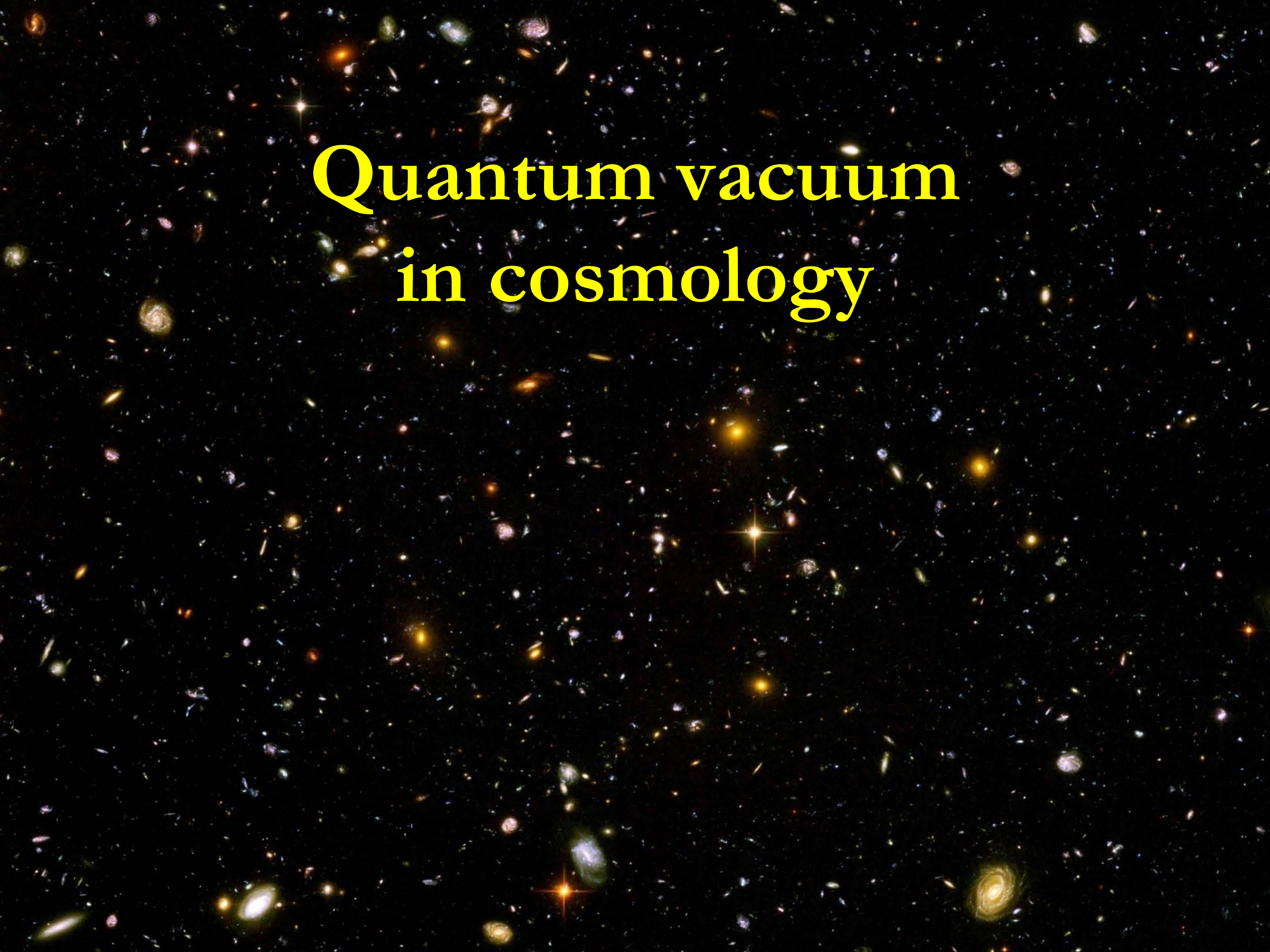
Quantum vacuum in cosmology



What is the vacuum in cosmology ?

Vacuum in cosmology

- state of the Universe in absence of matter and radiation ?
- gravity + scalar field
- characterized by mean fields and fluctuations
- no time translation invariance
- no conserved energy , state of minimal energy not meaningful
- attractor of time evolution

vacuum in cosmology
=
(space averaged)
state of Universe

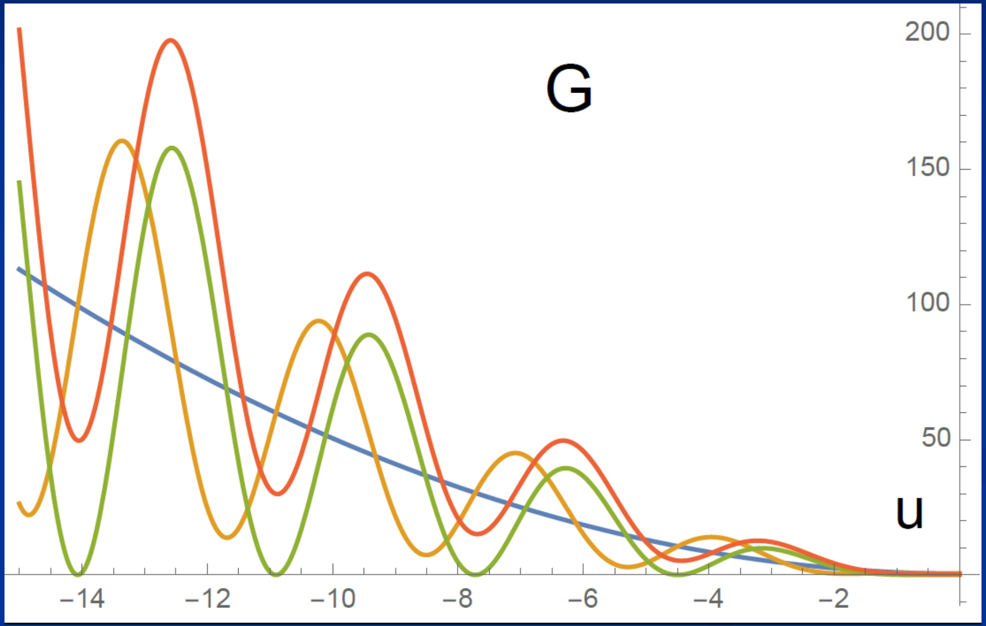
vacuum is quantum object and not “empty”

Three questions

- Can we observe quantum vacuum properties in cosmology ?
- Can we compute quantum vacuum properties in gravity ?
- What is the role of scale symmetry and its spontaneous breaking ?

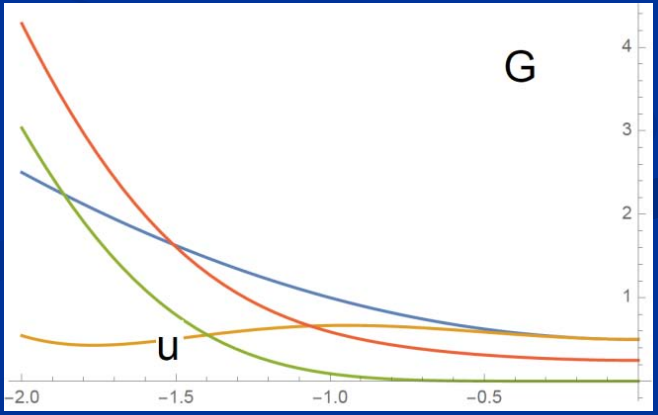
Does inflation allow us to observe quantum vacuum properties ?

correlation function for different initial conditions



$u = k\,\eta$

no loss of memory
 at horizon crossing
(u = -1)



inflation processes fluctuations

- fluctuations are not " generated " during inflation
- every statistical system has fluctuations
- fluctuations already present at "beginning" of inflation (or at extremely early stages if inflation lasts from the infinite past)
- fluctuations get **processed** by scale violating effects from inflaton potential near horizon crossing

memory of initial spectrum

$$\Delta^2 = \frac{(A_p + 1)V}{24\pi^2 \epsilon M^4}$$

$$n_s = 1 + n_p - 6\epsilon + 2\eta$$

equilibration ?

