Modified Gravity

Modification of Einstein equation

$$M^2(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = T_{\mu\nu}$$

replace

keep diffeomorphism symmetry !

at least unimodular diffeomorphisms

Modification of Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \ldots = \frac{1}{M^2} (T_{\mu\nu} + \ldots)$$

modified gravity

Dark Energy

Split is ambiguous !

example : cosmological constant

Quantum effective action

$$\Gamma = \int d^4x \sqrt{g}(\mathcal{L}_g + \mathcal{L}_m)$$

$$\mathcal{L}_g = -\frac{M^2}{2} R$$

$$\mathcal{L}_m = \mathcal{L}_{\text{Standard Model}}$$

gravitational part:
functional of metric

$$\mathcal{L}_g[g_{\mu\nu}]$$

New degrees of freedom

Modifications of gravity involve new degrees of freedom

not necessarily new fields beyond metric

what matters : degrees of freedom –
not choice of field variables to describe them

Cosmological scalar fields

Quintessence :

$$\Delta \mathcal{L}_m = \frac{1}{2}\partial^\mu \varphi \partial_\mu \varphi + V(\varphi)$$

Brans- Dicke theory :

$$\mathcal{L}_g = -\frac{\chi^2}{2}R \qquad \Delta \mathcal{L}_m = \frac{1}{2}K\partial^\mu \chi \partial_\mu \chi$$

turns out to be special form of scalar model coupled to matter

Weyl scaling

$$g_{\mu\nu} = w^2 g'_{\mu\nu}$$

w can depend on fields !

$$R = w^{-2}\{R' - 6(\ln w)_;{}^{\mu}(\ln w)_{;\mu} - 6(\ln w)_;{}^{\mu}{}_{\mu}\}$$

$$(\ln w)_{;\mu} = \partial_\mu \ln w$$

$$(\ln w)_;{}^{\mu} = g'^{\mu\nu}\partial_\nu \ln w$$

$$\sqrt{g} = w^4\sqrt{g'}$$

Weyl scaling in Brans-Dicke theory

$$w^2 = \frac{M^2}{\chi^2}$$

$$\sqrt{g}\chi^2 R \rightarrow \sqrt{g'} M^2 R' + \text{ derivatives of } \chi$$

$$\triangle \mathcal{L}'_m = \frac{M^2}{2}(K+6)\partial^\mu \ln\chi \, \partial_\mu \ln\chi$$

In this version (Einstein frame)
no modification of gravitational action !

Frames

- Jordan frame : field dependent gravitational constant (coefficient of curvature scalar)
- Einstein frame : fixed Planck mass M
- on level of quantum effective action:

  both frames are equivalent !
- simply different "field – coordinates " for solutions of differential equations
- no measurement can distinguish the two frames
- only dimensionless quantities can be measured

Weyl scaling in matter sector

fermions : $\psi = w^{-\frac{3}{2}}\psi'$

$$\sqrt{g}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow \sqrt{g'}\bar{\psi}'\gamma^{\mu}\partial'_{\mu}\psi' + \ldots$$

$$\sqrt{g}\bar{\psi}\psi \rightarrow \frac{M}{\chi}\sqrt{g'}\bar{\psi}'\psi'$$

constant mass in Jordan frame :
field dependent mass in Einstein frame !
similar for bosons

time variation of ratio nucleon mass / Planck mass:
strict limits !!!

How to obey constraints from time variation of particle masses

- field dependent mass in Jordan frame

mass ~ χ in Jordan frame :
constant mass in Einstein frame !
similar for bosons

$$\sqrt{g}\bar{\psi}\psi \rightarrow \frac{M}{\chi}\sqrt{g'}\bar{\psi}'\psi'$$

- only tiny variation of scalar field
- only tiny local variation of scalar field
  (chameleon mechanism etc.)

Quantum effective action with scale symmetry
(dilatation symmetry, “conformal symmetry”)

all mass scales replaced by χ
only dilatable dimensionless couplings

Fujii, CW

potential for Higgs scalar h

$$\mathcal{L}_h = \frac{\lambda_h}{2}(h^\dagger h - \epsilon_h \chi^2)^2$$

→ $h_0 \sim \chi$ $m_e \sim \chi$

fixed value of $\alpha_s(\chi)$ → $\Lambda_{QCD} \sim \chi$

scalar – tensor theories

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

violation of scale symmetry if V, F or K contain parameters with dimension of mass

Weyl scaling of scalar potential

$$\sqrt{g}V = \sqrt{g'}V'$$

V' = w^2 V

$$\sqrt{g} = w^4\sqrt{g'}$$

$$w^2 = \frac{M^2}{\chi^2}$$

cosmological constant
in Jordan frame λ_c

$$V' = \frac{M^4}{\chi^4}\lambda_c$$

Model

$$S = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$V(\chi) = \mu^2 \chi^2$$

$$K(\chi) = \frac{4}{\tilde{\alpha}^2} \frac{m^2}{m^2 + \chi^2} + \frac{4}{\alpha^2} \frac{\chi^2}{m^2 + \chi^2} - 6$$

$m \sim \mu$

only scale : $\mu = 2 \cdot 10^{-33}$ eV

Universe without Expansion

NATURE | NEWS

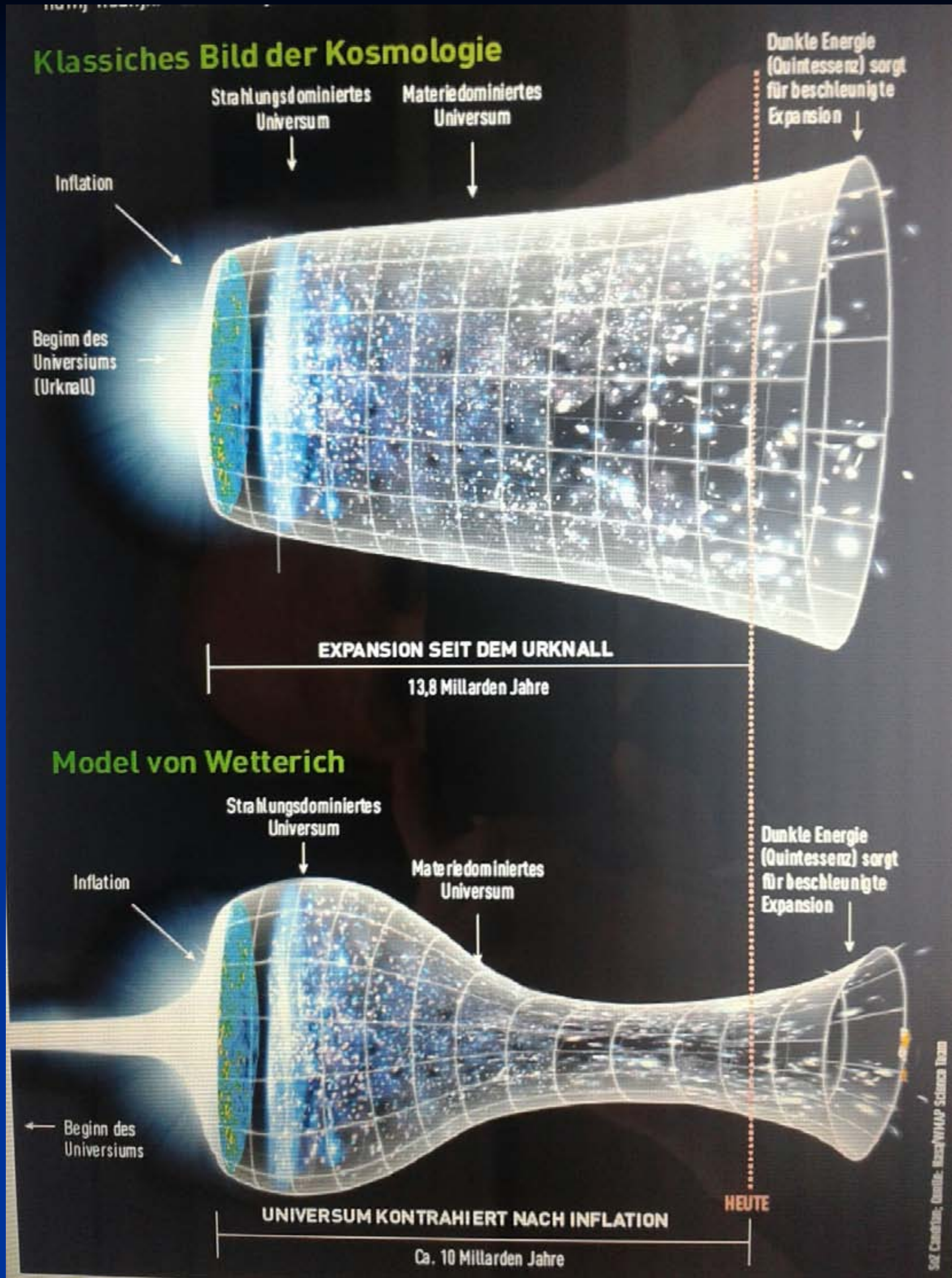
Cosmologist claims Universe may not be expanding

Particles' changing masses could explain why distant galaxies appear to be rushing away.

