

# Universe without Expansion



**The Universe is shrinking**

The Universe is shrinking ...

while Planck mass and particle  
masses are increasing

# Two models of “ Variable Gravity Universe ”

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple scalar potential :
  - quadratic ( model A )
  - cosmological constant ( model B )
- Nucleon and electron mass proportional to Planck mass
- Neutrino mass has different dependence on scalar field



# Model A

- Inflation : Universe expands
- Radiation : Universe shrinks
- Matter : Universe shrinks
- Dark Energy : Universe expands

# Model B

- Inflation : Universe expands
- Radiation : Static Minkowski space
- Matter : Universe expands
- Dark Energy : Universe expands

# Compatibility with observations

- Both models lead to same predictions for radiation, matter , and Dark Energy domination, despite the very different expansion history
- Different inflation models:  
A:  $n=0.97$ ,  $r=0.13$     B:  $n=0.95$ ,  $r=0.04$
- Almost same prediction for radiation, matter, and Dark Energy domination as  $\Lambda$ CDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

# Cosmon inflation

Unified picture of inflation and  
dynamical dark energy

Cosmon and inflaton are the same field



# Quintessence

Dynamical dark energy ,  
generated by scalar field  
(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

**Prediction :**

**homogeneous dark energy  
influences recent cosmology**

**- of same order as dark matter -**

Original models do not fit the present observations  
.... modifications

# Merits of variable gravity models

- Economical setting
- No big bang singularity
- Arrow of time
- Simple initial conditions for inflation

# Model A

$$S = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$V(\chi) = \mu^2 \chi^2$$

$$\mu = 2 \cdot 10^{-33} \text{ eV}$$

$$K(\chi) = \frac{4}{\tilde{\alpha}^2} \frac{m^2}{m^2 + \chi^2} + \frac{4}{\alpha^2} \frac{\chi^2}{m^2 + \chi^2} - 6$$



# Scalar field equation:

additional force from R counteracts  
potential gradient : increasing  $\chi$  !

scalar field eq.

$$-D_{\mu}(K \partial^{\mu} \chi) = -\frac{\partial V}{\partial \chi} + R\chi$$

Robertson-Walker metric

$$K \left( \ddot{\chi} + 3H\dot{\chi} + \frac{\partial \ln K}{\partial \chi} \dot{\chi}^2 \right) = -\frac{\partial V}{\partial \chi} + R\chi$$

# Modified Einstein equation

New term with derivatives of scalar field

gravitational field eq.

$$\chi^2 \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + (\chi^2)_{;\rho}^{\rho} g_{\mu\nu} - (\chi^2)_{;\mu\nu} \\ + \frac{1}{2} K \partial^{\rho} \chi \partial_{\rho} \chi g_{\mu\nu} - K \partial_{\mu} \chi \partial_{\nu} \chi + V g_{\mu\nu} = T_{\mu\nu}$$

$\Rightarrow$

$$\chi^2 R = 3(\chi^2)_{;\mu}^{\mu} + K \partial^{\mu} \chi \partial_{\mu} \chi + 4V - T_{\mu}^{\mu}$$

# Curvature scalar and Hubble parameter

Robertson Walker metric

$$\chi^2 R = 4V - (K+6)\dot{\chi}^2 - 6\chi\ddot{\chi} - 18H\chi\dot{\chi} - T_{\mu}^{\mu}$$

$$(\chi^2)_{;\rho}^{\rho} = -2\dot{\chi}^2 - 2\chi\ddot{\chi} - 6H\chi\dot{\chi}$$

0-0-component

$$3\chi^2 H^2 + 6H\chi\dot{\chi} = \frac{1}{2}K\dot{\chi}^2 + V + T_{00}$$

# Scaling solutions

( for constant  $K$  )

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Four different scaling solutions for  
inflation, radiation domination,  
matter domination and  
Dark Energy domination



# Scalar dominated epoch, inflation

$$c = \pm \frac{2}{\sqrt{(K+6)(3K+16)}}$$

$$K > -\frac{16}{3}.$$

$$\begin{aligned} b &= \pm \sqrt{\frac{1}{3} + \frac{K+6}{6}c^2} - c \\ &= \pm \frac{K+4}{\sqrt{(K+6)(3K+16)}} = \frac{K+4}{2}c. \end{aligned}$$

Universe expands for  $K > 4$ , shrinks for  $K < 4$ .

# No big bang singularity

$$R_{\mu\nu\rho\sigma} = b^2 \mu^2 (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$$

# Radiation domination

$$c = \frac{2}{\sqrt{K+6}}$$

$$b = -\frac{c}{2}$$

**Universe  
shrinks !**

$$T_{00} = \rho = \bar{\rho} \mu^2 \chi^2$$

$$\bar{\rho}_r = -3 \frac{K+5}{K+6}$$

$$K < -5$$

# scaling of particle masses

mass of electron or nucleon is proportional to variable Planck mass  $\chi$  !

effective potential for Higgs doublet  $h$

$$\tilde{V}_h = \frac{1}{2} \lambda_h (\tilde{h}^\dagger \tilde{h} - \epsilon_h \chi^2)^2.$$



# cosmon coupling to matter

$$-D_{\mu}(K\partial^{\mu}\chi) = -\frac{\partial V}{\partial\chi} + \frac{1}{2}\frac{\partial F}{\partial\chi}R + q_{\chi},$$

$$q_{\chi} = -(\rho - 3p)/\chi$$

# Matter domination

$$c = \sqrt{\frac{2}{K+6}},$$

$$b = -\frac{1}{3}\sqrt{\frac{2}{K+6}} = -\frac{1}{3}c,$$

**Universe  
shrinks !**

$$\bar{\rho}_m = -\frac{2(3K+14)}{3(K+6)}$$

$$K < -14/3$$

# Dark Energy domination

neutrino masses scale  
differently from electron mass

$$m_\nu = \bar{c}_\nu \chi^{2\tilde{\gamma}+1}$$

$$\chi q_\chi = -(2\tilde{\gamma} + 1)(\rho_\nu - 3p_\nu)$$

$$\frac{\rho_\nu}{\chi^2} = \bar{\rho}_\nu \mu^2$$

$$b = \frac{1}{3}(2\tilde{\gamma} - 1)c$$

new scaling solution. not yet reached.  
at present : transition period

# Model B

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$F(\chi) = \chi^2 + m^2, \quad V(\chi) = \bar{\lambda}_c$$

$$\frac{\bar{\lambda}_c}{M^4} \approx 7 \cdot 10^{-121}, \quad (\bar{\lambda}_c)^{1/4} = 2 \cdot 10^{-3} eV$$

$$K + 6 = \frac{16}{\tilde{\alpha}^2} \frac{m^2}{m^2 + \chi^2} + \frac{16}{\alpha^2} \frac{\chi^2}{m^2 + \chi^2}$$

# Radiation domination

Flat static Minkowski space !  $H=0$  !

$$\chi = 2\sqrt{\frac{\lambda_c}{K+6}}(t + t_0).$$

exact regular solution ! (constant  $K$ )

constant energy  
density

$$\frac{\bar{\rho}}{\bar{\lambda}_c} = -\frac{3(K+2)}{K+6}$$

$$K < -2.$$

# Matter domination

$$H = \frac{1}{3}\dot{s}.$$

$$\dot{\chi}^2 = \frac{2}{K+6}\bar{\lambda}_c$$

$$\frac{14-3K}{6}\dot{\chi}^2 = \bar{\lambda}_c + \bar{\rho}$$

$$\frac{\bar{\rho}}{\lambda_c} = -\frac{2(2+3K)}{3(K+6)}$$

$$K < -\frac{2}{3}$$

# Weyl scaling

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{k^2}{2} \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp \left( -\frac{\alpha \varphi}{M} \right) \right\}$$

$$k^2 = \frac{\alpha^2 (K + 6)}{4}.$$

$$\varphi = \frac{2M}{\alpha} \ln \left( \frac{\chi}{\mu} \right)$$

# Kinetial

$$k^2(\varphi) = \left( \frac{\alpha^2}{\tilde{\alpha}^2} - 1 \right) \frac{m^2}{m^2 + \mu^2 \exp(\alpha\varphi/M)} + 1.$$

scalar  $\sigma$  with  
standard normalization

$$\frac{d\sigma}{d\varphi} = k(\varphi).$$

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R \right. \\ \left. + \frac{k^2}{2} \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp\left(-\frac{\alpha\varphi}{M}\right) \right\}$$
$$k^2 = \frac{\alpha^2(K+6)}{4}.$$



# Inflation : Slow roll parameters

$$\epsilon = \frac{M^2}{2} \left( \frac{\partial \ln V}{\partial \sigma} \right)^2 = \frac{M^2}{2k^2} \left( \frac{\partial \ln V}{\partial \varphi} \right)^2 = \frac{\alpha^2}{2k^2}$$

$$\eta = \frac{M^2}{V} \frac{\partial^2 V}{\partial \sigma^2} = 2\epsilon - \frac{M}{\alpha} \frac{\partial \epsilon}{\partial \varphi}$$

For large  $\alpha \gg 1$  and small  $\tilde{\alpha} \ll 1$  we can approximate

$$\begin{aligned}\epsilon &= \frac{\tilde{\alpha}^2}{2} \left( 1 + \frac{\mu^2}{m^2} \exp(\alpha\varphi/M) \right), \\ \eta &= \epsilon + \frac{\tilde{\alpha}^2}{2}.\end{aligned}$$

End of inflation  
at  $\epsilon = 1$

$$\exp\left(\frac{\alpha\varphi_f}{M}\right) = \frac{2m^2}{\tilde{\alpha}^2\mu^2}$$

# Number of e-foldings before end of inflation

$$\begin{aligned} N(\varphi) &= \frac{1}{\alpha M} \int_{\varphi}^{\varphi_f} d\varphi' k^2(\varphi') \\ &= \frac{\alpha(\varphi_f - \varphi)}{\tilde{\alpha}^2 M} - \left( \frac{1}{\tilde{\alpha}^2} - \frac{1}{\alpha^2} \right) \ln \left( \frac{m^2 + \mu^2 \exp(\alpha\varphi_f/M)}{m^2 + \mu^2 \exp(\alpha\varphi/M)} \right) \end{aligned}$$

