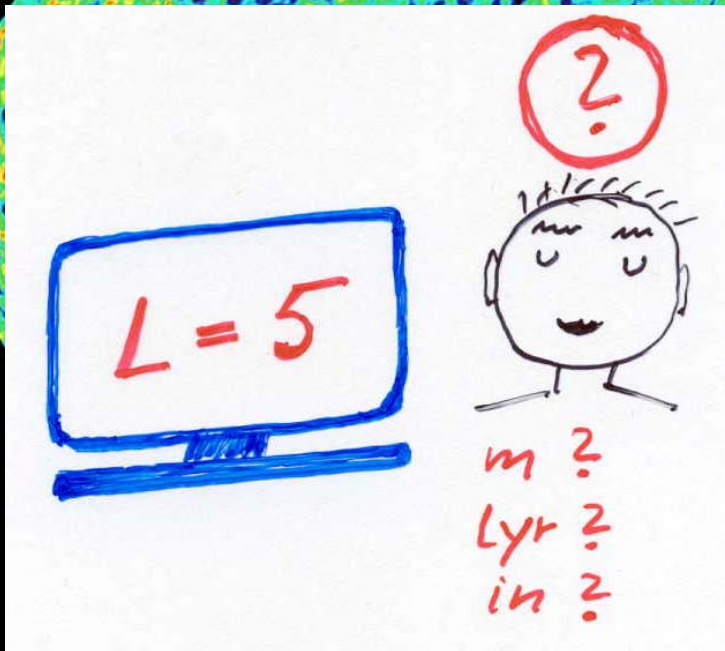


# Fixed points in quantum gravity and cosmology

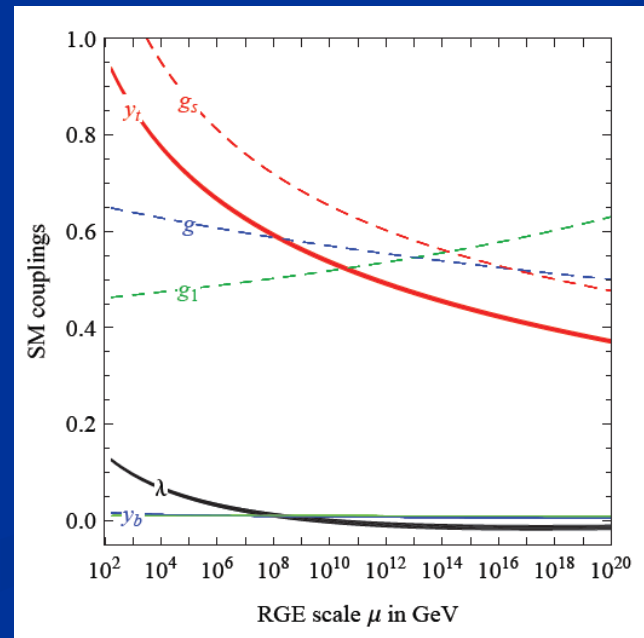
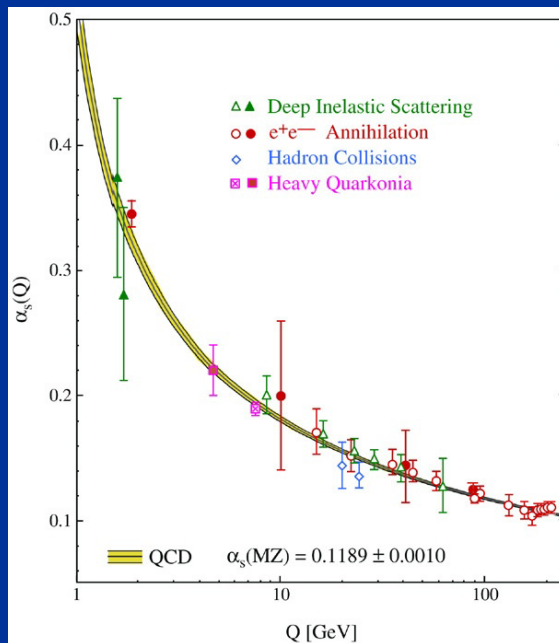


