QCD – from the vacuum to high temperature

an analytical approach

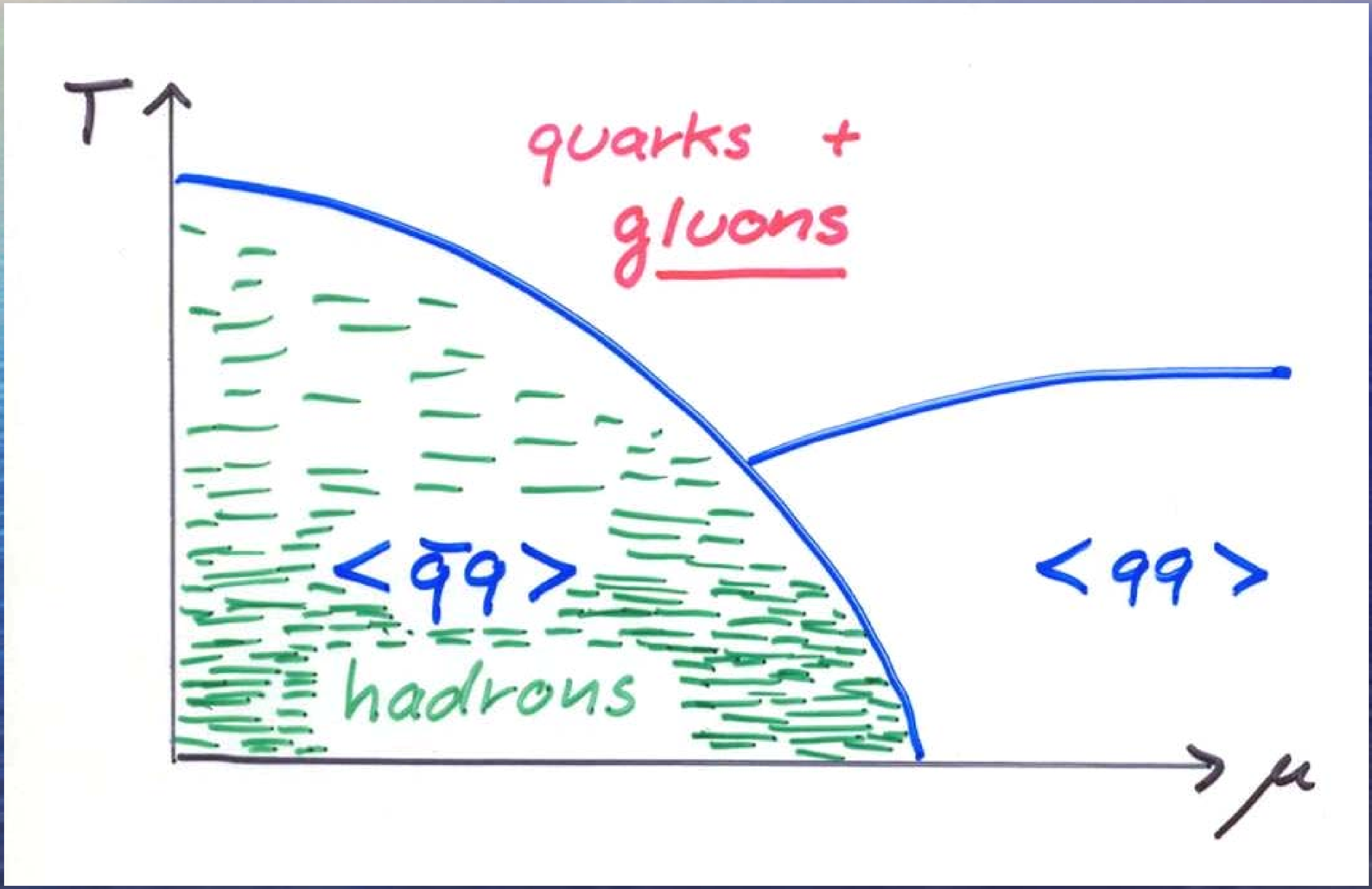
3 lectures

- Condensates and phase diagram
- Higgs picture of QCD
- Functional renormalization group

Condensates and Phase Diagram

- Action and functional integral
- Generating functionals
- Chiral symmetry
- Condensates and symmetry breaking
- Phase transition
- Has T_c been measured?

Understanding the phase diagram



Cosmological phase transition…

…when the universe cools below 175 MeV

10^{-5} seconds after the big bang

QCD at high density

Nuclear matter

Heavy nuclei

Neutron stars

Quark stars …

Why analytical approach ?

Complementary to numerical investigations (lattice – QCD)

Many questions cannot be addressed by lattice - QCD

QCD

Quantum numbers

	quark	quark	gluon	meson	baryon
particle	u	d	g	π^+	p
electric charge	2/3	-1/3	0	1	1
isospin	1/2	-1/2	0	1	1/2
color	3	3	8	1	1

Light particles

mesons

- pseudoscalar octet: $(\pi^+,\pi^-,\pi^0,K^+,K^-,K^0,\bar{K}^0,\eta)$
- vector meson octet + …

baryons

- octet : $(p,n,\Lambda,\Sigma^+,\Sigma^-,\Sigma^0,\Xi^0,\Xi^-)$
- decuplet + …

quarks and gluons at high momenta

- Visible as jets at high virtual momenta
- Perturbation theory valid

Short and long distance degrees of freedom are different !

Short distances : quarks and gluons
Long distances : baryons and mesons

How to make the transition?

confinement

Action

$$\mathcal{L}_{\mathrm{QCD}} = +\frac{1}{2}\,\mathrm{tr}\,G_{\mu\nu}G^{\mu\nu} + \sum_{a=1}^{N_f}\sum_{i,j=1}^{3} \bar{q}_{a,i}\left(i\gamma^{\mu}\mathcal{D}_{\mu}^{ij} - m_a\right) q_{a,j}$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$$

$$A_\mu \equiv \sum_{\alpha=1}^{8} A_\mu^\alpha \frac{\lambda^\alpha}{2}$$

$$\mathcal{D}_\mu^{ij} q_a \equiv \left[\partial_\mu \delta^{ij} - igA_\mu^{ij}\right] q_a$$

$a = 1, \ldots, N_f$ = quark flavor index
$i, j = 1, 2, 3$ = color indices corresponding to $\underline{\mathbf{3}}$ of $SU(3)$
λ^α = Gell–Mann matrices (generators of $SU(3)$), $\alpha = 1, \ldots, 8$
γ^μ ($\mu = 0, \ldots, 3$) are Dirac's gamma matrices (all spinor indices are suppressed)

QCD is microscopic theory of strong interactions

Similarly to QED the interaction between quarks is described through the exchange of massless vector mesons: photon $\rightarrow$ eight gluons A^{α}_{μ}

Crucial difference to QED: gluons are not neutral (like photons) but carry $SU(3)$ color charge $\rightarrow$ gluon self–interactions!

Local gauge symmetry

$$S_{\mathrm{QCD}} \equiv \int d^4x \mathcal{L}_{\mathrm{QCD}}$$

is invariant under the infinitesimal transformations

$$\begin{aligned}
q_a(x) &\rightarrow q'_a(x) = q_a(x) - i\frac{\lambda^\alpha}{2}\Theta^\alpha(x)q_a(x) \\
A^\alpha_\mu(x) &\rightarrow A'^\alpha_\mu(x) = A^\alpha_\mu(x) \\
&\qquad\qquad + C^{\alpha\beta\gamma}\Theta^\beta(x)A^\gamma_\mu(x) - \frac{1}{g}\partial_\mu\Theta^\alpha(x)
\end{aligned}$$

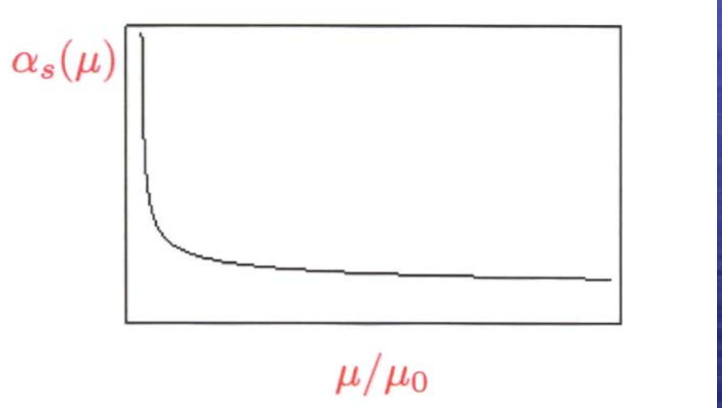
where the structure constants $C^{\alpha\beta\gamma}$ are defined through

$$\left[\frac{\lambda^\alpha}{2}, \frac{\lambda^\beta}{2}\right] = iC^{\alpha\beta\gamma}\frac{\lambda^\gamma}{2}$$

- Quarks q_a transform in the fundamental representation $\underline{\mathbf{3}}$ of $SU(3)$
- Gluons A_μ transform in the adjoint representation $\underline{\mathbf{8}}$ of $SU(3)$

Running coupling : QCD

effective gauge coupling depends on momentum scale μ



$$\beta_{\rm QCD} \equiv \mu \frac{d\alpha_s}{d\mu} = -\frac{1}{3\pi}\left[\frac{33}{2} - N_f\right]{\alpha_s}^2 \; ; \quad \alpha_s \equiv \frac{g^2}{4\pi}$$

$$\alpha_s(\mu) = \frac{\alpha_{s0}}{1 + \frac{\alpha_{s0}}{3\pi}\left[\frac{33}{2} - N_f\right]\ln(\mu/\mu_0)}$$

- Asymptotic freedom $\Rightarrow$ perturbative expansion in powers of α_s applicable for large momenta or short length scales
- Infrared slavery $\Rightarrow$ perturbation theory breaks down at large distances and there are no free quarks and gluons in nature!