Jon Cartwright 16 July 2013

German physicist stops Universe

25.07.2013



Sonntagszeitung
Zuerich
Laukenmann

The Universe is shrinking

The Universe is shrinking …

while Planck mass and particle masses are increasing

What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

general idea not new : Hoyle, Narlikar,…

Simple model of “ Variable Gravity Universe “

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential :
- Nucleon and electron mass proportional to Planck mass
- Neutrino mass has different dependence on scalar field

Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Time history of the Universe

- Inflation : Universe expands
- Radiation : Universe shrinks
- Matter : Universe shrinks
- Dark Energy : Universe expands

Compatibility with observations

- Almost same prediction for radiation, matter, and Dark Energy domination as ΛCDM

- Inflation with:

  $n=0.97$, $r=0.13$

- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Cosmon inflation

Unified picture of inflation and dynamical dark energy

Cosmon and inflaton are the same field

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications

Merits of variable gravity model

- Economical setting
- No big bang singularity
- Arrow of time
- Simple initial conditions for inflation

Model

$$S = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$V(\chi) = \mu^2 \chi^2$$

$\mu = 2 \cdot 10^{-33}$ eV

$$K(\chi) = \frac{4}{\tilde{\alpha}^2} \frac{m^2}{m^2 + \chi^2} + \frac{4}{\alpha^2} \frac{\chi^2}{m^2 + \chi^2} - 6$$

Scalar field equation:

additional force from R counteracts potential gradient : increasing χ !

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$-D_\mu (K \partial^\mu \chi) + \frac{1}{2} \frac{\partial K}{\partial \chi} \partial^\mu \chi \partial_\mu \chi = -\frac{\partial V}{\partial \chi} + \frac{1}{2} \frac{\partial F}{\partial \chi} R + q_\chi$$

$$K(\ddot{\chi} + 3H\dot{\chi}) + \frac{1}{2} \frac{\partial K}{\partial \chi} \dot{\chi}^2 = -\frac{\partial V}{\partial \chi} + \frac{1}{2} \frac{\partial F}{\partial \chi} R + q_\chi$$

Incoherent contribution to scalar field equation

$$K(\ddot{\chi} + 3H\dot{\chi}) + \frac{1}{2}\frac{\partial K}{\partial \chi}\dot{\chi}^2 = -\frac{\partial V}{\partial \chi} + \frac{1}{2}\frac{\partial F}{\partial \chi}R + q_\chi$$

$$q_\chi = -\frac{\partial \ln m_p}{\partial \chi}(\rho - 3p)$$

if particle mass proportional to χ :

$$q_\chi = -\frac{\rho - 3p}{\chi} = -\frac{m_p}{\chi} n_p$$

Modified Einstein equation

New term with derivatives of scalar field

gravitational field eq.

$$\chi^2\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)+(\chi^2)_{;\rho}^{;\rho}g_{\mu\nu}-(\chi^2)_{;\mu\nu}$$

$$+\frac{1}{2}K\partial^\rho\chi\partial_\rho\chi g_{\mu\nu}-K\partial_\mu\chi\partial_\nu\chi+Vg_{\mu\nu}=T_{\mu\nu}$$

$$\Rightarrow$$

$$\chi^2R=3(\chi^2)_{;\mu}^{;\mu}+K\partial^\mu\chi\partial_\mu\chi+4V-T_\mu^\mu$$

Curvature scalar and Hubble parameter

Robertson Walker metric

$$\chi^2 R = 4V - (K+6)\dot{\chi}^2 - 6\chi\ddot{\chi} - 18H\chi\dot{\chi} - T^{\mu}_{\mu}$$

$$(\chi^2)_{;\rho}^{;\rho} = -2\dot{\chi}^2 - 2\chi\ddot{\chi} - 6H\chi\dot{\chi}$$

0-0-component

$$3\chi^2 H^2 + 6H\chi\dot{\chi} = \frac{1}{2}K\dot{\chi}^2 + V + T_{00}$$

Scaling solutions

(for constant K)

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Four different scaling solutions for inflation, radiation domination, matter domination and Dark Energy domination

Scalar dominated epoch, inflation

$$c = \pm \frac{2}{\sqrt{(K+6)(3K+16)}}$$

$$K > -\frac{16}{3}.$$

$$\begin{aligned} b &= \pm\sqrt{\frac{1}{3} + \frac{K+6}{6}c^2} - c \\ &= \pm\frac{K+4}{\sqrt{(K+6)(3K+16)}} = \frac{K+4}{2}c. \end{aligned}$$

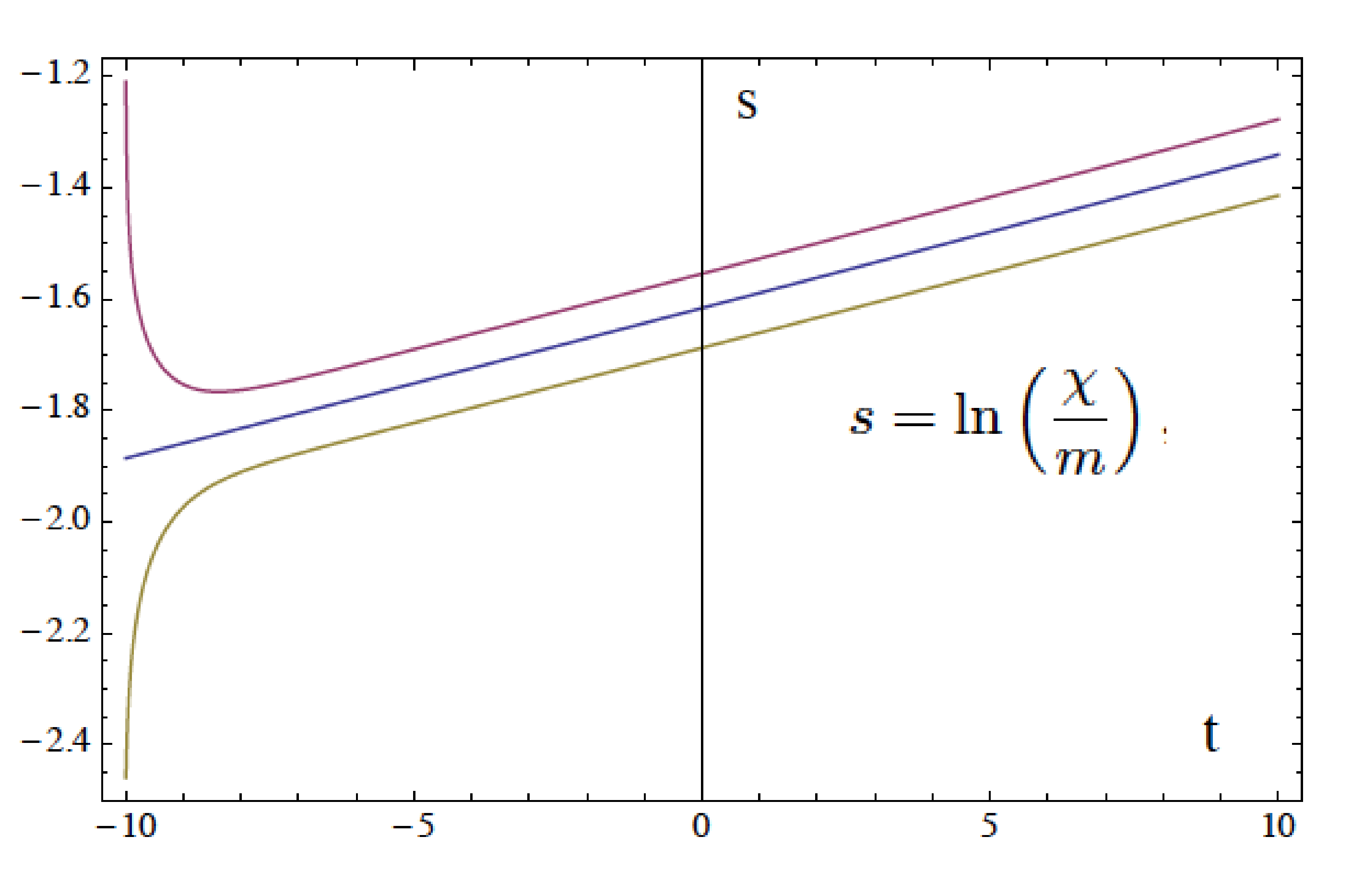
Universe expands for K > -4, shrinks for K < -4.