$\varepsilon$ ,  $\eta$ ,  $N$  can all be computed from kinetic alone

# Spectral index and tensor to scalar ratio

Model A

$$\begin{aligned}n &= 1 - 6\epsilon + 2\eta = 1 - \frac{2}{N} \\r &= 16\epsilon = \frac{8}{N} = 4(1 - n).\end{aligned}$$

$$n \approx 0.97 \ , \ r \approx 0.13$$

# Amplitude of density fluctuations

$$24\pi^2 \Delta^2 = \frac{V}{\epsilon M^4} = 2N \exp\left(-\frac{\alpha\varphi}{M}\right) \approx 5 \cdot 10^{-7}.$$

$$\begin{aligned} \exp\left(-\frac{\alpha\varphi}{M}\right) &\approx 4 \cdot 10^{-9}, \\ \frac{\tilde{\alpha}^2 \mu^2}{m^2} &\approx \frac{2}{3} \cdot 10^{-10} \end{aligned}$$

# Einstein frame , model B

$$\varphi = \frac{M}{\alpha} \ln \frac{(\chi^2 + m^2)^2}{\bar{\lambda}_c}$$

$$\mathbf{k}^2 = 1 + \alpha^2 \left( \frac{1}{\tilde{\alpha}^2} - \frac{3}{8} \right) \frac{m^2}{\chi^2}$$

for large  $\chi$  :  
no difference to model A

$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{k^2}{2} \partial^\mu \varphi \partial_\mu \varphi + M^4 \exp \left( -\frac{\alpha \varphi}{M} \right) \right\}$$

# inflation model B

approximate relation between  $r$  and  $n$

$$r = \frac{16(1 - n) \exp(-N(1 - n))}{1 - 3[N(1 - n) - 1] \exp(-N(1 - n))}$$

$$n=0.95 \quad , \quad r=0.035$$

# conclusion 1

- cosmon inflation :
- compatible with observation
- simple
- no big bang singularity
- stability of solution singles out arrow of time
- simple initial conditions

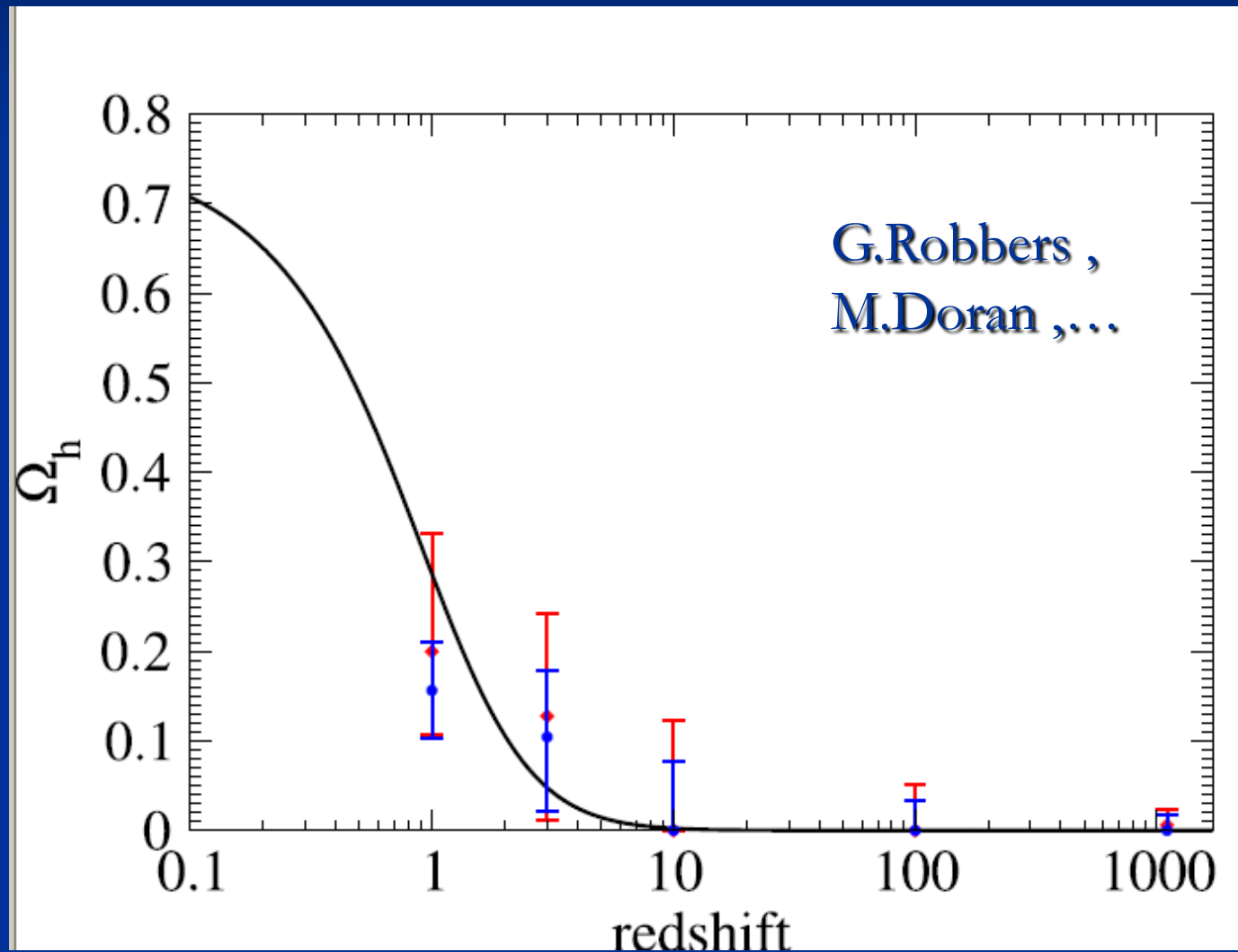
$$\Gamma = \int d^4x \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + V(\chi) \right\}$$

$$F(\chi) = \chi^2 + m^2, \quad V(\chi) = \bar{\lambda}_c$$

# Growing neutrino quintessence

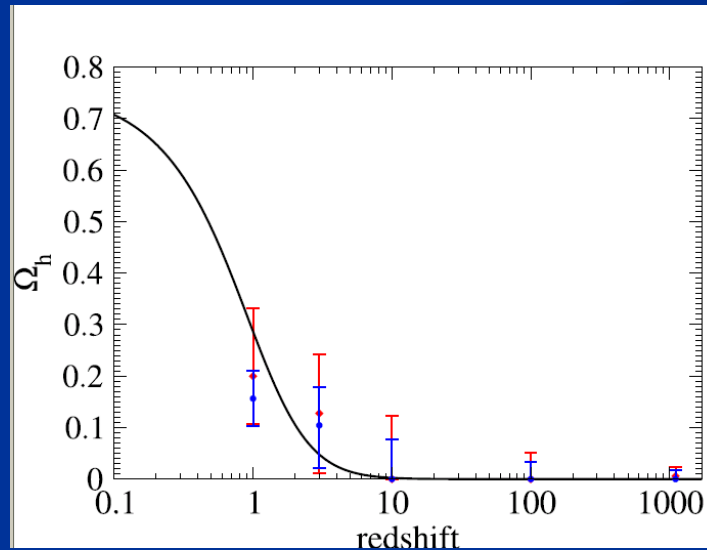


# Observational bounds on $\Omega_h$



# Why now problem

Why does fraction in Dark Energy increase in present cosmological epoch ,  
and not much earlier or much later ?



# Why neutrinos may play a role

## Mass scales :

Dark Energy density :  $\rho \sim (2 \times 10^{-3} \text{ eV})^{-4}$ .

Neutrino mass : eV or below.

**Cosmological trigger** : Neutrinos became non-relativistic only in the late Universe .

**Neutrino energy density** not much smaller than Dark Energy density .

Neutrinos can have substantial **coupling to Dark Energy**.

# connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left( \frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

present equation  
of state given by  
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

# Neutrinos in cosmology

only small fraction of energy density



only sub-leading role ?

# Neutrino cosmon coupling

- Strong bounds on atom-cosmon coupling from tests of equivalence principle or time variation of couplings.
- No such bounds for neutrino-cosmon coupling.
- In particle physics : Mass generation mechanism for neutrinos differs from charged fermions. Seesaw mechanism involves heavy particles whose mass may depend on the value of the cosmon field.

# neutrino mass

$$M_\nu = M_D M_R^{-1} M_D^T + M_L$$

$$M_L = h_L \gamma \frac{d^2}{M_t^2}$$

seesaw and  
cascade  
mechanism

triplet expectation value  $\sim$  doublet squared

$$m_\nu = \frac{h_\nu^2 d^2}{m_R} + \frac{h_L \gamma d^2}{M_t^2}$$

omit generation  
structure

# Neutrino cosmon coupling

- realized by dependence of neutrino mass on value of cosmon field

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi)$$

- $\beta \approx 1$  : cosmon mediated attractive force between neutrinos has similar strength as gravity