Generating functionals, Effective action

Partition function and functional integral

$$Z = \int \mathcal{D}\psi \mathcal{D}A \; e^{-S}$$

Integration over all configurations of gauge field A and quark fields ψ

Fermion fields are anticommuting Grassmann variables

$$\{\psi(x)\psi(y)\} = 0 \;\; , \quad \{\psi(q)\psi(q')\} = 0$$

$$\text{Functional measure:} \;\; \int \mathcal{D}\psi \mathcal{D}A$$

This is the difficult part ! Needs regularisation !

Lattice QCD ; gauge fixing,ghosts,Slavnov Taylor identities …

Correlation functions

Only gauge invariant correlation functions can differ from zero.

Local gauge symmetries are not spontaneously broken
(exact proof)

- Scalar correlation functions

$$G_s(x,y) = -\langle \left(\bar{\psi}_L(x)\psi_R(x)\right)\,\left(\bar{\psi}_R(y)\psi_L(y)\right)\rangle$$

describes propagation of (pseudo)scalar mesons

$$\text{large } |x-y| : \; G_s \sim \exp\left(-m_\pi |x-y|\right)$$

in suitable channel

- Vector meson correlation function

$$G_V^{\mu\nu}(x,y) = \langle \left(\bar{\psi}(x)\gamma^\mu\psi(x)\right)\,\left(\bar{\psi}(y)\gamma^\nu\psi(y)\right)\rangle$$

Generating functionals

- Add to the action a source term

$$S_j = -\int d^4x \, \{\bar{\psi}_L(x) j_s^\dagger(x) \psi_R(x) + h.c.$$
$$+\bar{\psi}(x)\gamma^\mu j_{v\mu}(x)\psi(x)\}$$

$$j_s : \text{complex} \quad N_f \times N_f \text{ matrix}$$
$$j_v : \text{hermitean } N_f \times N_f \text{ matrix}$$

- Partition function becomes a functional of the sources, Z[j]. Can be evaluated for arbitrary j(x)

Generating functional and correlation functions

Concentrate on scalars –

other channels can be treated in complete analogy

- Order parameter

$$\langle \bar{\psi}_{Lb}(x)\psi_{Ra}(x)\rangle = \sigma_{ab}(x) = Z^{-1}\frac{\delta Z}{\delta j^*_{ab}(x)}$$

- Correlation function

$$\begin{aligned} G_s(x,y) &= -\langle \left(\bar{\psi}_R(x)\psi_L(x)\right)\left(\bar{\psi}_L(y)\psi_R(y)\right)\rangle \\ &= Z^{-1}\frac{\delta^2 Z}{\delta j(x)\delta j^*(y)} \end{aligned}$$

Generating functional for connected Green's functions

W[j]=ln Z[j]

$$\sigma_{ab}(x) = \frac{\delta W}{\delta j^*_{ab}(x)}$$

$$G^c_s(x,y) = G_s(x,y) - \sigma^*(x)\sigma(y) = \frac{\delta^2 W}{\delta j(x)\delta j^*(y)}$$

G^c : propagator or

connected two point function

Effective action

Effective action is defined by Legendre transform

$$\Gamma[\sigma] = -W[j] + \int d^4x \, tr \left(j^\dagger(x)\sigma(x) + \sigma^\dagger(x) j(x) \right)$$

$$j = j[\sigma] \text{ by inversion of } \sigma[j] = \frac{\delta W}{\delta j^*}$$

$$\text{field equation} \quad \frac{\delta \Gamma}{\delta \sigma_{ab}(x)} = j^*_{ab}(x)$$

Matrix of second functional derivatives $\Gamma^{(2)}$ is the inverse propagator

$$\Gamma^{(2)} W^{(2)} = 1\!\!1$$

Physical sources

$$\text{physical value}: \; j_s = j_s^\dagger = m_q = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}, \; j_v = 0$$

With this convention for physical sources the quark mass term is not included in S anymore.

S: action for N_f massless quarks

Γ has the full chiral symmetry

"Solution" of QCD

Effective action (for suitable fields) contains all the relevant information of the solution of QCD

Gauge singlet fields, low momenta:

Order parameters, meson-(baryon-) propagators

Gluon and quark fields, high momenta:

Perturbative QCD

Aim: Computation of effective action

Symmetries

- If functional measure preserves the symmetries of S :

  Γ has the same symmetries as S

- If not : anomaly

Examples for symmetries of Γ :

- gauge symmetry,
- global chiral flavor symmetry $SU(N_f)xSU(N_f)$

Example for anomaly: global axial U(1)

Chiral symmetry breaking

Chiral symmetry

For massless quarks the action of QCD is invariant under independent phase rotations acting on left handed and right handed quarks.

$$SU(N_f)_L \times SU(N_f)_R$$

$$\sum_{a=1}^{N_f} \sum_{i,j=1}^{3} \bar{q}_{a,i} \left(i\gamma^\mu \mathcal{D}_\mu^{ij} - m_a \right) q_{a,j}$$

Quark masses

- Quark mass terms mix left handed and right handed quarks

$$m_q \bar{\mathrm{q}}_L \mathrm{q}_R + h.c.$$

- They break the axial symmetry
- Equal quark masses leave vectorlike "diagonal" SU(N) unbroken.
- Quark mass terms can be treated as sources for scalar quark-antiquark bilinears.

Effective potential

Evaluate effective action

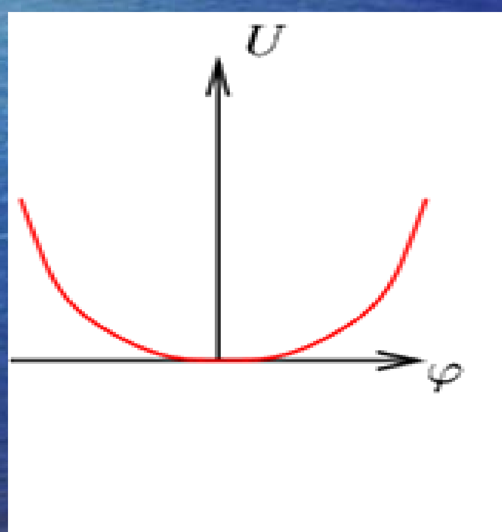
for homogenous field φ

$$\Gamma = \int d^4x\, U(\varphi)$$

Spontaneous symmetry breaking

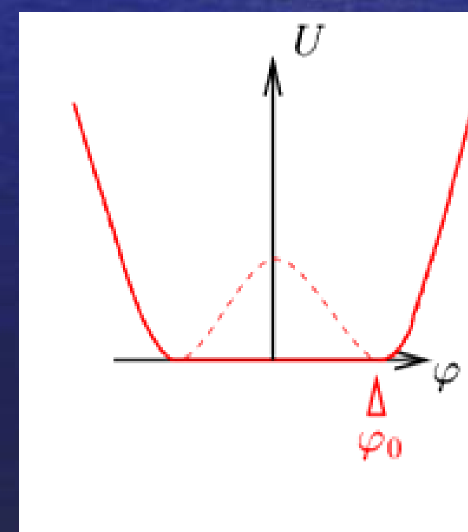
SYM

$\langle\varphi\rangle = 0$



SSB

$\langle\varphi\rangle = \varphi_0 \neq 0$



Pions

Two-flavor QCD , σ : 2x2 matrix

Effective potential

$$U(\sigma) = m^2\, tr(\sigma^\dagger\sigma) - \frac{\nu}{2}(\det\sigma + \det\sigma^\dagger) + \frac{\lambda_1}{2}\left(tr(\sigma^\dagger\sigma)\right)^2 + \frac{\lambda_2}{2} tr\left(\sigma^\dagger\sigma - \frac{1}{2}tr(\sigma^\dagger\sigma)\right)^2 + \ldots$$

$$\text{field equation:} \quad \frac{\partial U}{\partial\sigma} = j^* = m_q$$

$$\text{define } U_j = U - tr(j^\dagger\sigma + \sigma^\dagger j) \;, \; \frac{\partial U_j}{\partial\sigma} = 0$$

Spontaneous chiral symmetry breaking

For sufficiently small m^2 and positive $\lambda_{1,2}$:

Minimum of U_j at $\langle\sigma\rangle = \begin{pmatrix} \bar{\sigma} & \\ & \bar{\sigma} \end{pmatrix}$

breaks $SU(2)_L \times SU(2)_R \to SU(2)_V$

$$\bar{\sigma} = -\frac{1}{2}\langle\bar{\psi}\psi\rangle$$

(even for vanishing quark masses)