No big bang singularity

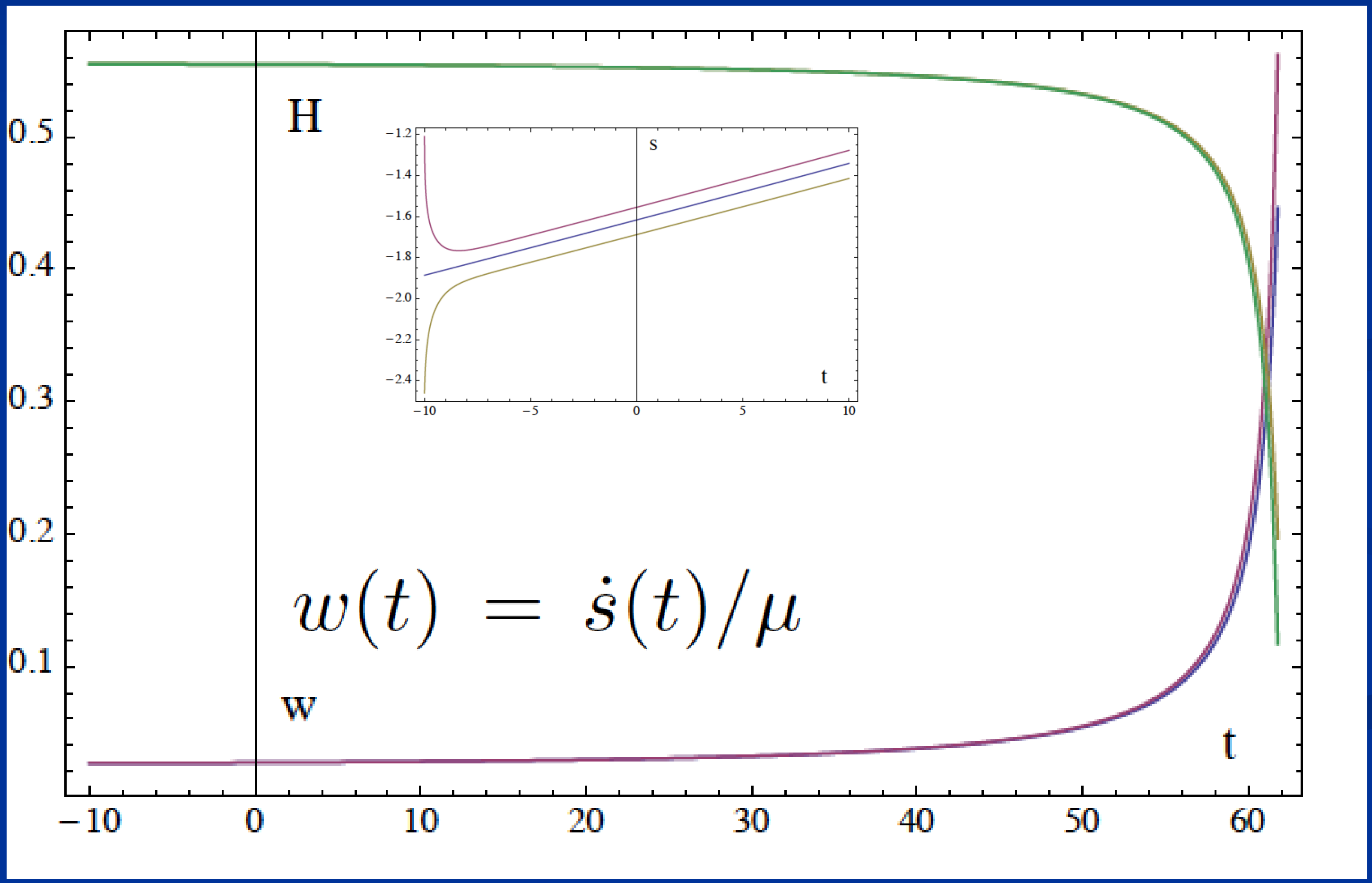
$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

$$R_{\mu\nu\rho\sigma} = b^2\mu^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$

Scaling solution is attractive



Scaling solution ends when K gets closer to -6



Radiation domination

$$c = \frac{2}{\sqrt{K+6}}$$

$$b = -\frac{c}{2}.$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2,$$

$$\bar{\rho}_r = -3\frac{K+5}{K+6}.$$

$$K < -5$$

$$H = b\mu \; , \; \chi = \chi_0 \exp(c\mu t).$$

Early Dark Energy

Energy density in radiation increases , proportional to cosmon potential

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2$$

$$V(\chi) = \mu^2\chi^2$$

fraction in early dark energy

$$\Omega_h = \frac{\rho_h}{\rho_r + \rho_h} = \frac{1}{K+6} = \frac{4}{\alpha^2}$$

requires large α >10

$$K(\chi) = \frac{4}{\tilde{\alpha}^2}\frac{m^2}{m^2+\chi^2} + \frac{4}{\alpha^2}\frac{\chi^2}{m^2+\chi^2} - 6$$

scaling of particle masses

mass of electron or nucleon is proportional to variable Planck mass χ !

effective potential for Higgs doublet h

$$\tilde{V}_h = \frac{1}{2}\lambda_h(\tilde{h}^\dagger\tilde{h} - \epsilon_h\chi^2)^2$$

cosmon coupling to matter

$$K(\ddot{\chi} + 3H\dot{\chi}) + \frac{1}{2}\frac{\partial K}{\partial \chi}\dot{\chi}^2 = -\frac{\partial V}{\partial \chi} + \frac{1}{2}\frac{\partial F}{\partial \chi}R + q_\chi$$

$q_\chi = -(\rho - 3p)/\chi$

Matter domination

$$c = \sqrt{\frac{2}{K+6}},$$

$$b = -\frac{1}{3}\sqrt{\frac{2}{K+6}} = -\frac{1}{3}c,$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2,$$

$$\bar{\rho}_m = -\frac{2(3K+14)}{3(K+6)}$$

$$K < -14/3$$

Neutrino mass

$$M_\nu = M_D M_R^{-1} M_D^T + M_L$$

$$M_L = h_L \gamma \frac{d^2}{M_t^2}$$

seesaw and cascade mechanism

triplet expectation value ~ doublet squared

$$m_\nu = \frac{h_\nu^2 d^2}{m_R} + \frac{h_L \gamma d^2}{M_t^2}$$

omit generation structure

Neutrino mass

assume that singlet scale has not yet reached scaling limit $\sim \chi$

$$\frac{M_{B-L}(\chi)}{\chi} = F_{B-L} - G_{B-L} \ln\left(\frac{\chi^2}{\mu^2}\right)$$

$$m_\nu \sim \frac{\tilde{h}^2}{M_{B-L}} \sim \frac{\epsilon_h \chi^2}{M_{B-L}(\chi)}$$