$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

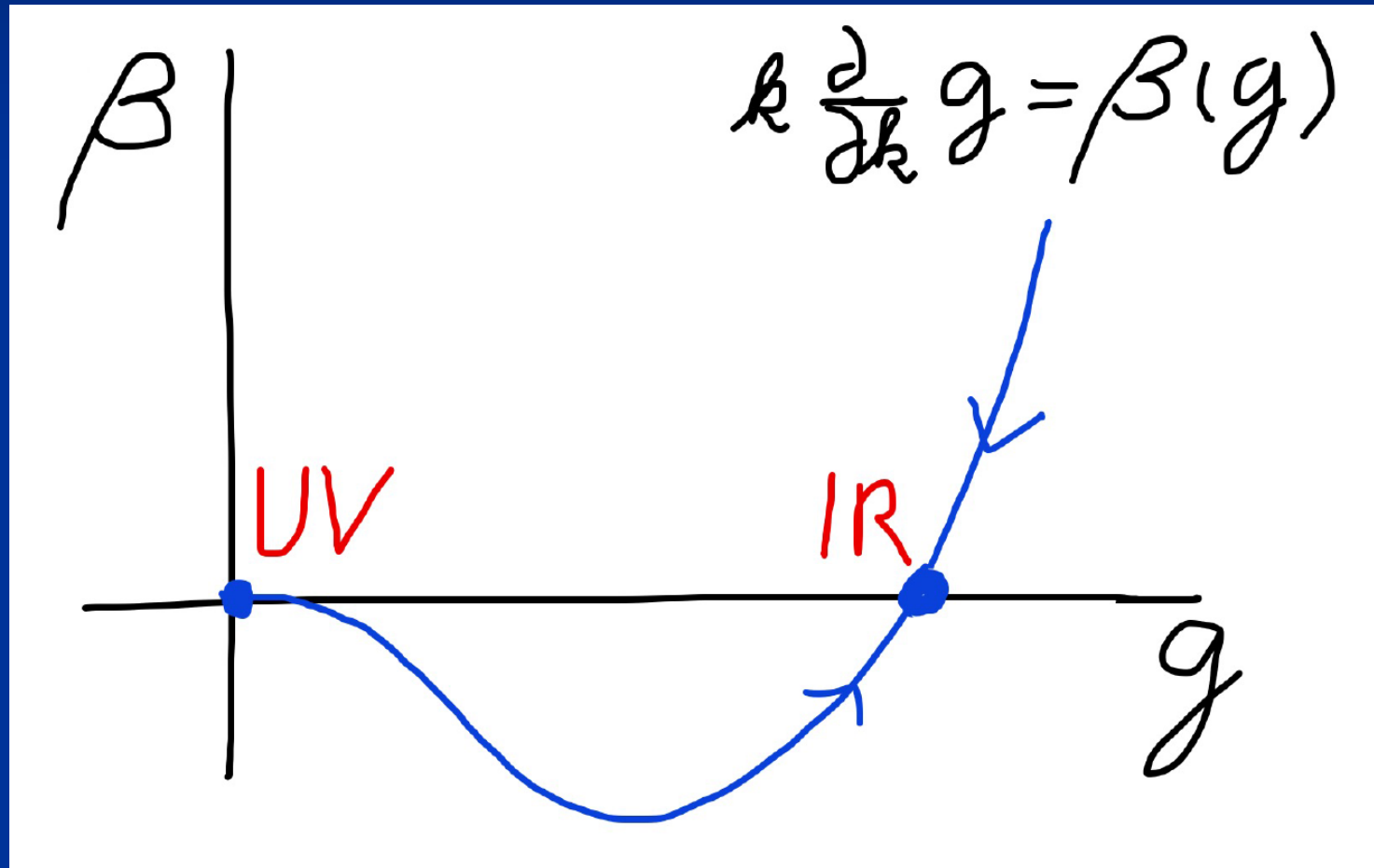
quantum gravity with  
scalar field –  
the role of scale symmetry

# fluctuations induce running couplings

- violation of scale symmetry
- well known in QCD or standard model



# Fixed Points

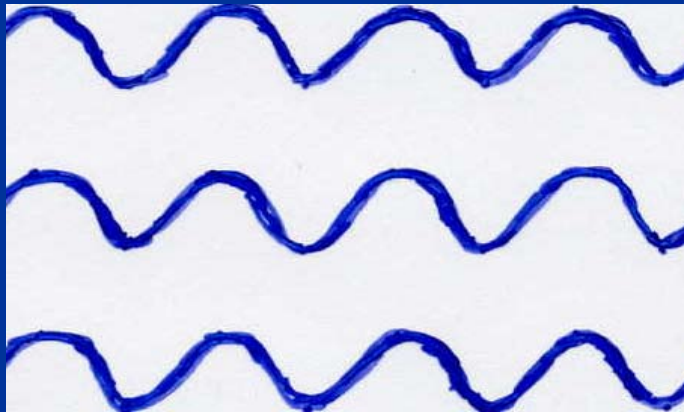
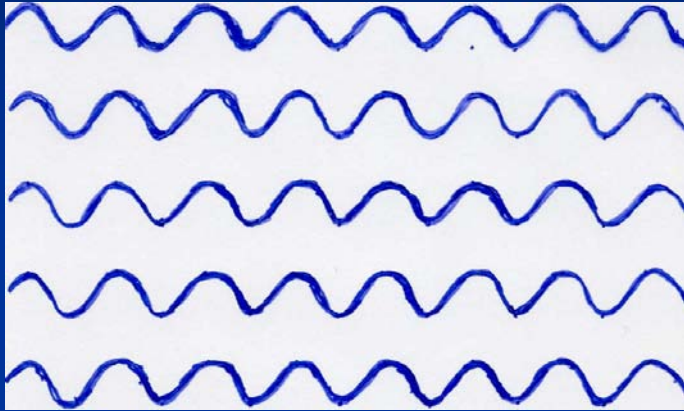