- for interacting theories : there could be processes that bring arbitrary initial fluctuations close to universal form
- no such loss of memory found within present approximations
- equilibration time will be very long since interactions are tiny

scalar correlation function

$$G(x,y) = \langle\tilde{\phi}(x)\tilde{\phi}(y)\rangle - \langle\tilde{\phi}(x)\rangle\langle\tilde{\phi}(y)\rangle$$

$$G(y,x) = G(x,y)$$

$$G(\eta,\vec{x};\eta',\vec{y}) = G(\vec{r},\eta,\eta')$$

$$G(\vec{r},\eta,\eta') = \int_k G(\vec{k},\eta,\eta')e^{i\vec{k}\vec{r}}$$

$$G(k,\eta) = G(k,\eta,\eta)$$

$$\Delta^2(k) \approx \frac{k^3H^2}{4\pi^2\dot{\varphi}^2}G(k,\eta)_{|hc} = A_s\left(\frac{k}{k_s}\right)^{n_s-1}$$

correlation function from quantum effective action

- no operators needed
- no explicit construction of vacuum state
- no distinction between quantum and classical fluctuations
- one relevant quantity : correlation function

quantum theory as functional integral

$$Z[j] = \int \mathcal{D}\tilde{\phi} \exp\left(- S + \int_x J\tilde{\phi} \right)$$

$$S = \int_x eL[\tilde{\phi}, e_\mu^m].$$

background geometry given by vierbein or metric

$$e = \det(e_\mu^m)$$

$$\bar{g}_{\mu\nu} = e_\mu^m e_\nu^n \delta_{mn}$$

homogeneous and isotropic cosmology

$$e_k^m = a(\eta)\delta_k^m \ , \ e_0^m = ia(\eta)\delta_0^m$$

$$e = \bar{e}a^4$$

M: $\bar{e} = i$

E: $\bar{e} = 1$

quantum effective action

$$W[J] = \ln Z[J]$$

$$\frac{\partial W}{\partial J(x)} = \langle \phi(x) \rangle = \phi(x)$$

$$\Gamma[\phi] = -W[J] + \int_x J\phi$$

exact field equation

$$\frac{\partial \Gamma}{\partial \phi(x)} = J(x)$$

exact propagator equation

$$\Gamma^{(2)} W^{(2)} = 1$$

$$\Gamma^{(2)} = \partial^2 \Gamma / \partial \phi(x) \partial \phi(y)$$

$$W^{(2)} = \partial^2 W / \partial J(x) \partial J(y)$$

correlation function and quantum effective action

$$G(x,y) = \langle \tilde{\phi}(x)\tilde{\phi}(y)\rangle - \langle \tilde{\phi}(x)\rangle\langle \tilde{\phi}(y)\rangle = \frac{\partial^2 W}{\partial J(x)\partial J(y)}$$

$$G(x,y) = \left(\frac{\partial^2 \Gamma}{\partial \phi(x)\partial \phi(y)}\right)^{-1}$$

propagator equation for interacting scalar field in homogeneous and isotropic cosmology

$$\Gamma^{(2)}\ \mathrm{G} = 1$$

$$\tilde{D}_\eta G(k,\eta,\eta') = -\frac{i}{a^2}\delta(\eta-\eta'),$$

$$\tilde{D}_\eta = \partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 + m^2 a^2$$

$$\mathcal{H}(\eta) = \frac{\partial \ln a(\eta)}{\partial \eta}\ ,\ \ m^2(\eta) = \frac{\partial^2 \mathcal{U}}{\partial \varphi^2}_{|\bar{\varphi}(\eta)}$$

general solution and mode functions

$$G_>(k,\eta,\eta') = \frac{\alpha(k)+1}{2} w_k^-(\eta) w_k^+(\eta') + \frac{\alpha(k)-1}{2} w_k^+(\eta) w_k^-(\eta') + \zeta(k) w_k^+(\eta) w_k^+(\eta') + \zeta^*(k) w_k^-(\eta) w_k^-(\eta')$$

$$\tilde{D}_\eta \psi_k(\eta) = 0 \ , \ \ \psi_k(\eta) = c_+ w_k^+(\eta) + c_- w_k^-(\eta)$$

$$w_k^-(\eta) = \left(w_k^+(\eta)\right)^*$$

$$\partial_\eta \left[w_k^-(\eta) w_k^+(\eta') - w_k^+(\eta) w_k^-(\eta')\right]_{|\eta=\eta'} = -\frac{i}{a^2(\eta)}$$

Bunch – Davies vacuum

$\alpha = 1$, $\zeta = 0$ for all k

$$
\begin{aligned}
G_>(k,\eta,\eta') &= \frac{\alpha(k)+1}{2} w_k^-(\eta) w_k^+(\eta') \\
&+ \frac{\alpha(k)-1}{2} w_k^+(\eta) w_k^-(\eta') \\
&+ \zeta(k) w_k^+(\eta) w_k^+(\eta') + \zeta^*(k) w_k^-(\eta) w_k^-(\eta')
\end{aligned}
$$

initial conditions

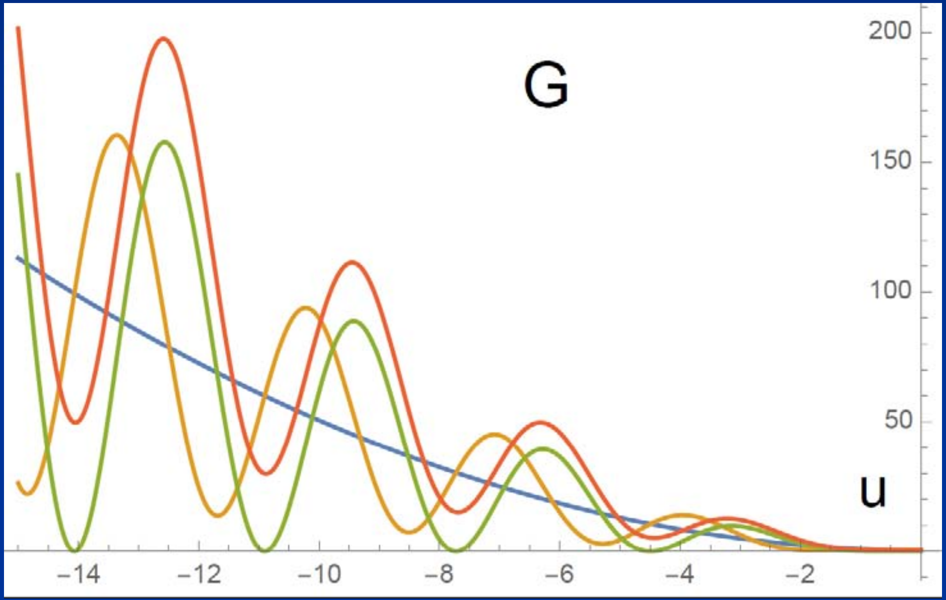
$$G_>(k,\eta,\eta') = \sum_i p_i \psi_k^{(i)}(\eta)\left(\psi_k^{(i)}(\eta')\right)^*$$

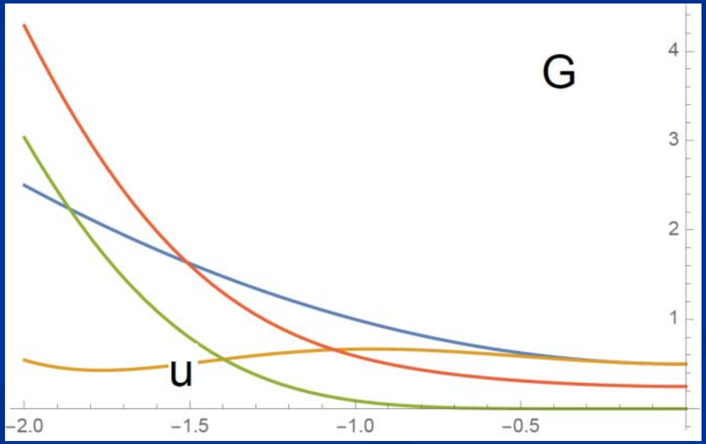
mixed state : positive probabilities p_i

$$\alpha(k) \geq 1\ ,\ \ \beta^2(k) + \gamma^2(k) \leq \alpha^2(k) - 1$$

$\beta^2 + \gamma^2 = 4\,\zeta\,\zeta^*$

absence of loss of memory of initial correlations





each k-mode has three free integration constants !

memory of initial spectrum

$\alpha = 1 + A_p$

$$\Delta^2 = \frac{(A_p+1)V}{24\pi^2 \epsilon M^4}$$

example :

$$A_p(k) = \frac{A}{2}\left(1 - \frac{2}{\pi} arctg\ x\right)\ ,\ x = \Delta^{-1} \ln\left(\frac{k}{k_0}\right)$$

$$\tilde{p} M \frac{a_{in}}{a_{hc}} = H_0, \quad \tilde{p} = \frac{H_0 a_{hc}}{M a_{in}} = e^{N_{in}} \frac{H_0}{M}$$
$$x = \Delta^{-1}\left(N_{in} - \ln\left(\frac{M}{H_0}\right)\right)$$

spectral index :

$$n_s = 1 + n_p - 6\epsilon + 2\eta$$

$$\begin{aligned} n_p &= \frac{\partial \ln\left(1 + A_p(k)\right)}{\partial \ln k} \\ &= -\left[\Delta(1+x^2)\left(\frac{\pi}{2} - arctg\ x + \frac{\pi}{A}\right)\right]^{-1} \end{aligned}$$

$$n_p(x = 0(1)) = -\frac{2}{\pi\Delta}\left(1 + \frac{2(4)}{A}\right)^{-1}$$

predictivity of inflation ?

- initial spectrum at beginning of inflation : gets only processed by inflaton potential
- small tilt in initial spectrum is not distinguishable from small scale violation due to inflaton potential
- long duration of inflation before horizon crossing of observable modes : one sees UV-tail of initial spectrum .
- **If flat , predictivity retained !**

vacuum in cosmology

- simply the (averaged) state of the Universe
- result of time evolution
- minimal “particle number” ?

  depends on definition of particle number

  as observable

Can we compute the vacuum for gravity ?