Dark Energy domination

neutrino masses scales differently from electron mass

$$m_\nu = \bar{c}_\nu \chi^{2\tilde{\gamma}+1}$$

$$\chi q_\chi = -(2\tilde{\gamma}+1)(\rho_\nu - 3p_\nu)$$

$$\frac{\rho_\nu}{\chi^2} = \bar{\rho}_\nu \mu^2$$

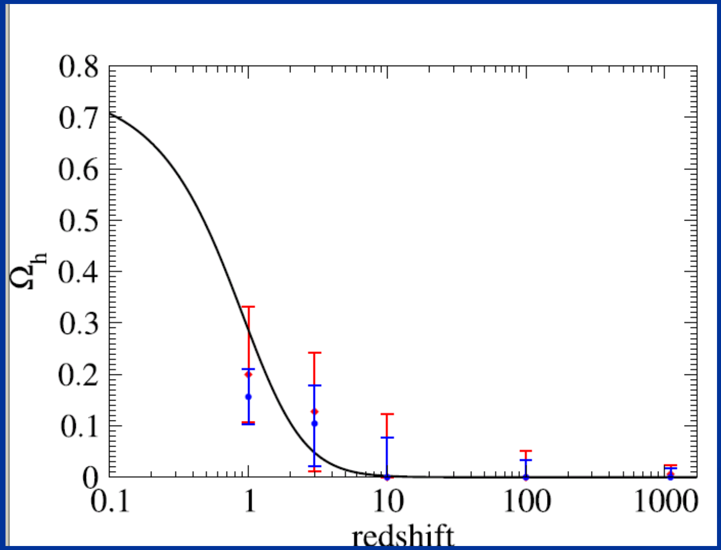
$$b = \frac{1}{3}(2\tilde{\gamma}-1)c.$$

new scaling solution. not yet reached.

at present : transition period

Why now problem

Why does fraction in Dark Energy increase in present cosmological epoch , and not much earlier or much later ?



neutrinos become non-relativistic at z = 5

Observations

simplest description in Einstein frame

Weyl scaling

$$g_{\mu\nu} = \frac{M^2}{F(\chi)} g'_{\mu\nu}.$$

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{k^2}{2} \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp\left(-\frac{\alpha\varphi}{M}\right) \right\}$$

$$k^2 = \frac{\alpha^2 (K+6)}{4}.$$

$$\varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

Kinetial

$$k^2(\varphi) = \left(\frac{\alpha^2}{\tilde{\alpha}^2} - 1\right) \frac{m^2}{m^2 + \mu^2 \exp(\alpha\varphi/M)} + 1.$$

scalar σ with
standard normalization

$$\frac{d\sigma}{d\varphi} = k(\varphi).$$

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{k^2}{2} \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp\left(-\frac{\alpha\varphi}{M}\right) \right\}$$

$$k^2 = \frac{\alpha^2 (K+6)}{4}.$$

conclusions

- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Different cosmon dependence of neutrino mass can explain why Universe makes a transition to Dark Energy domination now
- characteristic signal : neutrino lumps

f (R) - theories

$$\Gamma[g_{\mu\nu}] = -\frac{M^4}{2}\int_x \sqrt{g} f(x)$$

$$x = \frac{R}{M^2}$$

f (R) – theories , example

$$\Gamma[g_{\mu\nu}] = \int_x \sqrt{g} \left\{ -\frac{cM^2}{2} R - \frac{\alpha}{2} R^2 \right\}$$

$$\Gamma[g_{\mu\nu}] = -\frac{M^4}{2} \int_x \sqrt{g} f(x)$$

$$x = \frac{R}{M^2}$$

$$f(x) = cx + \alpha x^2$$

Equivalent scalar model

$$\Gamma[\phi, g_{\mu\nu}] = \int_x \sqrt{g} \left\{ -\frac{cM^2}{2} R - \frac{\alpha}{2} R^2 + \frac{\alpha}{2} \left(\frac{\phi}{\alpha} - R \right)^2 \right\}$$

solve scalar field equation

$$\frac{\delta \Gamma}{\delta \phi} = 0$$

$$\phi = \alpha R$$

insert solution into effective action

$$\Gamma[g_{\mu\nu}] = \int_x \sqrt{g} \left\{ -\frac{cM^2}{2} R - \frac{\alpha}{2} R^2 \right\}$$

Equivalent scalar model

$$\Gamma[\phi, g_{\mu\nu}] = \int_x \sqrt{g}\left\{V(\phi) - \frac{M^2}{2}\left(c + \frac{2\phi}{M^2}\right)R\right\}$$

$$V(\phi) = \frac{1}{2\alpha}\phi^2$$

$$\Gamma[\phi, g_{\mu\nu}] = \int_x \sqrt{g}\left\{-\frac{cM^2}{2}R - \frac{\alpha}{2}R^2 + \frac{\alpha}{2}\left(\frac{\phi}{\alpha} - R\right)^2\right\}$$

Weyl scaling

$$w^2 = \frac{1}{c + \frac{2\phi}{M^2}}$$

$$\Gamma[\phi', g'_{\mu\nu}] = \int_x \sqrt{g'} \left\{ V' - \frac{M^2}{2} \left(R' - \frac{3}{2} (\ln w^2)_{;}{}^{\mu} (\ln w^2)_{;\mu} \right) \right\}$$

$$V' = w^4 V = \frac{\phi^2}{2\alpha \left(c + \frac{2\phi}{M^2} \right)^2}$$

Canonical scalar kinetic term

$$w^2 = exp\left\{-\sqrt{\frac{2}{3}}\frac{\varphi}{M}\right\}$$

$$\varphi = \sqrt{\frac{3}{2}}M\ln\left(c + \frac{2\phi}{M^2}\right)$$

$$\Gamma[\varphi, g'_{\mu\nu}] = \int_x \sqrt{g'}\left\{V' + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{M^2}{2}R'\right\}$$

$$V'(\varphi) = \frac{M^4}{8\alpha}\left(1 - c\exp\left(-\sqrt{\frac{2}{3}}\frac{\varphi}{M}\right)\right)^2$$

Expansion for small φ

$$V'(\varphi) = \frac{M^4}{8\alpha}\left\{(1-c)^2 + \sqrt{\frac{8}{3}}c(1-c)\frac{\varphi}{M} + \frac{2}{3}c(2c-1)\frac{\varphi^2}{M^2} + \dots\right\}$$

c = 1 :

$$V'(\varphi) = \frac{M^4}{12\alpha}\varphi^2 + \dots$$

scalar mass

$$m_\varphi = \frac{M}{\sqrt{6\alpha}}$$

order Planck mass , unless α is huge !!!!

Higher order terms in effective gravitational action

- similar situation if f(R) admits Taylor expansion around R=0
- additional fields with mass close to Planck mass are not relevant for late cosmology
(but inflation…)
- holds also for more complicated effective actions

Universal coupling to massive particles

$$m_n' = w m_n = \exp\left\{-\frac{1}{\sqrt{6}}\frac{\varphi}{M}\right\} m_n$$

$$w^2 = exp\left\{-\sqrt{\frac{2}{3}}\frac{\varphi}{M}\right\}$$

$$\beta_n = -M \frac{\partial}{\partial \varphi} \ln m_n' = \frac{1}{\sqrt{6}}$$

Scalar field is allowed to change only by tiny amount on cosmological and local scales !