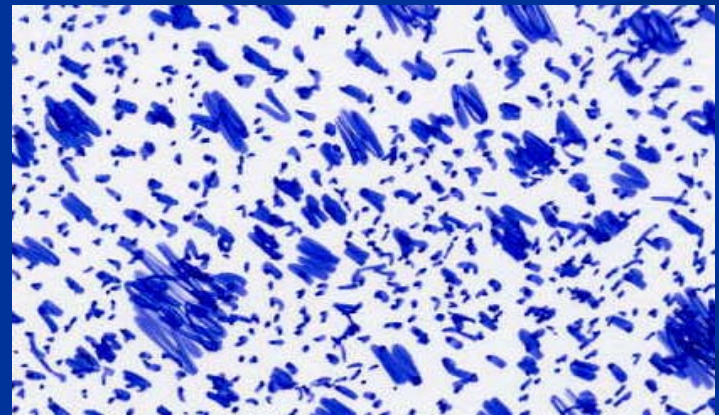
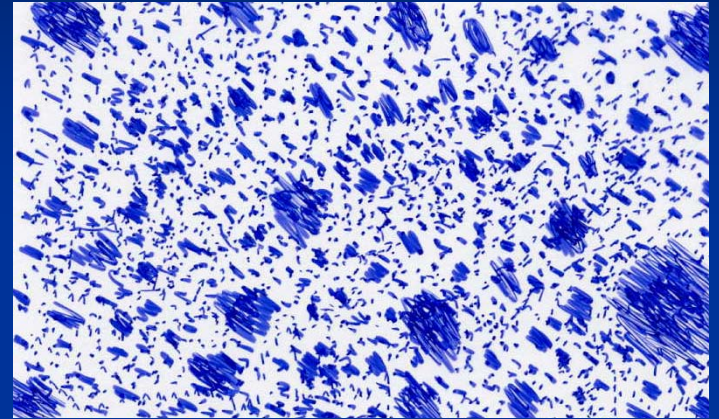
# Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