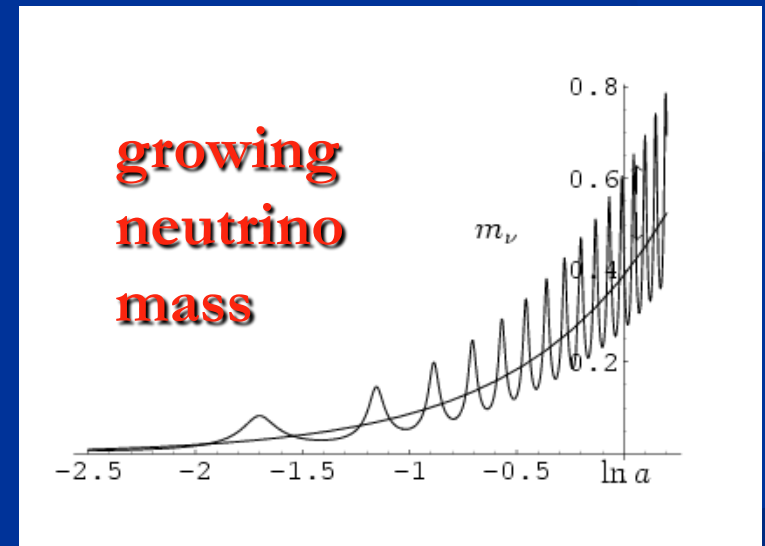
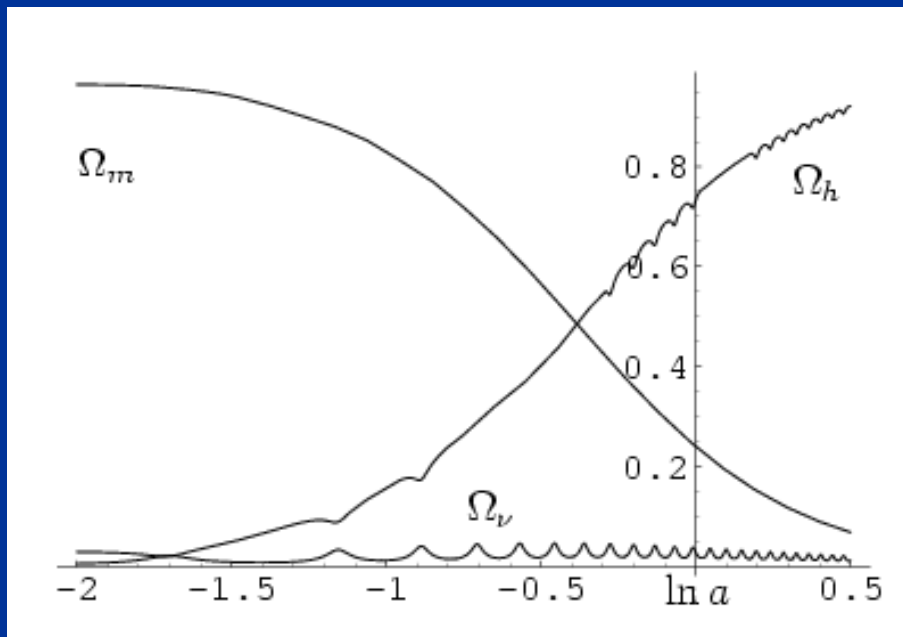
# growing neutrinos change cosmological evolution

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu),$$
$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

modification of conservation equation for neutrinos

$$\begin{aligned}\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) &= -\frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)\dot{\varphi} \\ &= -\frac{\dot{\varphi}}{\varphi - \varphi_t}(\rho_\nu - 3p_\nu)\end{aligned}$$

# growing neutrino mass triggers transition to almost static dark energy



L. Amendola, M. Baldi, ...

effective cosmological trigger  
for stop of cosmon evolution :  
neutrinos get non-relativistic

- this has happened recently !
- sets scales for dark energy !

# connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left( \frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

present dark energy density given by neutrino mass

present equation  
of state given by  
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

# cosmological selection

- present value of dark energy density set by cosmological event :  
neutrinos become non – relativistic
- not given by ground state properties !

basic ingredient :

**cosmon coupling to neutrinos**

# Cosmon coupling to neutrinos

- can be large !

Fardon, Nelson, Weiner

- interesting effects for cosmology if neutrino mass is growing
- growing neutrinos can stop the evolution of the cosmon
- transition from early scaling solution to cosmological constant dominated cosmology

L. Amendola, M. Baldi, ...

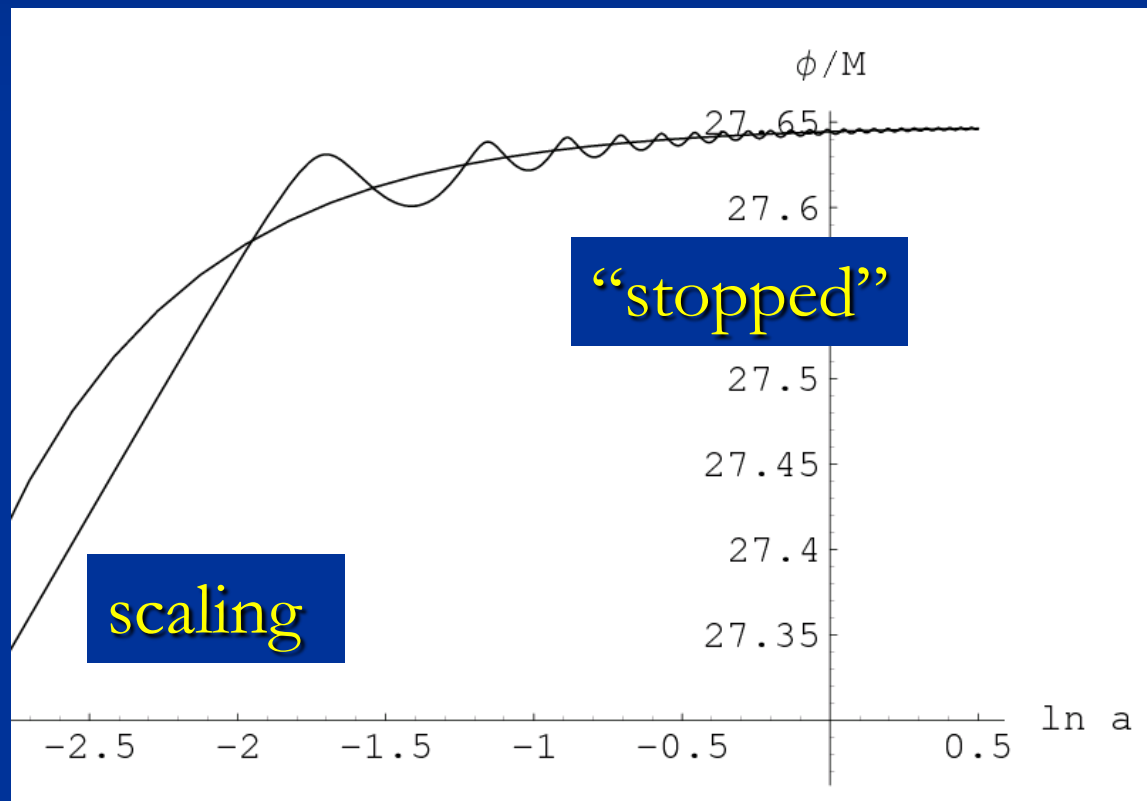
stopped scalar field  
mimicks a  
cosmological constant  
( almost ...)

rough approximation for dark energy :

- before redshift 5-6 : scaling ( dynamical )
- after redshift 5-6 : almost static  
( cosmological constant )



# cosmon evolution

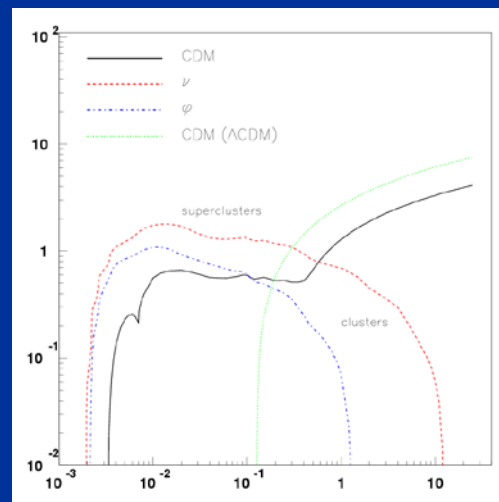
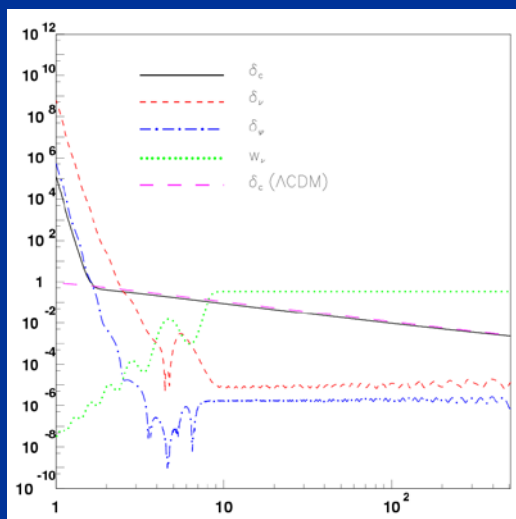