General f(R) theories as scalar models

$$\sqrt{g} f(R) = \sqrt{g'} \left\{ -\frac{M^2}{2} R' + \frac{1}{2} \partial^\nu \varphi \partial_\nu \varphi + V(\varphi) \right\}$$

$$V(\varphi) = \frac{M^2}{2} \frac{R f' - f}{(f')^2}$$

$$f'(R) = \frac{\partial f(R)}{\partial R}$$

$$f' = exp \left\{ \sqrt{\frac{2}{3}} \frac{\varphi}{M} \right\}$$

$$\frac{\varphi}{M} = \sqrt{\frac{3}{2}} \ln f'$$

$$w^2 = \frac{1}{f'}$$

$$w^2 = exp \left\{ -\sqrt{\frac{2}{3}} \frac{\varphi}{M} \right\}$$

Non – local gravity

$$\Gamma = \frac{M_p^2}{16\pi}\int d^dx\left\{\sqrt{g}R + \frac{1}{2}\tau^2\mathcal{L}_{nl}\right\}$$

$$\mathcal{L}_{nl} = \sqrt{g}RD^{-1}R = -\sqrt{g}R(\varepsilon R + D^2)^{-1}R$$

$$D^2\varphi = (\tau - \varepsilon\varphi)R$$

$$\Gamma = \frac{M_p^2}{16\pi}\int d^dx\sqrt{g}\left\{-\frac{1}{2}D^\mu\varphi D_\mu\varphi + (1 - \tau\varphi + \frac{1}{2}\varepsilon\varphi^2)R\right\}$$

Effective Nonlocal Euclidean Gravity

C. Wetterich , Gen.Rel.Grav. 30 (1998) 159

more general modifications of gravity

- can often be written in form where effective degrees of freedom are more easily visible

- massive gravity
- two- metric theories
- “MOND”

conclusions

- Modified gravity often easier understood in terms of additional fields
- No basic distinction between modified gravity and Dark energy (except massive gravity)
- Is the picture of modified gravity useful ?
- Sometimes , if important features are more easily visible (scale symmetry, absence of singularities)

End

Cosmon inflation

Inflation : Slow roll parameters

$$\epsilon = \frac{M^2}{2}\left(\frac{\partial \ln V}{\partial \sigma}\right)^2 = \frac{M^2}{2k^2}\left(\frac{\partial \ln V}{\partial \varphi}\right)^2 = \frac{\alpha^2}{2k^2}$$

$$\eta = \frac{M^2}{V}\frac{\partial^2 V}{\partial \sigma^2} = 2\epsilon - \frac{M}{\alpha}\frac{\partial \epsilon}{\partial \varphi}$$

For large $\alpha \gg 1$ and small $\tilde{\alpha} \ll 1$ we can approximate

$$\epsilon = \frac{\tilde{\alpha}^2}{2}\left(1 + \frac{\mu^2}{m^2}\exp(\alpha\varphi/M)\right),$$

$$\eta = \epsilon + \frac{\tilde{\alpha}^2}{2}.$$

End of inflation

at $\varepsilon = 1$

$$\exp\left(\frac{\alpha\varphi_f}{M}\right) = \frac{2m^2}{\tilde{\alpha}^2\mu^2}$$

Number of e-foldings before end of inflation

$$N(\varphi) = \frac{1}{\alpha M} \int_{\varphi}^{\varphi_f} d\varphi' k^2(\varphi')$$

$$= \frac{\alpha(\varphi_f - \varphi)}{\tilde{\alpha}^2 M} - \left(\frac{1}{\tilde{\alpha}^2} - \frac{1}{\alpha^2}\right) \ln\left(\frac{m^2 + \mu^2 \exp(\alpha\varphi_f/M)}{m^2 + \mu^2 \exp(\alpha\varphi/M)}\right)$$

ε, η, N can all be computed from kinetial alone

Spectral index and tensor to scalar ratio

$$n = 1 - 6\epsilon + 2\eta = 1 - \frac{2}{N}$$

$$r = 16\epsilon = \frac{8}{N} = 4(1-n).$$

$$n \approx 0.97 \;,\; r \approx 0.13$$

Amplitude of density fluctuations

$$24\pi^2\Delta^2 = \frac{V}{\epsilon M^4} = 2N\exp\left(-\frac{\alpha\varphi}{M}\right) \approx 5\cdot 10^{-7}.$$

$$\exp\left(-\frac{\alpha\varphi}{M}\right) \approx 4\cdot 10^{-9},$$

$$\frac{\tilde{\alpha}^2\mu^2}{m^2} \approx \frac{2}{3}\cdot 10^{-10}$$

Properties of density fluctuations

$\tilde{\alpha}$	0.001	0.02	0.1
n	0.975 (0.97)	0.975 (0.97)	0.972 (0.967)
r	0.13 (0.16)	0.13 (0.16)	0.18 (0.2)
$\frac{m}{\mu}$	120 (100)	2400 (2000)	12 000(10 000)

conclusion

cosmon inflation :

- compatible with observation
- simple
- no big bang singularity
- stability of solution singles out arrow of time
- simple initial conditions

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

Growing neutrino quintessence

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$

Neutrino cosmon coupling

- realized by dependence of neutrino mass on value of cosmon field

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi)$$

- $\beta \approx 1$: cosmon mediated attractive force between neutrinos has similar strength as gravity

growing neutrinos change cosmon evolution

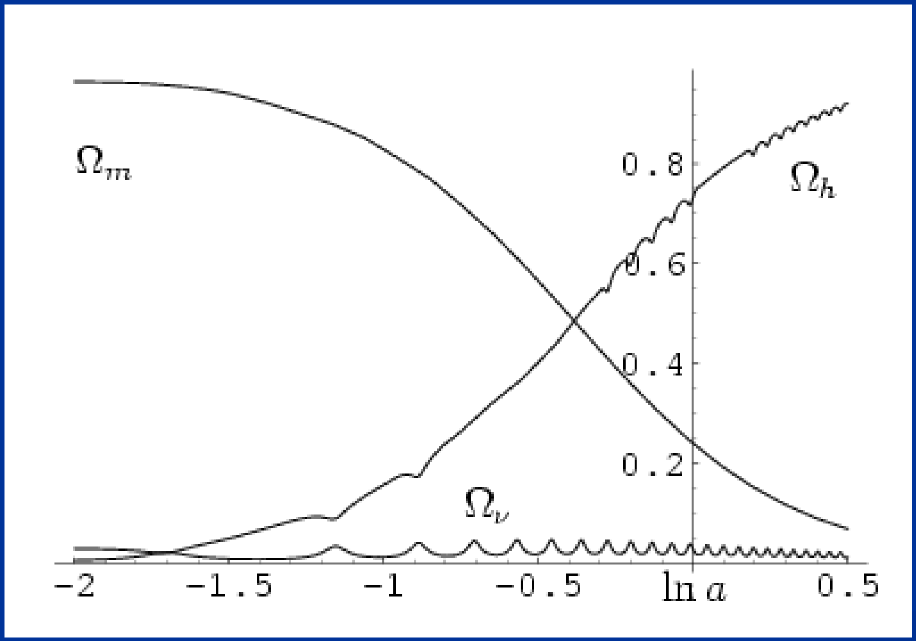
$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu),$$