Goldstone bosons

Flat directions in potential,
dictated by symmetry breaking

$\longrightarrow$ massless (pseudo)scalar fields : pions

Presence of quark mass term shifts minimum and gives mass to pions

Nonlinear σ - model

Nonlinear description

(neglects the scalar excitations beyond π,η)

$$\sigma(x) = \bar{\sigma} U(x)\ ,\ U^{\dagger} U = 1$$

$$U_j(U) = \text{const} - \underbrace{m_q \bar{\sigma}\, tr(U + U^{\dagger})}_{\text{mass term for pions}} - \underbrace{\frac{\nu}{2}\bar{\sigma}^2(\det U + \det U^{\dagger})}_{\text{potential for } \eta \Rightarrow M_\eta^2}$$

$$\text{neglect } \eta : \quad \det U = 1\ ,\ U(x) = \exp\left\{\frac{i\vec{\tau}\vec{\pi}(x)}{f_\pi}\right\}$$

$$U(\sigma) = m^2\, tr(\sigma^{\dagger}\sigma) - \frac{\nu}{2}(\det\sigma + \det\sigma^{\dagger}) + \frac{\lambda_1}{2}\left(tr(\sigma^{\dagger}\sigma)\right)^2 + \frac{\lambda_2}{2} tr\left(\sigma^{\dagger}\sigma - \frac{1}{2}tr(\sigma^{\dagger}\sigma)\right)^2$$

Kinetic term

$$\bar{Z} tr(\partial^\mu \sigma^\dagger \partial_\mu \sigma) \to \bar{Z}\bar{\sigma}^2 tr(\partial^\mu U^\dagger \partial_\mu U) = \frac{2\bar{Z}\bar{\sigma}^2}{f_\pi^2}\partial^\mu \vec{\pi}\partial_\mu \vec{\pi} + \ldots$$

$$\text{standard normalization: } f_\pi = 2\sqrt{\bar{Z}}\bar{\sigma}$$

$$U(x) = \exp\left\{\frac{i\vec{\tau}\vec{\pi}(x)}{f_\pi}\right\}$$

Chiral perturbation theory

$$\mathcal{L}[U] = \frac{f_\pi^2}{4}\{tr(\partial^\mu U^\dagger \partial_\mu U)$$
$$-2Bm_q tr(U + U^\dagger) + \ldots\}$$

$$B = \frac{2\bar{\sigma}}{f_\pi^2} = -\frac{\langle\bar{\psi}\psi\rangle}{f_\pi^2} B = \frac{2\bar{\sigma}}{f_\pi^2} - \frac{\langle\bar{\psi}\psi\rangle}{f_\pi^2}$$

$$U(x) = \exp\left\{\frac{i\vec{\tau}\vec{\pi}(x)}{f_\pi}\right\}$$

The meaning of F and B_0

Under weak interactions $\begin{pmatrix} \nu \\ \mu \end{pmatrix}_L$ and $Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L$ are both doublets with respect to the $SU_L(2)$ gauge symmetry. Virtual W–boson exchange leads to an effective four–Fermi interaction:

$$\mathcal{L}_{\text{eff}}^{\text{weak}} = \frac{G_F}{\sqrt{2}} \left[\jmath^{\mu -} - \jmath_5^{\mu -} \right] \bar{\nu} \gamma_\mu (1 - \gamma_5) \mu$$

where G_F is the Fermi constant.
If the non–linear sigma model is a valid description of low–energy QCD, the $SU_V(2)$ vector and $SU_A(2)$ axial–vector currents, $\jmath^{\mu -}$ and $\jmath_5^{\mu -}$, respectively, may be computed from $\mathcal{L}_{\chi PT}$. From there one determines, e.g., the width of the decay $\pi \to \mu\nu$:

$$\Gamma_{\pi \to \mu\nu} = \frac{G_F^2}{4\pi} m_\mu^2 m_\pi F^2 \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)^2$$

with the help of the PCAC (partially conserved axial–vector current)

$$\langle 0 | \jmath_5^{\mu a}(0) | \pi^b(p) | \rangle = i p^\mu \delta^{ab} F$$

$\Gamma_{\pi \to \mu\nu}$ can be determined from experiment and one finds the value of the pion decay constant

$$\boxed{f_\pi \equiv F = 92.4\,\text{MeV}}$$

Next, expand the symmetry breaking part $\sim M_q$ of $\mathcal{L}_{\chi PT}$:

$$\mathcal{L}_{\chi PT}^{(\mathrm{SB})} = (m_u + m_d) B_0 \left\{ F^2 - \frac{1}{2}\vec{\pi}^2 + \frac{\vec{\pi}^4}{24F^2} + \ldots \right\} = -\mathcal{H}_{\chi PT}^{(\mathrm{SB})}$$

This implies

$$M_\pi^2 = (m_u + m_d) B_0 + \mathcal{O}(M_q^2)$$

($M_\pi^2 \to 0$ for $M_q \to 0$ as it should for Goldstone bosons!)

Furthermore:

$$\langle \overline{q}q \rangle = \partial_{m_q} \mathcal{H}_{\chi PT} = -F^2 B_0 + \mathcal{O}(M_q)$$

Eliminating B_0 yields the Gell–Mann–Oakes–Renner relation

$$F^2 M_\pi^2 = -(m_u + m_d) \langle \overline{q}q \rangle + \mathcal{O} M_q^2$$

For $N_f = 3$ one can also easily derive the Gell–Mann–Okubo formula

$$M_\eta^2 = \frac{1}{3} \left[4M_K^2 - M_\pi^2 \right] + \mathcal{O}(M_q^2)$$

Meson fluctuations

fermion bilinear: $\tilde{\sigma}_{ab} = \bar{\psi}_{Lb}\psi_{Ra}$, $\langle\tilde{\sigma}_{ab}\rangle = \sigma_{ab}$

- Insert factor unity

Hubbard,Stratonovich

$$\mathbb{1} = N \int \mathcal{D}\sigma' \exp\left\{ - \int \frac{d^4q}{(2\pi)^4} tr\{(\sigma' - \lambda_\sigma\tilde{\sigma} - j)^\dagger \lambda_\sigma^{-1}(\sigma' - \lambda_\sigma\tilde{\sigma} - j)\}\right\}$$

Fourier basis $\sigma' \widehat{=} \sigma'(q)$ etc., $\lambda_\sigma \widehat{=} \lambda_\sigma(q^2)$

- New action and functional integral involves also σ - field

$$S_j^{(\sigma)} = S_0 + \int \frac{d^4q}{(2\pi)^4} tr\{\sigma'^\dagger\lambda_\sigma^{-1}\sigma' - (\sigma'^\dagger\tilde{\sigma} + \tilde{\sigma}^\dagger\sigma') + \lambda_\sigma\tilde{\sigma}^\dagger\tilde{\sigma}$$
$$-(j^\dagger\lambda_\sigma^{-1}\sigma' + \sigma'^\dagger\lambda_\sigma^{-1}j) + j^\dagger\lambda_\sigma^{-1}j\}$$

σ - interactions

- Four fermion interaction can cancel corresponding piece induced by loops in QCD
- Typical form

$$\lambda_\sigma = \frac{1}{M^2 + Zq^2}$$

- Mass and kinetic term for σ
- Yukawa interaction between σ and quarks
- Source now for σ

$$\begin{aligned} S_j^{(\sigma)} &= S_0 + \int \frac{d^4q}{(2\pi)^4} tr\{\sigma'^\dagger \lambda_\sigma^{-1} \sigma' - (\sigma'^\dagger \tilde{\sigma} + \tilde{\sigma}^\dagger \sigma') + \lambda_\sigma \tilde{\sigma}^\dagger \tilde{\sigma} \\ &\quad -(j^\dagger \lambda_\sigma^{-1} \sigma' + \sigma'^\dagger \lambda_\sigma^{-1} j) + j^\dagger \lambda_\sigma^{-1} j\} \end{aligned}$$

Connection with quark bilinears

$$\frac{\delta W}{\delta j^*} = \langle \lambda_\sigma^{-1}(\sigma' - j) \rangle = \langle \tilde{\sigma} \rangle = \sigma$$

$$\varphi = \langle \sigma' \rangle = \lambda_\sigma \sigma + j$$

Effective action with scalar fluctuations

$$\varphi = \langle \sigma' \rangle = \lambda_\sigma \sigma + j$$

$$j_\varphi = \lambda_\sigma^{-1} j \ , \ \ S_j^{(\varphi)} = S_j^{(\sigma)} - \int j^\dagger \lambda_\sigma^{-1} j$$

