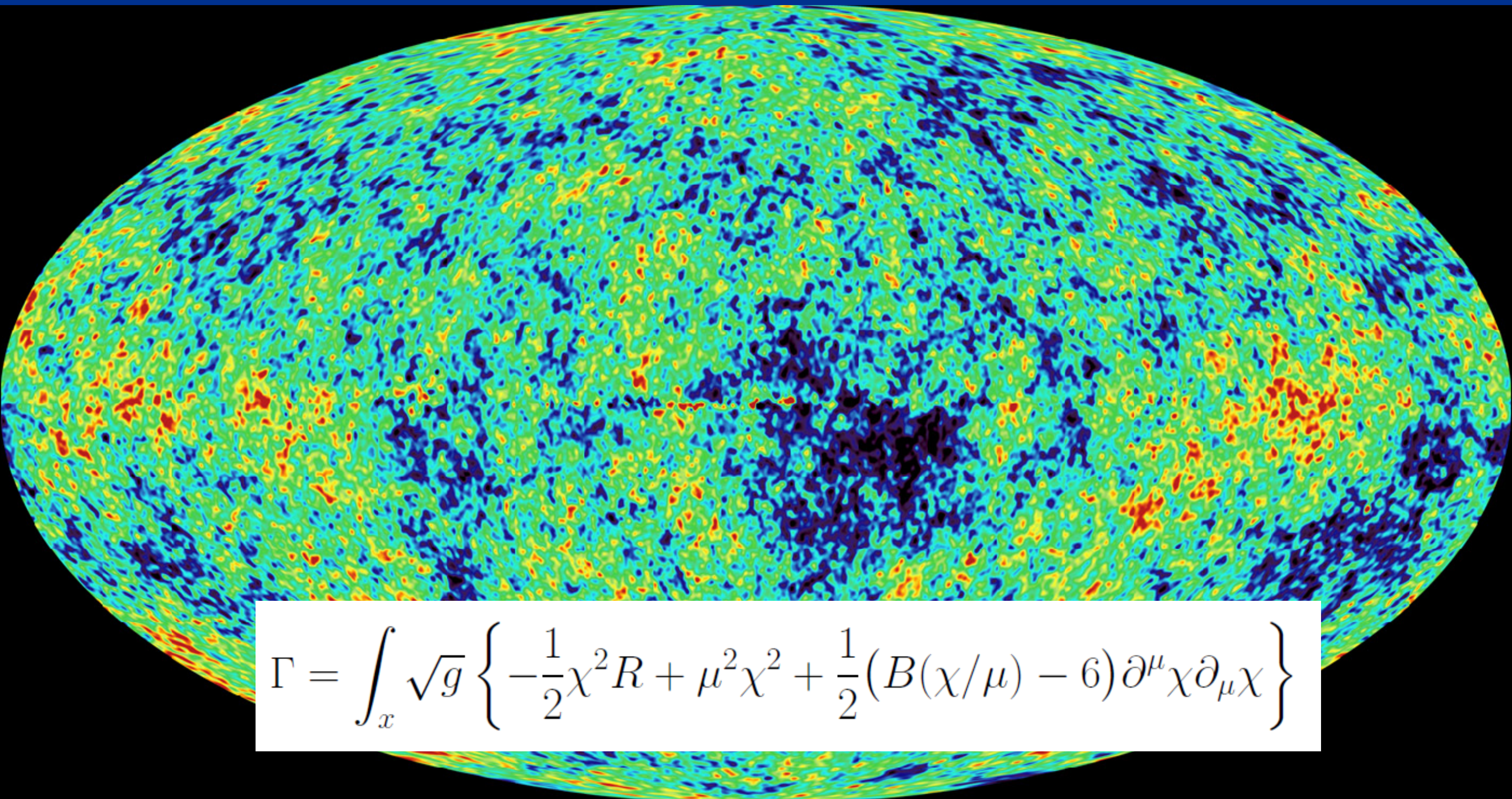
Graviton fluctuations erase the cosmological constant



$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

In quantum gravity,
the graviton fluctuations can
play an important role on
distances as large as the
size of the Universe

Graviton propagator

effective action $$\Gamma = \int_x \sqrt{g}\left(-\frac{M^2}{2}R + V\right)$$

flat space: $$G^{-1} = \frac{M^2\,q^2}{4} - \frac{V}{2}$$

Instability for V>0 : ”tachyonic mass term” $-\frac{2V}{M^2}$

curved space: $$G^{-1} = \sqrt{g}\left\{\frac{M^2}{4}\left(-D^2 + \frac{2R}{3}\right) - \frac{V}{2}\right\}$$

On shell graviton propagator

$$G^{-1} = \sqrt{g}\left\{\frac{M^2}{4}\left(-D^2 + \frac{2R}{3}\right) - \frac{V}{2}\right\}$$

on shell :

(for solution of field equations)

$$R = \frac{4V}{M^2}$$

homogenous isotropic metric,
conformal time

$$g_{\mu\nu} = a^2(\eta)\,\delta_{\mu\nu}$$

$$\mathcal{H} = \frac{\partial \ln a}{\partial \eta}$$

inverse graviton
propagator in
de Sitter space

$$a^{-2}\left(-D^2 + \frac{R}{6}\right)a^2 = \frac{1}{a^2}\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + \vec{q}^{\,2}\right)$$

milder instability, not tachyonic, absent for
cosmologies close to de Sitter space

IR – instability for graviton fluctuations

problem solved ?

- yes for primordial cosmic fluctuations (on shell)
- no for quantum gravity (off shell)
- Computation of effective action is an off-shell problem.
- example : one needs the effective potential for the Higgs field in the vicinity of its minimum (off shell), not only at the minimum (on shell)

Quantum gravity with scalar field

M^2 and V depend on scalar field χ

$$M^2 = c_1 + c_2\,\chi^2$$

$$V = d_1 + d_2\,\chi^2 + d_3\,\chi^4$$

question : behavior of V for $\chi \to \infty$

- $d_3 \neq 0$ excluded!
- $d_3 < 0$ unstable potential
- $d_3 > 0$ instability of graviton propagator

Graviton barrier

Quantum gravity computation :

For $\chi \to \infty$

V cannot increase stronger than M^2 !

Instability of graviton propagator is avoided

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ, and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

Normalization of scalar field

If M increases monotonically with χ :

choose normalization of scalar

$$M = \chi$$

asymptotically vanishing cosmological „constant"

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

Einstein frame

- "Weyl scaling" maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !

- Standard gravity coupled to scalar field.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k, such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

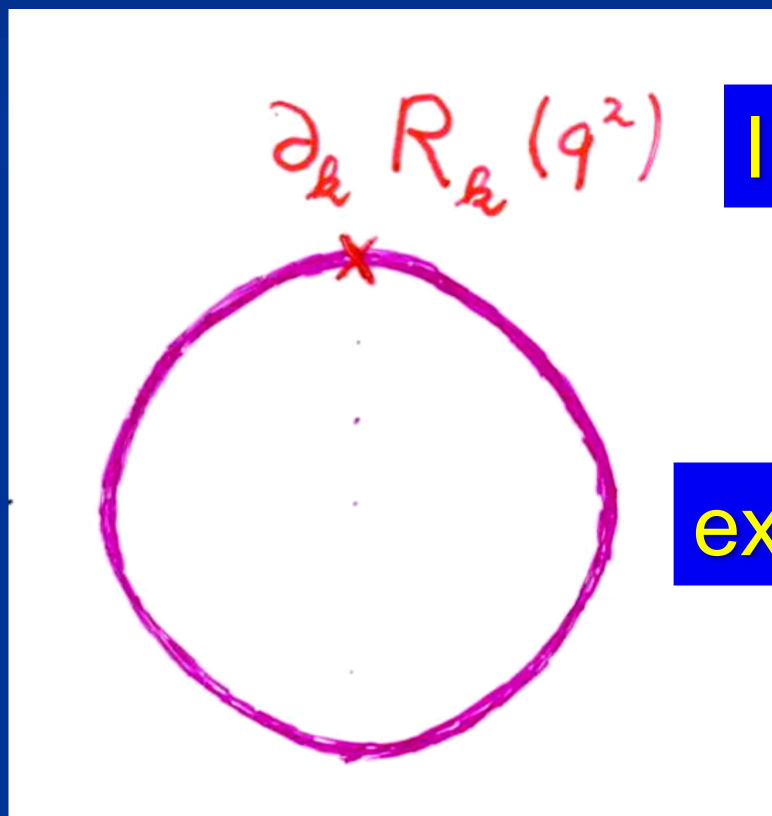
'92

$$\left(\Gamma_k^{(2)}\right)_{ab}(q,q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

From

Microscopic Laws
(Interactions, classical action)