Can we compute the vacuum for gravity ?

Can we compute the quantum effective action ?

Functional renormalization : Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

'92

$$\left(\Gamma_k^{(2)}\right)_{ab}(q,q') = \frac{\delta^2\Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$\mathrm{Tr} : \sum_a \int \frac{d^dq}{(2\pi)^d}$

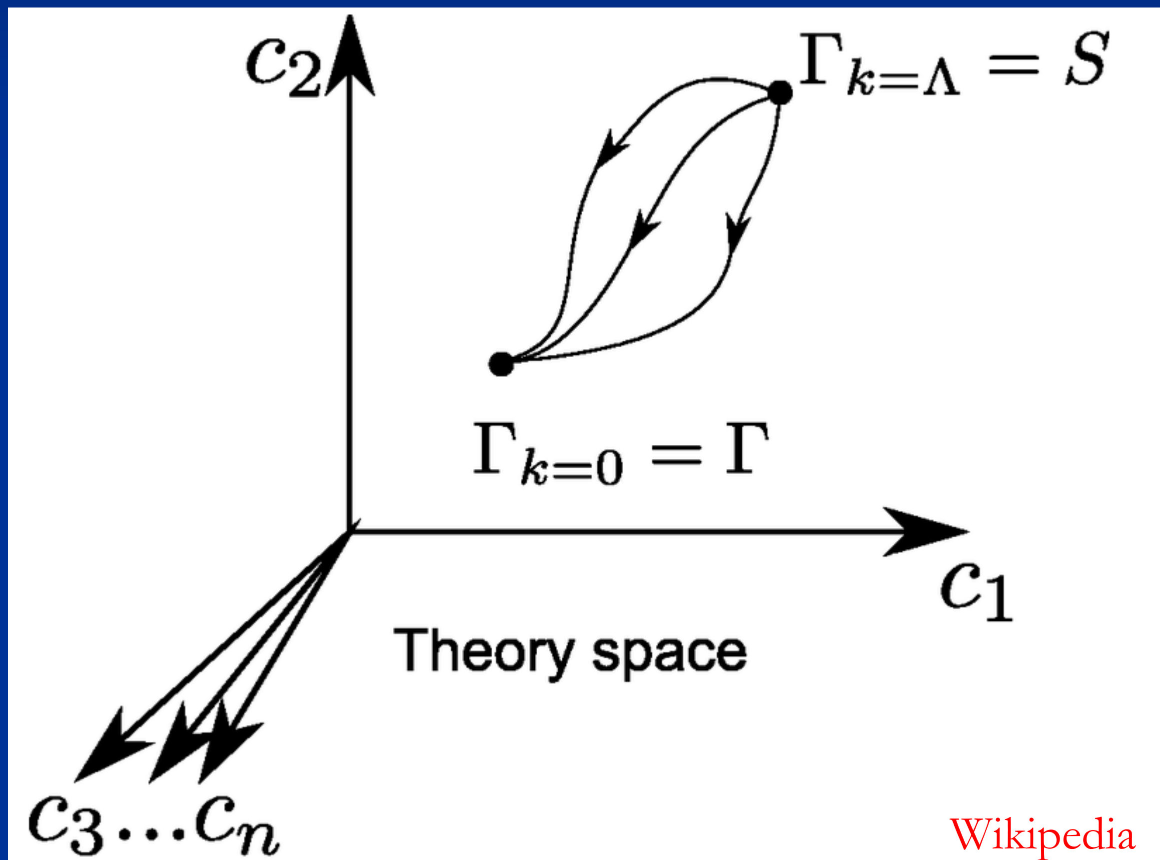
(fermions : STr)

M.Reuter

+…

for gravity

flowing action



Wikipedia

asymptotic safety for gravity ?

S. Weinberg, M. Reuter, …

Dilaton quantum gravity

Dilaton Quantum Gravity

T. Henz, J. M. Pawlowski, A. Rodigast, and C. Wetterich

Functional renormalization flow,

with truncation :

$$\Gamma_k = \int d^4x \sqrt{g} \left(V_k(\chi^2) - \frac{1}{2} F_k(\chi^2)\, R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

Functional renormalization equations

$$V_k = k^4\, y^2\, v_k(y),\;\; F_k = k^2\, y\, f_k(y)$$

$$y = \frac{\chi^2}{k^2}$$

$$\partial_t v_k(y) \;=\; 2\,y\,v_k'(y) + \frac{1}{y^2}\,\zeta_V$$

$$\partial_t f_k(y) \;=\; 2\,y\,f_k'(y) + \frac{1}{y}\,\zeta_F\,.$$

$$\zeta_V = \frac{1}{192\pi^2}\Big\{6 + \frac{30\,\tilde V}{\Sigma_0} + \frac{3(2\Sigma_0 + 24\,y\,\tilde F'\,\Sigma_0' + \tilde F\Sigma_1)}{\Delta} + \delta_V\Big\},$$

$$\begin{aligned}\zeta_F = \frac{1}{1152\pi^2}\Big\{&150 + \frac{30\,\tilde F\,(3\,\tilde F - 2\tilde V)}{\Sigma_0^2} \\ &-\frac{12}{\Delta}\Big(24\,y\,\tilde F'\,\Sigma_0' + 2\Sigma_0 + \tilde F\Sigma_1\Big) - 6y\,(3\,\tilde F'^2 + 2\Sigma_0'^2) \\ &-\frac{36}{\Delta^2}\Big[2y\,\Sigma_0\,\Sigma_0'\,(7\,\tilde F' - 2\tilde V')\,(\Sigma_1 - 1) + 2\,\Sigma_0^2\,\Sigma_2 \\ &+2\,y\Sigma_1\,(7\,\tilde F' - 2\tilde V')\,(2\,\Sigma_0\,\tilde V' - \tilde V\,\Sigma_0') \\ &+24\,y\,\tilde F'\,\Sigma_0\,\Sigma_0'\,\Sigma_2 - 12\,y\,\tilde F\,\Sigma_0'^2\,\Sigma_2\Big] + \delta_F\Big\}.\end{aligned} \qquad (10)$$

$$\tilde V = y^2\,v_k(y)\;,\;\; \tilde F \;=\; y\,f_k(y),$$

$$\Sigma_0 = \frac{1}{2}\tilde F - \tilde V\;,\;\; \Delta \;=\; (12\,y\,\Sigma_0'^2 + \Sigma_0\,\Sigma_1)$$

$$\Sigma_1 = 1 + 2\,\tilde V' + 4\,y\,\tilde V''\;,\;\; \Sigma_2 \;=\; \tilde F' + 2\,y\,\tilde F''.$$