neutrino lumps

# neutrino fluctuations

neutrino structures become nonlinear at  $z \sim 1$  for  
supercluster scales

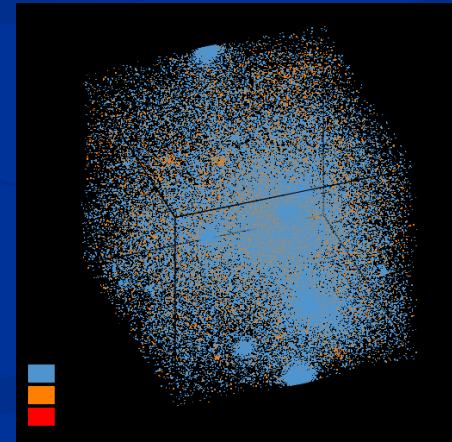
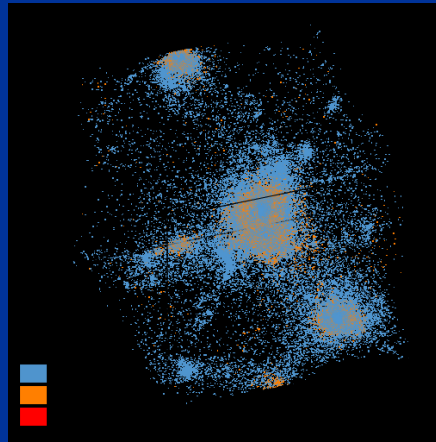
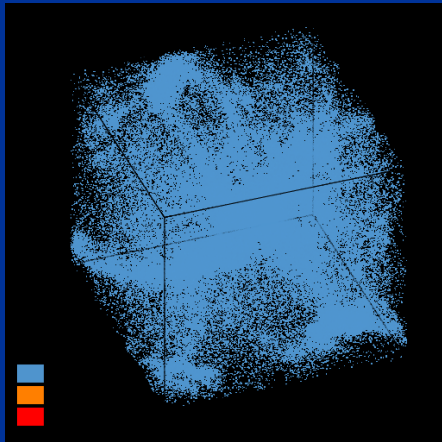
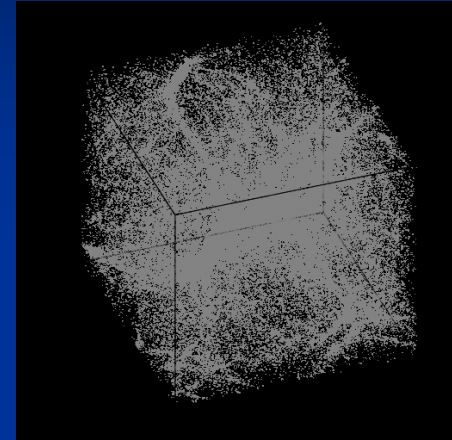
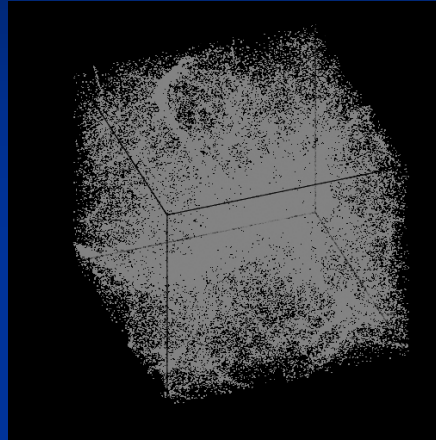
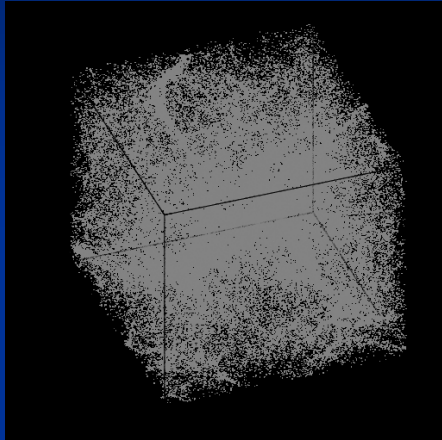
D.Mota , G.Robbers , V.Pettorino , ...



stable neutrino-cosmon lumps exist

N.Brouzakis , N.Tetradis , ... ; O.Bertolami ; Y.Ayaita , M.Weber, ...

# Formation of neutrino lumps



N- body simulation M.Baldi et al

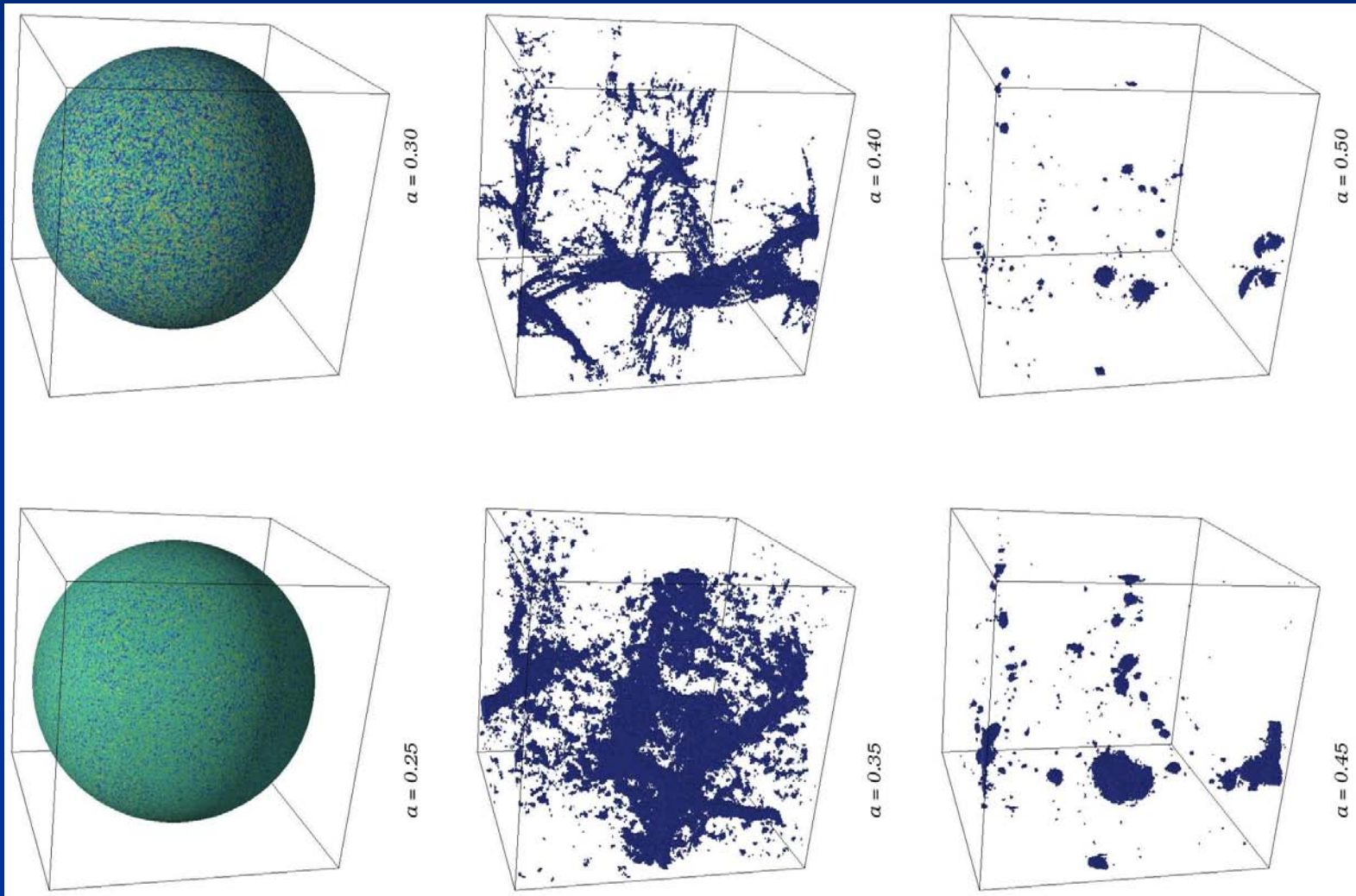
# N-body code with fully relativistic neutrinos and backreaction

one has to resolve local value of cosmon field  
and then form cosmological average;  
similar for neutrino density, dark matter and  
gravitational field

Y.Ayaita, M.Weber, ...

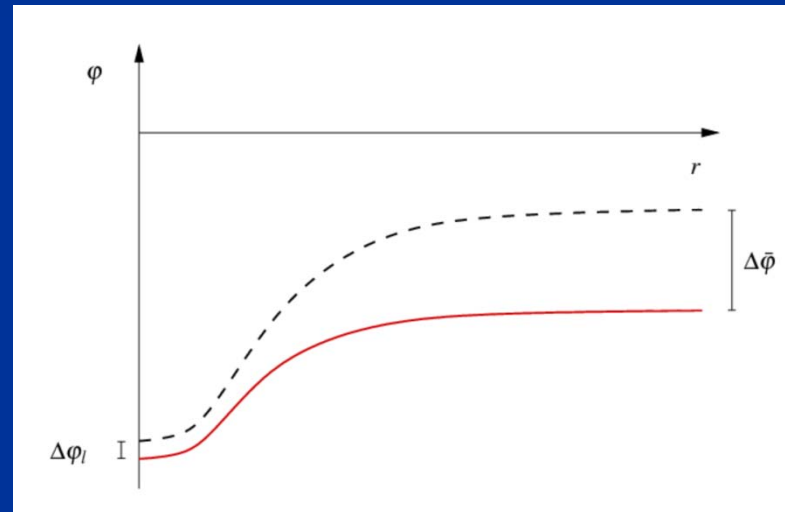
# Formation of neutrino lumps

Y.Ayaita,M.Weber,...



# backreaction

cosmon field inside lumps does not follow cosmological evolution

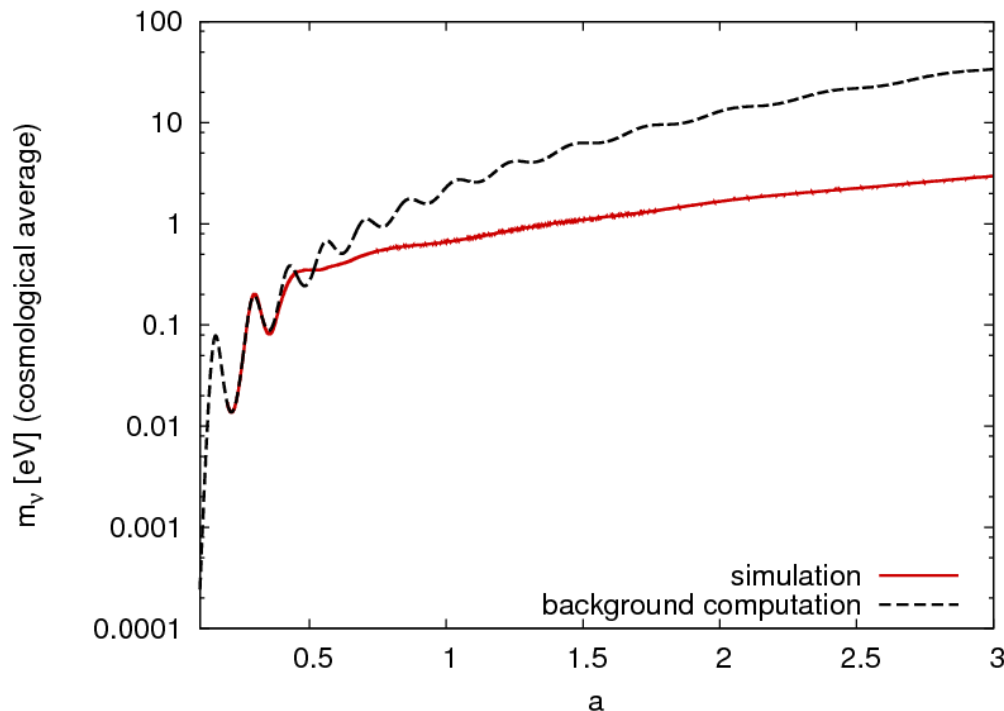


neutrino mass inside lumps smaller than  
in environment L.Schrempp, N.Nunes,...