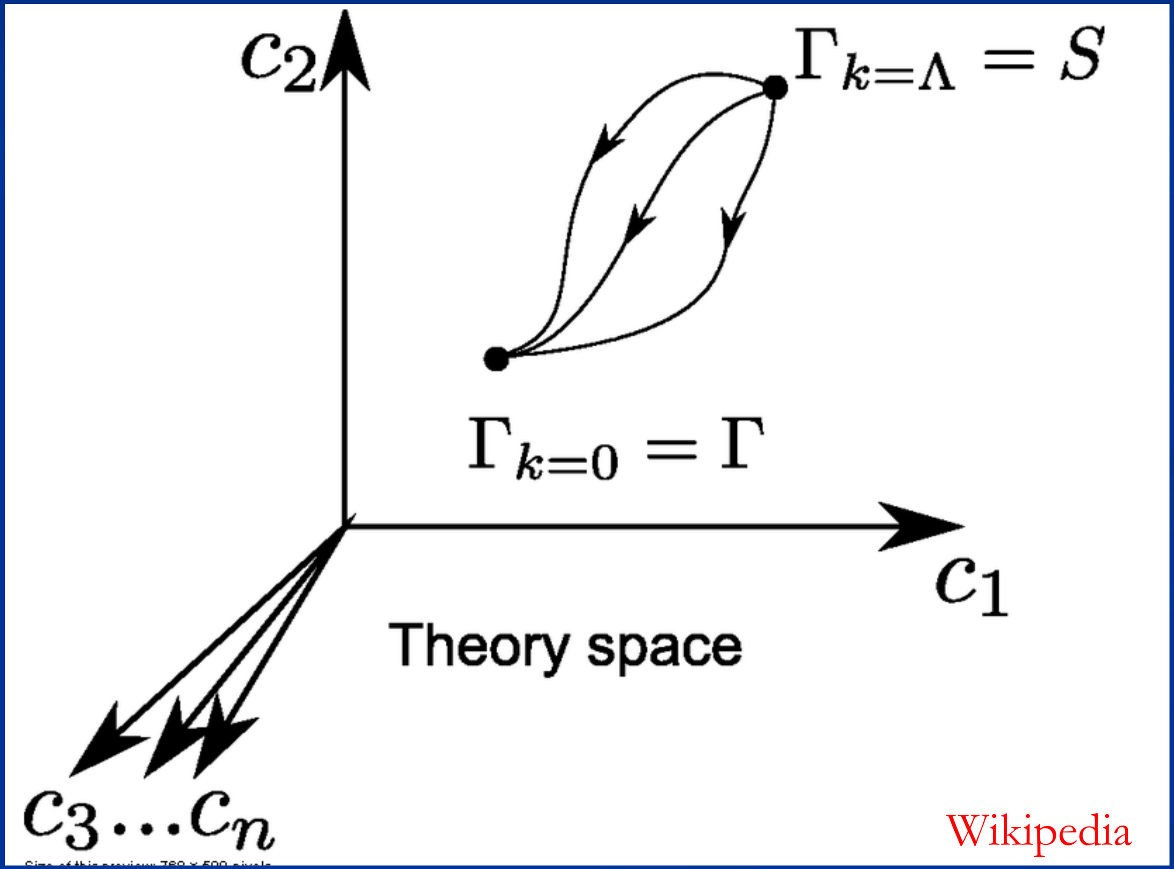
↓

to

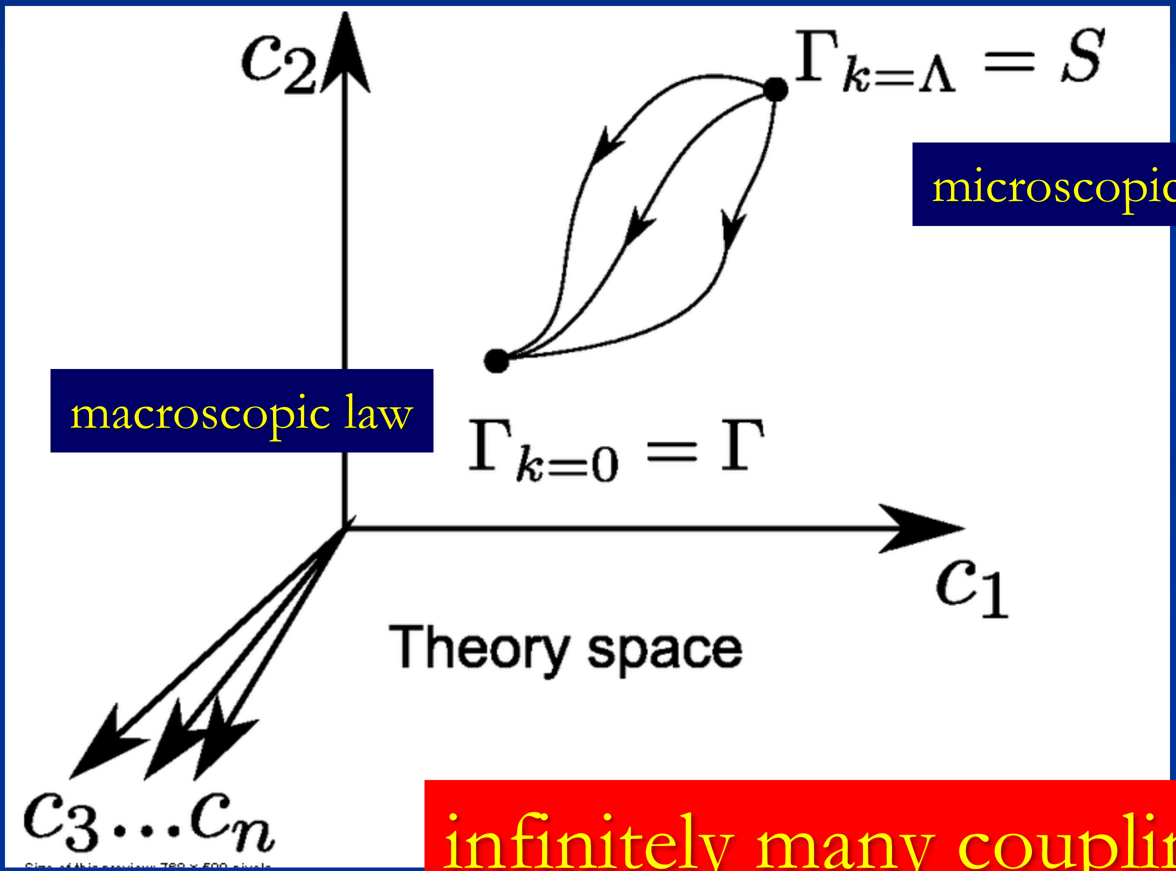
Fluctuations!

Macroscopic Observation
(Free energy functional, effective action)

functional renormalization : flowing action



flowing action



microscopic law

macroscopic law

infinitely many couplings

flow of functions

Effective potential includes all fluctuations

Average potential U_k

$\equiv$ scale dependent effective potential
$\equiv$ coarse grained free energy

Only fluctuations with momenta $q^2 > k^2$ included

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

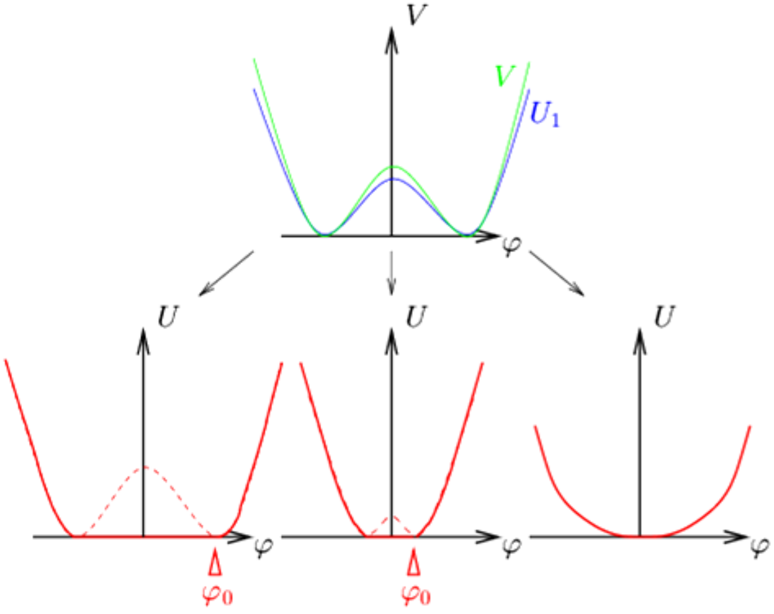
$$U_\Lambda \approx S \rightarrow U_0 \equiv U$$

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^dx \left\{ \frac{1}{2}\partial_\mu\varphi_a\partial_\mu\varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2}\varphi_a\varphi_a$$



Flow equation for average potential

cutoff

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

propagator with cutoff

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

R_k : IR-cutoff

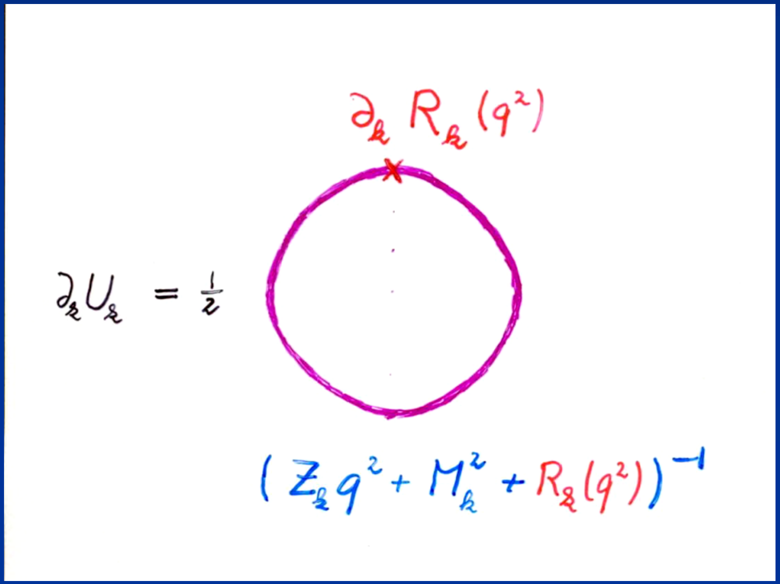
e.g $R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$ (Litim)