Percacci, Narain

Fixed point for large scalar field

$$\lim_{y\to\infty} f(y) = \xi$$

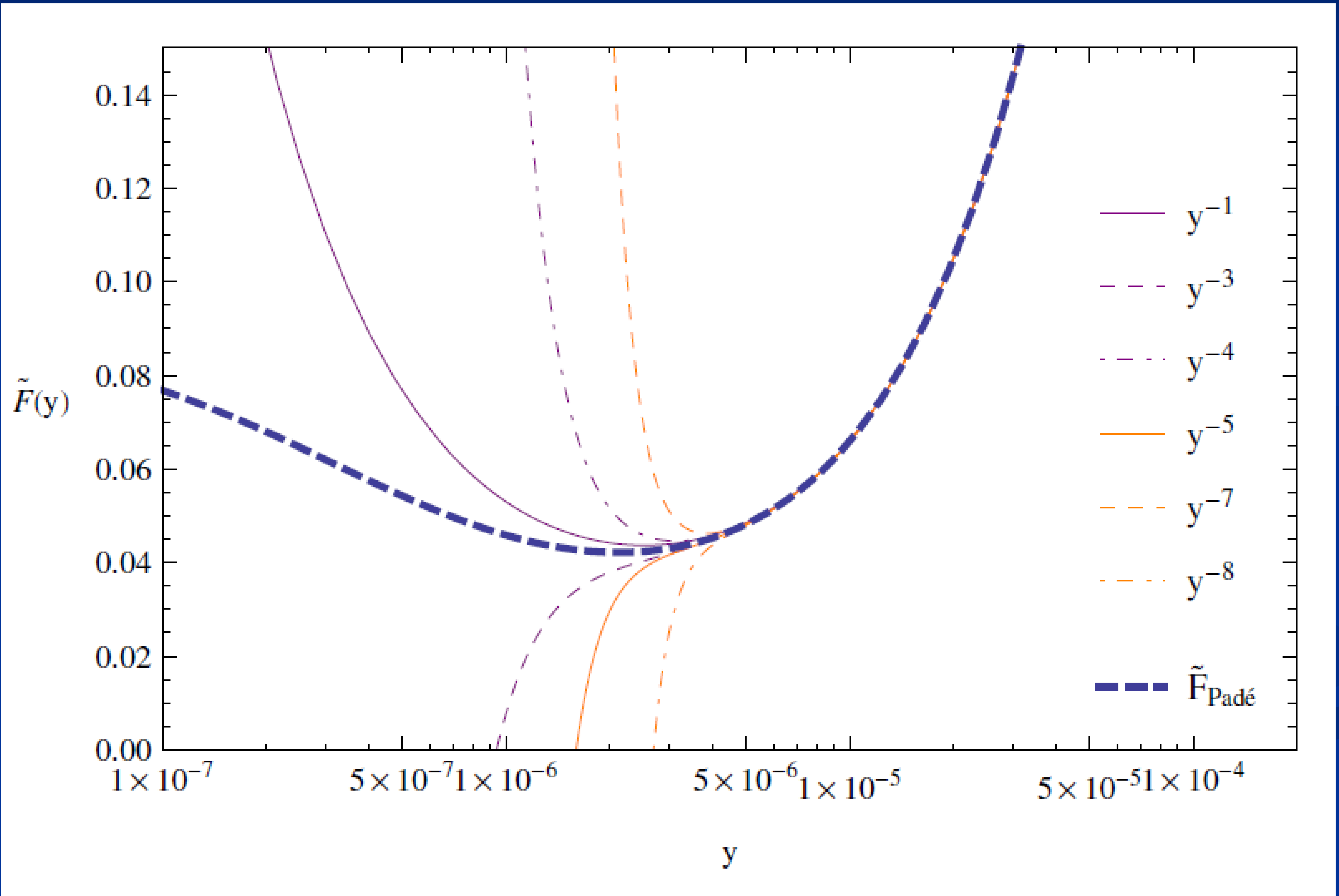
$$\lim_{y\to\infty} v(y) = 0$$

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - \frac{1}{2} \xi \chi^2 \, R \right)$$

This fixed point describes already realistic gravity !

Limit $k \to 0$ can be taken !

Fixed point for large scalar field



Vicinity of fixed point

$$\partial_t V = \bar{\zeta}_V k^4 \ , \ \partial_t F = \bar{\zeta}_F k^2$$

$$\bar{\zeta}_V = -\frac{1}{48\pi^2}\left(6 - \frac{\partial_t f_0}{f_0}\right),$$
$$\bar{\zeta}_F = \frac{1}{1728\pi^2}\left(249 - 41\frac{\partial_t f_0}{f_0}\right)$$

solution :

$$V = \frac{\bar{\zeta}_V}{4} k^4 + \bar{V},$$
$$F = \xi\chi^2 + \frac{\bar{\zeta}_F}{2} k^2 + \bar{F}$$

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2}(\xi\chi^2 + \bar{F}) R + \bar{V} \right)$$

Cosmology with dynamical dark energy !

Cosmological constant vanishes asymptotically !

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4}$$

- vanishes for $\chi \to \infty$!

similar for mass term : $\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$

How well motivated are guesses on the "natural value" of the cosmological constant ?

Quantum fluctuations induce cosmological constant

Zero point energies for normal modes of field with mass m,

for wave numbers $|k| < \Lambda$ $(m^2 \ll \Lambda^2)$

$$\langle \rho \rangle_{vac} = \int_0^{\Lambda} \frac{4\pi k^2 dk}{(2\pi)^3} \cdot \frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

$$V = \frac{\bar{\zeta}_V}{4} k^4 + \bar{V},$$

$$F = \xi \chi^2 + \frac{\bar{\zeta}_F}{2} k^2 + \bar{F}$$

Same argument leads to very different physical effects when applied in different frames

V / M^4 or $\frac{V}{\chi^4}$

<u>Zero point energies</u> for normal modes of field with mass m, for wave numbers $|k| < \Lambda$ $\quad (m^2 \ll \Lambda^2)$

$$\langle \rho \rangle_{vac} = \int_0^{\Lambda} \frac{4\pi k^2 dk}{(2\pi)^3} \cdot \frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

small dimensionless number ?

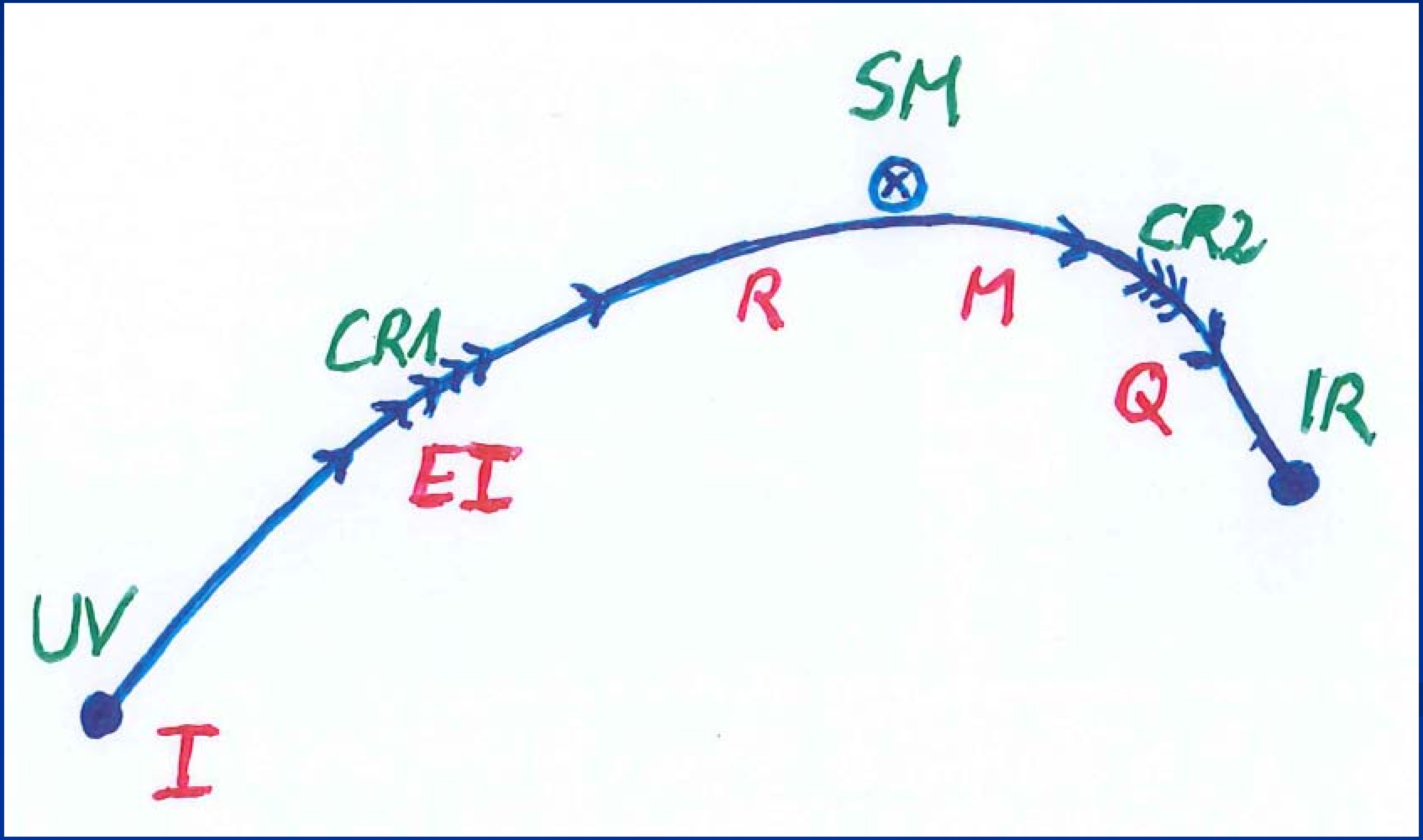
- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V: ratio vanishes asymptotically !

How well motivated are guesses on the "natural values" of masses and couplings ?

properties of fixed points
will not be seen by
naïve order of magnitude estimates

Scale symmetry and its spontaneous breaking

Crossover in quantum gravity



Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Asymptotic safety

if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

Quantum scale symmetry

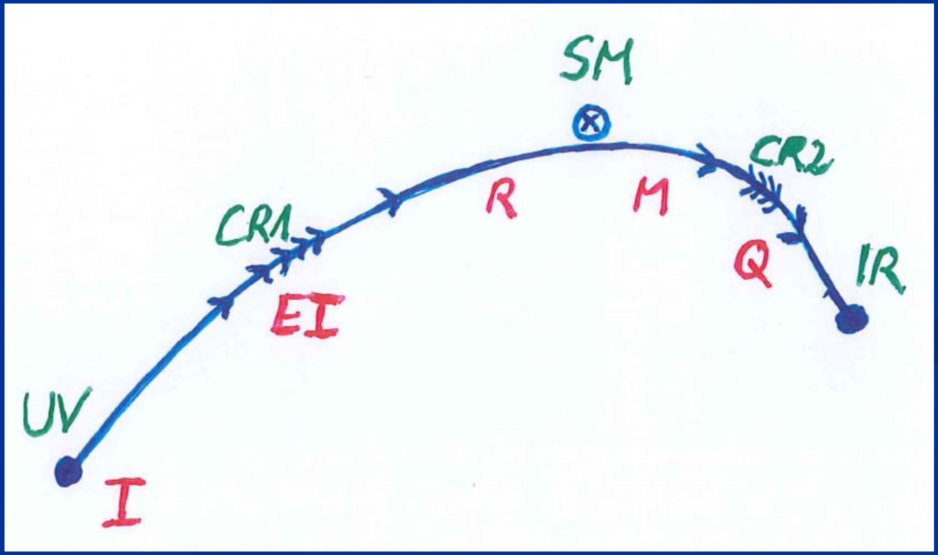
- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

- scale symmetry includes gravity (different from approximate scale symmetry of particle physics with fixed Planck mass)

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions depend only on ratio μ/χ .
- IR: $\mu \rightarrow 0$, $\chi \rightarrow \infty$
- UV: $\mu \rightarrow \infty$, $\chi \rightarrow 0$

Cosmology makes crossover between fixed points by variation of χ .



Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right) \partial^\mu \chi \partial_\mu \chi \right\}$$

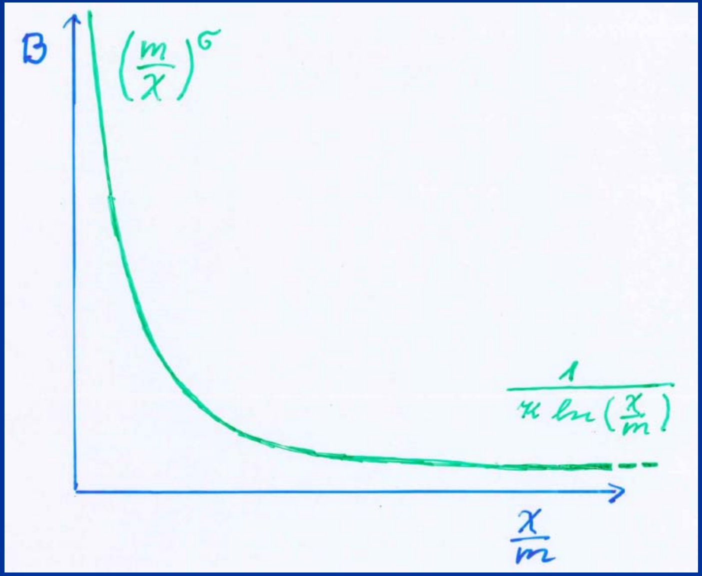
quantum effective action,
variation yields field equations

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass
- Neutrino mass has different dependence on scalar field

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

Kinetial B : Crossover between two fixed points



running coupling obeys flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t \right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

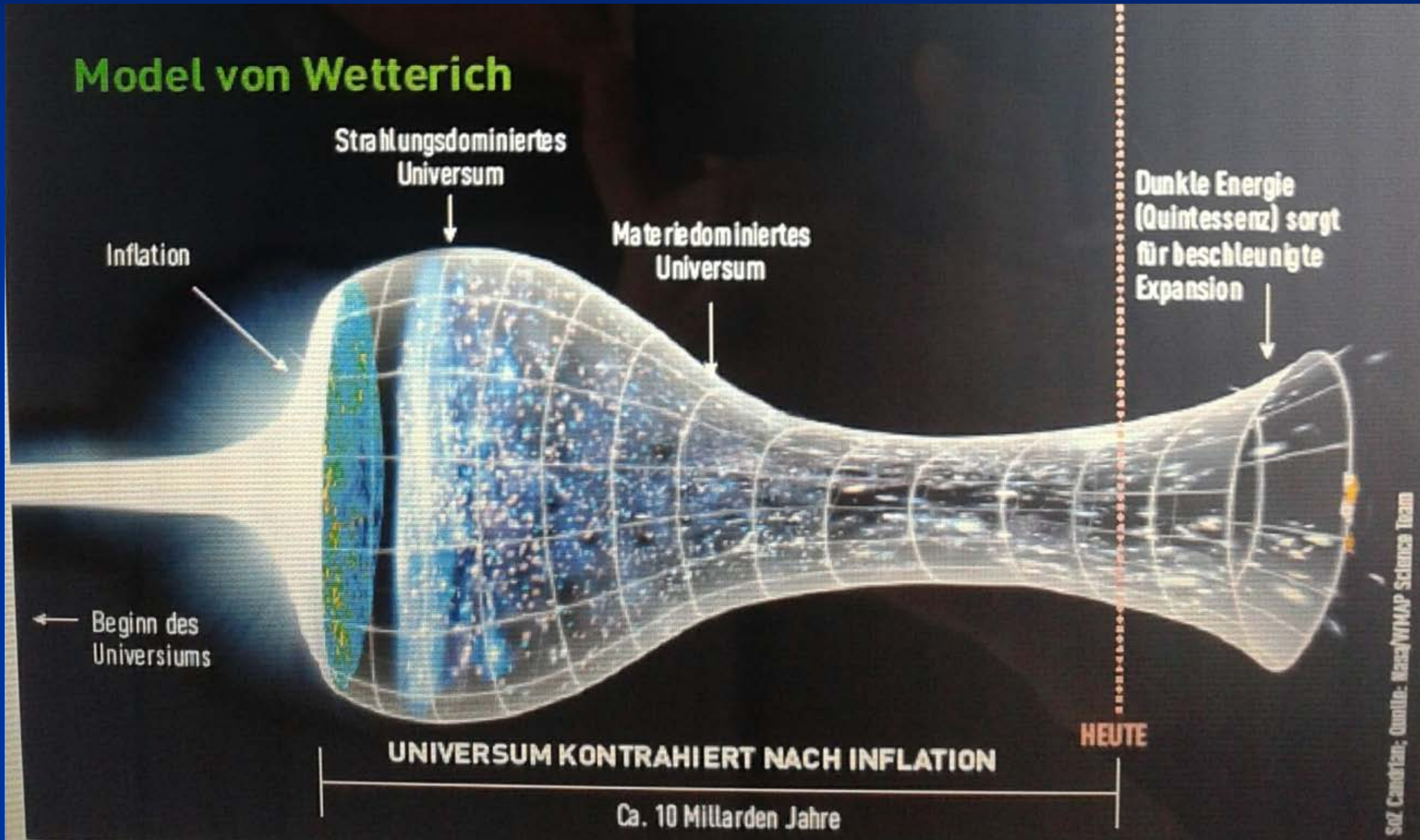
m : scale of crossover

can be exponentially larger than intrinsic scale μ

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Model is compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations

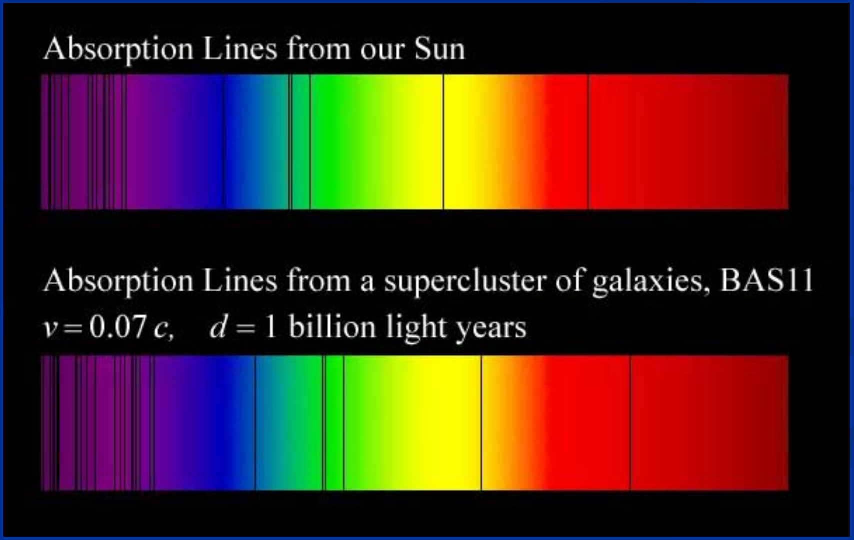
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past, because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

conclusions from variable gravity

- Big bang singularity is artefact of inappropriate choice of field variables – no physical singularity
- Quantum gravity is obervable in dynamics of present Universe

No tiny dimensionless parameters
(except gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33}$ eV
- one time scale $\mu^{-1} = 10^{10}$ yr
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Slow Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \; , \; \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33}\ \mathrm{eV}$$

Expansion or shrinking always slow , characteristic time scale of the order of the age of the Universe : $t_{ch} \sim \mu^{-1} \sim$ 10 billion years !

Hubble parameter of the order of present Hubble parameter for all times , including inflation and big bang !

Slow increase of particle masses !