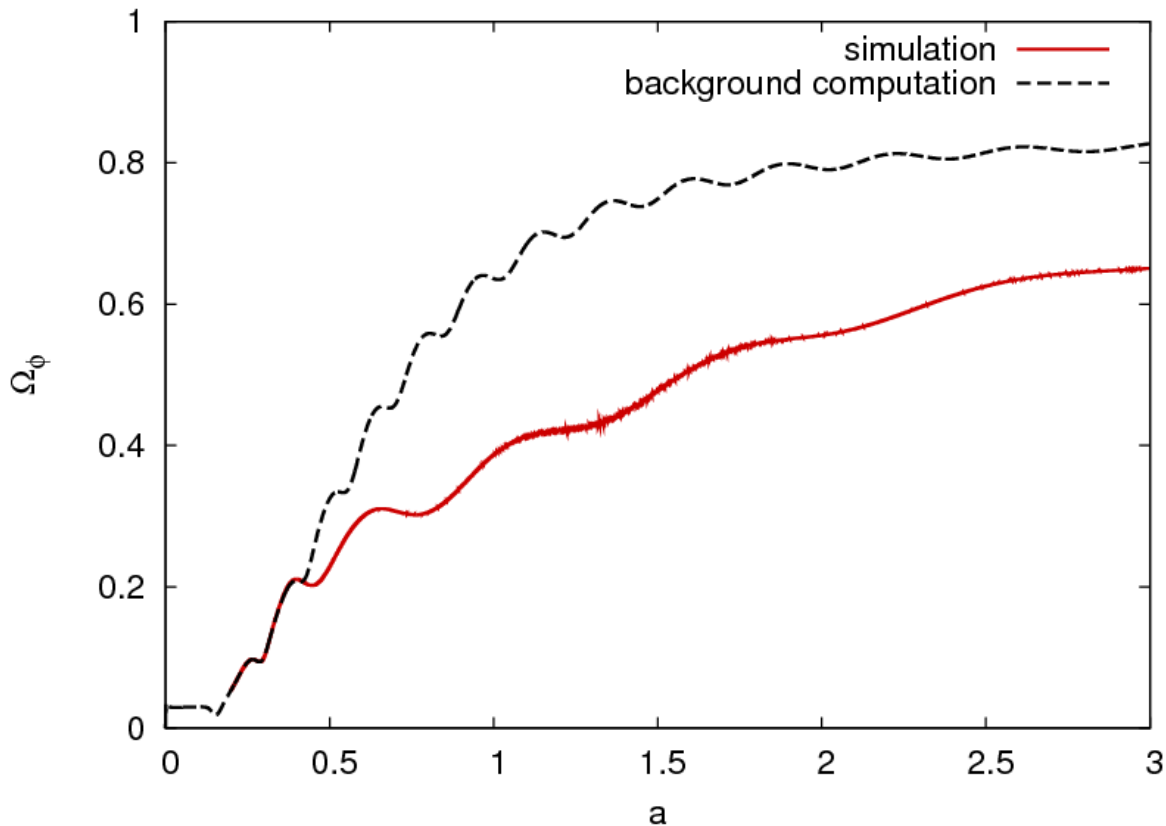
# importance of backreaction : cosmological average of neutrino mass

Y.Ayaita , E.Puchwein,...



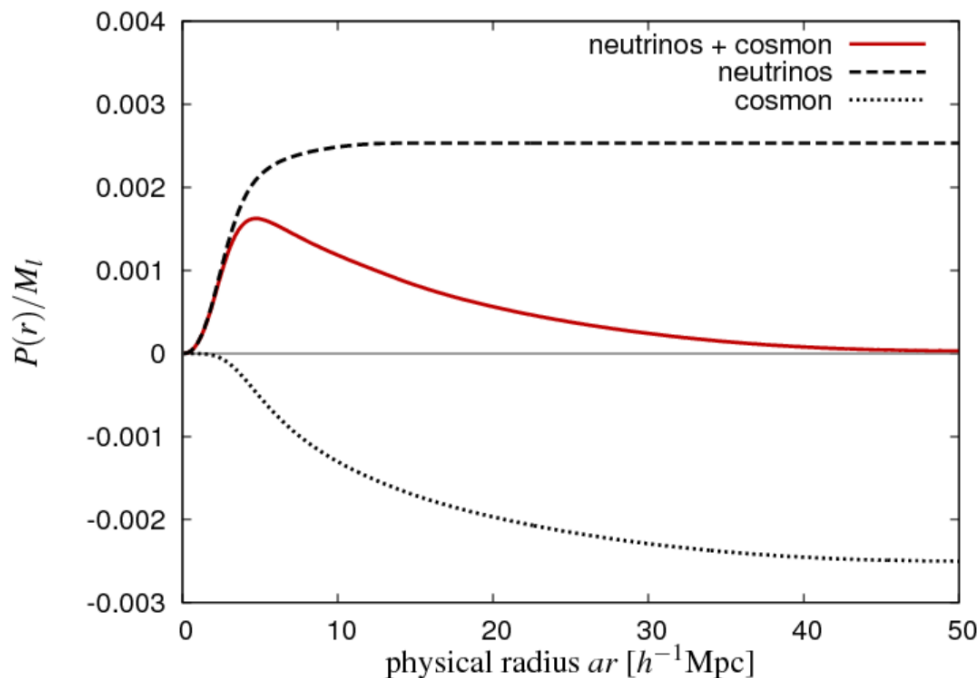


# importance of backreaction : fraction in Dark Energy

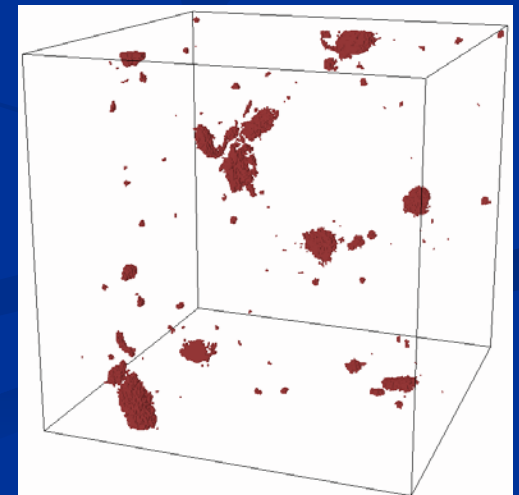


# neutrino lumps

behave as non-relativistic fluid with  
effective coupling to cosmon



Y. Ayaita,  
M. Weber, ...

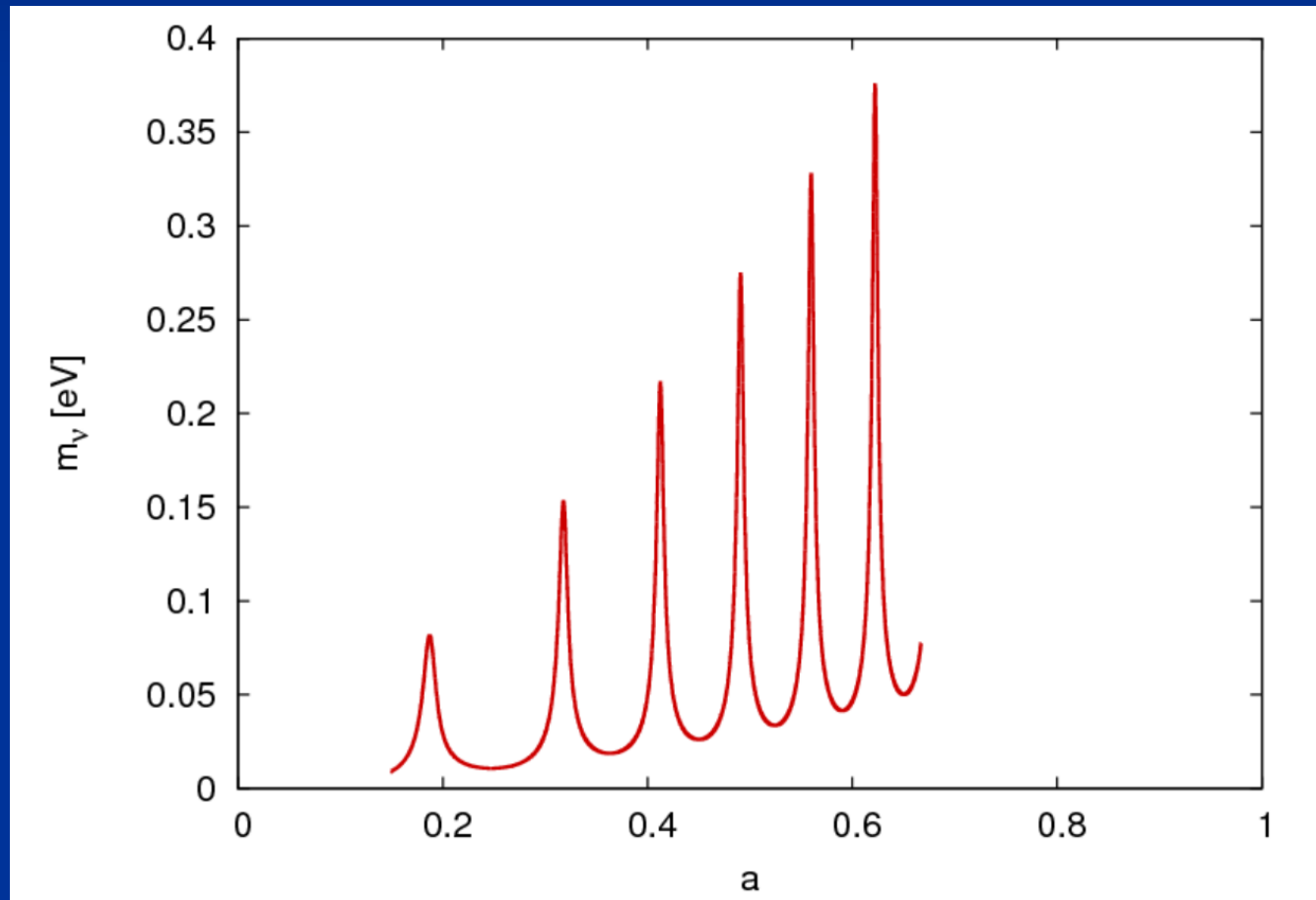


# $\varphi$ - dependent neutrino – cosmon coupling

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

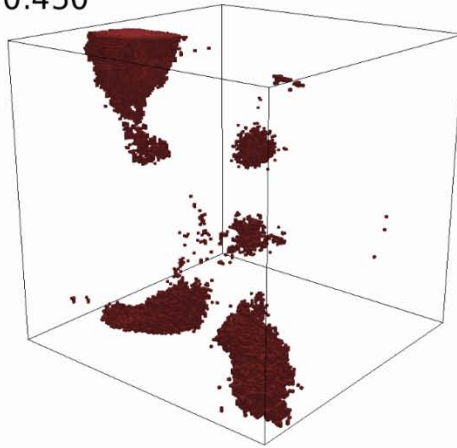
neutrino lumps form and are disrupted by  
oscillations in neutrino mass  
smaller backreaction