$\lim_{k\to 0} R_k = 0$

$\lim_{k\to\infty} R_k \to \infty$

Simple one loop structure – nevertheless (almost) exact



$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

Well tested for non-perturbative phenomena:
critical exponents, Kosterlitz-Thouless phase transition, etc

Graviton contribution to flow of scalar potential

$$\partial_t V = k\,\partial_k V = 5 I_k\left(-\tfrac{2V}{M^2}\right)$$

$$I_k(m^2) = \frac{1}{2}\int_q \left(q^2 + R_k(q) + m^2\right)^{-1} \partial_t R_k(q)$$

Litim cutoff: $$R_k(q) = (k^2 - q^2)\,\theta(k^2 - q^2)$$

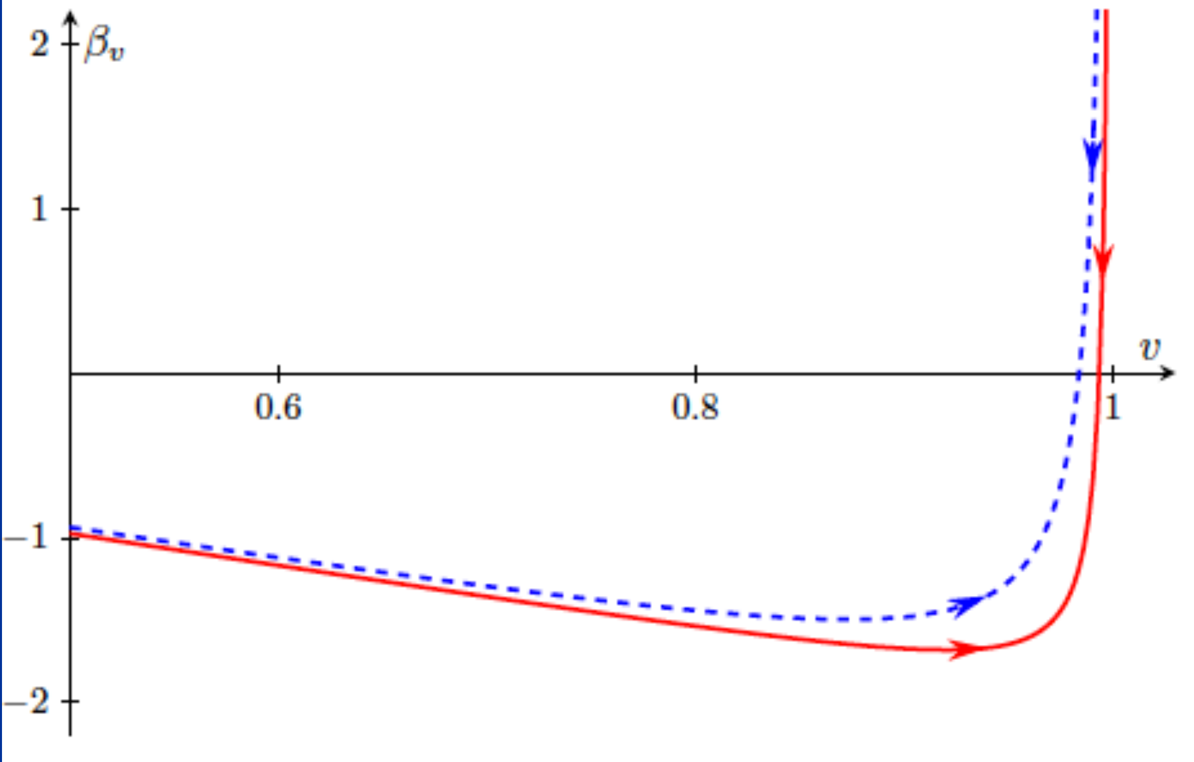
$$I_k(m^2) = \frac{1}{32\pi^2}\frac{k^6}{k^2+m^2}$$ $$m^2 = -\frac{2V}{M^2}$$

crucial dimensionless quantity $$v = \frac{2V}{M^2 k^2}$$

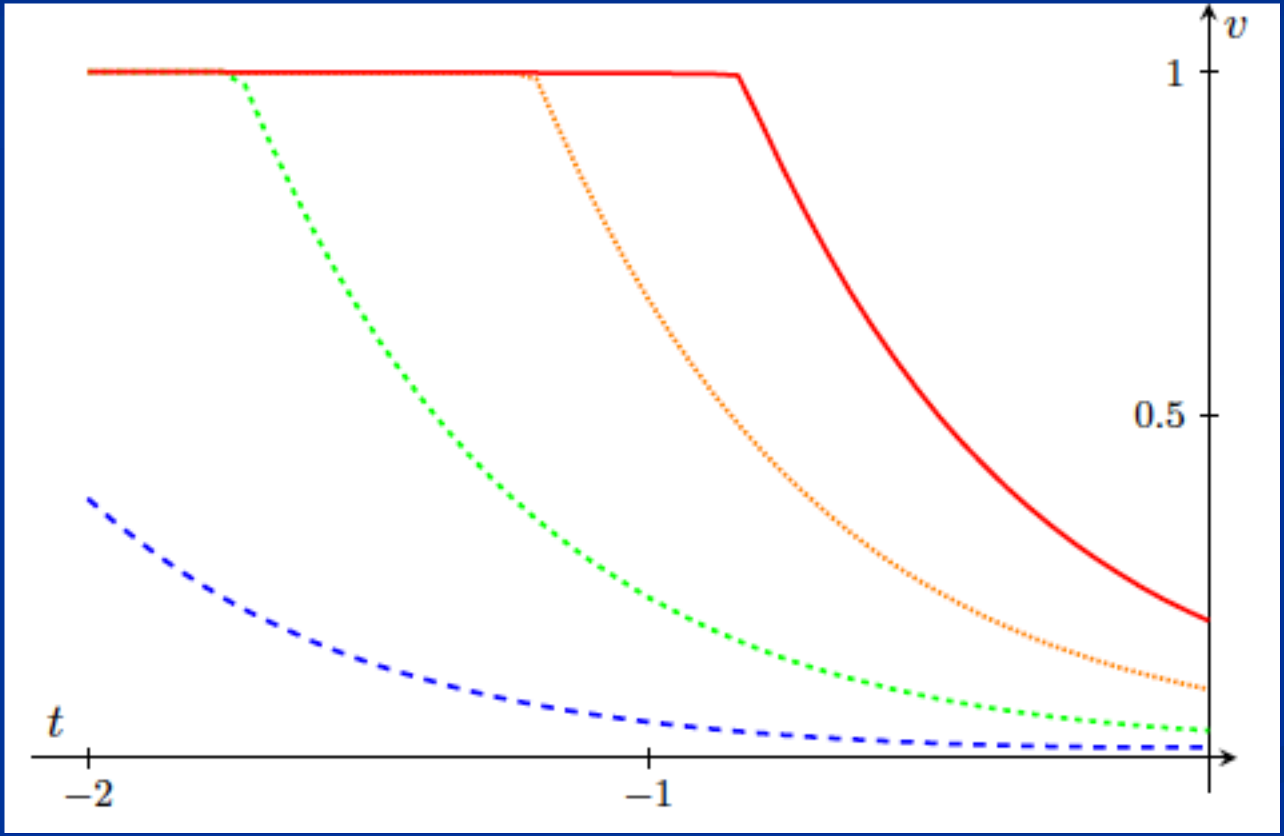
Flow equation for v

$$\partial_t v = \beta_v = -2v + \frac{5k^2}{16\pi^2 M^2}(1-v)^{-1}.$$

$$v = \frac{2V}{M^2 k^2}$$



Flow of v for different initial conditions



Infrared value of effective scalar potential for k/$\chi \to 0$

v=1

$$U = \frac{\bar{k}^2}{2} M^2(\chi)$$

graviton barrier !

Ultraviolet behavior for $k/\chi \to \infty$

$$M^2 = f k^2$$

$$\partial_t v = -4v + \frac{5}{16\pi^2 f}(1-v)^{-1}$$

UV – and IR – fixed points

$$v_*(1-v_*) = \frac{5}{64\pi^2 f}$$

Graviton contribution to flow of quartic scalar coupling : positive and substantial anomalous dimension

$$\partial_t U = \frac{k^6}{32\pi^2}\left(\frac{5}{k^2 - 2U/M^2} + \frac{1}{k^2 + \partial^2 U/\partial^2\chi}\right)$$

$$\partial_t \lambda = \frac{9\lambda^2}{16\pi^2} + \frac{5\lambda k^2}{16\pi^2 M^2}$$

$$\partial_t \lambda = A_\lambda \lambda + \frac{9\lambda^2}{16\pi^2}, \qquad A_\lambda = \frac{5}{16\pi^2 f} = \frac{5 g_*}{2\pi}$$

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\text{min}} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\text{min}} = 126$ GeV, with o

