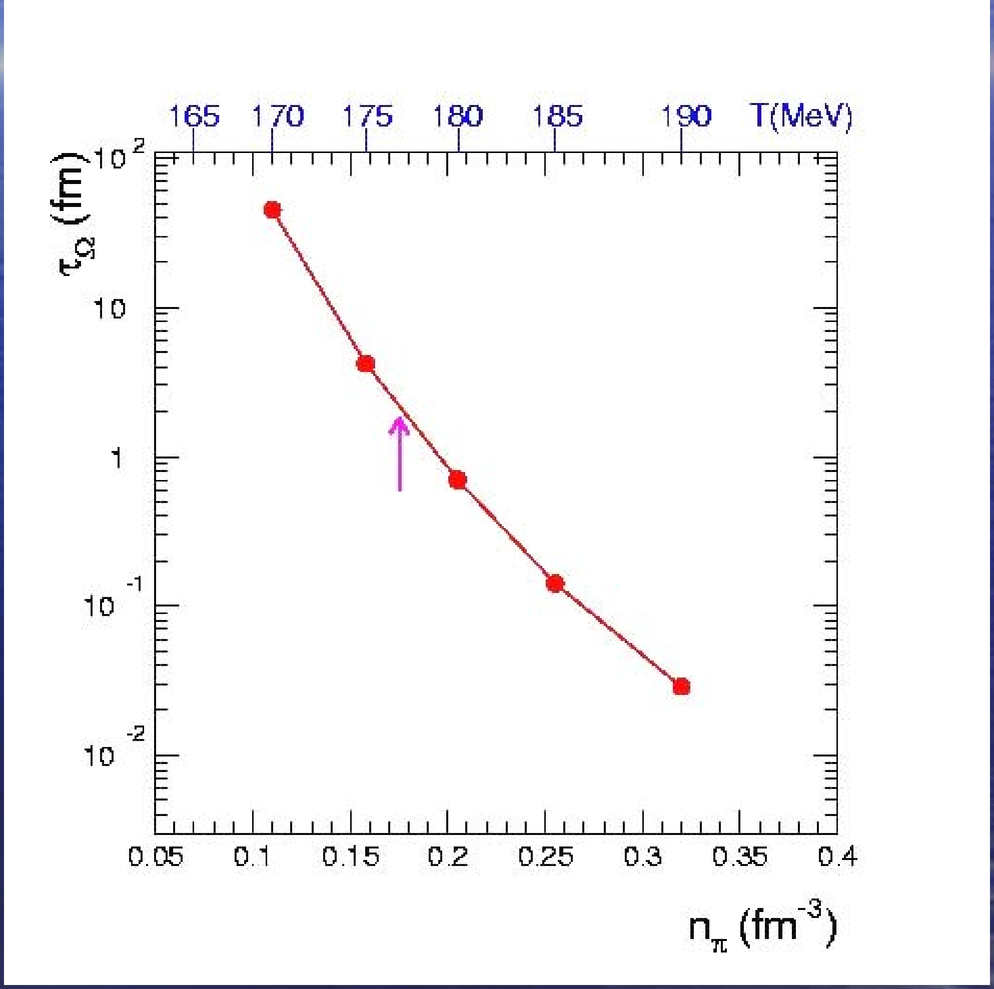
This will even be more dramatic if transition is closer to first order phase transition

Production time for Ω

multi-meson scattering

п+п+п+K+K ->
Ω+p

strong dependence on pion density



P.Braun-Munzinger,J.Stachel,CW

enough time for Ω - production

at T=176 MeV :

$$\tau_\Omega \sim 2.3 \text{ fm}$$

consistency !

extremely rapid change

lowering T by 5 MeV below critical temperature :

rate of Ω – production decreases by factor 10

This restricts chemical freeze out to close vicinity of critical temperature

$$0.95 < T_{ch} / T_c < 1$$

Relevant time scale in hadronic phase

rates needed for equilibration of Ω and kaons:

$$\bar{r}_j = \frac{\dot{N}_j}{V} = \dot{n}_j + n_j \dot{V}/V.$$

$$\left| \frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K} \right| = \frac{\ln F_{\Omega K}}{\tau_T} \frac{\mathrm{T_{ch}}}{\Delta \mathrm{T}} = (1.10\text{–}0.55)/\mathrm{fm}$$