$$W_\varphi = W + \int j^\dagger \lambda_\sigma^{-1} j$$

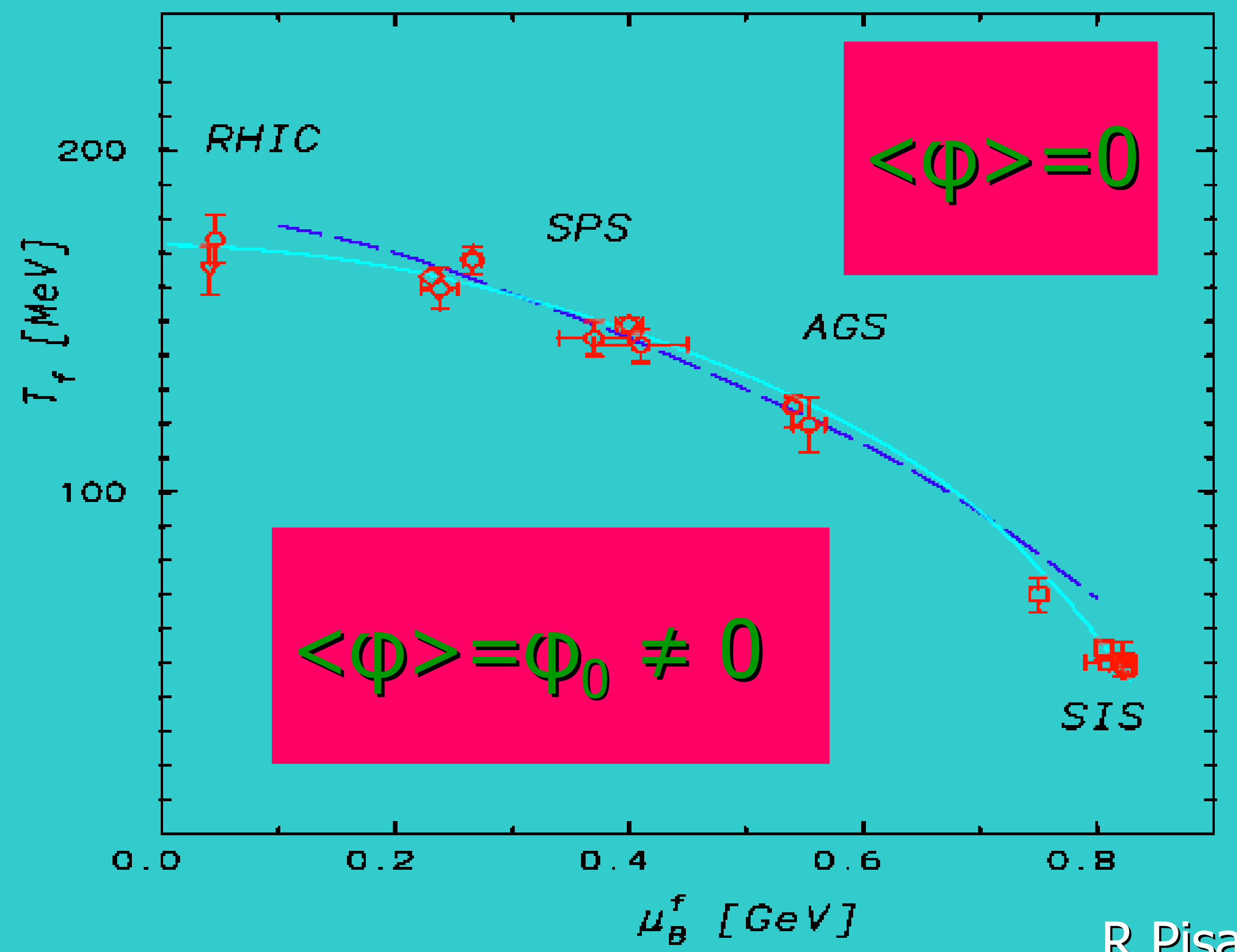
$$\Gamma[\varphi] = -W_\varphi + \int (j_\varphi^\dagger \varphi + \varphi^\dagger j_\varphi)$$

$$\Gamma[\varphi] = \Gamma[\sigma] + \int \frac{\delta \Gamma}{\delta \sigma} \lambda_\sigma^{-1} \frac{\delta \Gamma}{\delta \sigma}$$

$$\frac{\delta \Gamma}{\delta \varphi} = j_\varphi^\dagger = \lambda_\sigma^{-1} j^\dagger$$

QCD phase transition

Phase diagram



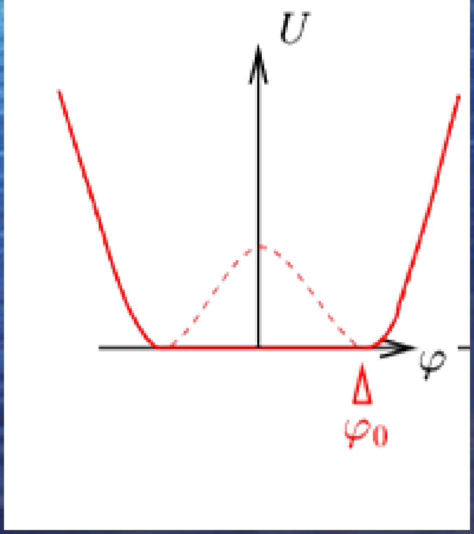
R.Pisarski

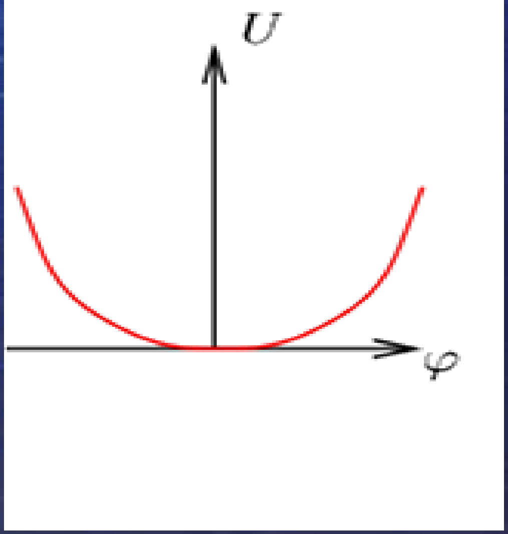
Chiral symmetry restoration at high temperature

Low T
SSB
$\langle\varphi\rangle = \varphi_0 \neq 0$

High T
SYM
$\langle\varphi\rangle = 0$

at high T :
less order
more symmetry





examples:

magnets, crystals

QCD at high temperature

- Quark – gluon plasma
- Chiral symmetry restored
- Deconfinement (no linear heavy quark potential at large distances)
- Lattice simulations : both effects happen at the same temperature

Lattice results

e.g. Karsch,Laermann,Peikert

Critical temperature in chiral limit :

$N_f = 3$: $T_c = (154 \pm 8)$ MeV
$N_f = 2$: $T_c = (173 \pm 8)$ MeV

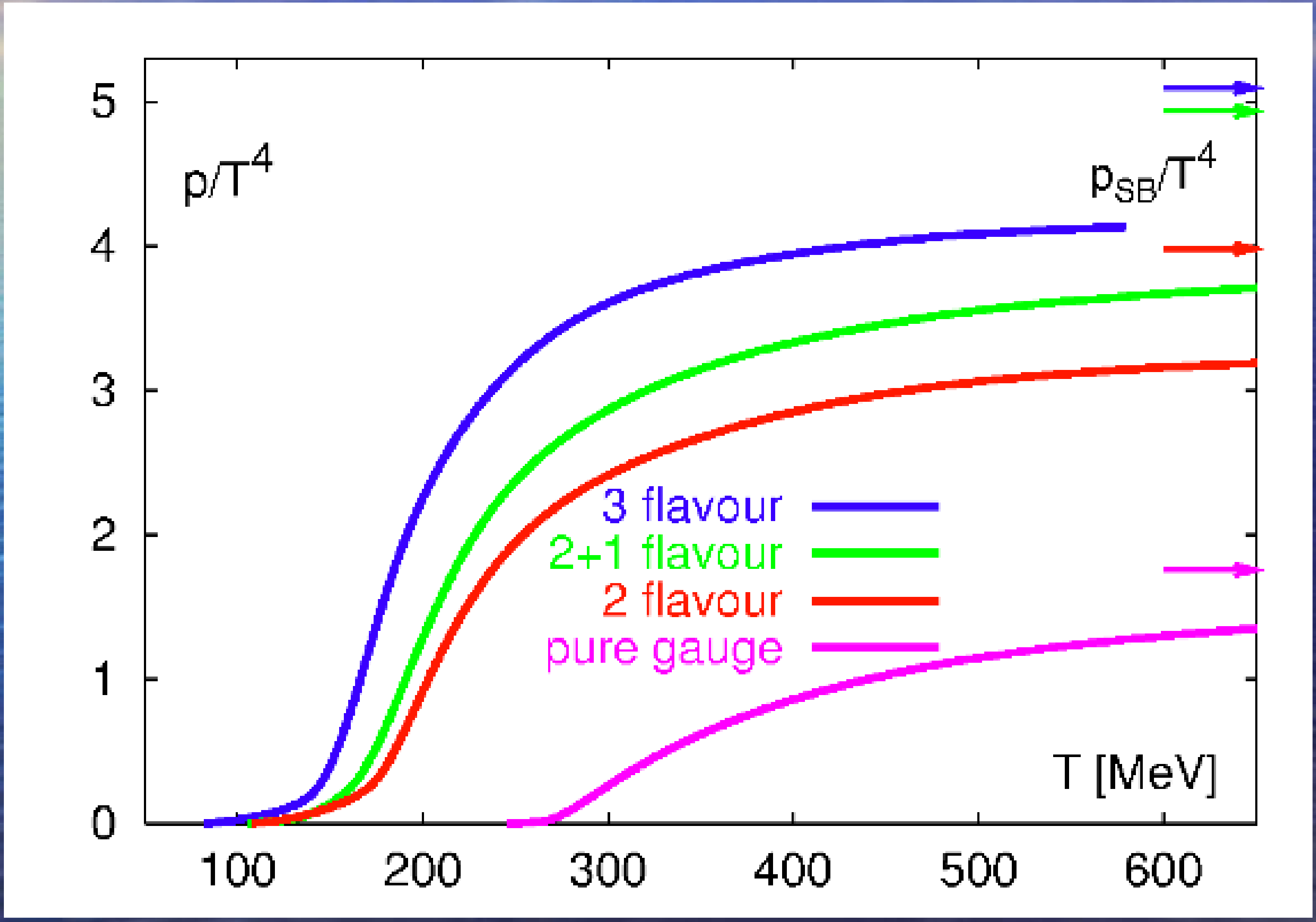
Chiral symmetry restoration and deconfinement at same T_c

QCD – phase transition

Quark –gluon plasma	**Hadron gas**
• Gluons : 8 x 2 = 16	• Light mesons : 8
• Quarks : 9 x 7/2 =12.5	• (pions : 3)
• Dof : 28.5	• Dof : 8
Chiral symmetry	Chiral sym. broken

Large difference in number of degrees of freedom !
Strong increase of density and energy density at T_c !