Asymptotic safety

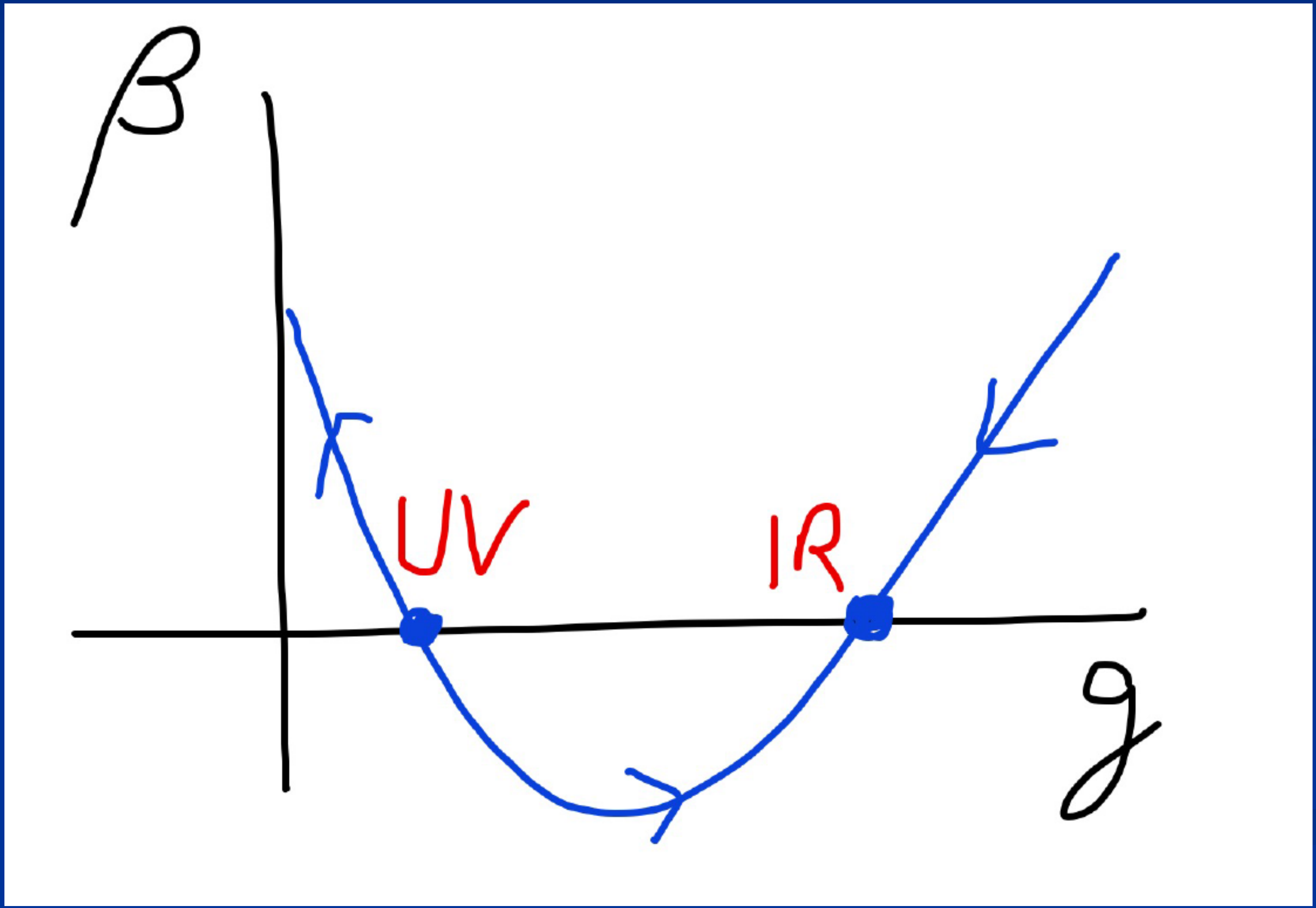
if UV fixed point exists :

quantum gravity is

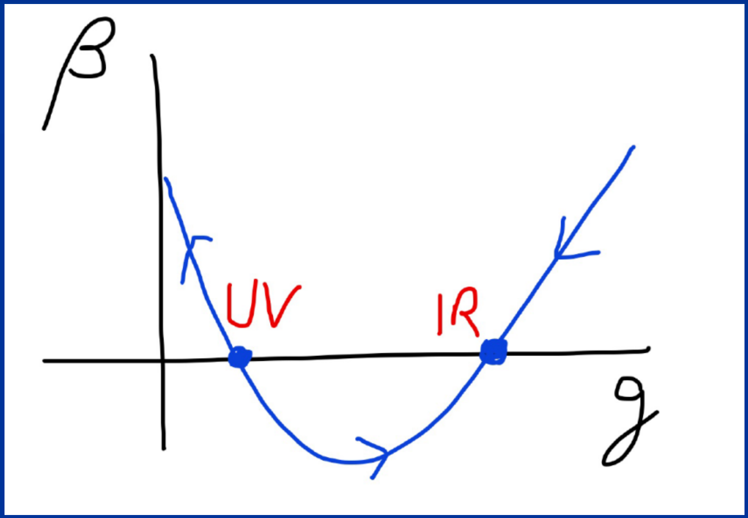
non-perturbatively renormalizable !

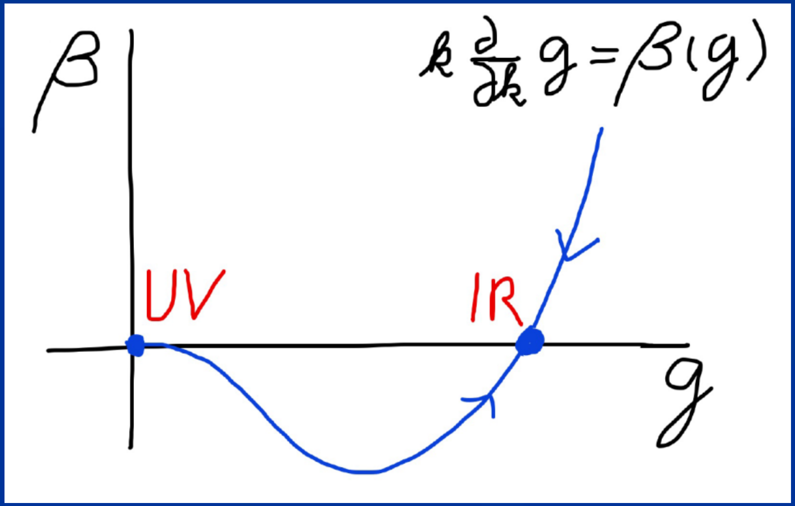
S. Weinberg , M. Reuter

Asymptotic safety



Asymptotic safety   Asymptotic freedom





Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

Spontaneous breaking of scale symmetry

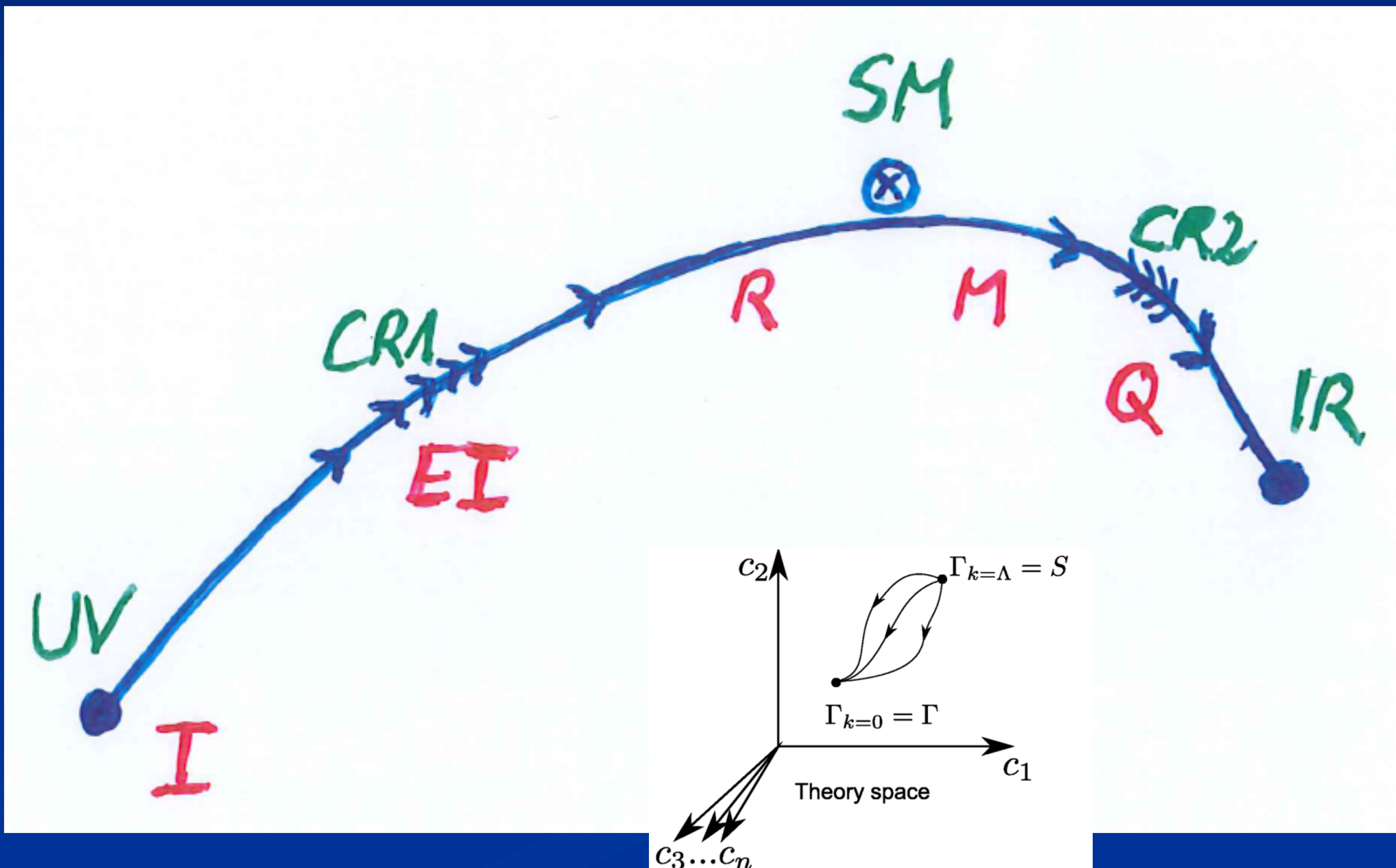
- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- all mass scales proportional to scalar field χ : electron mass, proton mass, Planck mass
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