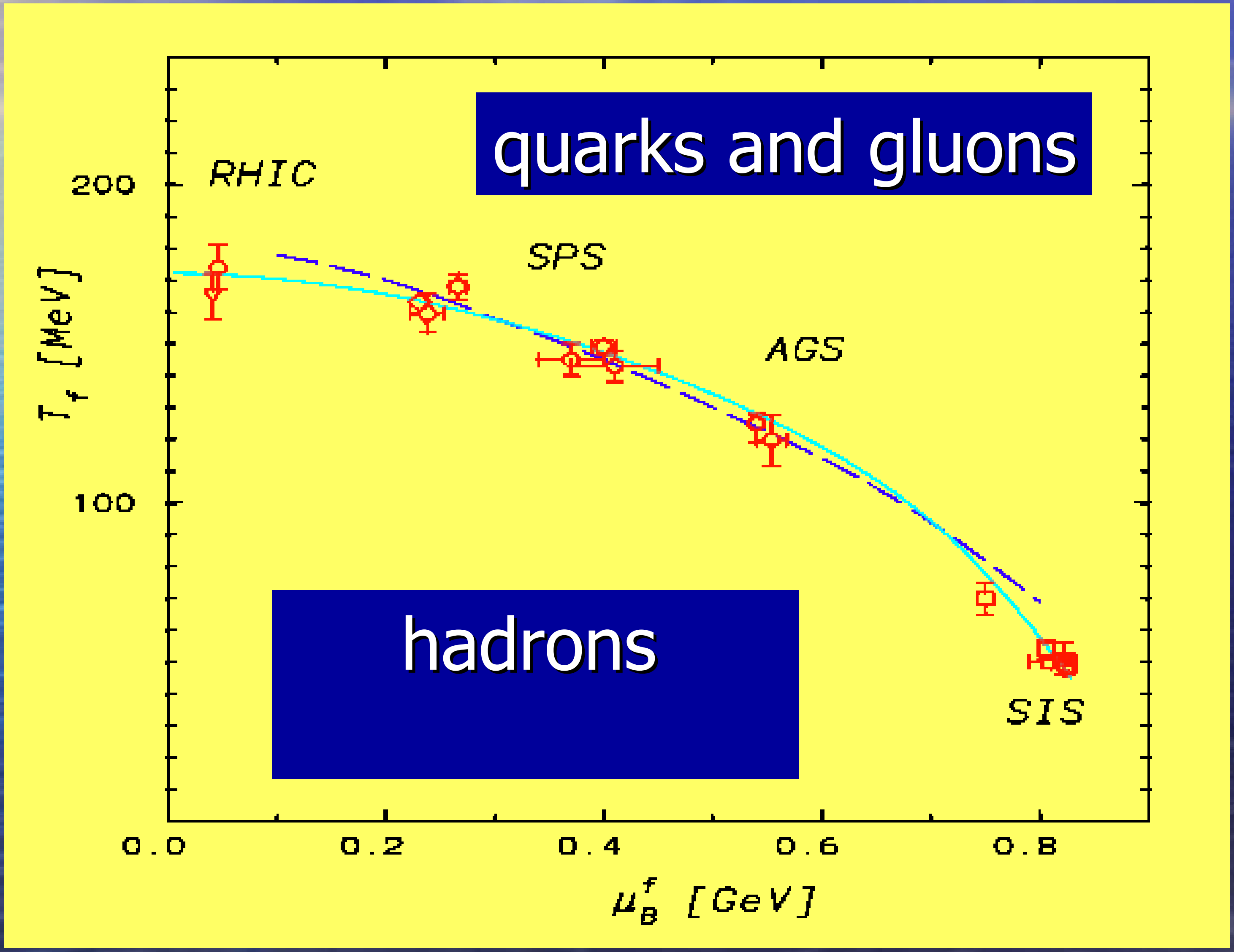
ΔT = 5 MeV,
$F_{\Omega K}$ = 1.13 ,
τ_T =8 fm

two –particle – scattering :

$$\left| \frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K} \right| = (0.02\text{-}0.2)/\mathrm{fm}$$

$$T_{ch} \approx T_c$$

Phase diagram

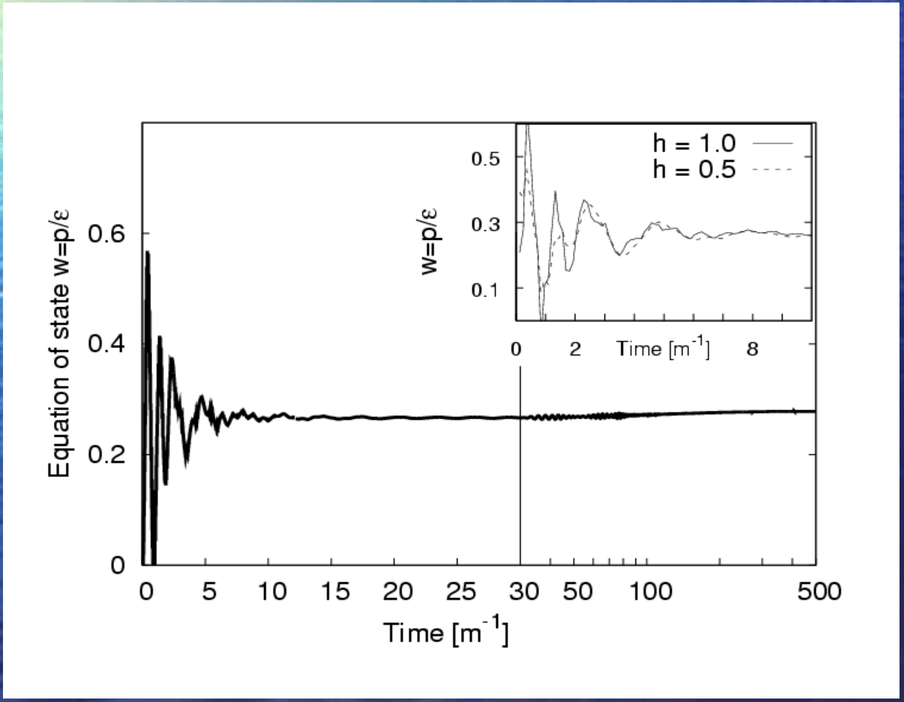


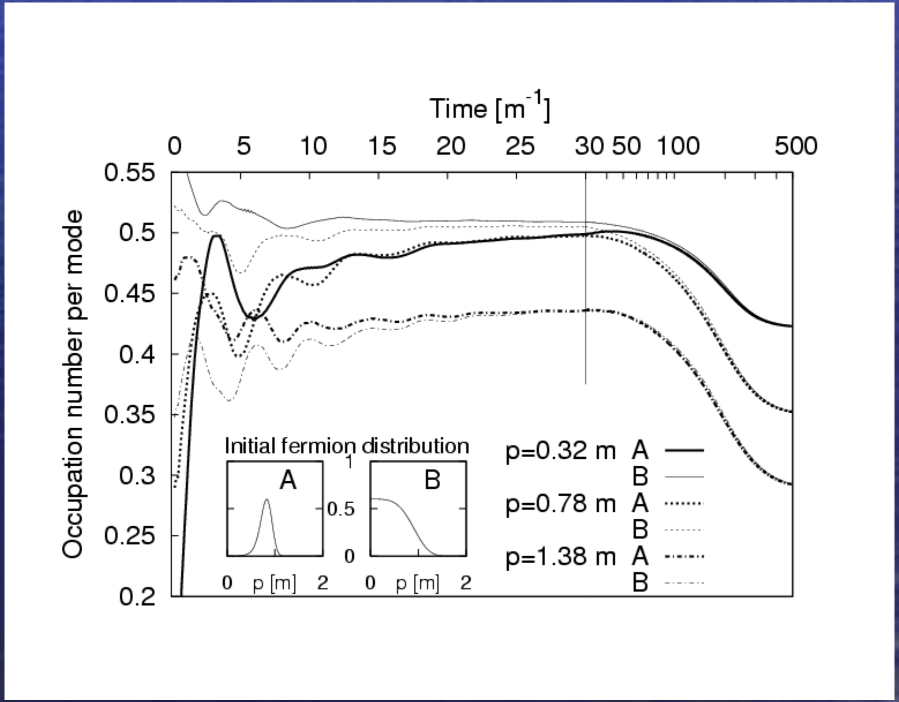
Is temperature defined ?