Pressure



5
4
3
2
1
0
p/T4
pSB/T4
3 flavour
2+1 flavour
2 flavour
pure gauge
T [MeV]
100
200
300
400
500
600

Analytical description of phase transition

- Needs model that can account simultaneously for the correct degrees of freedom below and above the transition temperature.
- Partial aspects can be described by more limited models, e.g. chiral properties at small momenta.

Quark descriptions (NJL-model) fail to describe the high temperature and high density phase transitions correctly

High T : chiral aspects could be ok , but glue …
(pion gas to quark gas)

High density transition : different Fermi surface for quarks and baryons (T=0)
– in mean field theory factor 27 for density at given chemical potential –

Confinement is important : baryon enhancement

Berges,Jungnickel,…

Chiral perturbation theory even less complete

Universe cools below 170 MeV…

Both gluons and quarks disappear from thermal equilibrium : mass generation

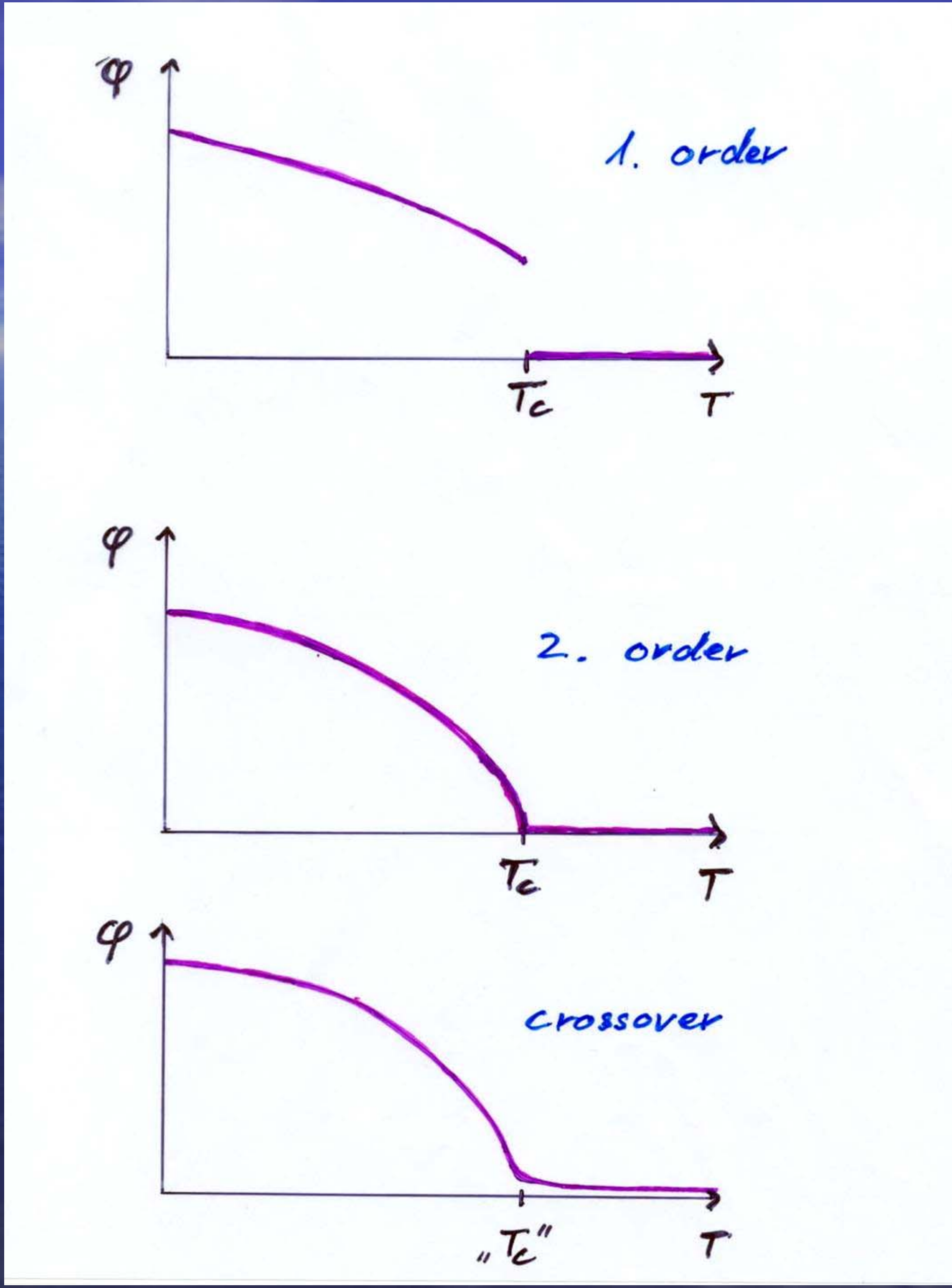
- Chiral symmetry breaking
  - → mass for fermions
- Gluons ?

Analogous situation in electroweak phase transition understood by Higgs mechanism

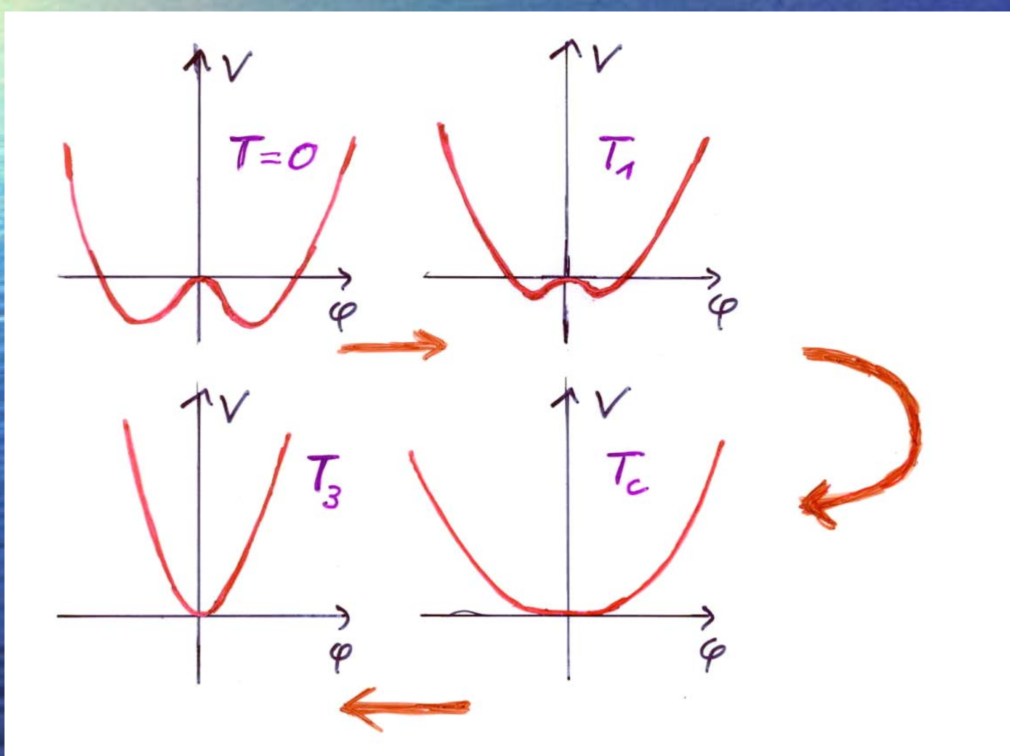
Higgs description of QCD vacuum ?