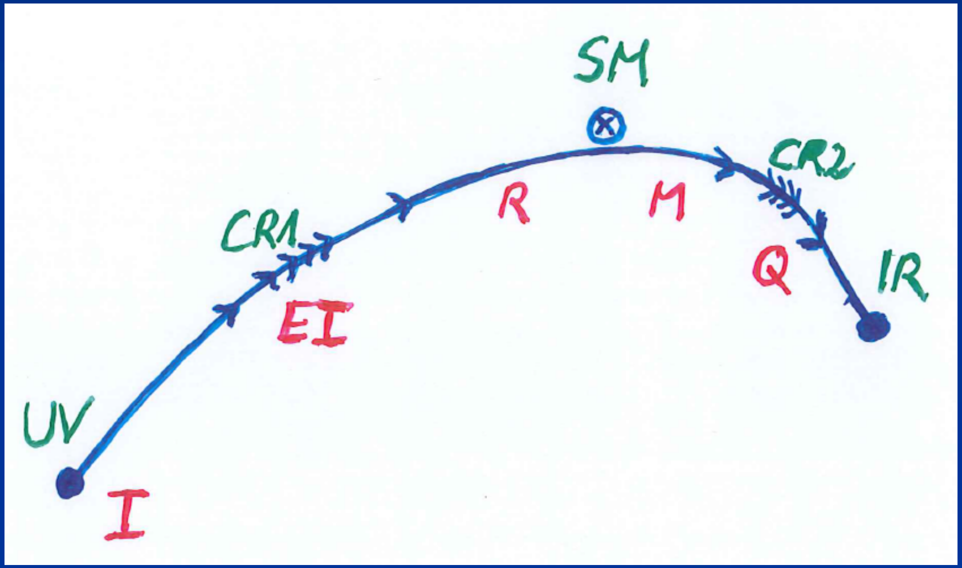
- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Crossover in quantum gravity



Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy with single scalar field

Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

intrinsic scale μ replaces k

quantum effective action,
variation yields field equations,
solve for cosmology

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi\right\}$$

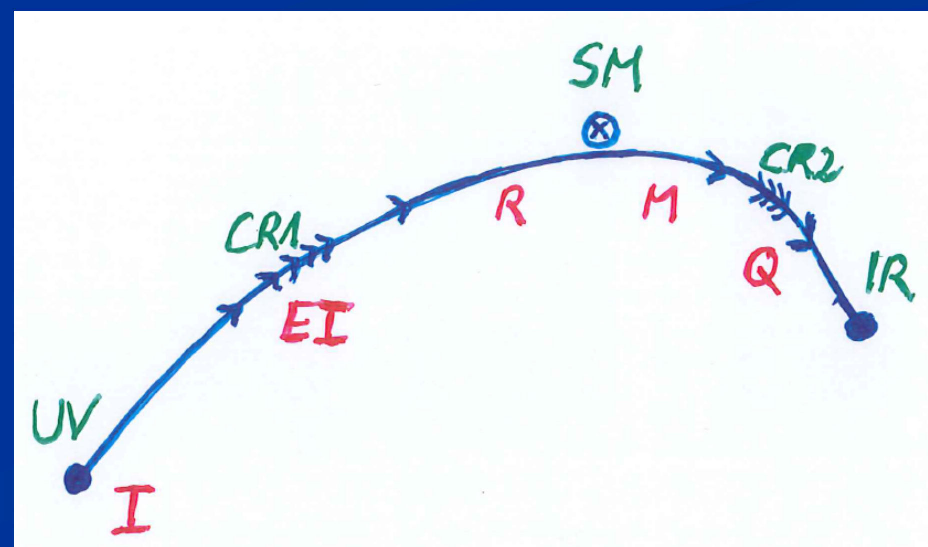
Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu)-6\big)\partial^\mu\chi\,\partial_\mu\chi\right\}$$

- IR: $\mu \to 0$, $\chi \to \infty$
- UV: $\mu \to \infty$, $\chi \to 0$

Cosmology makes crossover between fixed points by variation of χ .



renormalization flow and cosmological evolution

- renormalization flow as function of μ

  is mapped by dimensionless functions to

- field dependence of effective action on scalar field χ

  translates by solution of field equation to

- dependence of cosmology on time t or η

Simplicity

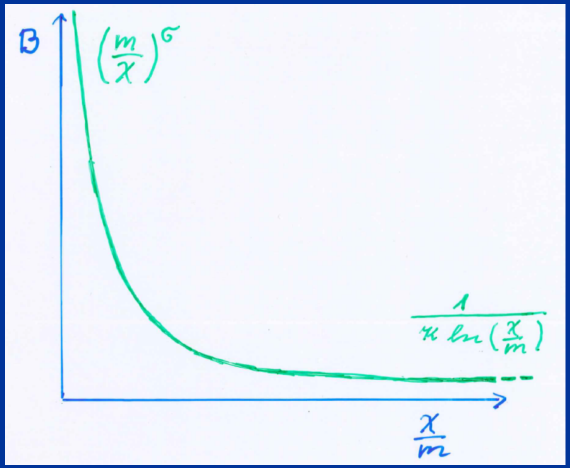
simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Kinetial B :
Crossover between two fixed points

Ansatz , not computed !



running coupling obeys flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover

can be exponentially larger than intrinsic scale μ

Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi\right\}$$

No tiny dimensionless parameters
(except matter sector, e.g. gauge hierarchy)

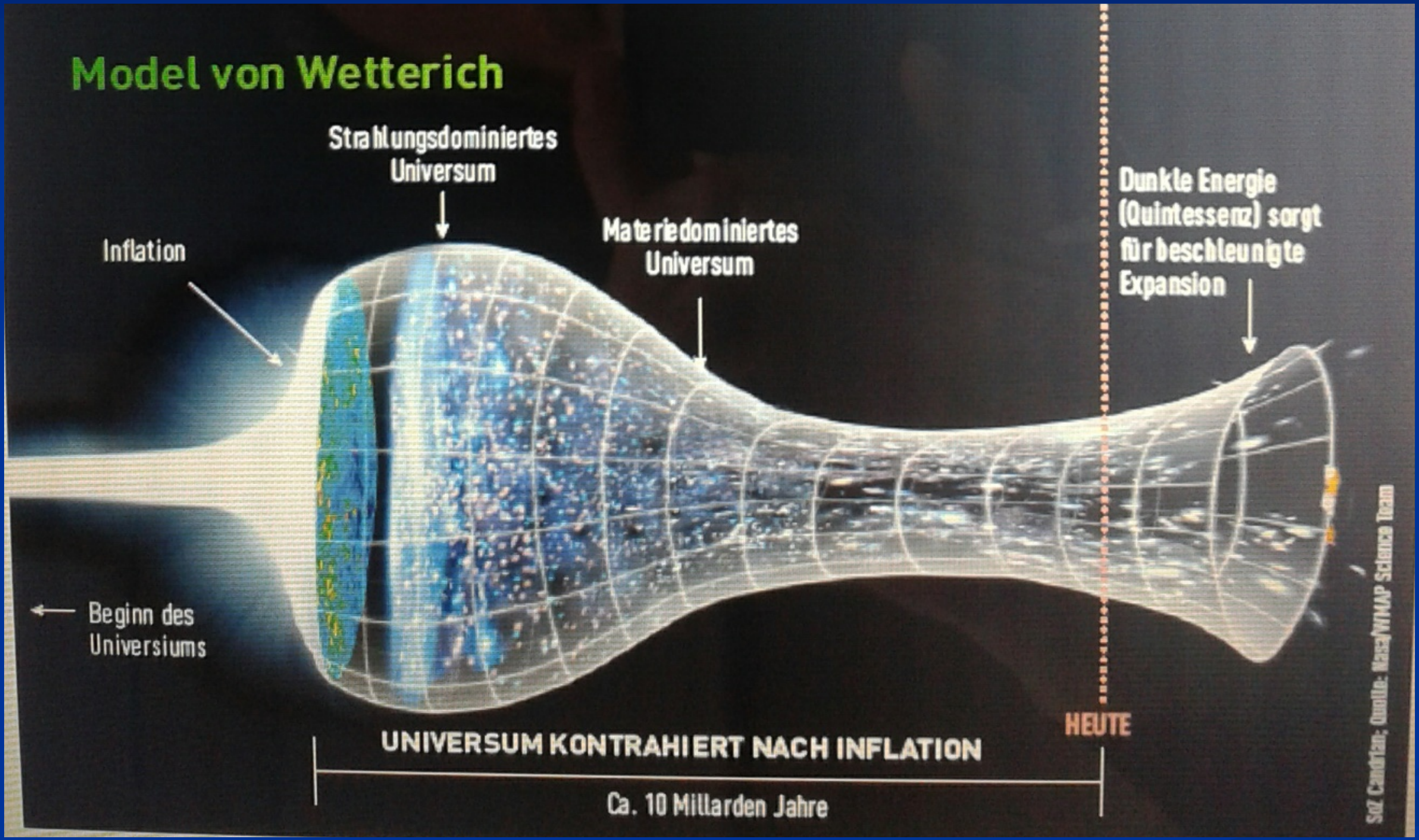
- one mass scale $\mu = 2 \cdot 10^{-33}$ eV
- one time scale $\mu^{-1} = 10^{10}$ yr
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Particle masses change with time

At SM fixed point :

- All particle masses (except for neutrinos) are proportional to scalar field χ .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Slow Universe

Asymptotic solution : $H = \frac{\mu}{\sqrt{3}}\ ,\ \chi = \frac{3^{\frac{1}{4}}m}{2\sqrt{\mu}}(t_c - t)^{-\frac{1}{2}}$

$\mu = 2 \cdot 10^{-33}$ eV

Expansion or shrinking always slow , characteristic time scale of the order of the age of the Universe : $t_{ch} \sim \mu^{-1} \sim$ 10 billion years !