Does comparison with equilibrium critical temperature make sense ?

Prethermalization

J.Berges,Sz.Borsanyi,CW





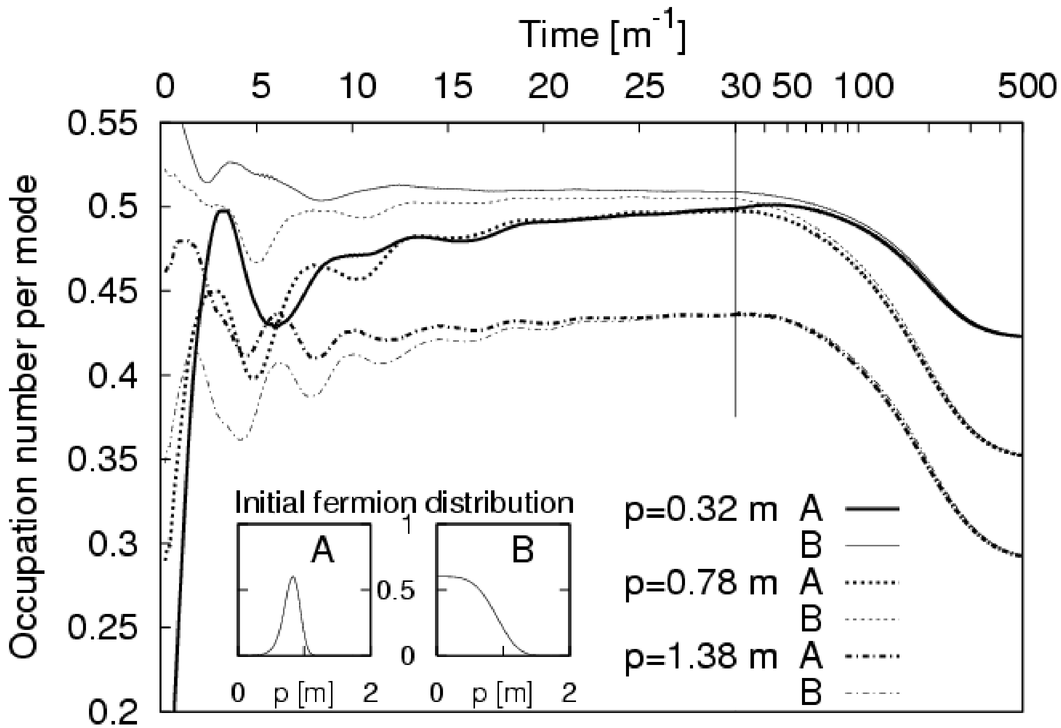
Vastly different time scales

for “thermalization” of different quantities

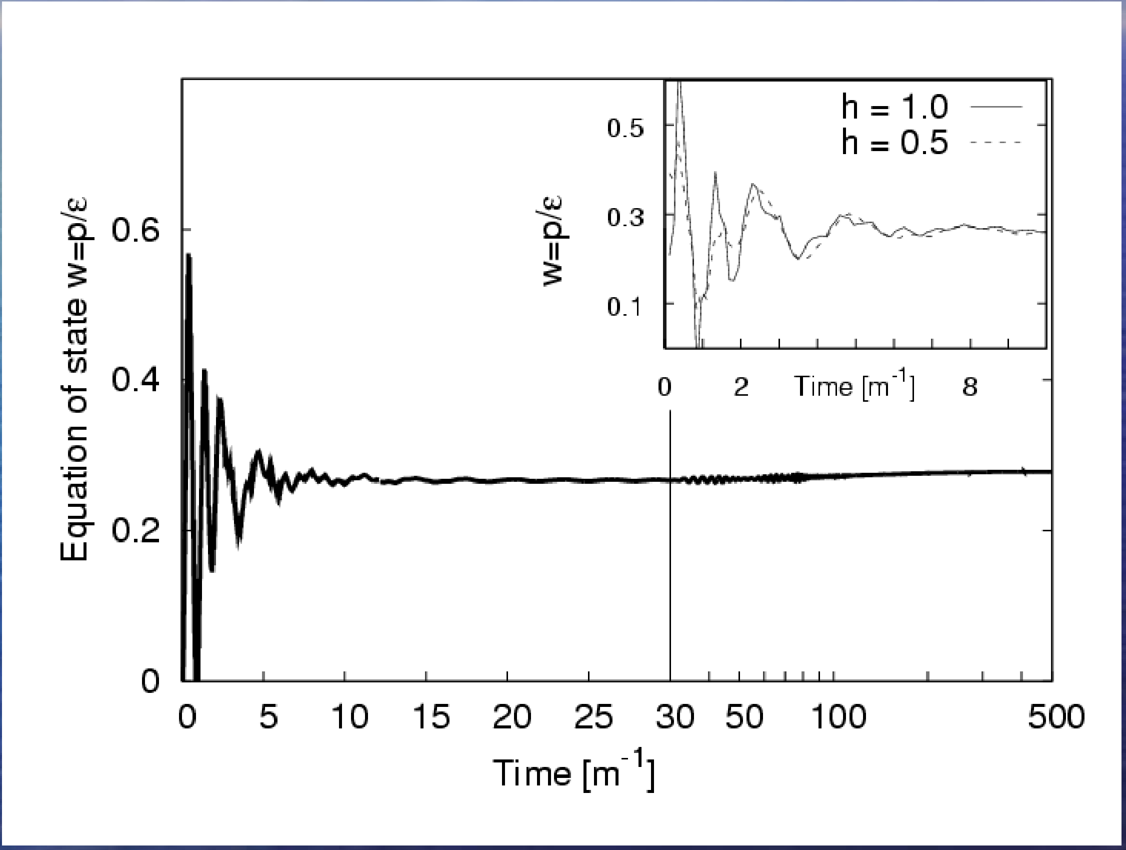
here : scalar with mass m coupled to fermions
(linear quark-meson-model)

method : two particle irreducible non-equilibrium effective action (J.Berges et al)