# Scale symmetry

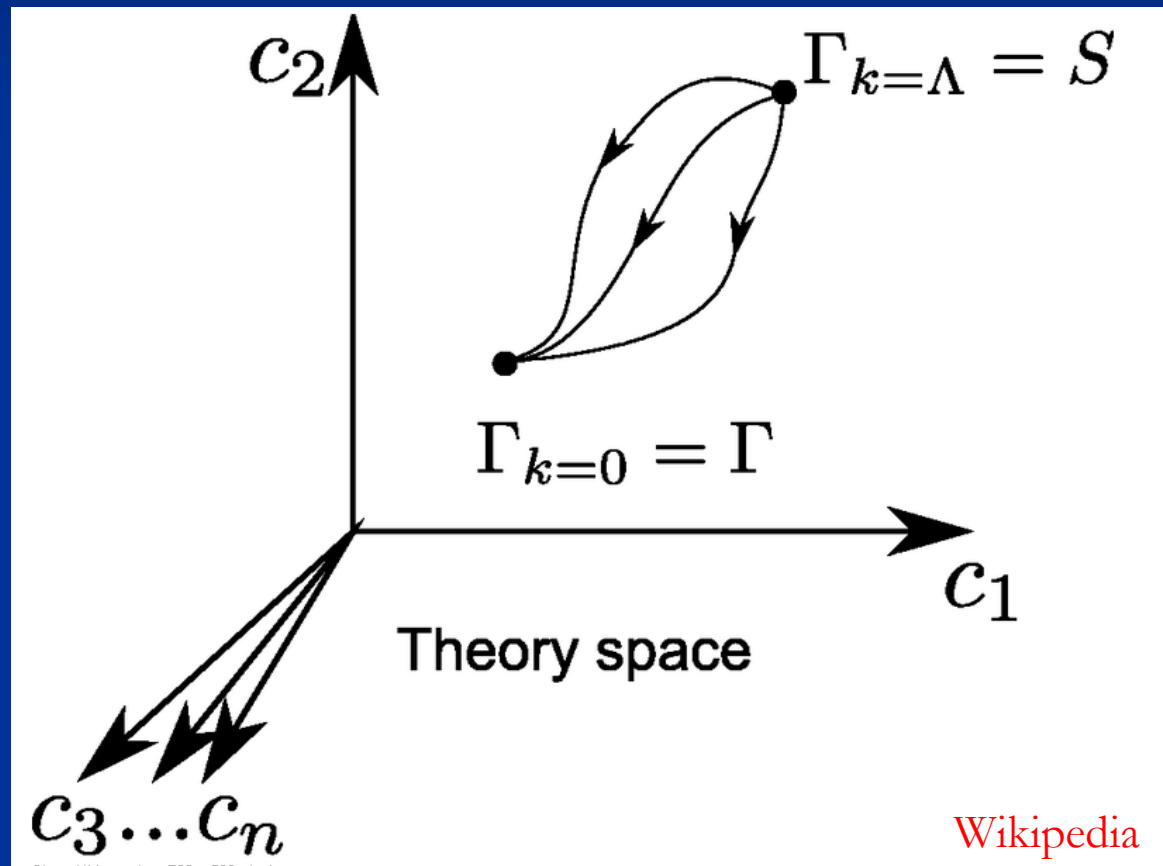


no scale symmetry

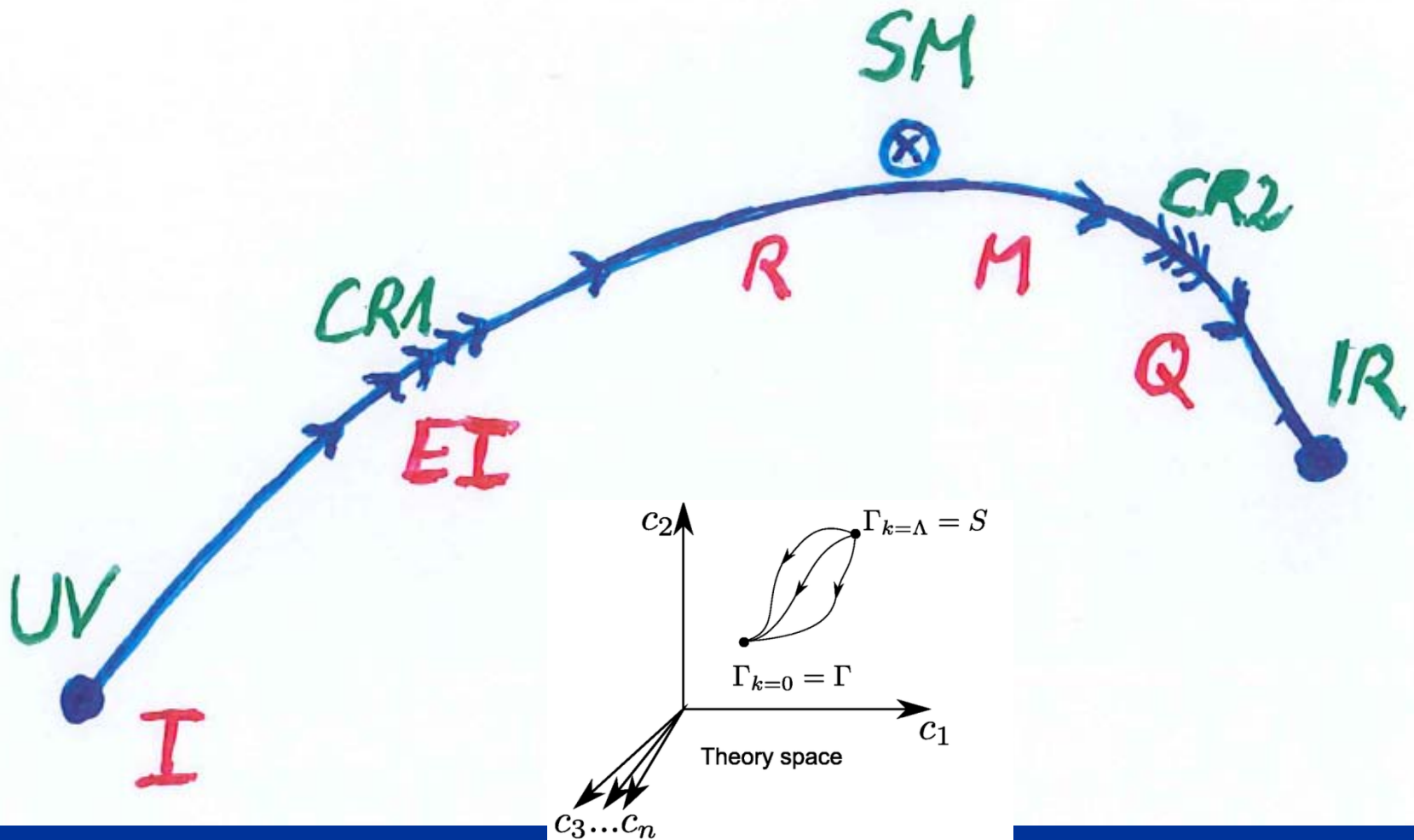


scale symmetry

# functional renormalization : flowing action

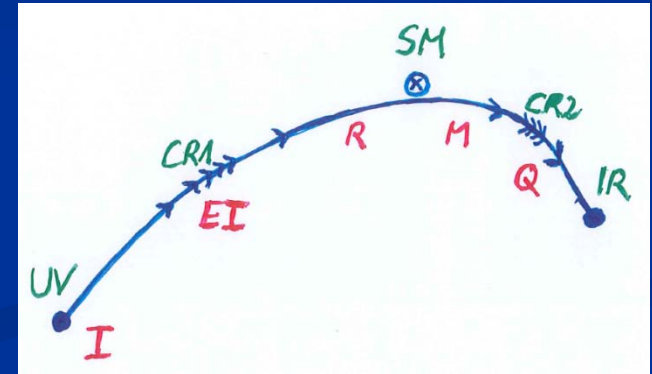


# Crossover in quantum gravity



# Origin of mass

- UV fixed point : scale symmetry unbroken  
all particles are massless
- IR fixed point :  
scale symmetry spontaneously broken,  
massive particles , massless dilaton
- crossover : explicit mass scale  $\mu$  important
- approximate SM fixed point : approximate scale symmetry  
spontaneously broken, massive particles , almost massless  
cosmon, tiny cosmon potential





# Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly **massless Goldstone boson** – the dilaton

# Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

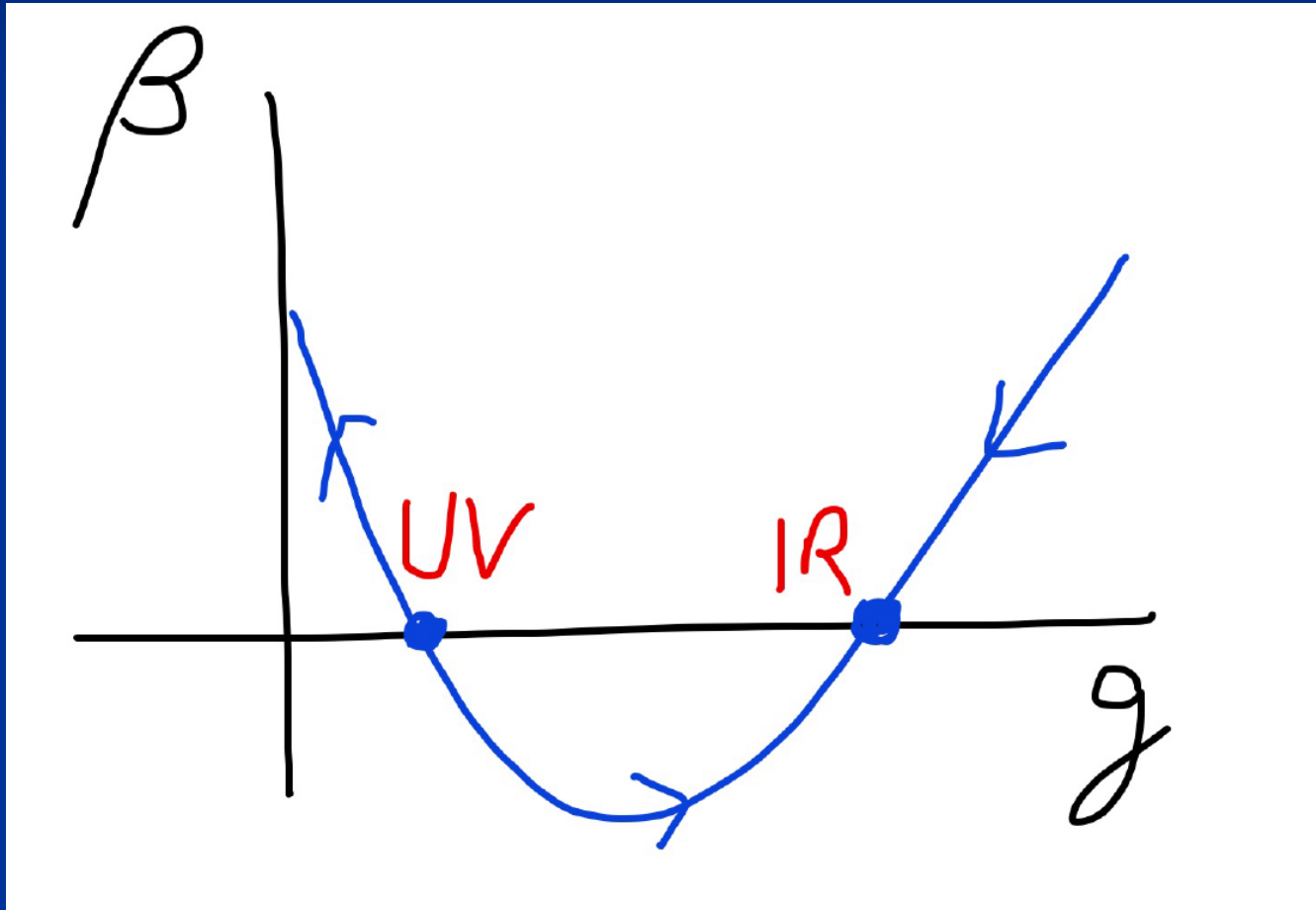
# Asymptotic safety

if UV fixed point exists :

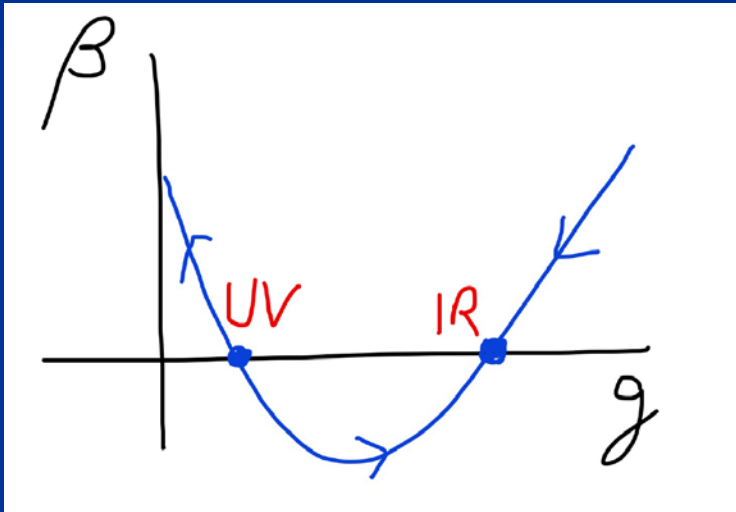
*quantum gravity is  
non-perturbatively renormalizable !*

S. Weinberg , M. Reuter

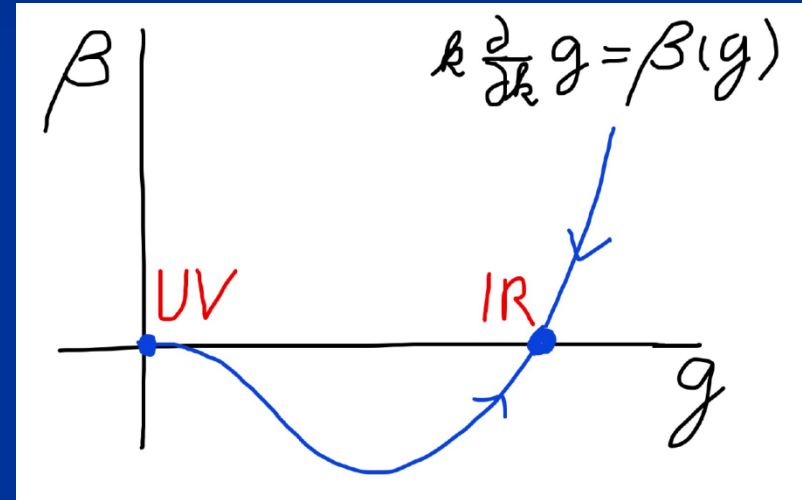
# Asymptotic safety



## Asymptotic safety



## Asymptotic freedom





# a prediction...

## Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

*Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*

Christof Wetterich

*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany*

12 January 2010

### Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson  $m_H$  can be predicted. For a positive gravity induced anomalous dimension  $A_\lambda > 0$  the running of the quartic scalar self interaction  $\lambda$  at scales beyond the Planck mass is determined by a fixed point at zero. This results in  $m_H = m_{\min} = 126$  GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in  $m_H = m_{\min} = 126$  GeV, with o