Hubble parameter of the order of present Hubble parameter for all times , including inflation and big bang !

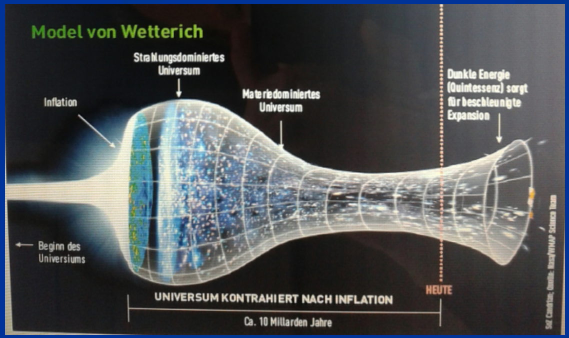
Slow increase of particle masses !

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \ , \ \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity



- physical time to infinite past is infinite

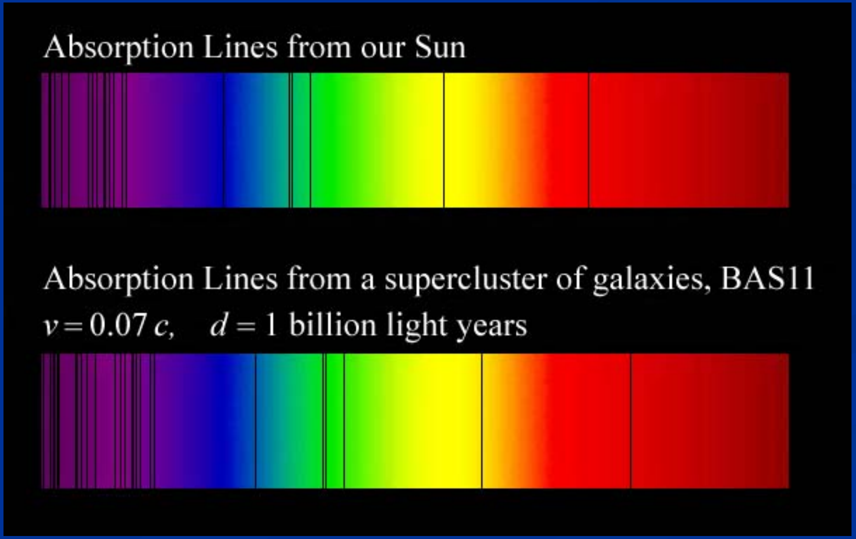
small dimensionless number ?

- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V: ratio vanishes asymptotically !

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past, because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

Hot plasma ?

- Temperature in radiation dominated Universe :
  $T \sim \chi^{1/2}$ smaller than today
- Ratio temperature / particle mass :
  $T / m_p \sim \chi^{-1/2}$ larger than today
- T/m_p counts ! This ratio decreases with time.

- Nucleosynthesis , CMB emission as in standard cosmology !

Model is compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

Einstein frame

- "Weyl scaling" maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

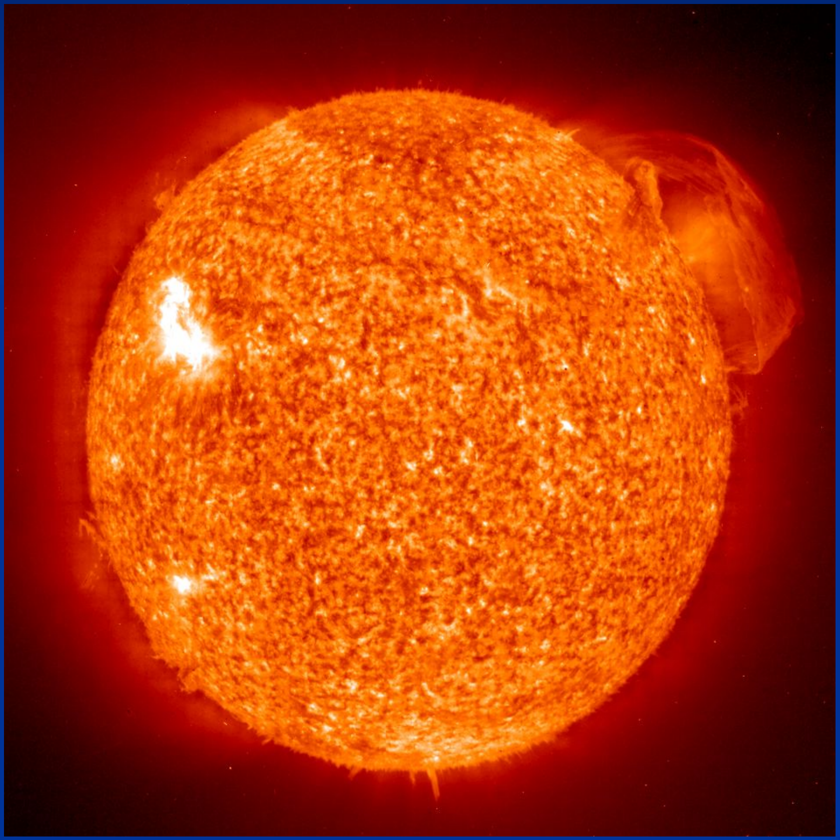
$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

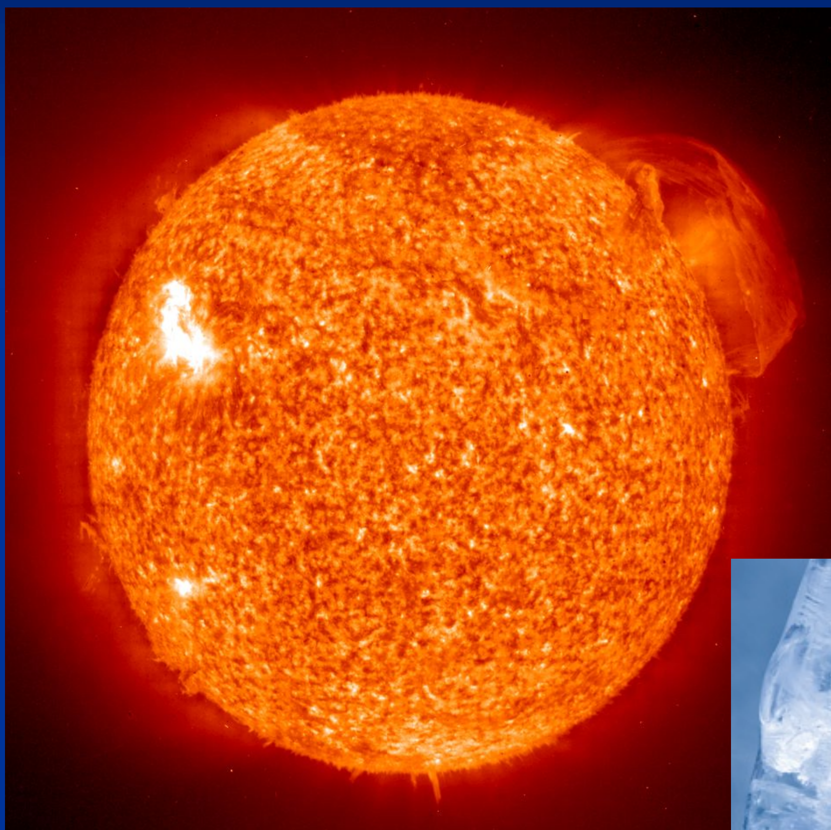
Field relativity :
different pictures of cosmology