Thermal equilibration : occupation numbers



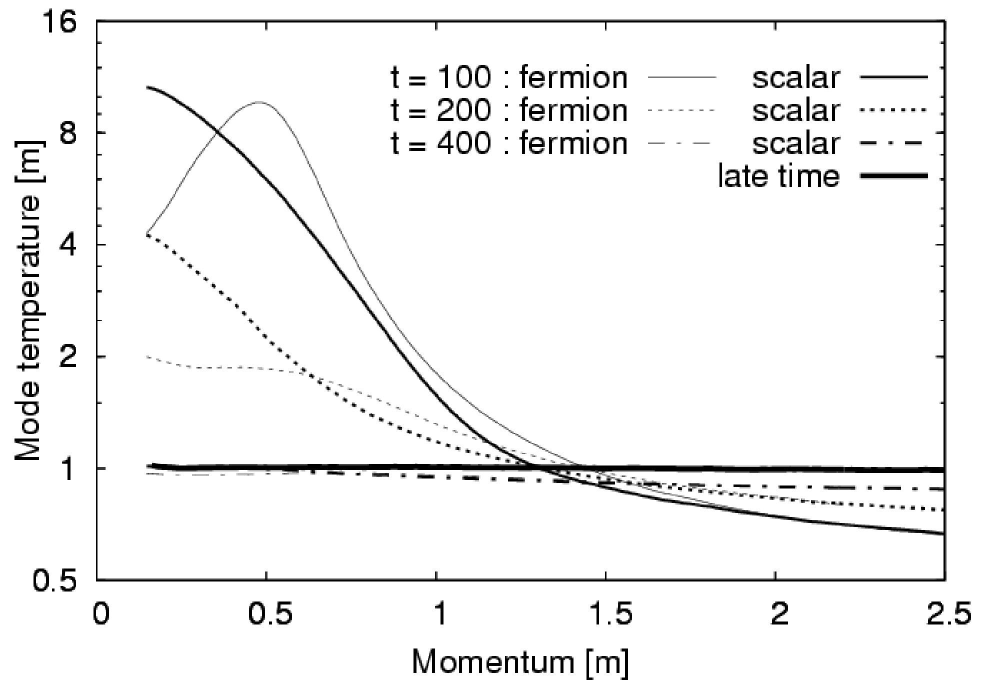
Prethermalization equation of state p/ε



similar for kinetic temperature

different “temperatures”

Mode temperature



$$n_p(t) \stackrel{!}{=} \frac{1}{\exp\left[\omega_p(t)/T_p(t)\right] \pm 1}$$

$\omega_p^{(f,s)}(t)$ determined by peak of spectral function

n_p :occupation number for momentum p

late time:
Bose-Einstein or Fermi-Dirac distribution

Global kinetic temperature $T_{\rm kin}$

Practical definition:

- association of temperature with average kinetic energy per d.o.f.

$$T_{\rm kin}(t) = E_{\rm kin}(t)/c_{\rm eq}$$

- $c_{\rm eq} = E_{\rm kin,eq}/T_{\rm eq}$ is given solely in terms of equilibrium quantities
  (E.g. relativistic plasma: $E_{\rm kin}/N = \epsilon/n = \alpha T$)

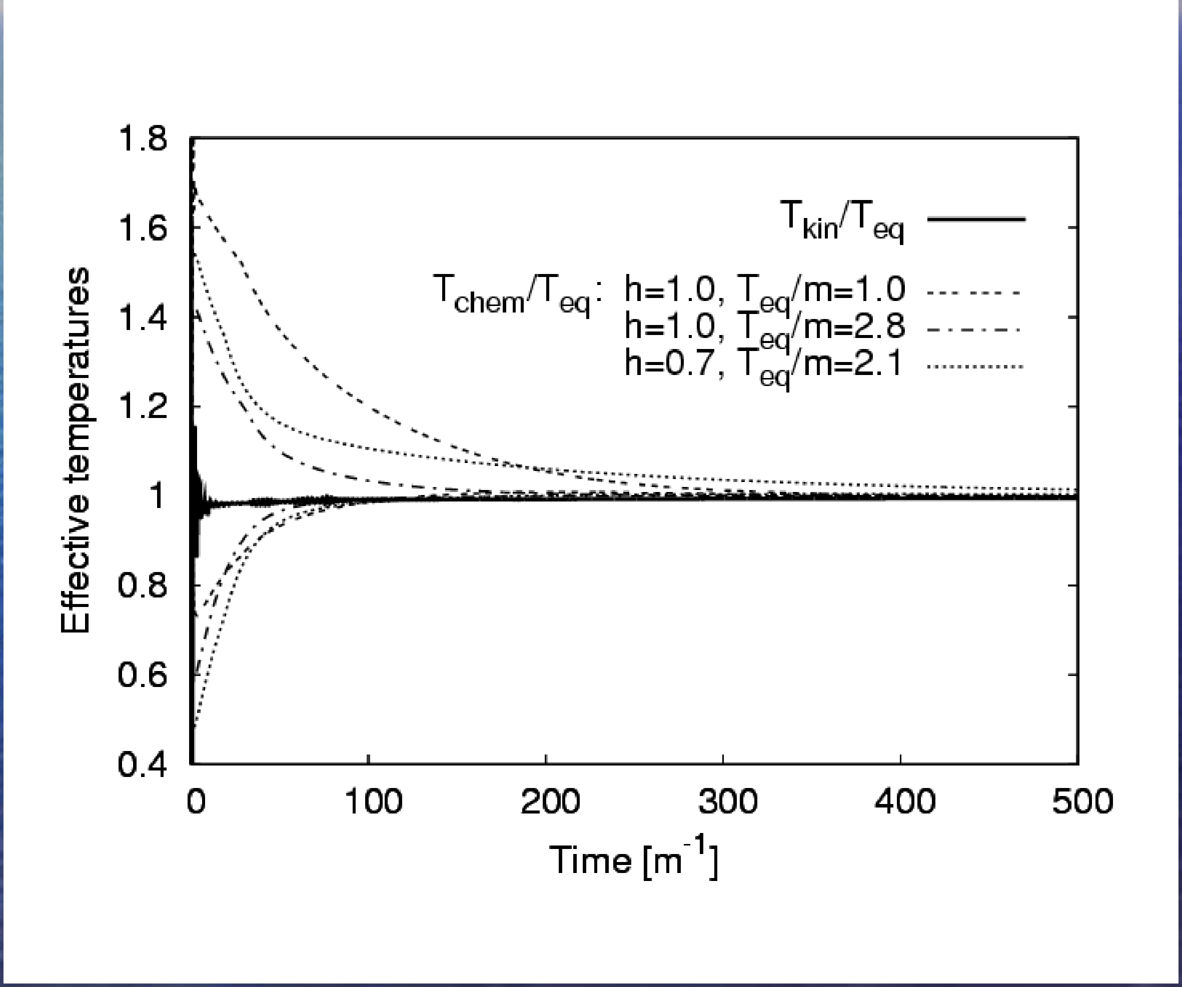
$$\boxed{\text{Kinetic equilibration:} \quad T_{\rm kin}(t) \; = \; T_{\rm eq}}$$