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Cosmon inflation

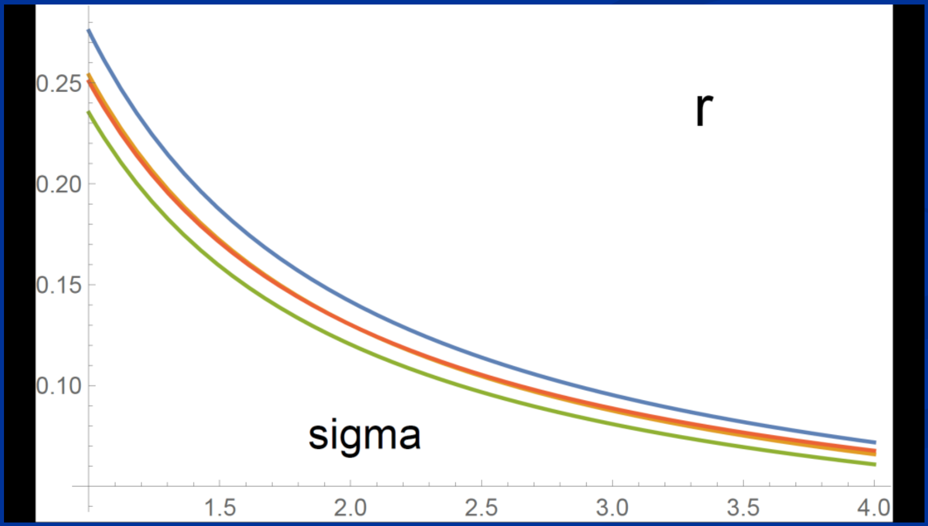
Unified picture of inflation and dynamical dark energy

Cosmon and inflaton are the same scalar field

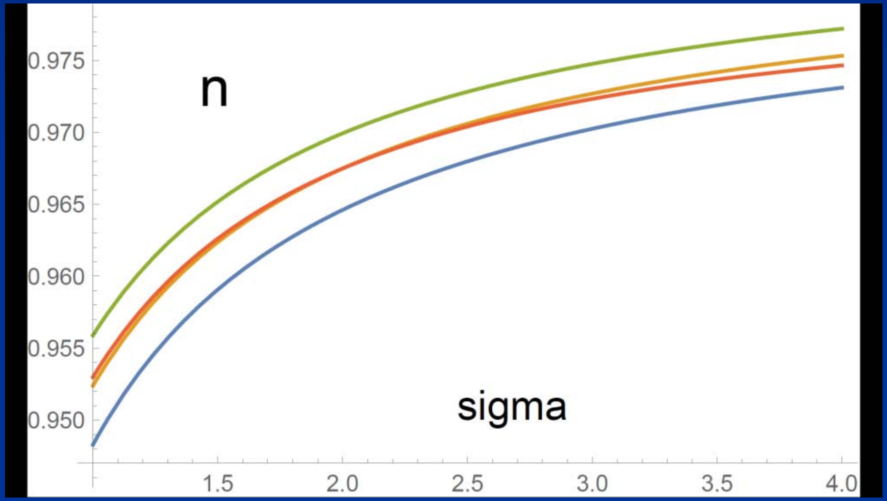
Anomalous dimension determines spectrum of primordial fluctuations

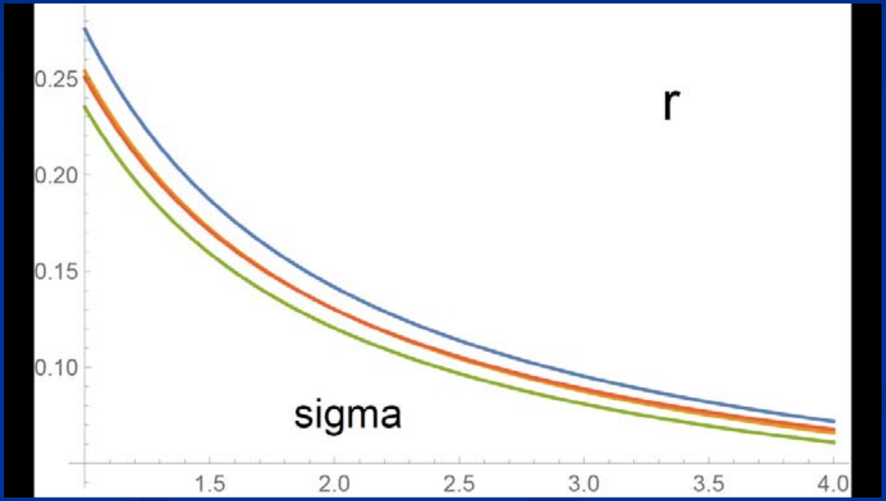
$$r = \frac{0.26}{\sigma}$$

$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$



relation between n and r





$$r = 8.19\,(1 - n) - 0.1365$$

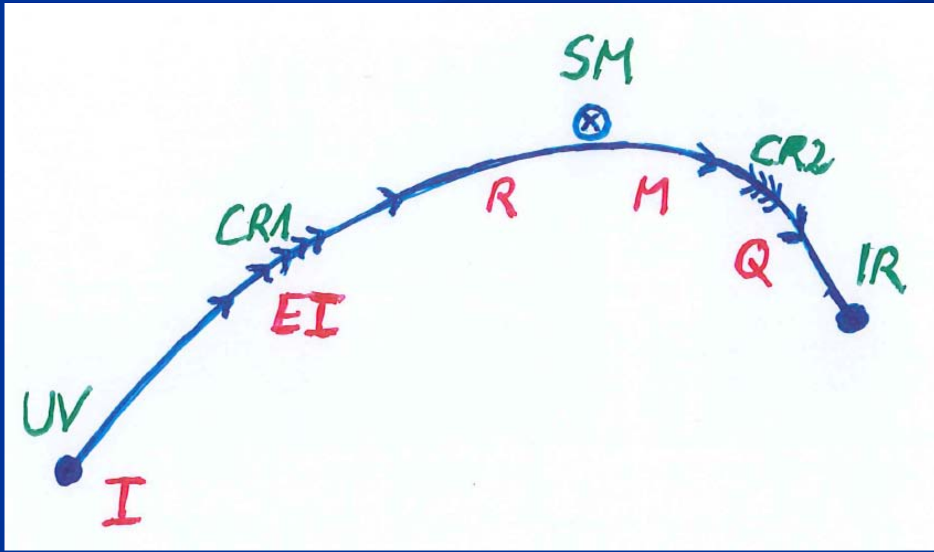
Compatibility with observations and possible tests

- Realistic inflation model:

  $n = 0.97$, $r = 0.1$
- Almost same prediction for radiation, matter, and Dark Energy domination as ΛCDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that **ratio neutrino mass over electron mass grows** .