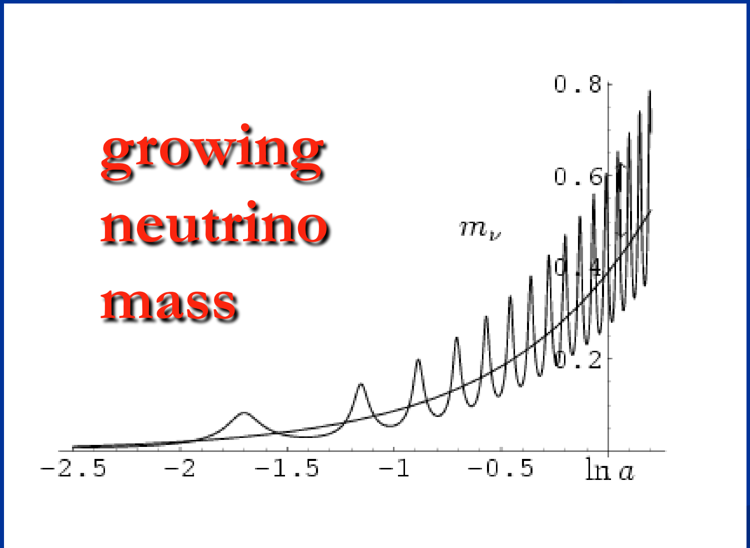
$$\beta(\varphi) = -M\frac{\partial}{\partial \varphi}\ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

modification of conservation equation for neutrinos

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = -\frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)\dot{\varphi}$$
$$= -\frac{\dot{\varphi}}{\varphi - \varphi_t}(\rho_\nu - 3p_\nu)$$

growing neutrino mass triggers transition to almost static dark energy



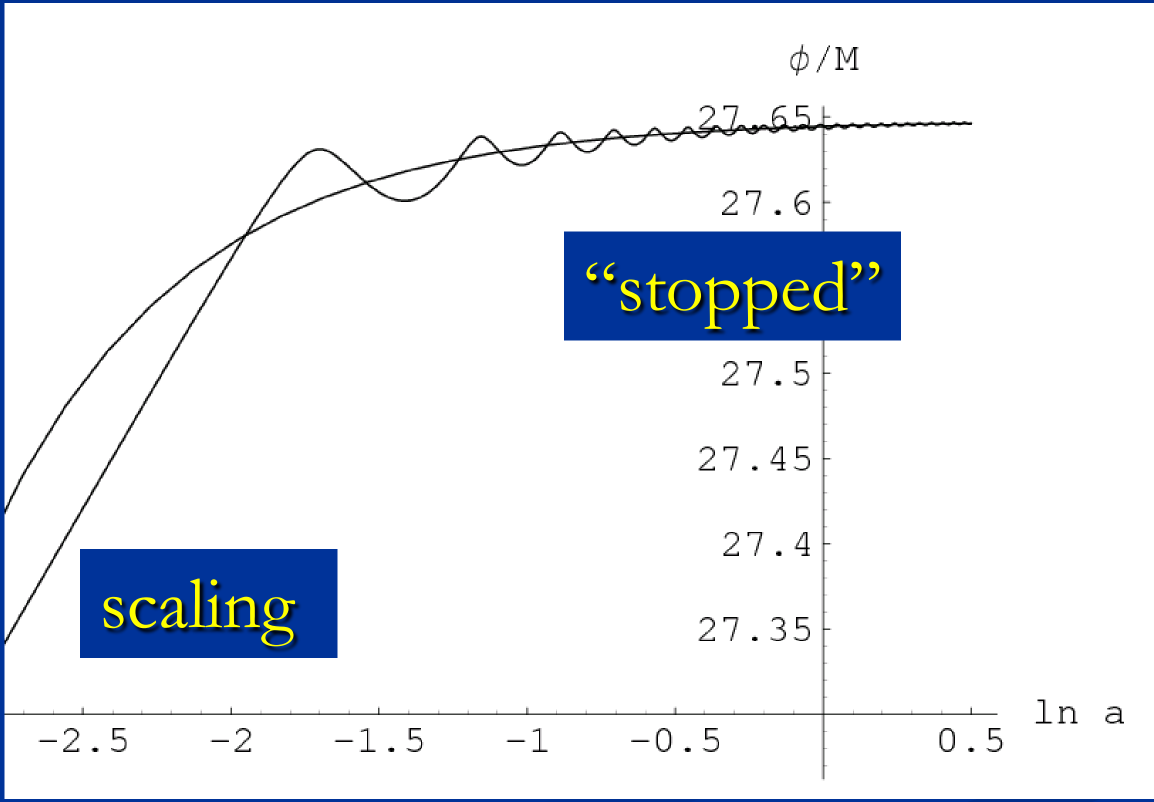


L.Amendola, M.Baldi,…

effective cosmological trigger for stop of cosmon evolution : neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