Consider also *chemical temperatures* $T_{\rm ch}^{(f,s)}$ from integrated number density of each species, $n^{(f,s)}(t) = g^{(f,s)} \int {\rm d}^3p/(2\pi)^3 \, n_p^{(f,s)}(t)$:

$$n(t) \stackrel{!}{=} \frac{g}{2\pi^2} \int_0^\infty {\rm d}p p^2 \left[\exp\left(\omega_p(t)/T_{\rm ch}(t)\right) \pm 1\right]^{-1}$$

$$\boxed{\text{Chemical equilibration:} \quad T_{\rm ch}^{(f)}(t) \; = \; T_{\rm ch}^{(s)}(t)}$$

Kinetic equilibration before chemical equilibration



Once a temperature becomes stationary it takes the value of the equilibrium temperature.

Once chemical equilibration has been reached the chemical temperature equals the kinetic temperature and can be associated with the overall equilibrium temperature.

Comparison of chemical freeze out temperature with critical temperature of phase transition makes sense

A possible source of error : temperature-dependent particle masses

Chiral order parameter σ depends on T

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

$$\frac{\sigma(\mathrm{T}_{\mathrm{ch}},\mu)}{\mathrm{T}_{\mathrm{ch}}} = \frac{\sigma(0,0)}{\mathrm{T}_{\mathrm{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$

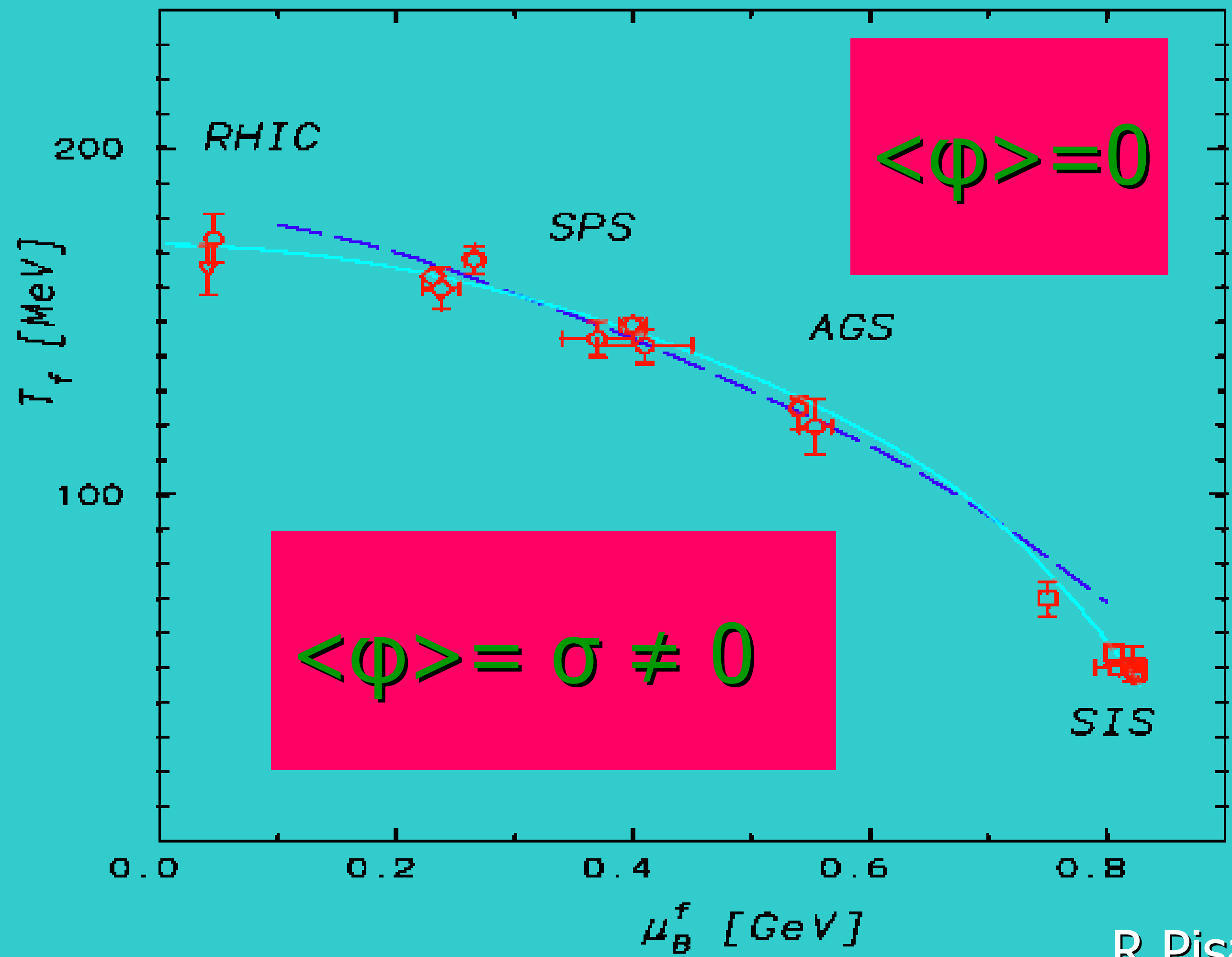
chemical freeze out measures T/m !