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

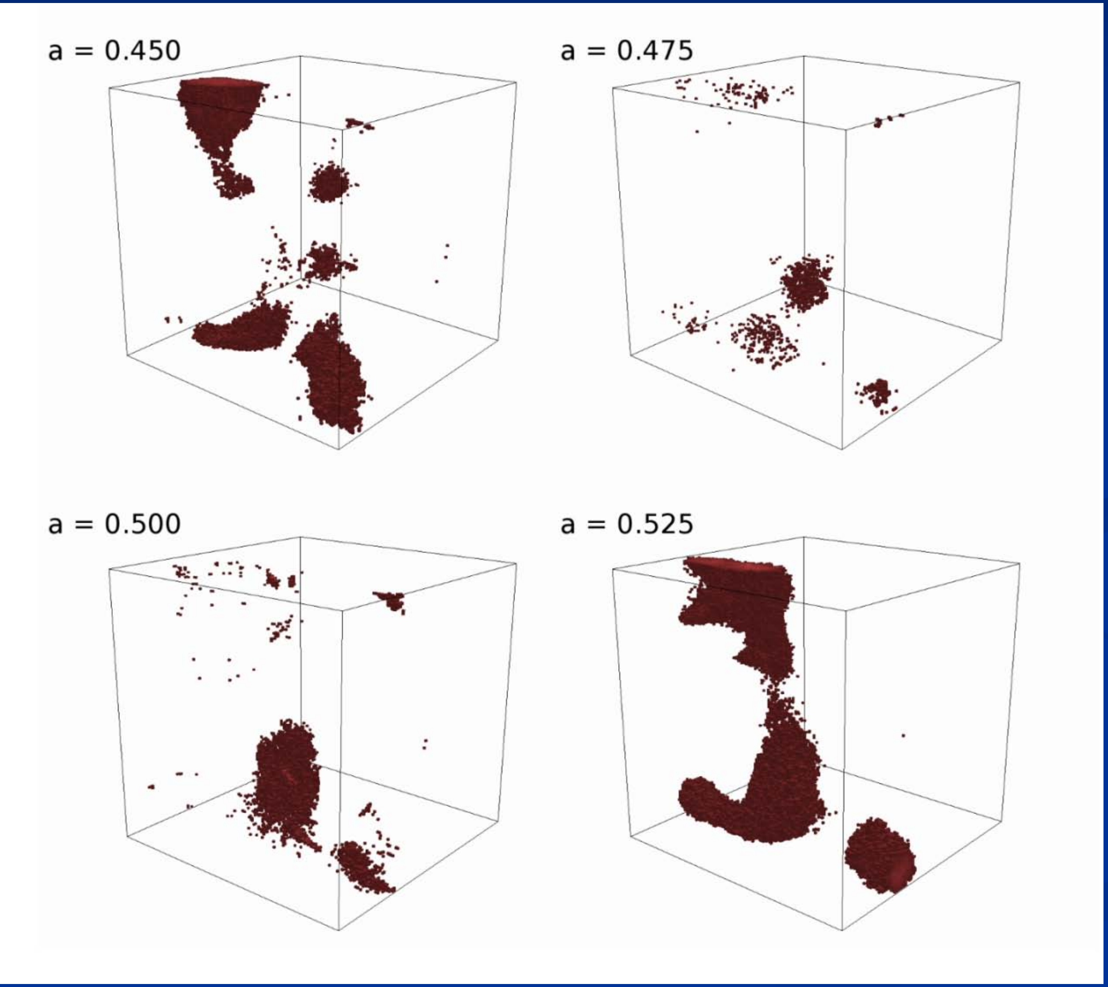
L.Amendola, M.Baldi, …

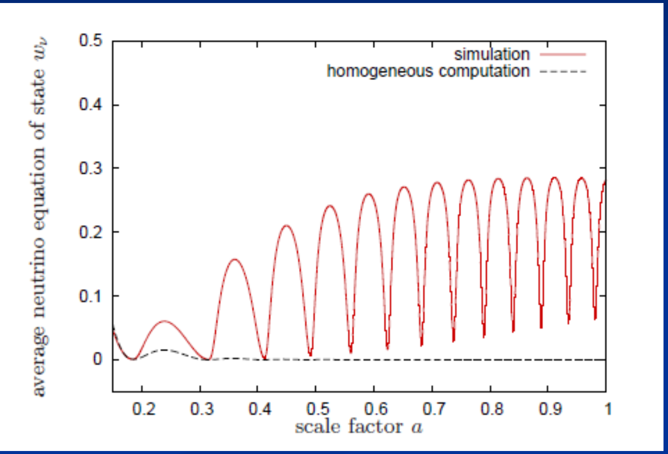
present dark energy density given by neutrino mass

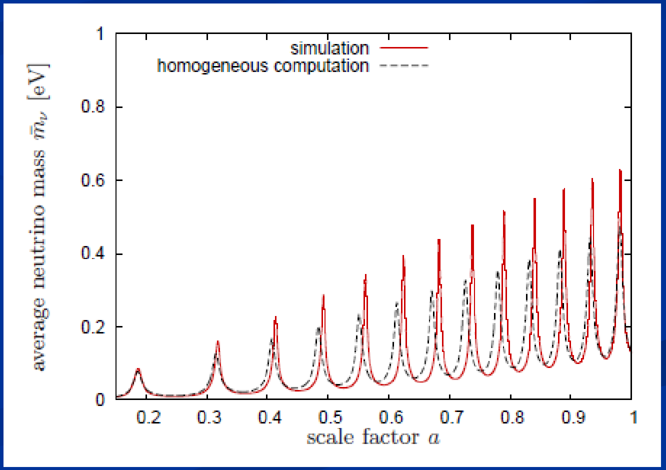
present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$

Oscillating neutrino lumps



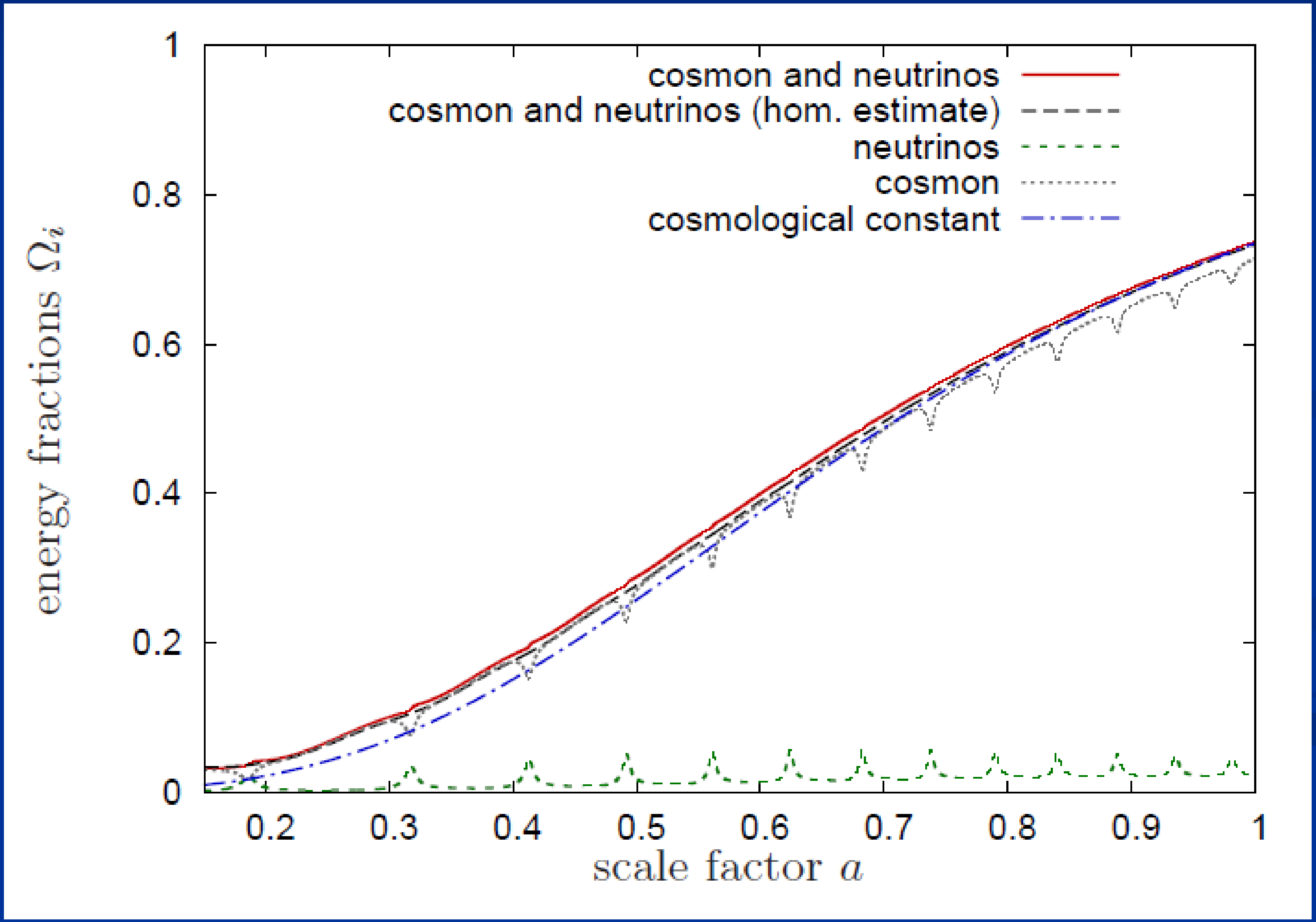




Y.Ayaita, M.Weber,…

Ayaita, Baldi, Fuehrer, Puchwein,…

Evolution of dark energy similar to ΛCDM



Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

conclusions

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than ΛCDM : tests possible

end

conclusions (2)

- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Different cosmon dependence of neutrino mass can explain why Universe makes a transition to Dark Energy domination now
- characteristic signal : neutrino lumps

propagator equation in homogeneous and isotropic cosmology

$$\tilde{D}_\eta G(k,\eta,\eta') = -\frac{i}{a^2}\delta(\eta-\eta'),$$
$$\tilde{D}_\eta = \partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 + m^2 a^2$$

$\eta > \eta'$: $G_> = G_s + G_a$

$$G_s(\eta',\vec{y};\eta,\vec{x}) = G_s(\eta,\vec{x};\eta',\vec{y}),$$
$$G_a(\eta',\vec{y};\eta,\vec{x}) = -G_a(\eta,\vec{x};\eta',\vec{y})$$

$$\tilde{D}_\eta G_s = 0\ ,\ \tilde{D}_\eta G_a = 0\ ,\ \partial_\eta G_{a_{|\eta=\eta'}} = -\frac{i}{2a^2}$$

evolution equation for equal time correlation function

$$\begin{aligned}
\langle\varphi(\eta,\vec{k})\varphi^*(\eta,\vec{k}')\rangle_c &= G_{\varphi\varphi}(k,\eta)\delta(k-k') \\
Re\big(\langle\partial_\eta\varphi(\eta,\vec{k})\varphi^*(\eta,\vec{k}')\rangle_c\big) &= G_{\pi\varphi}(k,\eta)\delta(k-k') \\
\langle\partial_\eta\varphi(\eta,\vec{k})\partial_\eta\varphi^*(\eta,\vec{k}')\rangle_c &= G_{\pi\pi}(k,\eta)\delta(k-k')
\end{aligned}$$

$$\tilde{G}_{\varphi\varphi} = 2a^2kG_{\varphi\varphi}\ ,\ \tilde{G}_{\pi\varphi} = 2a^2G_{\pi\varphi}\ ,\ \tilde{G}_{\pi\pi} = \frac{2a^2}{k}G_{\pi\pi}$$

$$\begin{aligned}
\partial_u\tilde{G}_{\varphi\varphi} &= -\frac{2\tilde{h}}{u}\tilde{G}_{\varphi\varphi} + 2\tilde{G}_{\pi\varphi}, \\
\partial_u\tilde{G}_{\pi\varphi} &= \tilde{G}_{\pi\pi} - \left(1+\frac{\hat{m}^2}{u^2}\right)\tilde{G}_{\varphi\varphi}, \\
\partial_u\tilde{G}_{\pi\pi} &= \frac{2\tilde{h}}{u}\tilde{G}_{\pi\pi} - 2\left(1+\frac{\hat{m}^2}{u^2}\right)\tilde{G}_{\pi\varphi}
\end{aligned}$$