# IR fixed point in quantum gravity

## Dilaton Quantum Gravity

T. Henz, J. M. Pawłowski, A. Rodigast, and C. Wetterich

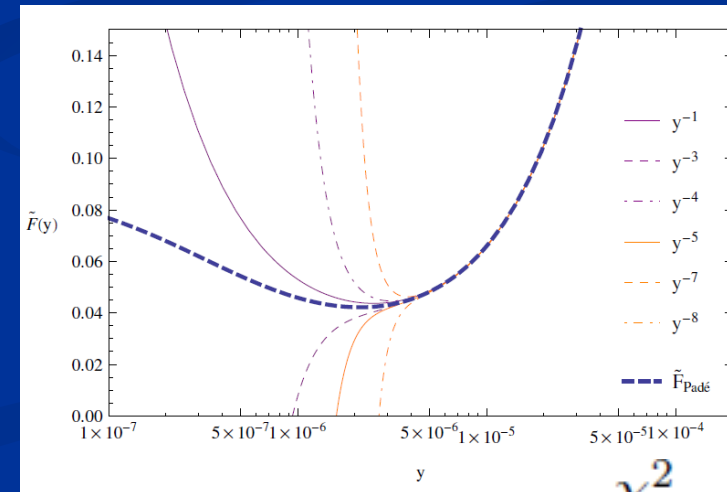
First positive indication from  
functional renormalization  
flow with truncation :

$$\Gamma_k = \int d^4x \sqrt{g} \left( V_k(\chi^2) - \frac{1}{2} F_k(\chi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

fixed point effective action :

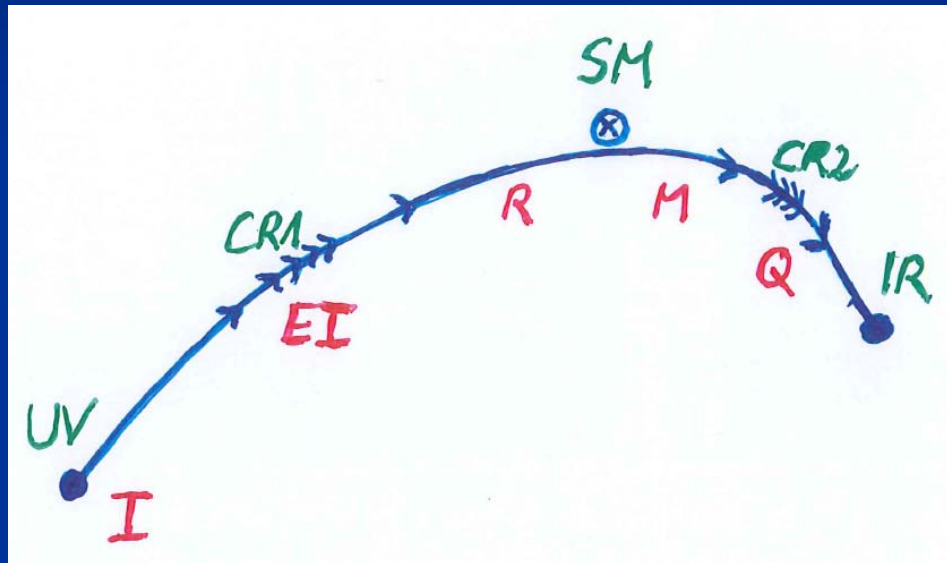
$$\Gamma = \int d^4x \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} \xi \chi^2 R \right)$$

large field  
behavior of  $F$



$$y = \frac{\chi^2}{k^2}$$

# Possible consequences of crossover in quantum gravity

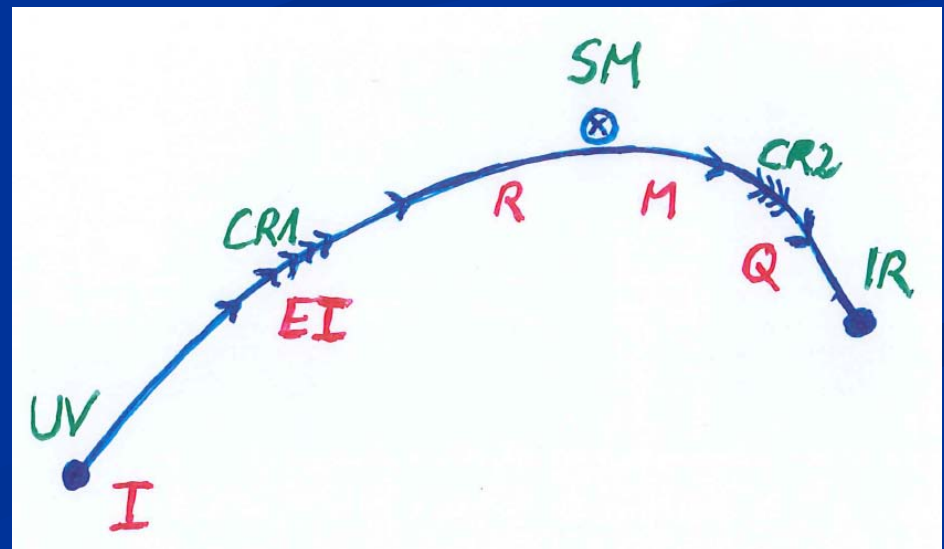


Realistic model for inflation and dark energy  
with single scalar field

# Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as  $B$  depend only on ratio  $\mu/\chi$ .
- IR:  $\mu \rightarrow 0$  ,  $\chi \rightarrow \infty$
- UV:  $\mu \rightarrow \infty$  ,  $\chi \rightarrow 0$