uncertainty in m(T)

→

uncertainty in critical temperature

Phase diagram

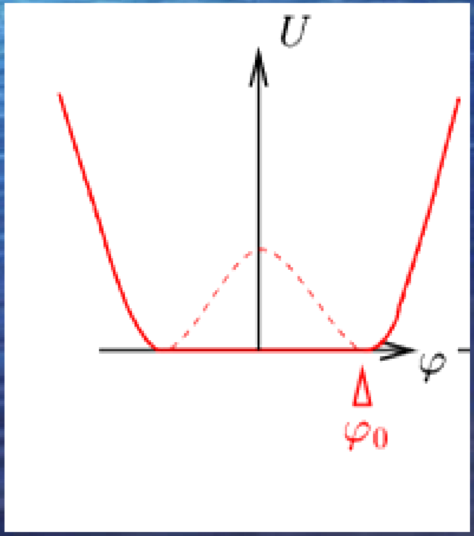


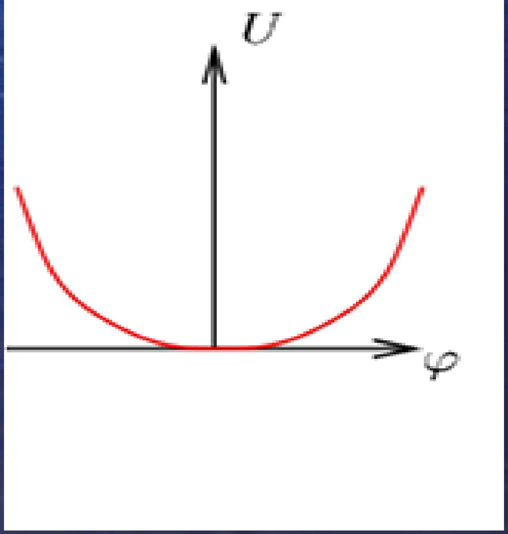
R.Pisarski

Chiral symmetry restoration at high temperature

Low T
SSB
$<\varphi>=\varphi_0 \neq 0$

High T
SYM
$<\varphi>=0$





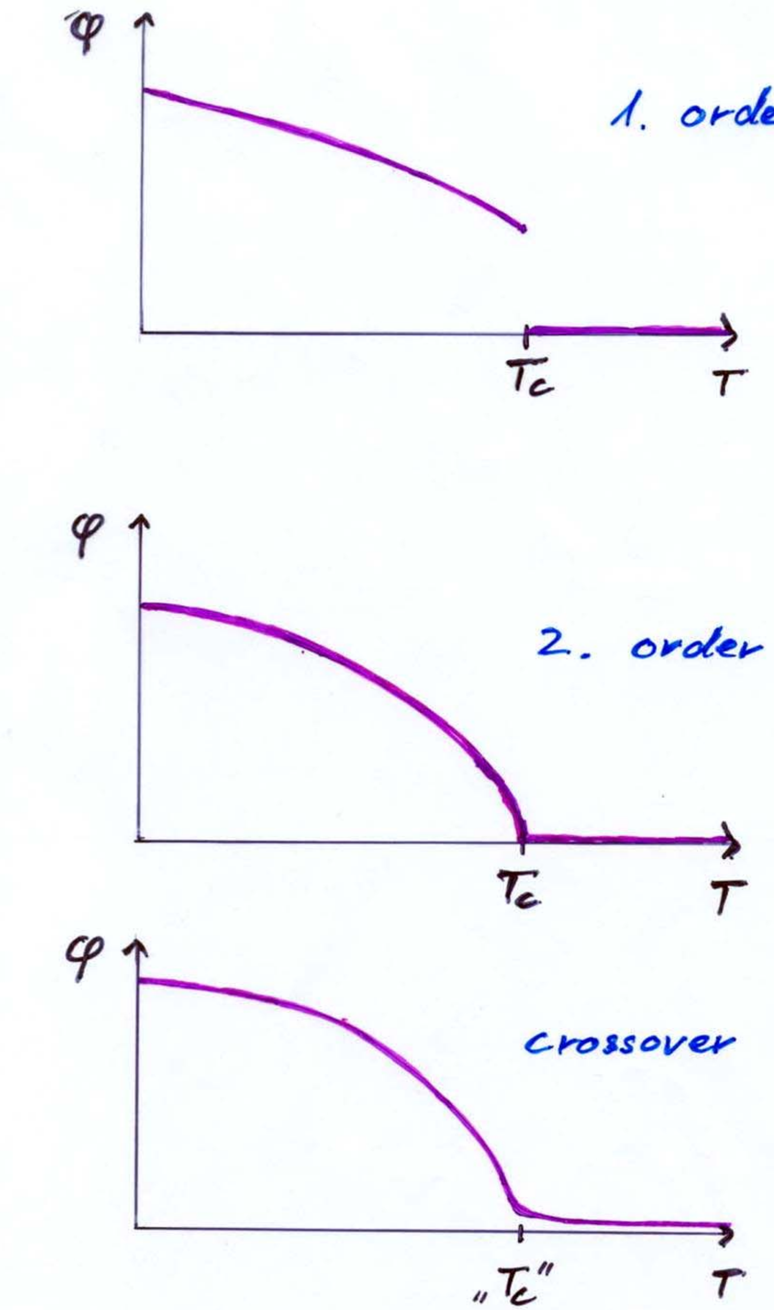
at high T :
less order
more symmetry

examples:

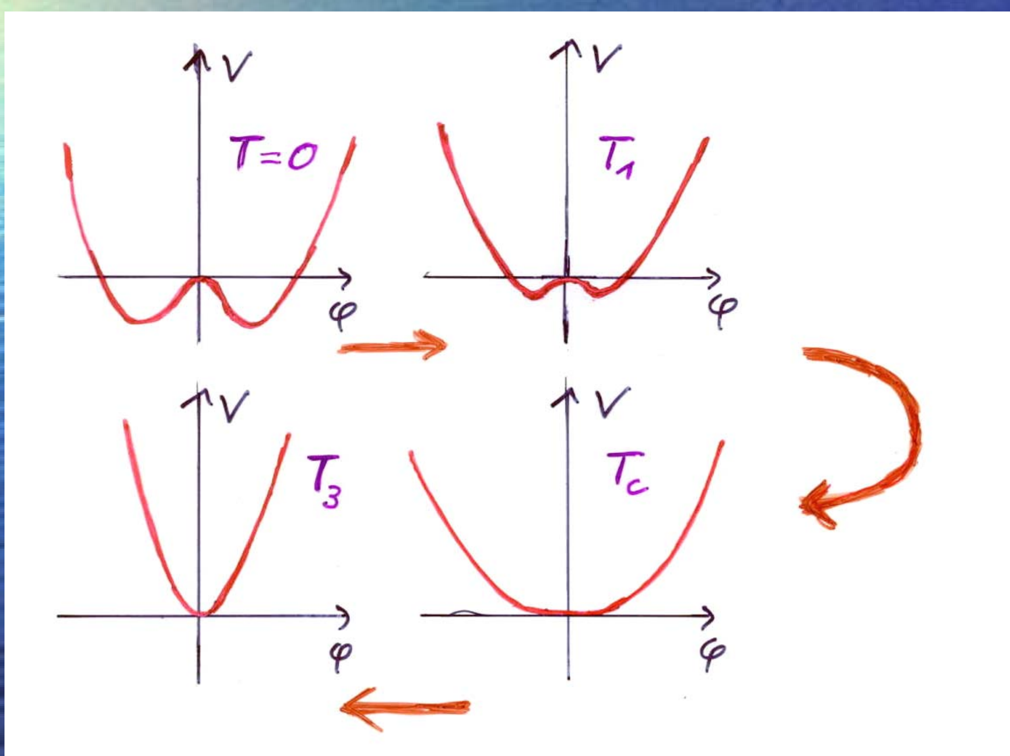
magnets, crystals

Order of the phase transition is crucial ingredient for experiments (heavy ion collisions)
and cosmological phase transition

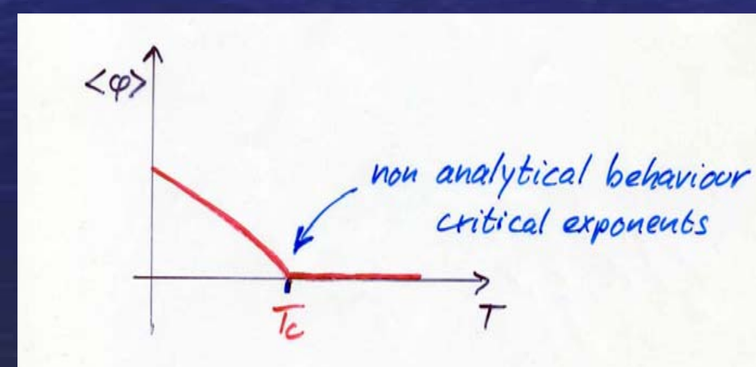
Order of the phase transition



Second order phase transition



$$V = -\mu_0^2\,\varphi^\dagger\varphi + cT^2\,\varphi^\dagger\varphi + \frac{\lambda}{2}(\varphi^\dagger\varphi)$$



second order phase transition

for T only somewhat below T_c :

the order parameter σ is expected to deviate substantially from its vacuum value

This seems to be disfavored by observation of chemical freeze out !

Ratios of particle masses and chemical freeze out

at chemical freeze out :

- ratios of hadron masses seem to be close to vacuum values
- nucleon and meson masses have different characteristic dependence on σ
- $m_{nucleon} \sim \sigma$, $m_{\pi} \sim \sigma^{-1/2}$
- $\Delta\sigma/\sigma < 0.1$ (conservative)

systematic uncertainty :