Order of the phase transition is crucial ingredient for experiments (heavy ion collisions) and cosmological phase transition

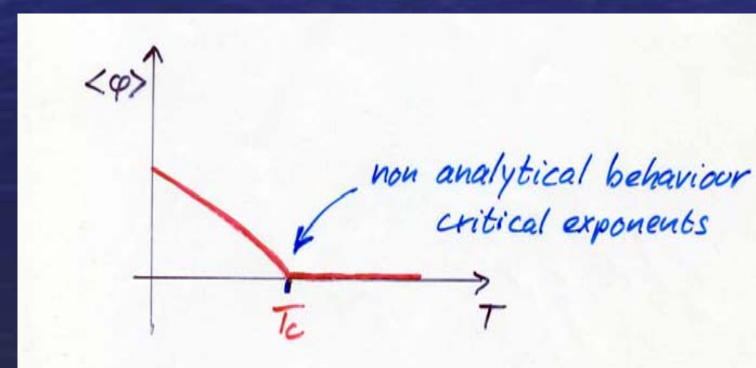
Order of the phase transition



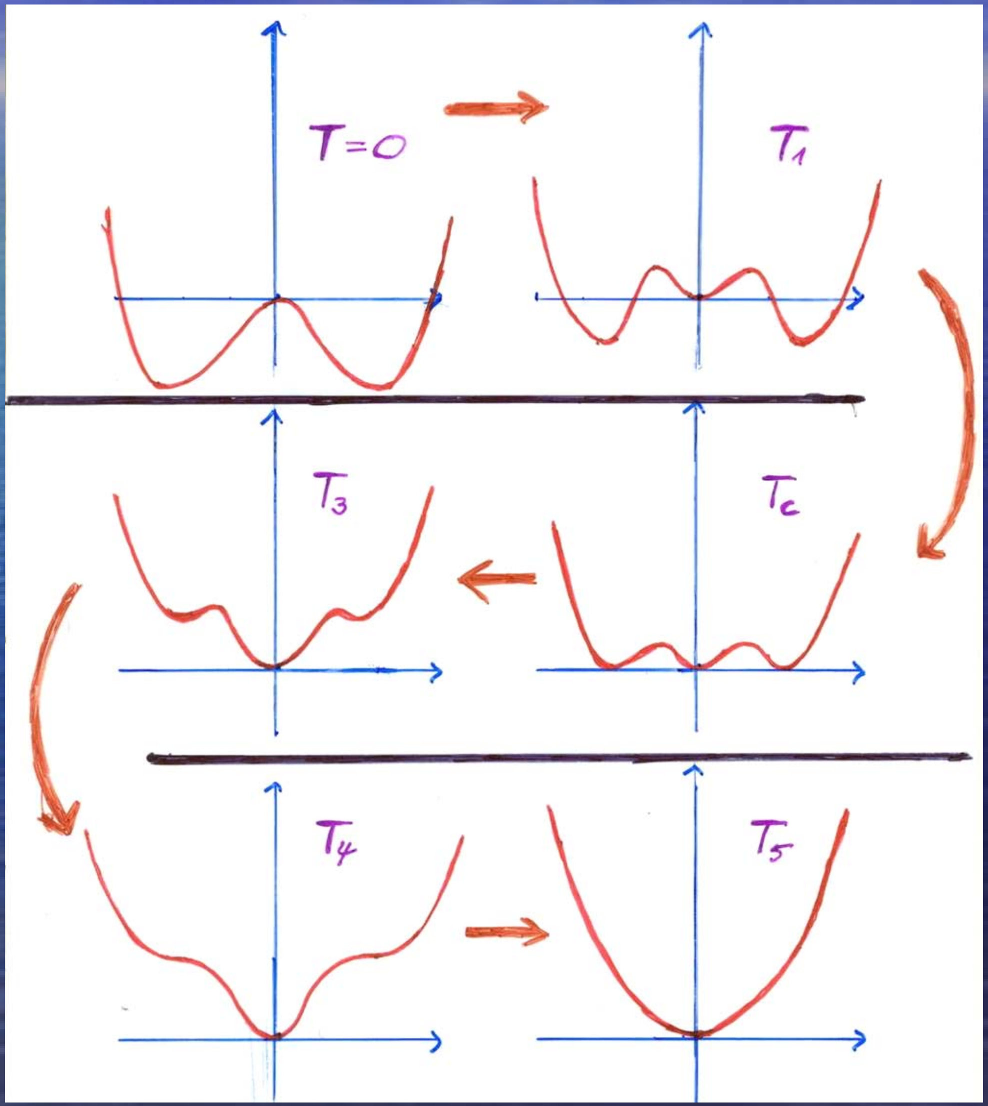
Second order phase transition



$$V = -\mu_0^2\varphi^\dagger\varphi + cT^2\varphi^\dagger\varphi + \frac{\lambda}{2}(\varphi^\dagger\varphi)$$



First order phase transition



Matsubara formalism

- Only change for T ≠ 0 :
- Euclidean time on torus with circumference β =1/T

This implies the rule of thumb for loop integrals:

$$\int \frac{d^4q}{(2\pi)^4} f(q^2) \to T \sum_{l \in \mathbb{Z}} \int \frac{d^3\overline{q}}{(2\pi)^3} f(q_0^2 + \overline{q}^2)$$

with

$$q_0(l) = \begin{cases} 2l\pi T & \text{for bosons} \\ (2l+1)\pi T & \text{for fermions} \end{cases} \quad ; \quad l \in \mathbb{Z}$$

Matsubara formalism for fermions

Fermions are treated quite similarly with an important difference in the boundary condition:

$$\Psi(\vec{x}, \tau = 0) = -\Psi(\vec{x}, \tau = \beta)$$

Thermodynamic potentials and observables

Evaluate effective potential U at minimum

- free energy $$F = T\Gamma$$
- pressure $$p = -U$$
- energy density $$\epsilon = U - T\partial U / \partial T$$

vanishing chemical potential and sources

Thermodynamics

Effective action , when evaluated on torus for euclidean time with circumference 1/T, (and in presence of chemical potential) contains full information about thermodynamic behavior of QCD.

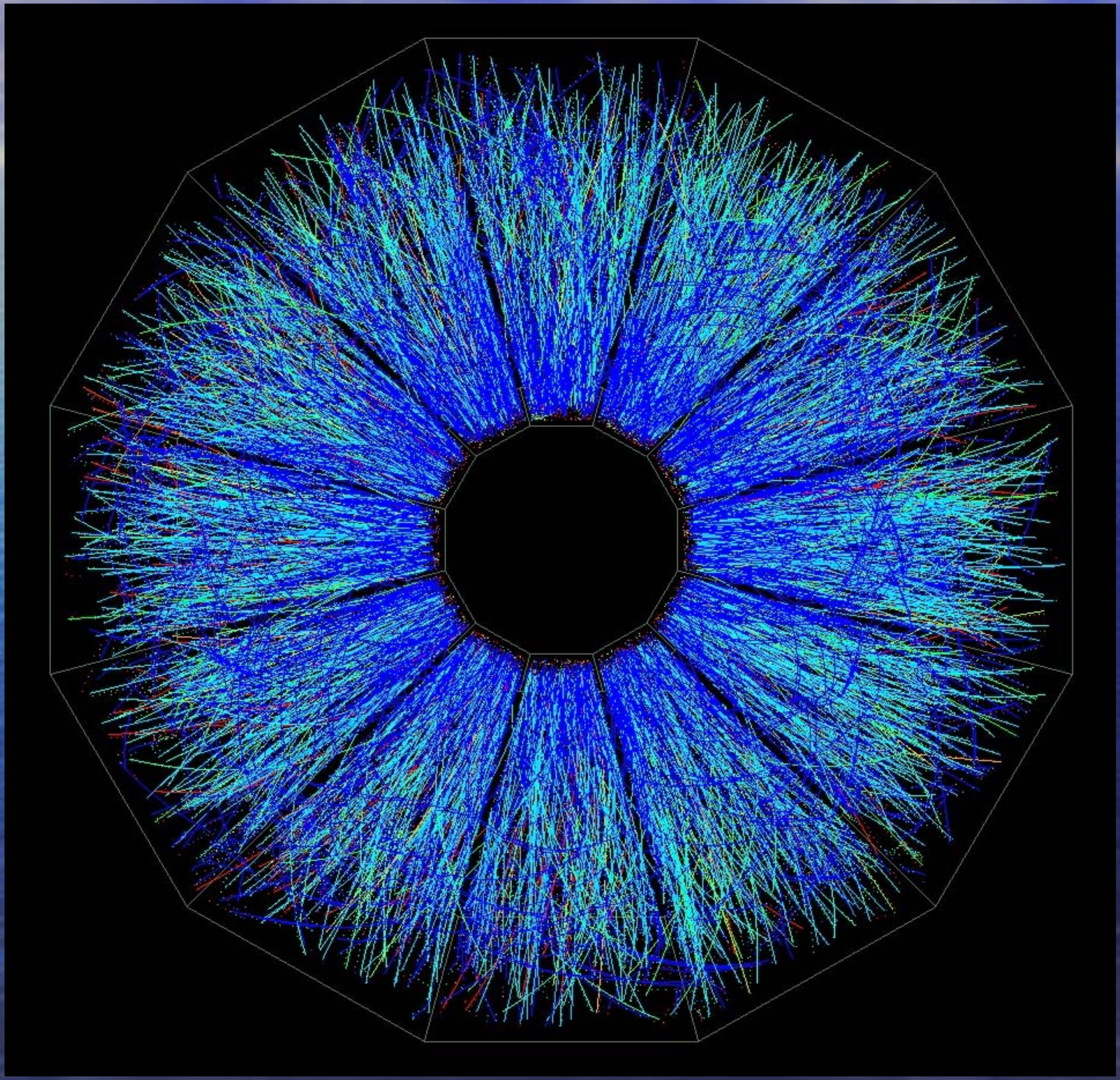
Has T_c been measured ?