# oscillating neutrino mass

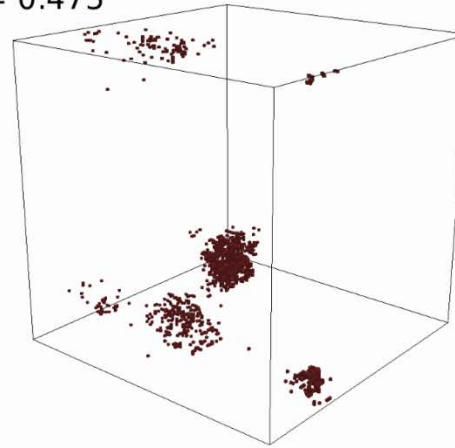


# oscillating neutrino lumps

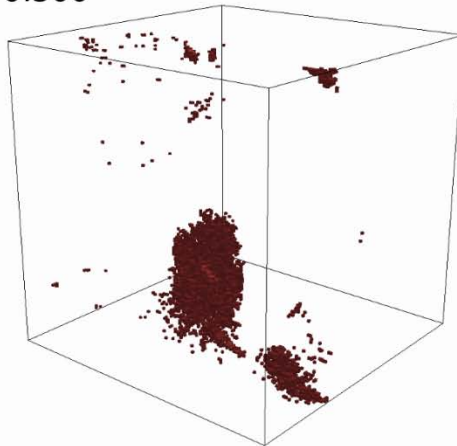
$a = 0.450$



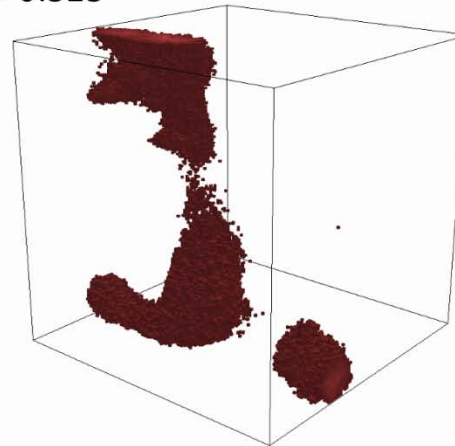
$a = 0.475$



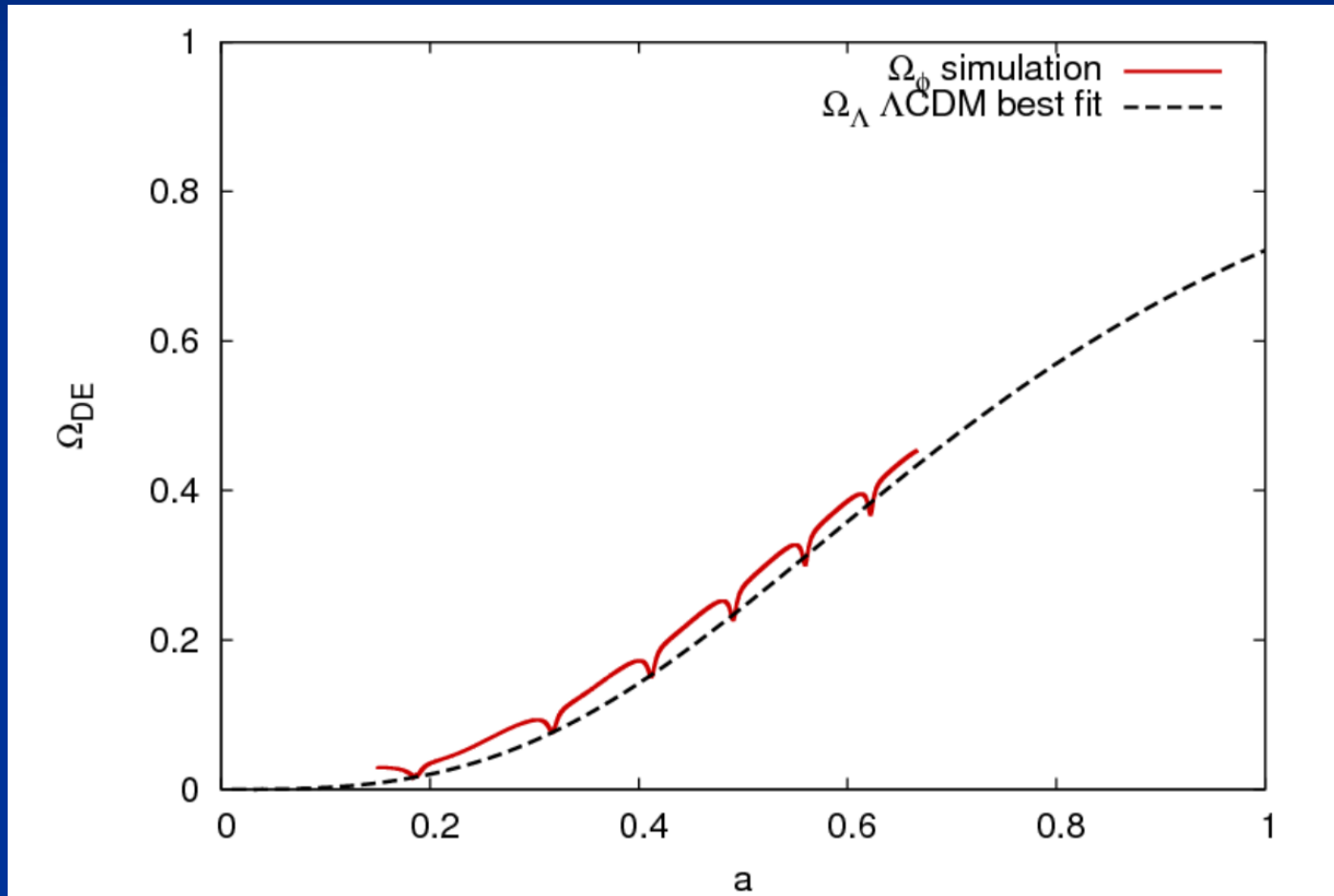
$a = 0.500$



$a = 0.525$



# small oscillations in dark energy



# conclusions

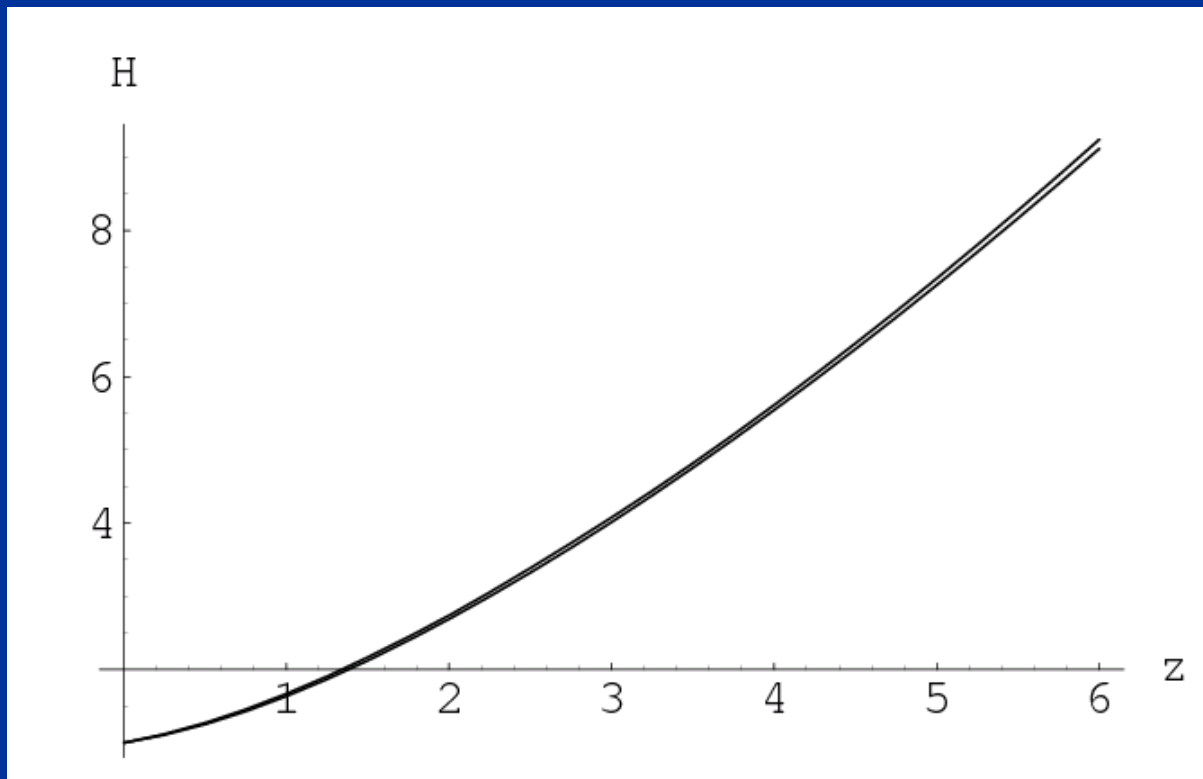
- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Different cosmological dependence of neutrino mass can explain why Universe makes a transition to Dark Energy domination **now**
- **characteristic signal : neutrino lumps**