$u = k\,\eta$

massless scalar in de Sitter space :

$$\tilde{G}_{\varphi\varphi} = \alpha(k)\left(1+\frac{1}{u^2}\right) + \beta(k)\left[\left(1-\frac{1}{u^2}\right)\cos(2u) + \frac{2}{u}\sin(2u)\right] + \gamma(k)\left[\frac{2}{u}\cos(2u) - \left(1-\frac{1}{u^2}\right)\sin(2u)\right]$$

Amplitude of density fluctuations

small because of logarithmic running near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln\left(\frac{m}{\mu}\right) = 14.1.$$

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60}\right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

N : number of e – foldings at horizon crossing

Einstein frame

- Weyl scaling maps variable gravity model to Universe with fixed masses and standard expansion history.
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Field relativity : different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions , e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Primordial flat frame

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \bar{\lambda}\chi^4 \ln\left(\frac{\bar{m}}{\chi}\right) + \left[\ln^{-1}\left(\frac{\bar{m}}{\chi}\right) - 3\right] \partial^\mu\chi\partial_\mu\chi \right\}$$

$$a = a_\infty \exp\left\{ -\frac{\tilde{c}_H}{\ln\left(\frac{\bar{m}}{\chi}\right)} \right\}$$

- Minkowski space in infinite past
- absence of any singularity
- geodesic completeness

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \ , \ \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite

Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \qquad \left\{\partial_\eta^2 + k^2 + a^2\left(m^2 - \frac{R}{6}\right)\right\}\tilde{\varphi}_k = 0$$

determine physical time by counting number of oscillations

$$\tilde{t}_p = n_k \qquad n_k = \frac{k\eta}{\pi}$$

m=0

Big bang singularity in Einstein frame is field singularity !

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

Origin of mass

- UV fixed point : scale symmetry unbroken

  all particles are massless

- IR fixed point : scale symmetry spontaneously broken,

  massive particles , massless dilaton

- crossover : explicit mass scale μ or m important

- SM fixed point : approximate scale symmetry spontaneously broken, massive particles , almost massless cosmon, tiny cosmon potential

Hot plasma ?

- Temperature in radiation dominated Universe :
  $T \sim \chi^{1/2}$ smaller than today
- Ratio temperature / particle mass :
  $T/m_p \sim \chi^{-1/2}$ larger than today
- T/m_p counts ! This ratio decreases with time.

- Nucleosynthesis , CMB emission as in standard cosmology !

Infinite past : slow inflation

σ = 2 : field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2}\frac{\dot{\chi}}{\chi}\right)\dot{\chi} = \frac{2\mu^2\chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

solution

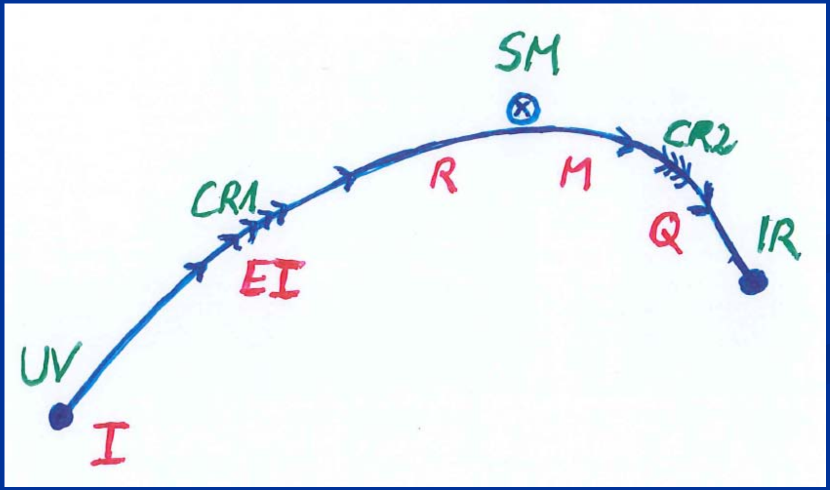
$$H = \frac{\mu}{\sqrt{3}}\,,\ \chi = \frac{3^{\frac{1}{4}}m}{2\sqrt{\mu}}(t_c - t)^{-\frac{1}{2}}$$

First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

- after crossover B changes only very slowly



Scaling solutions near SM fixed point

(approximation for constant B)

$$H = b\mu \;,\;\; \chi = \chi_0 \exp(c\mu t).$$

Different scaling solutions for radiation domination and matter domination

Radiation domination

$$c = \frac{2}{\sqrt{K+6}}$$ $$b = -\frac{c}{2}.$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2.$$ $$\bar{\rho}_r = -3\frac{K+5}{K+6}.$$

K = B - 6

solution exists for B < 1 or K< -5

$$S = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + V(\chi)\right\}$$

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Varying particle masses near SM fixed point

- All particle masses are proportional to χ .

  (scale symmetry)
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.

Scaling of particle masses

mass of electron or nucleon is proportional to variable Planck mass χ !

effective potential for Higgs doublet h

$$\tilde{V}_h = \frac{1}{2}\lambda_h(\tilde{h}^\dagger\tilde{h} - \epsilon_h\chi^2)^2$$

cosmon coupling to matter

$$K(\ddot{\chi} \ + \ 3H\dot{\chi}) + \frac{1}{2}\frac{\partial K}{\partial \chi}\dot{\chi}^2 = -\frac{\partial V}{\partial \chi} + \frac{1}{2}\frac{\partial F}{\partial \chi}R + q_\chi$$

$q_\chi = -(\rho - 3p)/\chi$

$F = \chi^2$

Matter domination

$$c = \sqrt{\frac{2}{K+6}},$$

$$b = -\frac{1}{3}\sqrt{\frac{2}{K+6}} = -\frac{1}{3}c,$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2,$$

$$\bar{\rho}_m = -\frac{2(3K+14)}{3(K+6)}$$

solution exists for

B < 4/3 , $K < -14/3$

K = B - 6

Early Dark Energy

Energy density in radiation increases , proportional to cosmon potential

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2$$

$$V(\chi) = \mu^2\chi^2$$

fraction in early dark energy

$$\Omega_h = \frac{\rho_h}{\rho_r + \rho_h} = \frac{nB(\chi)}{4}$$

or m

observation requires B < 0.02 (at CMB emission)

Dark Energy domination

neutrino masses scale differently from electron mass

$$\frac{\partial \ln m_\nu}{\partial \ln \chi}_{|\text{today}} = 2\tilde{\gamma} + 1$$

$$m_\nu = \bar{c}_\nu \chi^{2\tilde{\gamma}+1}$$

$$\chi q_\chi = -(2\tilde{\gamma} + 1)(\rho_\nu - 3p_\nu)$$

new scaling solution. not yet reached.

at present : transition period

$$\frac{\rho_\nu}{\chi^2} = \bar{\rho}_\nu \mu^2$$

$$b = \frac{1}{3}(2\tilde{\gamma} - 1)c.$$

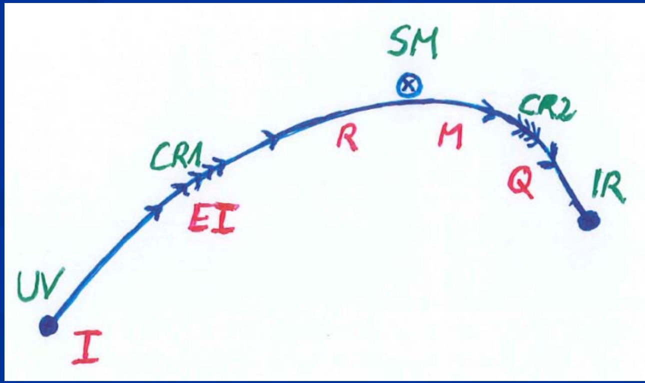
Infrared fixed point

- $\mu \longrightarrow 0$
- $B \longrightarrow 0$

$$\mu\partial_\mu B = \kappa B^2 \qquad \text{for} \qquad B \to 0$$

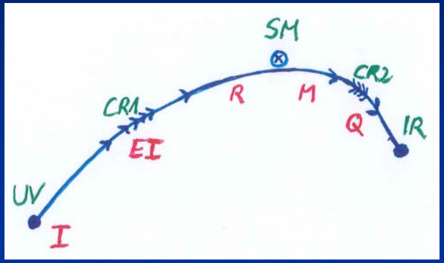
$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

- no intrinsic mass scale
- scale symmetry



Ultraviolet fixed point

- $\mu \longrightarrow \infty$



- kinetial diverges

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

- scale symmetry with anomalous dimension σ

$$g_{\mu\nu} \to \alpha^2 g_{\mu\nu}\ ,\ \chi \to \alpha^{-\frac{2}{2-\sigma}}\chi$$

Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$1 < \sigma < 2$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left(\mu^2 - \frac{R}{2}\right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

deviation from fixed point vanishes for $\mu \longrightarrow \infty$