- Observation : statistical distribution of hadron species with "chemical freeze out temperature " T_{ch}=176 MeV
- T_{ch} cannot be much smaller than T_c : hadronic rates for $T< T_c$ are too small to produce multistrange hadrons (Ω,..)
- Only near T_c multiparticle scattering becomes important (collective excitations …) – proportional to high power of density

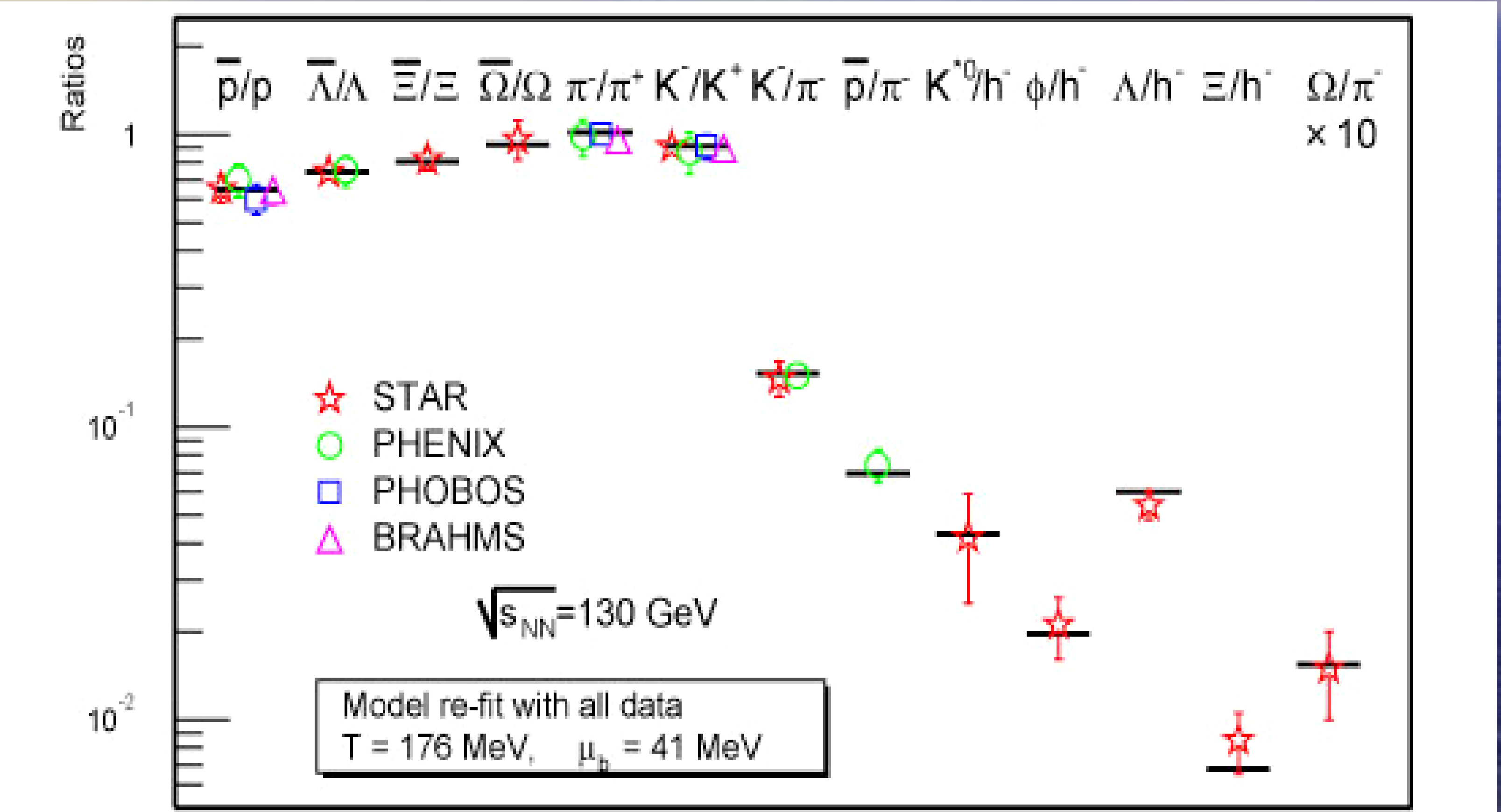
→ $T_{ch} \approx T_c$

P.Braun-Munzinger,J.Stachel,CW

Heavy ion collision



Hadron abundancies



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Very rapid density increase

…in vicinity of critical temperature

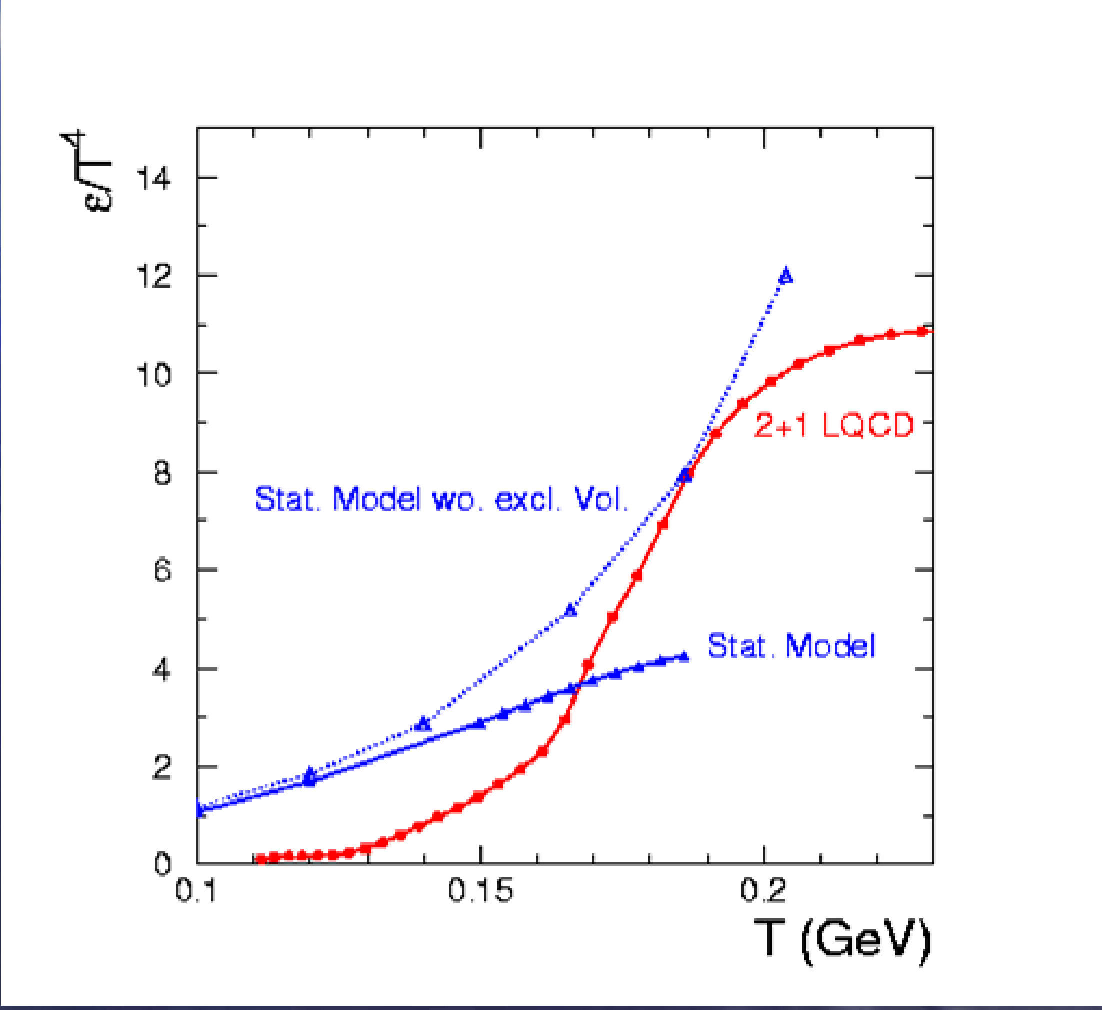
Extremely rapid increase of rate of multiparticle scattering processes

(proporional to very high power of density)