# Tests for growing neutrino quintessence



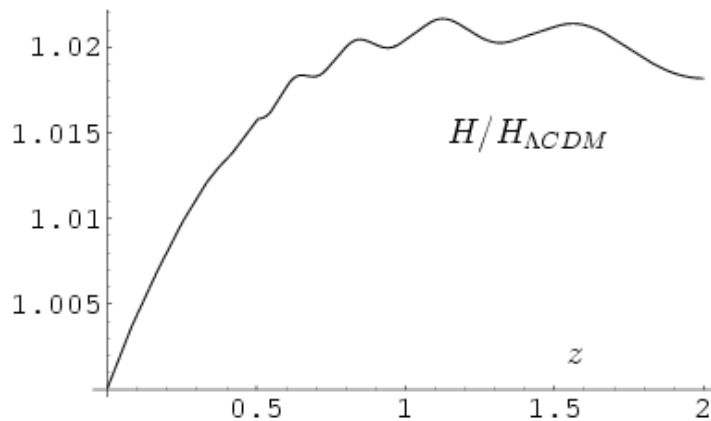
# Hubble parameter

as compared to  $\Lambda$ CDM



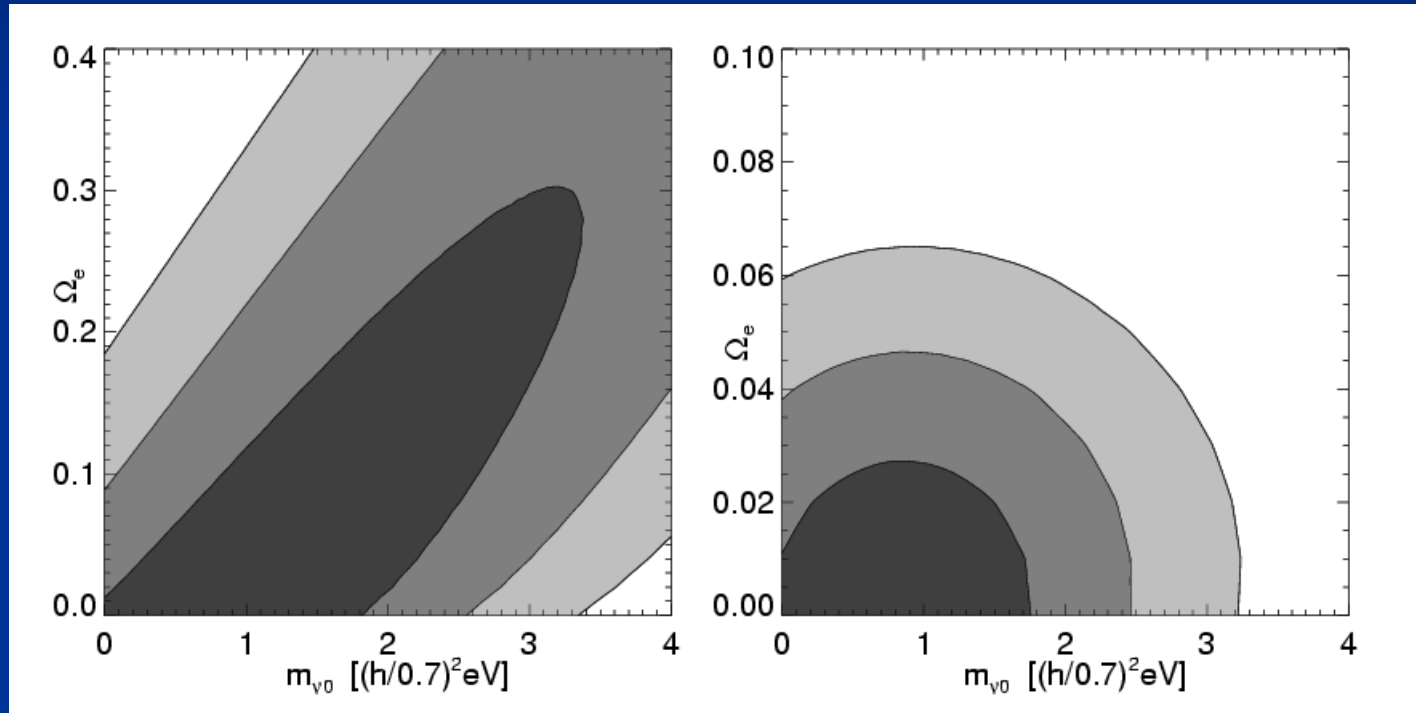
# Hubble parameter ( $z < z_c$ )

$$H^2 = \frac{1}{3M^2} \left\{ V_t + \rho_{m,0} a^{-3} + 2\tilde{\rho}_\nu a^{-\frac{3}{2}} \right\}$$



only small  
difference  
from  
 $\Lambda\text{CDM}$  !