cosmon evolution

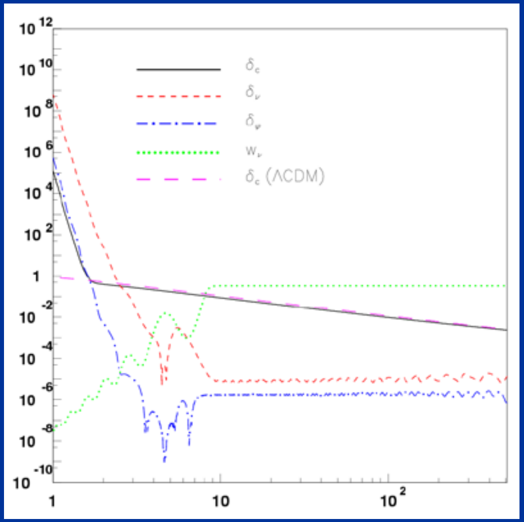


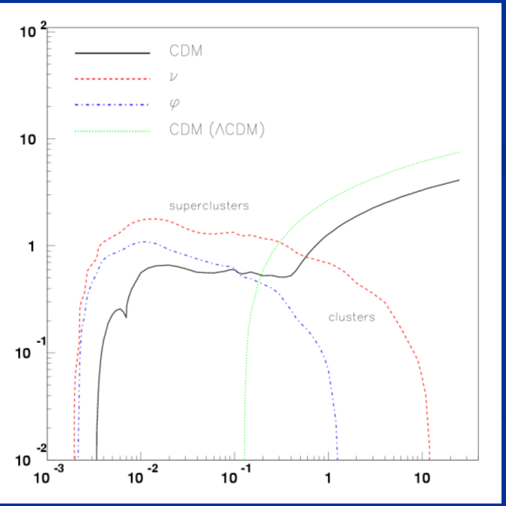
neutrino lumps

neutrino fluctuations

neutrino structures become nonlinear at z~1 for supercluster scales

D.Mota , G.Robbers , V.Pettorino , …





stable neutrino-cosmon lumps exist

N.Brouzakis , N.Tetradis ,… ; O.Bertolami ; Y.Ayaita , M.Weber,…

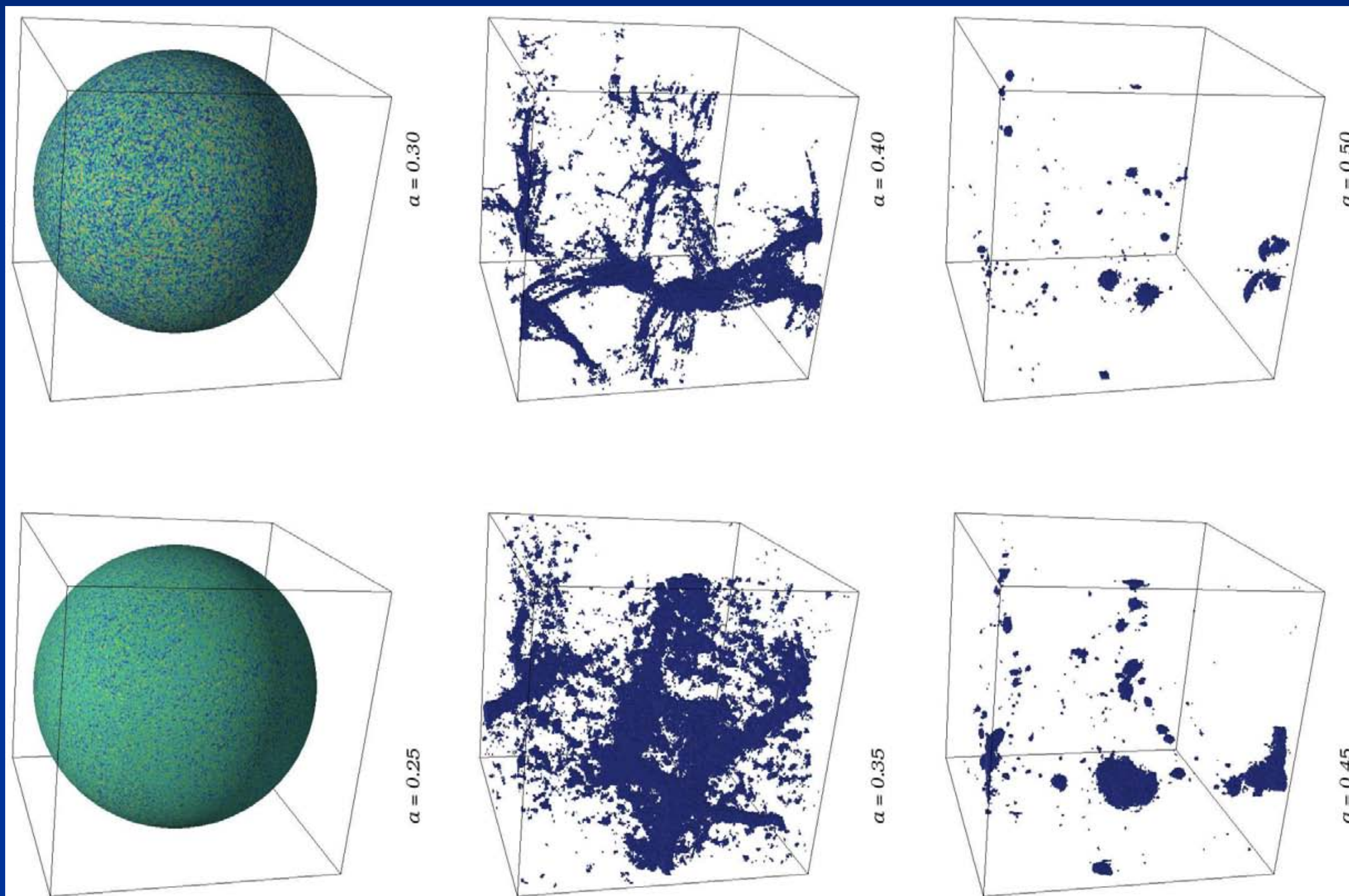
N-body code with fully relativistic neutrinos and backreaction

one has to resolve local value of cosmon field and then form cosmological average;
similar for neutrino density, dark matter and gravitational field

Y.Ayaita,M.Weber,…

Formation of neutrino lumps

Y.Ayaita,M.Weber,…



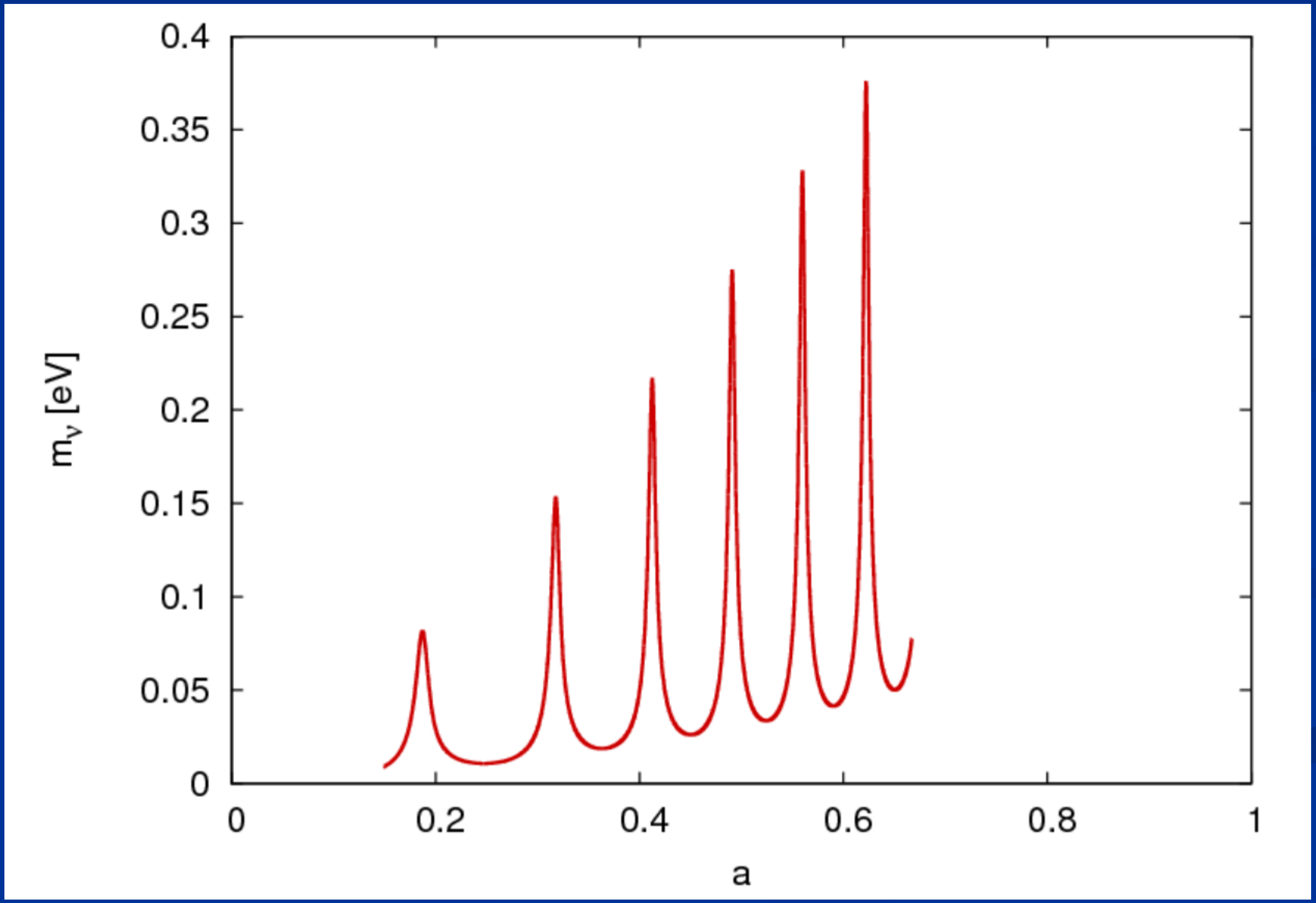
φ - dependent neutrino – cosmon coupling

$$\beta(\varphi) = -M\frac{\partial}{\partial\varphi}\ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