**Cosmology makes  
crossover between  
fixed points by  
variation of  $\chi$ .**



# renormalization flow and cosmological evolution

- renormalization flow as function of  $\mu$

is mapped by dimensionless functions to

- field dependence of effective action on scalar field  $\chi$

translates by solution of field equation to

- dependence of cosmology on time  $t$  or  $\eta$



# Simplicity

simple description of **all** cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

# Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,  
variation yields field equations

Einstein gravity :  $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

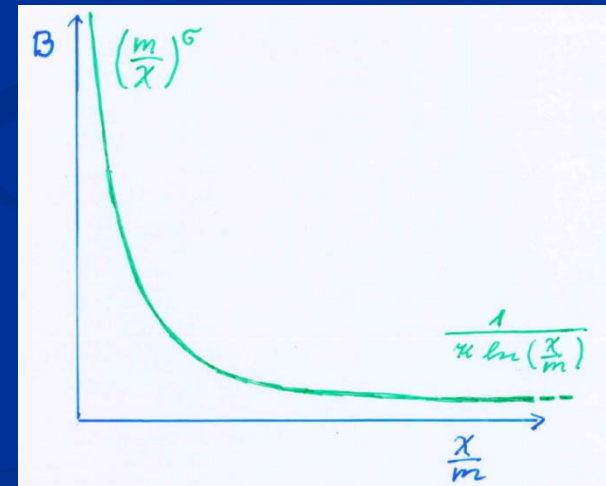
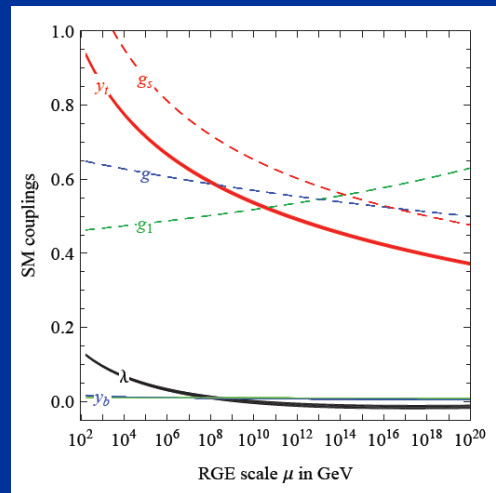
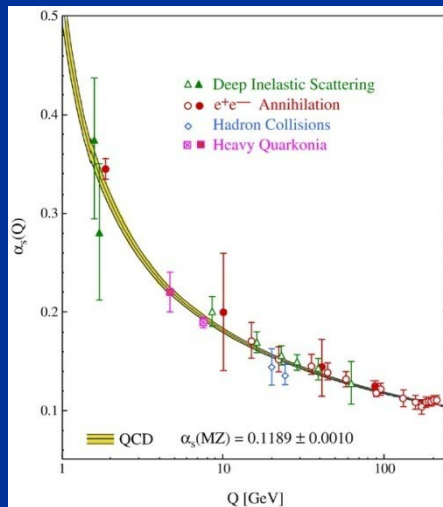
# Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass  $\mu$
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

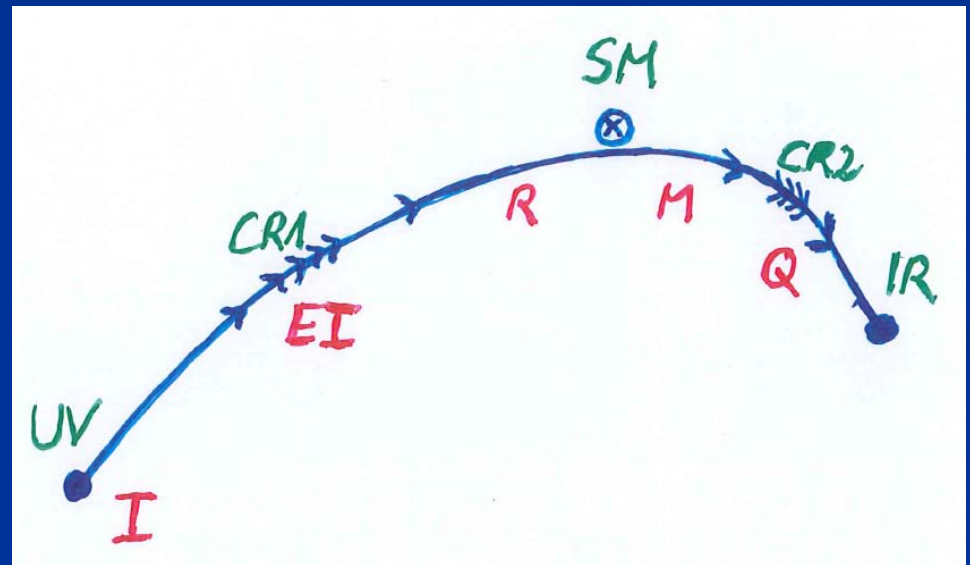
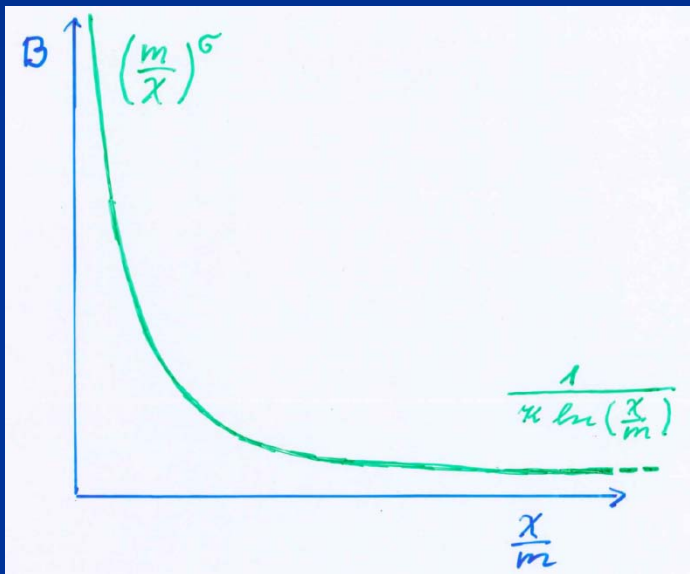
# running coupling B