$$\Delta\sigma/\sigma=\Delta T_c/T_c$$

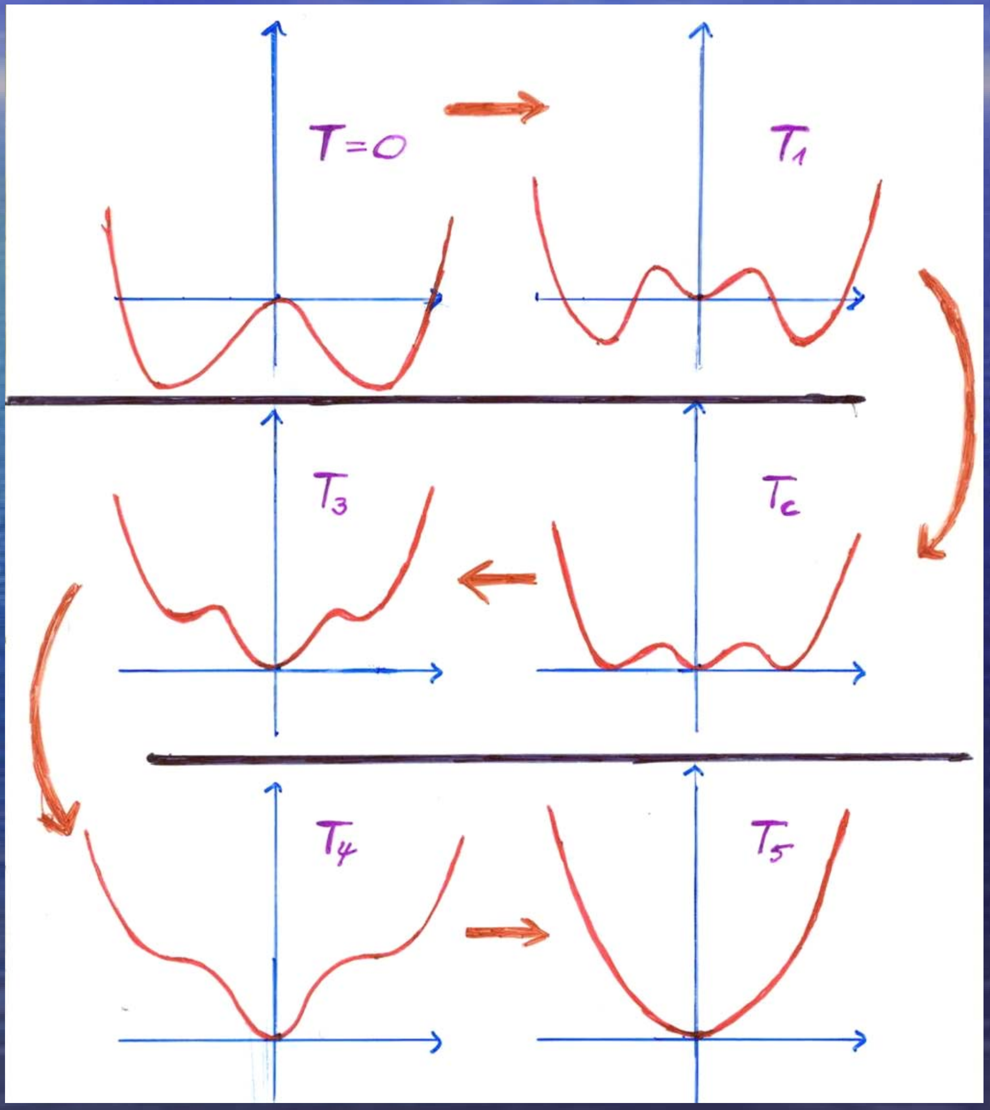
$\Delta\sigma$ is negative

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

$$\frac{\sigma(\mathrm{T_{ch}},\mu)}{\mathrm{T_{ch}}} = \frac{\sigma(0,0)}{\mathrm{T_{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$

First order phase transition



first order phase transition seems to be favored by chemical freeze out

Lattice results

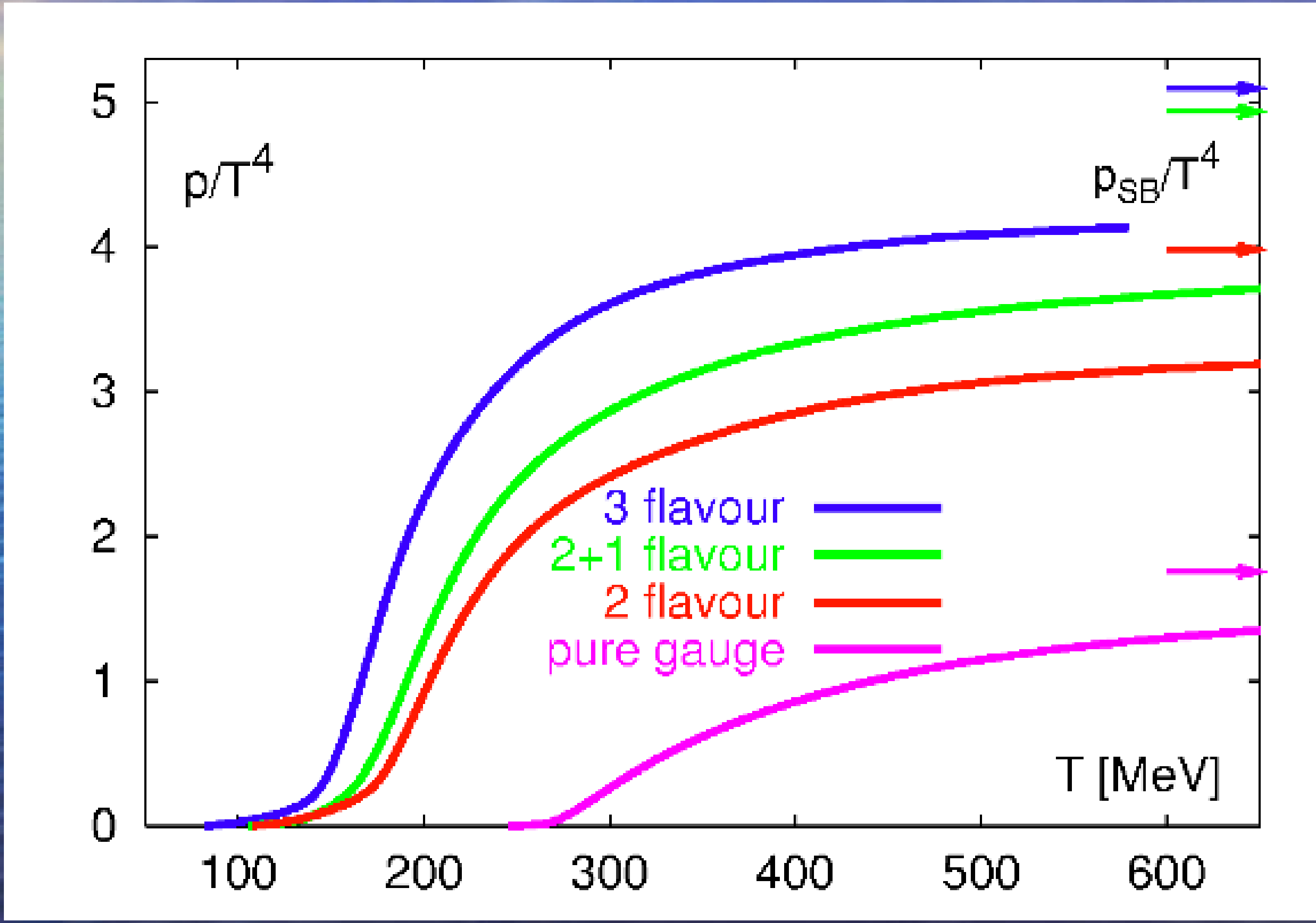
e.g. Karsch,Laermann,Peikert

Critical temperature in chiral limit :

$N_f = 3$: $T_c = (154 \pm 8)$ MeV
$N_f = 2$: $T_c = (173 \pm 8)$ MeV

Chiral symmetry restoration and deconfinement at same T_c

pressure



5
4
3
2
1
0
p/T^4
p_{SB}/T^4
3 flavour
2+1 flavour
2 flavour
pure gauge
T [MeV]
100
200
300
400
500
600

realistic QCD

- precise lattice results not yet available for first order transition vs. crossover
- also uncertainties in determination of critical temperature (chiral limit …)
- extension to nonvanishing baryon number only for QCD with relatively heavy quarks

conclusion

- experimental determination of critical temperature may be more precise than lattice results
- error estimate becomes crucial

end