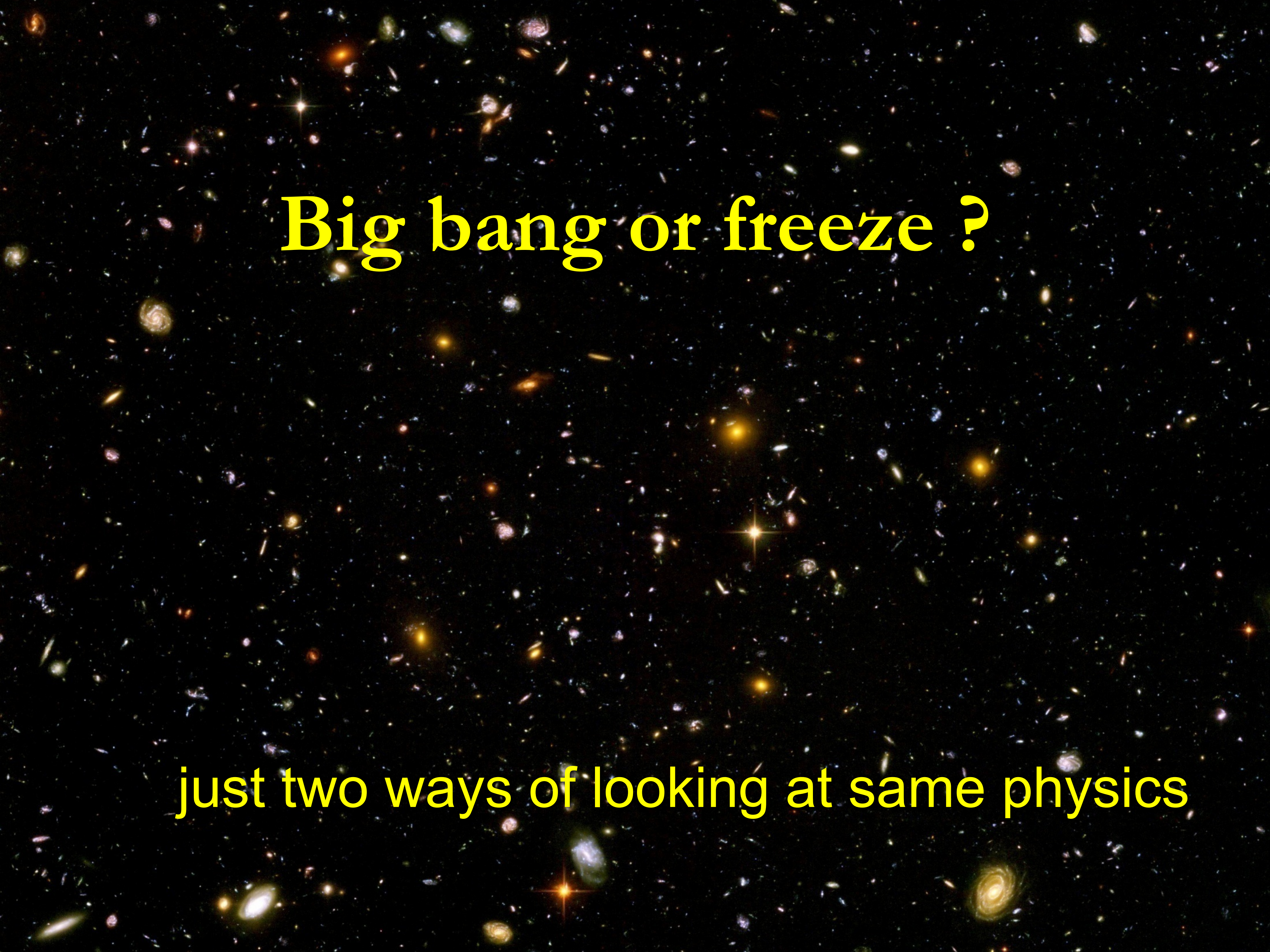
- same physical content can be described by different pictures
- related by field – redefinitions ,
  e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Big bang or freeze ?



Big bang or freeze ?




Big bang or freeze ?
just two ways of looking at same physics

Conclusions

- Graviton fluctuations play a role at large distances
- Graviton barrier for increase of scalar potential entails solution of cosmological constant problem
- Cosmology is described by crossover from UV-fixed point (inflation) to IR-fixed point (dynamical dark energy)

end

Four-parameter model

- model has four dimensionless parameters
- three in kinetial B :

  $\sigma \sim 2.5$

  $\varkappa \sim 0.5$

  $c_t \sim 14$ (or m/μ)
- one parameter for present growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than ΛCDM

Origin of mass

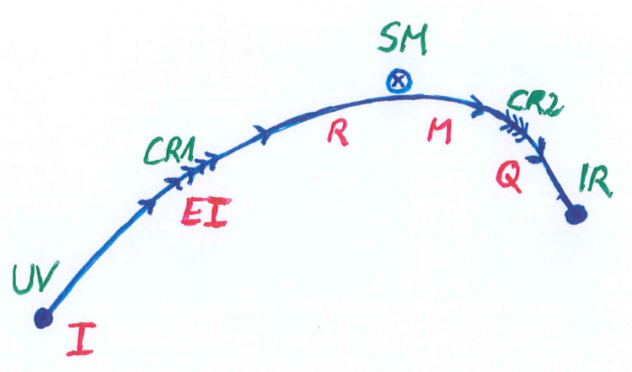
- UV fixed point : scale symmetry unbroken

  all particles are massless

- IR fixed point :

  scale symmetry spontaneously broken,

  massive particles , massless dilaton



- crossover : explicit mass scale μ important

- approximate SM fixed point : approximate scale symmetry spontaneously broken, massive particles , almost massless cosmon, tiny cosmon potential

Infinite past : slow inflation

$\sigma = 2$: field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2}\frac{\dot{\chi}}{\chi}\right)\dot{\chi} = \frac{2\mu^2\chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

approximate solution

$$H = \frac{\mu}{\sqrt{3}}\ ,\ \chi = \frac{3^{\frac{1}{4}}m}{2\sqrt{\mu}}(t_c - t)^{-\frac{1}{2}}$$

Eternal Universe

Asymptotic solution in freeze frame :

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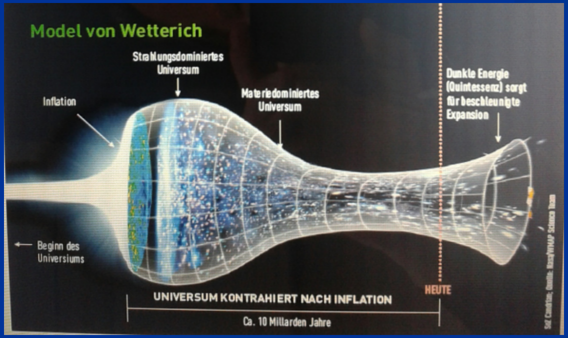
- solution valid back to the infinite past in physical time
- no singularity



- physical time to infinite past is infinite

Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2 m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \qquad \left\{\partial_\eta^2 + k^2 + a^2\left(m^2 - \frac{R}{6}\right)\right\}\tilde{\varphi}_k = 0$$

determine physical time by counting number of oscillations

$$\tilde{t}_p = n_k \qquad n_k = \frac{k\eta}{\pi}$$

(m=0)

Big bang singularity in Einstein frame is field singularity !

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

conclusions (2)

Fixed points and scale symmetry crucial

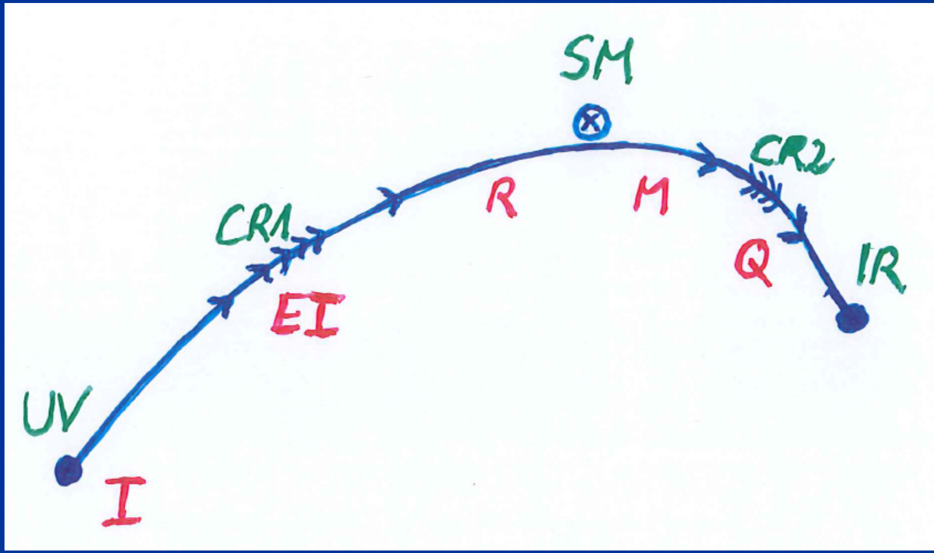
Big bang singularity is artefact
of inappropriate choice of field variables –
no physical singularity

conclusions (3)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than ΛCDM : tests possible

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that **ratio neutrino mass over electron mass grows** .

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

L.Amendola, M.Baldi, …

present dark energy density given by neutrino mass

present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$

Inflation

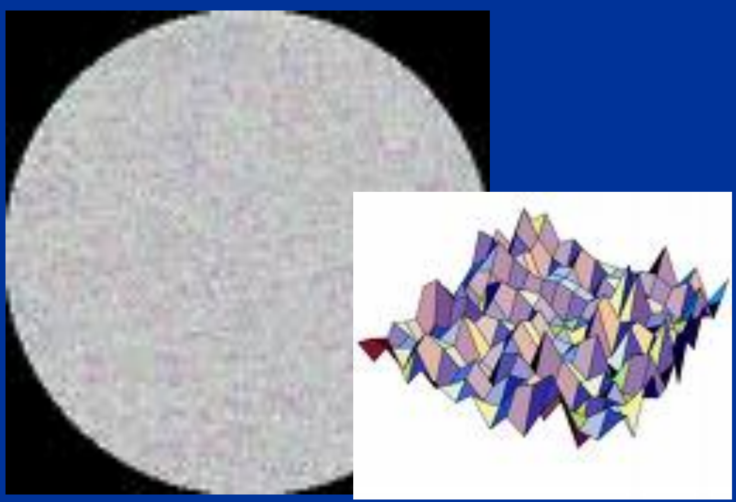
solution for small χ : inflationary epoch