# bounds on average neutrino mass

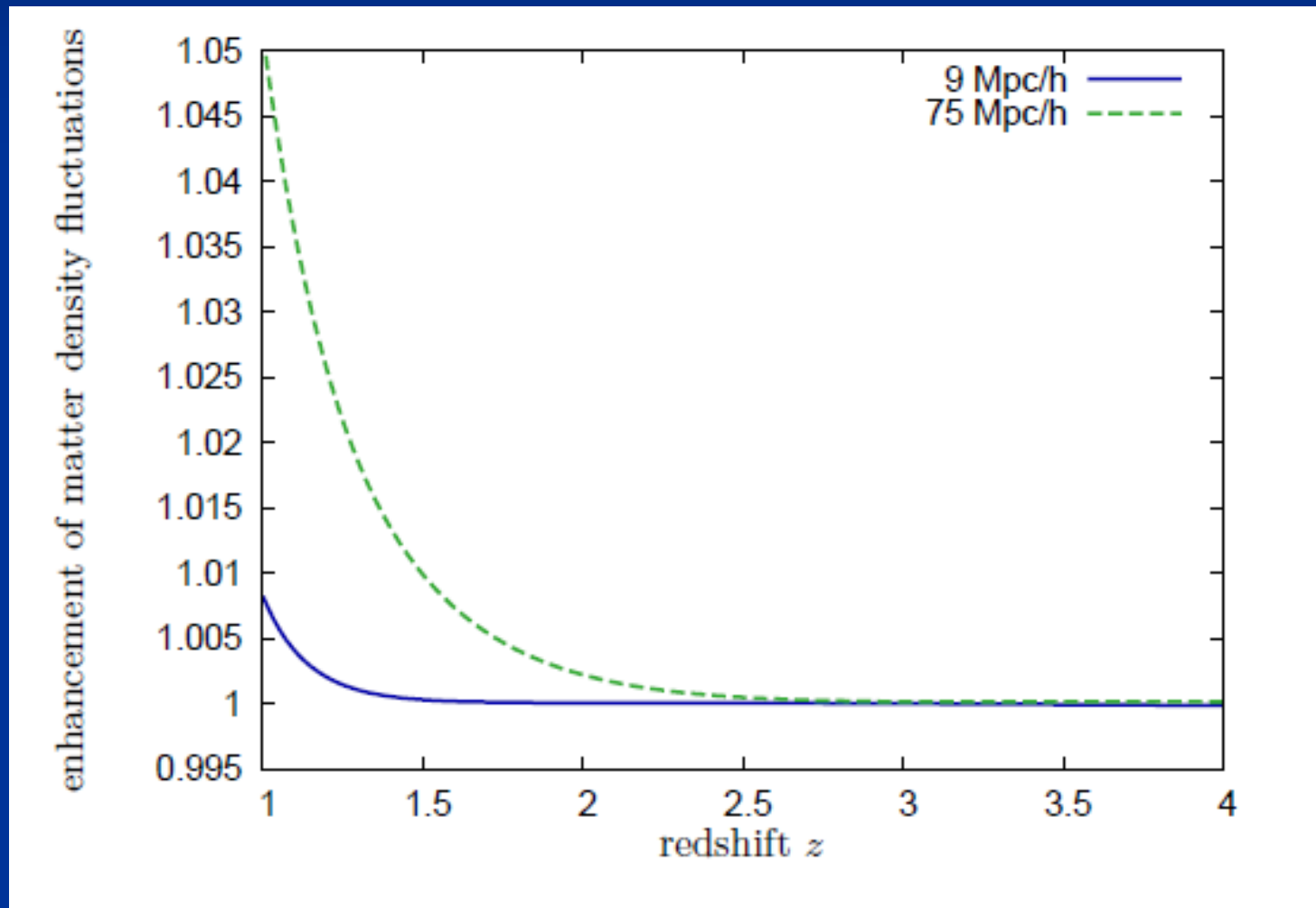


## Looking Beyond Lambda with the Union Supernova Compilation

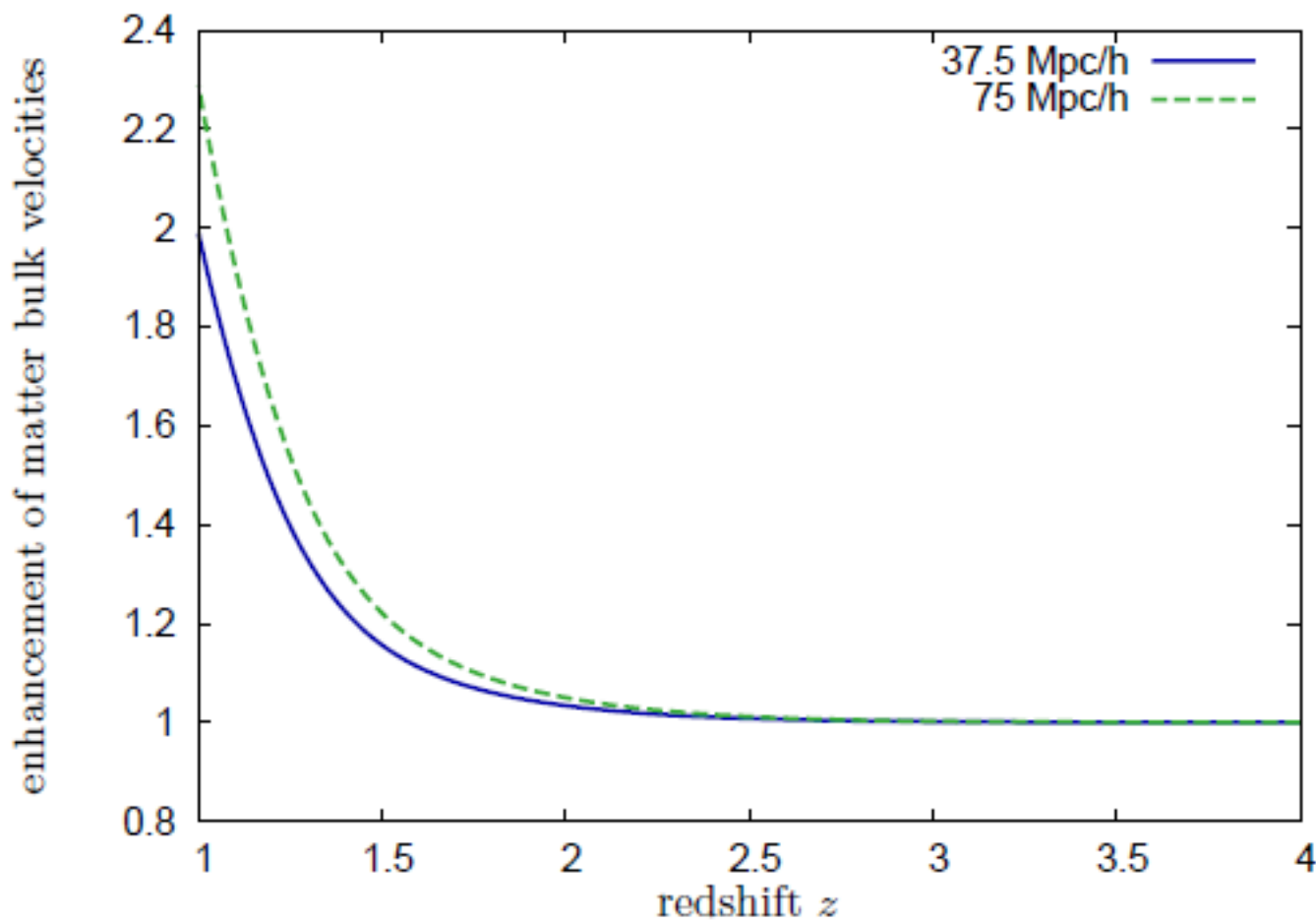
D. Rubin<sup>1,2</sup>, E. V. Linder<sup>1,3</sup>, M. Kowalski<sup>4</sup>, G. Aldering<sup>1</sup>, R. Amanullah<sup>1,3</sup>, K. Barbary<sup>1,2</sup>,  
N. V. Connolly<sup>5</sup>, K. S. Dawson<sup>1</sup>, L. Faccioli<sup>1,3</sup>, V. Fadeyev<sup>6</sup>, G. Goldhaber<sup>1,2</sup>, A. Goobar<sup>7</sup>,  
I. Hook<sup>8</sup>, C. Lidman<sup>9</sup>, J. Meyers<sup>1,2</sup>, S. Nobili<sup>7</sup>, P. E. Nugent<sup>1</sup>, R. Pain<sup>10</sup>, S. Perlmutter<sup>1,2</sup>,  
P. Ruiz-Lapuente<sup>11</sup>, A. L. Spadafora<sup>1</sup>, M. Strovink<sup>1,2</sup>, N. Suzuki<sup>1</sup>, and H. Swift<sup>1,2</sup>

(Supernova Cosmology Project)

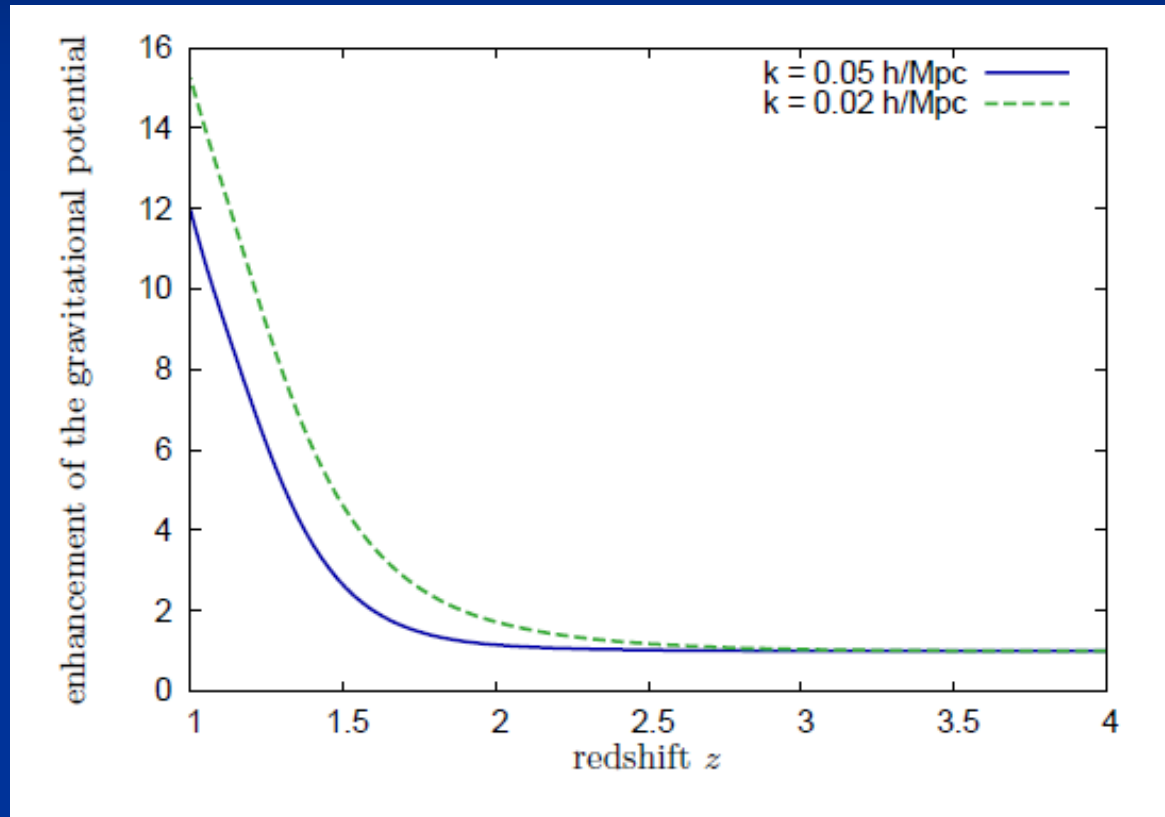
# Small induced enhancement of dark matter power spectrum at large scales



# Enhanced bulk velocities



# Enhancement of gravitational potential

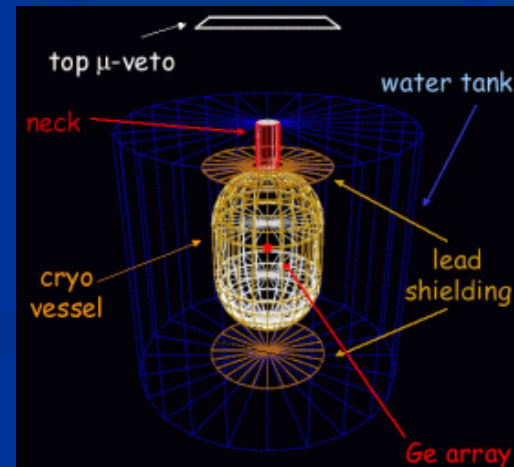


Test of allowed parameter space by ISW effect

# Can time evolution of neutrino mass be observed ?

Experimental determination of neutrino mass may turn out higher than cosmological upper bound in model with constant neutrino mass

( KATRIN, neutrino-less double beta decay )



GERDA

# Conclusions

- Cosmic event triggers qualitative change in evolution of cosmon
- Cosmon stops changing after neutrinos become non-relativistic
- Explains why now
- Cosmological selection
- Model can be distinguished from cosmological constant




Three decorative, wavy, light blue lines that sweep across the bottom right portion of the slide, starting from the right edge and moving towards the center.

End

strong effective  
neutrino – cosmon coupling  
for  $\varphi \rightarrow \varphi_t$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

typical present value :  $\beta \approx 50$    
cosmon mediated attraction between neutrinos  
is about  $50^2$  stronger than gravitational attraction

## early scaling solution ( tracker solution )

$$V(\varphi) = M^4 \exp \left( -\alpha \frac{\varphi}{M} \right)$$

$$\varphi = \varphi_0 + (2M/\alpha) \ln(t/t_0)$$

$$\Omega_{h,e} = \frac{n}{\alpha^2}$$

neutrino mass unimportant in early cosmology

# dark energy fraction determined by neutrino mass

$$\Omega_h(t_0) \approx \frac{\gamma m_\nu(t_0)}{16eV}$$

$$\gamma = -\frac{\beta}{\alpha}$$

constant neutrino - cosmon coupling  $\beta$

$$\Omega_h(t_0) \approx -\frac{\epsilon}{\alpha} \frac{m_\nu(t_0)}{\bar{m}_\nu} \frac{m_\nu(t_0)}{16eV}$$

variable neutrino - cosmon coupling

# effective stop of cosmon evolution

cosmon evolution almost stops once

- neutrinos get non-relativistic
- $\beta$  gets large

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} + \frac{\beta(\varphi)}{M}(\rho_\nu - 3p_\nu)$$

$$\beta(\varphi) = -M \frac{\partial}{\partial \varphi} \ln m_\nu(\varphi) = \frac{M}{\varphi - \varphi_t}$$

$$m_\nu(\varphi) = \frac{\beta(\varphi)}{\epsilon} \bar{m}_\nu$$

**This always  
happens  
for  $\varphi \rightarrow \varphi_t$  !**

A few early references on quintessence

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