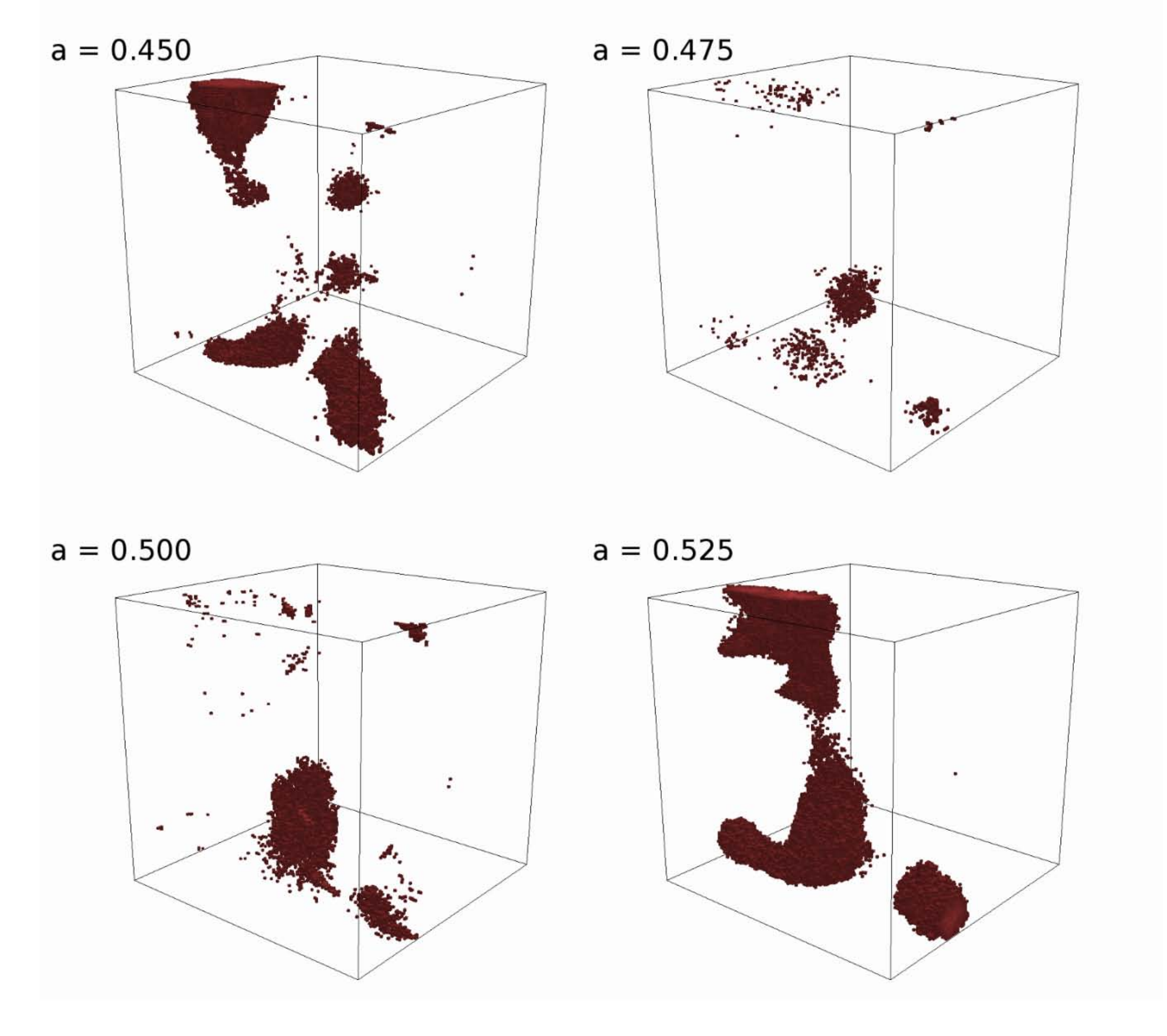
neutrino lumps form and are disrupted by oscillations in neutrino mass

smaller backreaction

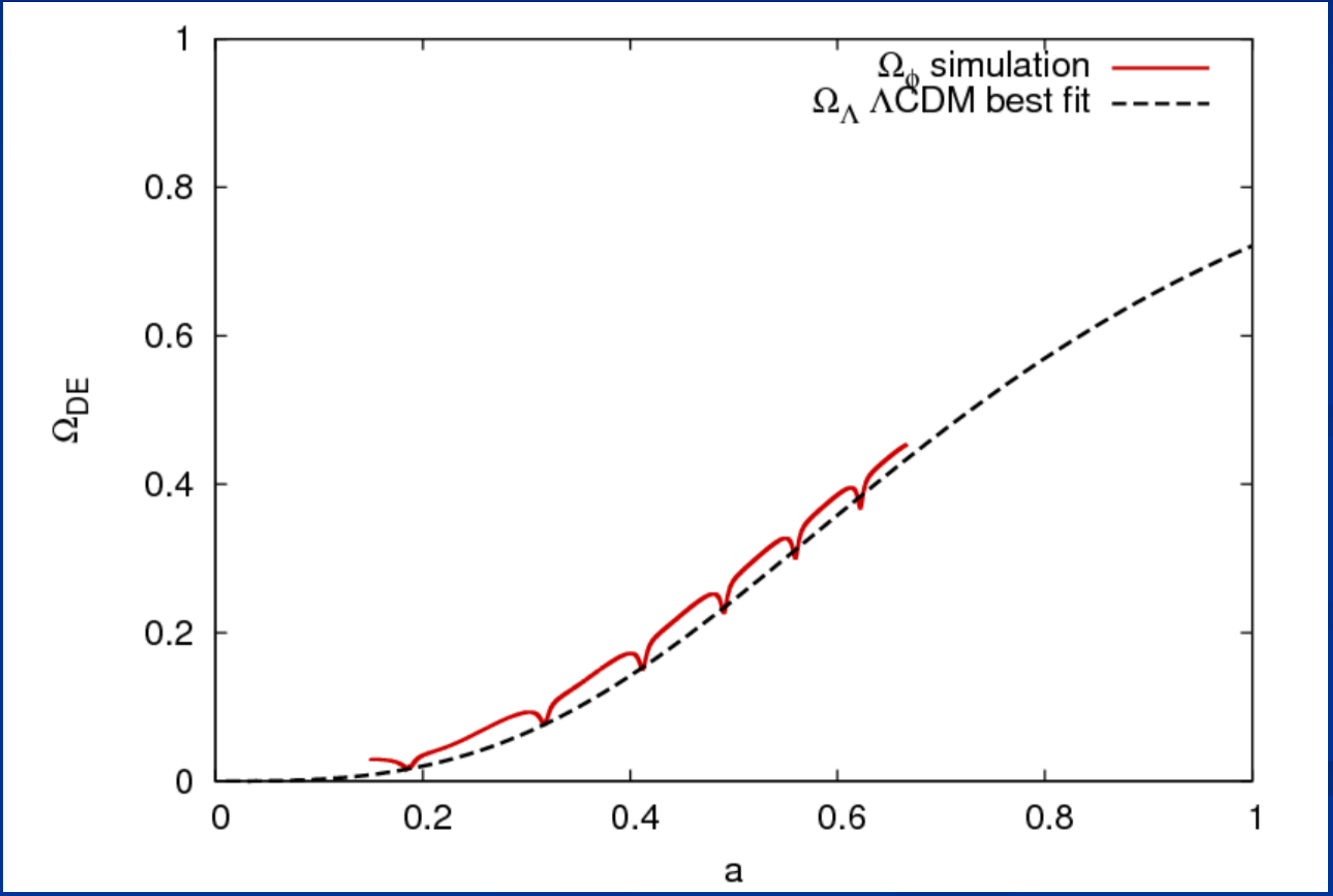
oscillating neutrino mass



oscillating neutrino lumps



small oscillations in dark energy



quantum fluctuations and dilatation anomaly

Dilatation anomaly

- Quantum fluctuations responsible both for fixed point and dilatation anomaly close to fxed point
- Running couplings: **hypothesis**

$$\partial\lambda/\partial\ln\chi = -A\lambda$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$

Asymptotic behavior of effective potential

- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\ \chi^4$

$$\mathbf{V \sim \chi^{4-A}}$$

crucial : behavior for large χ !

Without dilatation – anomaly :
V= const.
Massless Goldstone boson = dilaton

Dilatation – anomaly :
V (φ)
Scalar with tiny time dependent mass :
cosmon

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta $\sim\chi$!
- No prejudice on "natural value " of location of fixed point or anomalous dimension should be inferred from tiny contributions at QCD-momentum scale !

quantum fluctuations and naturalness

- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (**after** computation of quantum fluctuations !)
- Treatment of quantum fluctuations depends on frame : Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame

quantum fluctuations and frames

- Einstein frame : quantum fluctuations make zero cosmological constant look unnatural
- Jordan frame : quantum fluctuations can be the origin of dilatation anomaly;
- may be key ingredient for solution of cosmological constant problem !

fixed points and fluctuation contributions of individual components

If running couplings influenced by fixed points:
individual fluctuation contribution can be huge overestimate !

here : fixed point at vanishing quartic coupling and anomalous dimension ➡ $V \sim \chi^{4-A}$

it makes no sense to use naïve scaling argument to infer individual contribution $V \sim h\,\chi^{4}$

conclusions

- naturalness of cosmological constant and cosmon potential should be discussed in the light of dilatation symmetry and its anomalies
- Jordan frame
- higher dimensional setting
- four dimensional Einstein frame and naïve estimate of individual contributions can be very misleading !

conclusions

cosmic runaway towards fixed point may

solve the cosmological constant problem

and

account for dynamical Dark Energy