Energy density

- Lattice simulations

  Karsch et al

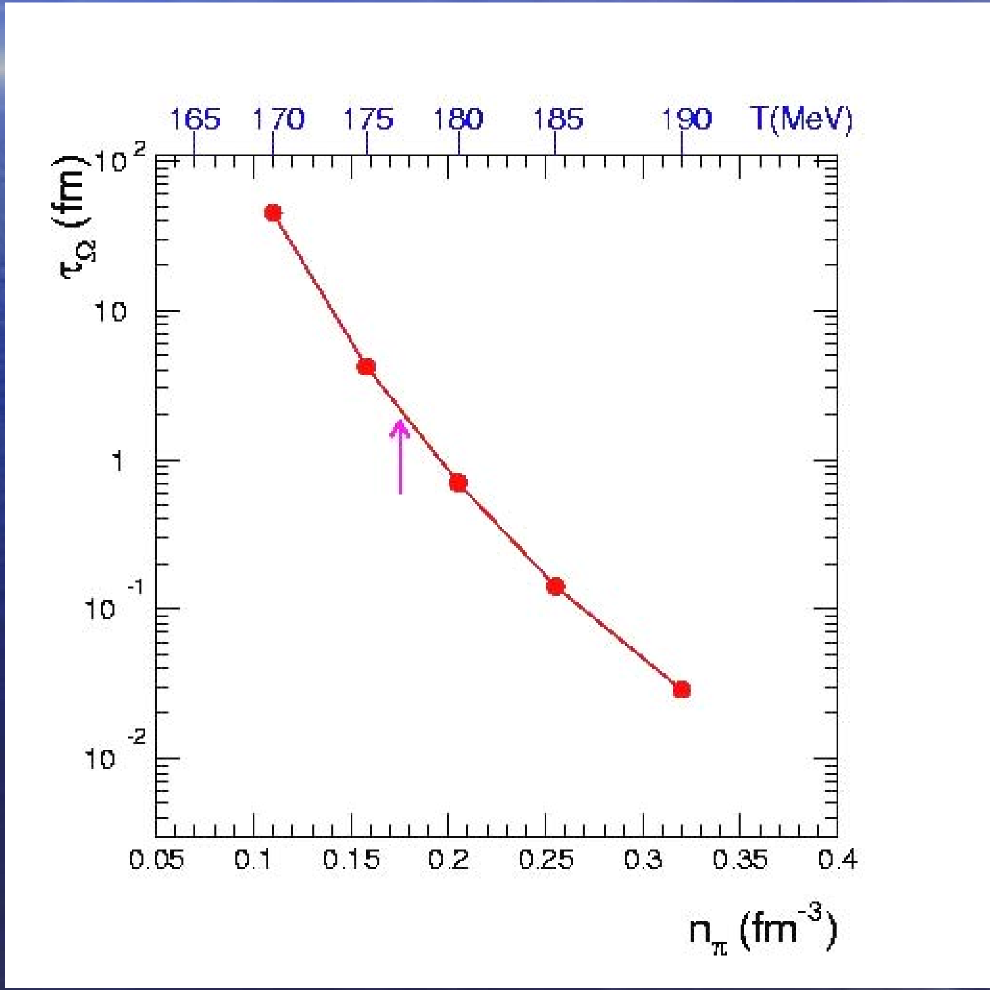


Production time for Ω

multi-meson scattering

π+π+π+K+K -> Ω+p

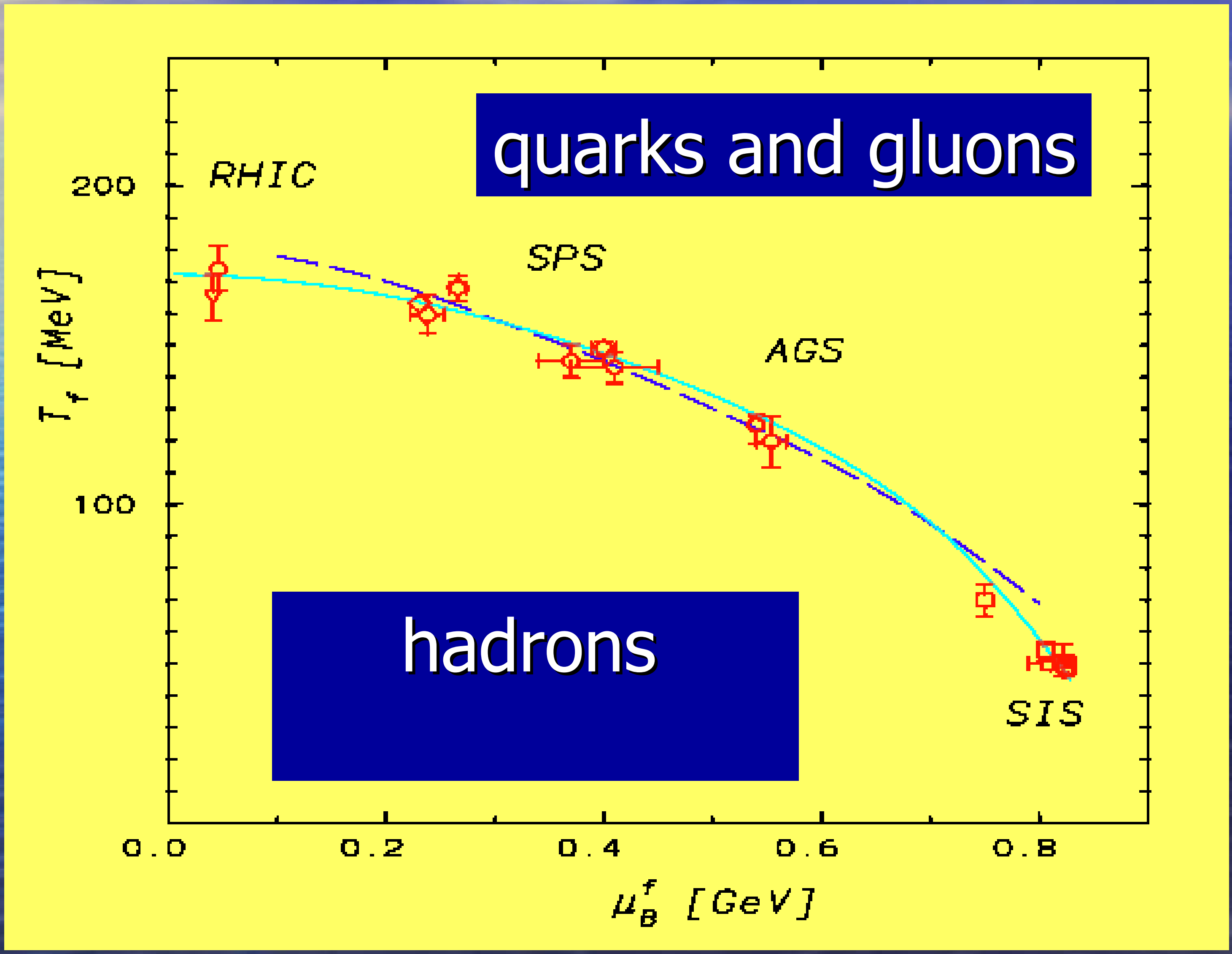
strong dependence on pion density



P.Braun-Munzinger,J.Stachel,CW

$$T_{ch} \approx T_c$$

Phase diagram



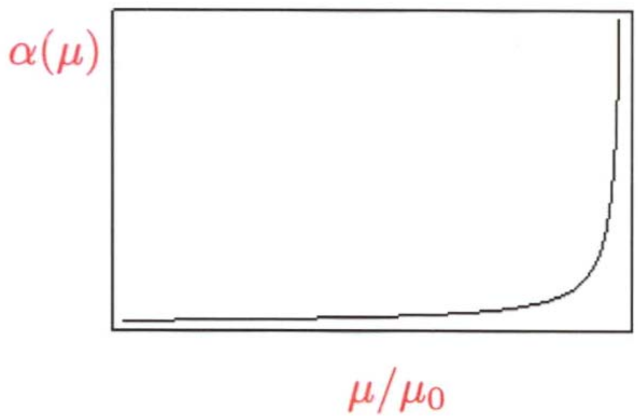
Running coupling : QED

$$\beta_{\mathrm{QED}} \equiv \mu \frac{d\alpha}{d\mu} = +\frac{2}{3\pi}\alpha^2 \; ; \quad \alpha \equiv \frac{e^2}{4\pi}$$

This yields

$$\alpha(\mu) = \frac{\alpha_0}{1 - \frac{2}{3\pi}\alpha_0 \ln(\mu/\mu_0)}$$

with $\alpha_0 \equiv \alpha(\mu_0)$ and μ_0 a normalization scale



Consequences:

- QED is infrared free
- the UV behavior of $\alpha(\mu)$ displays a Landau pole

$\Rightarrow$ QED is presumably a trivial theory for $\Lambda \to \infty$!

Chiral perturbation theory

The effective model of (very) low–energy QCD is the non–linear sigma model within the framework of chiral perturbation theory. It describes the dynamics of the Goldstone bosons of spontaneous chiral symmetry breaking through the chiral Lagrangian

$$\begin{aligned}\mathcal{L}_{\chi PT} &= \frac{1}{4}F^2\left\{\mathrm{tr}\,\partial_\mu U\partial^\mu U + 2B_0\,\mathrm{tr}\,M_q\left(U+U^\dagger\right)\right\}\\ &+ \mathcal{O}(\partial^4, M_q^2, M_q\partial^2)\end{aligned}$$

with

$$U \equiv \exp\left\{\frac{i}{F}\sum_{a=1}^{N_f^2-1}\pi^a T^a\right\} \in \frac{SU_L(N_f)\times SU_R(N_f)}{SU_V(N_f)}$$

T^a = generators of the spontaneously broken $SU_A(N_f)$
E.g., $\pi^\pm \equiv \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2)$, $\pi^0 \equiv \pi_3$, etc.

Chiral perturbation theory: systematic expansion of physical observables in powers of quark masses M_q. For higher and higher orders in this expansion a rapidly growing number of terms in $\mathcal{L}_{\chi PT}$ ($\rightarrow$ growing number of unknown parameters) has to be included and higher and higher loop–orders have to be computed. Already low orders yield excellent agreement with experiment for $N_f = 2, 3$.

Matsubara formalism

$$Z \equiv \mathrm{Tr}\, e^{-\beta H} = \int \mathcal{D}\Pi\, \mathcal{D}\Phi_{\text{periodic}}$$
$$\times \exp\left\{ \int_0^{\beta} d\tau \int d^3x \left(i\Pi\partial_\tau\Phi - \mathcal{H}(\Pi, \Phi) \right) \right\}$$

with $\beta = 1/T$ and the periodicity condition

$$\Phi(\vec{x}, \tau = 0) = \Phi(\vec{x}, \tau = \beta)$$