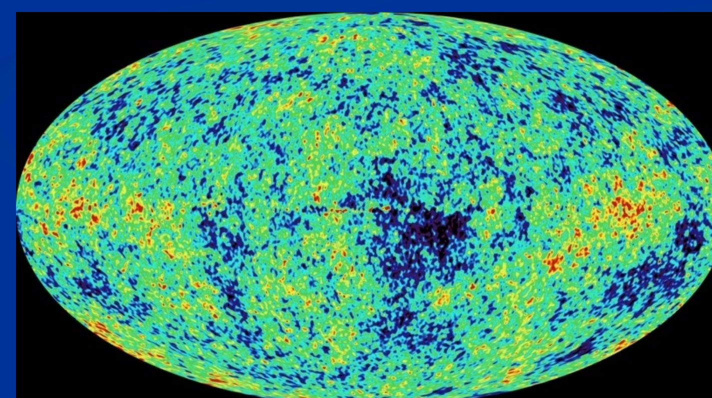
kinetial characterized by anomalous dimension σ

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

Primordial fluctuations

- inflaton field : χ
- primordial fluctuations of inflaton become observable in cosmic microwave background





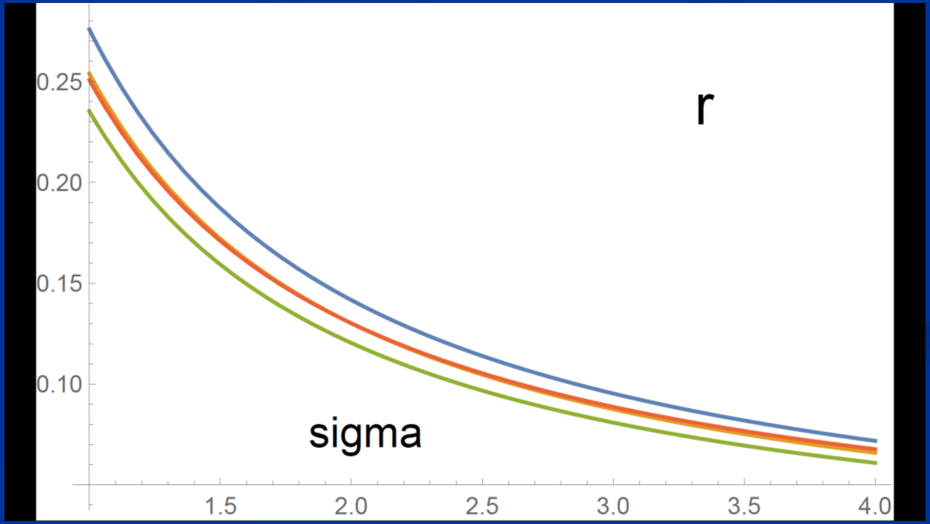
Anomalous dimension determines spectrum of primordial fluctuations

$$r = \frac{0.26}{\sigma}$$

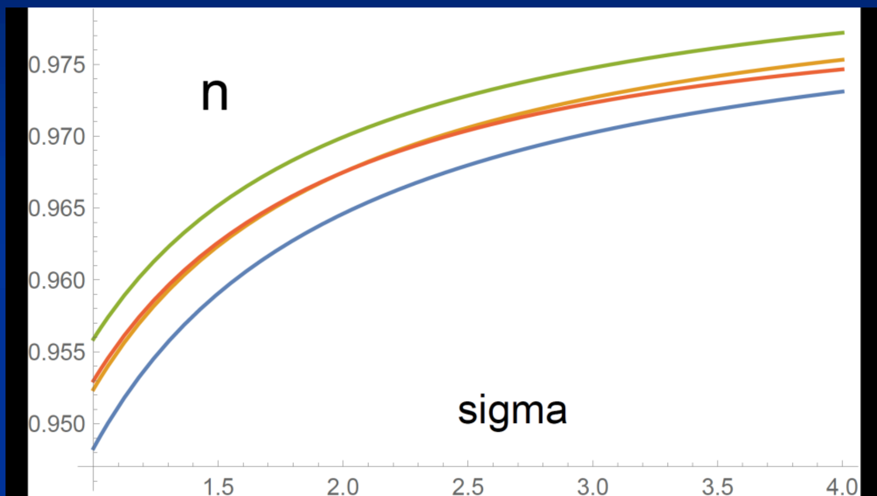
$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$

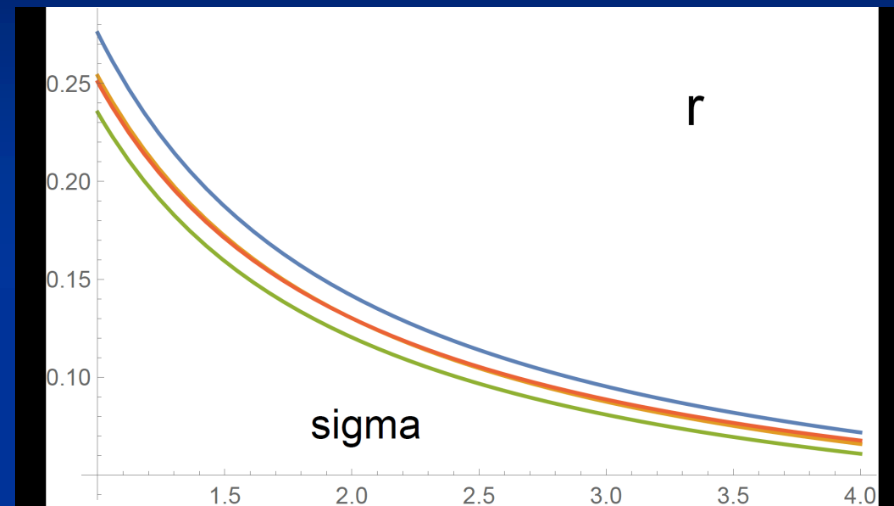
spectral index n

tensor amplitude r



relation between n and r





$$r = 8.19\,(1 - n) - 0.1365$$

Amplitude of density fluctuations

small because of logarithmic running near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln\left(\frac{m}{\mu}\right) = 14.1$$

σ=1

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60}\right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

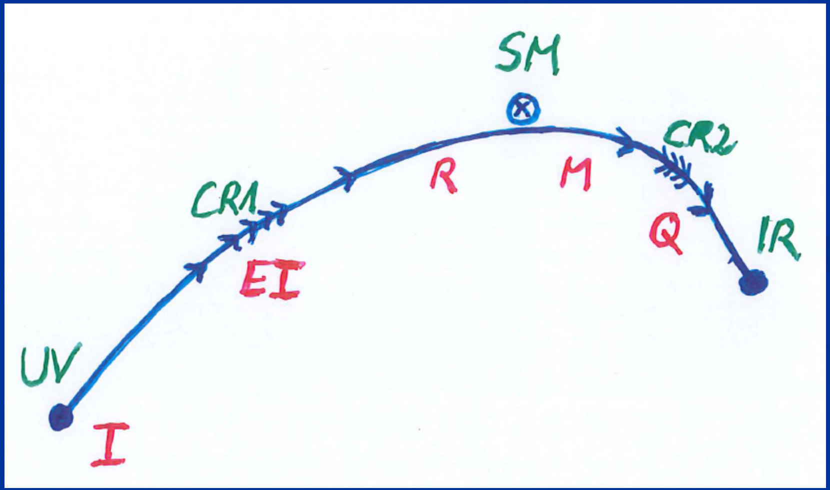
N : number of e – foldings at horizon crossing

First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

- after crossover B changes only very slowly



Scaling solution

- Heating of the Universe after inflation
- Scaling solution with almost fixed fraction of Early Dark Energy

Compatibility with observations and possible tests

- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as ΛCDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

conclusions (4)

- Variable gravity cosmologies can give a simple and realistic description of the Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Cosmon dependence of ratio neutrino mass/ electron mass can explain why Universe makes a transition to Dark Energy domination now
- characteristic signal : neutrino lumps

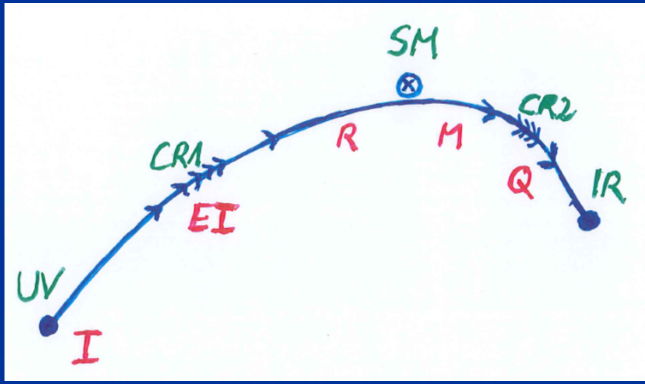
Infrared fixed point

- $\mu \longrightarrow 0$
- $B \longrightarrow 0$

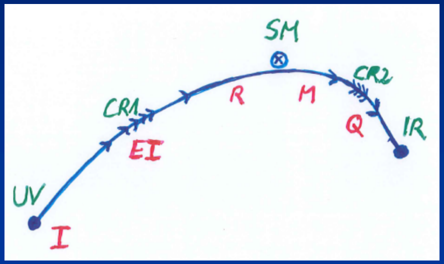
$$\mu \partial_\mu B = \kappa B^2 \qquad \text{for} \qquad B \to 0$$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right) \partial^\mu \chi \partial_\mu \chi \right\}$$

- no intrinsic mass scale
- scale symmetry



Ultraviolet fixed point



- $\mu \longrightarrow \infty$
- kinetial diverges

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

- scale symmetry with anomalous dimension σ

$$g_{\mu\nu} \to \alpha^2 g_{\mu\nu}\ ,\ \chi \to \alpha^{-\frac{2}{2-\sigma}}\chi$$

Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$1 < \sigma$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left(\mu^2 - \frac{R}{2}\right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

deviation from fixed point vanishes for $\mu \longrightarrow \infty$