- B varies if intrinsic scale  $\mu$  changes
- similar to QCD or standard model

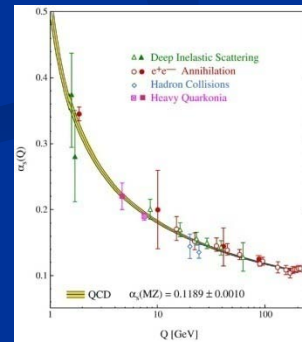


# Kinetic B :

## Crossover between two fixed points



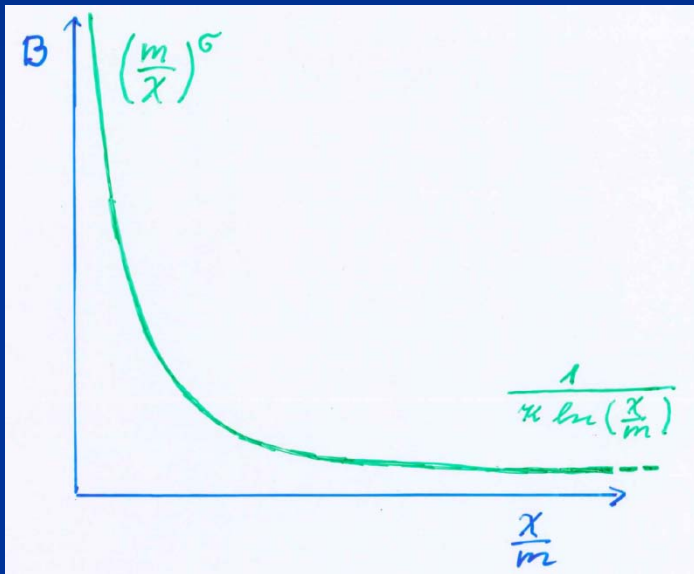
$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$





# Kinetic B :

## Crossover between two fixed points



running  
coupling obeys  
flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[ \ln \left( \frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left( \frac{\chi}{m} \right)$$

$m$  : scale of crossover

can be exponentially larger than intrinsic scale  $\mu$

# Infrared fixed point

■  $\mu \rightarrow 0$

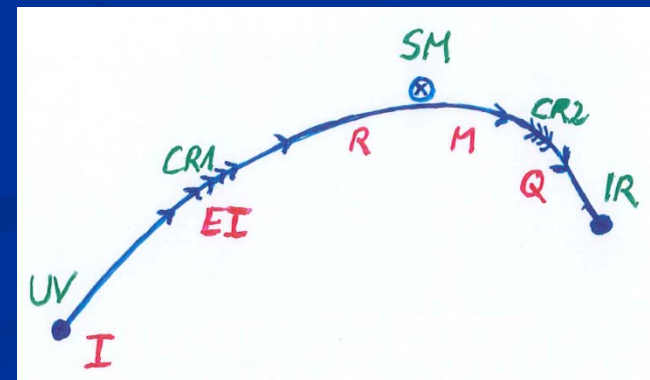
■  $B \rightarrow 0$

$$\mu \partial_\mu B = \kappa B^2 \quad \text{for} \quad B \rightarrow 0$$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

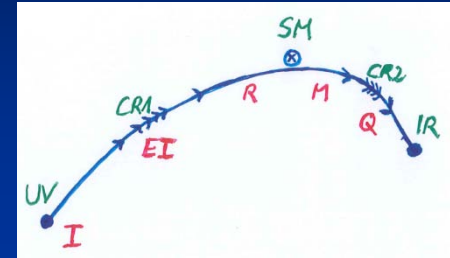
■ no intrinsic mass scale

■ scale symmetry



# Ultraviolet fixed point

■  $\mu \rightarrow \infty$



■ kinetic diverges

$$B = b \left( \frac{\mu}{\chi} \right)^{\sigma} = \left( \frac{m}{\chi} \right)^{\sigma}$$

■ scale symmetry with anomalous dimension  $\sigma$

$$g_{\mu\nu} \rightarrow \alpha^2 g_{\mu\nu} , \quad \chi \rightarrow \alpha^{-\frac{2}{2-\sigma}} \chi$$

# Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$$1 < \sigma$$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass  
scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left( \mu^2 - \frac{R}{2} \right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

deviation from  
fixed point  
vanishes for

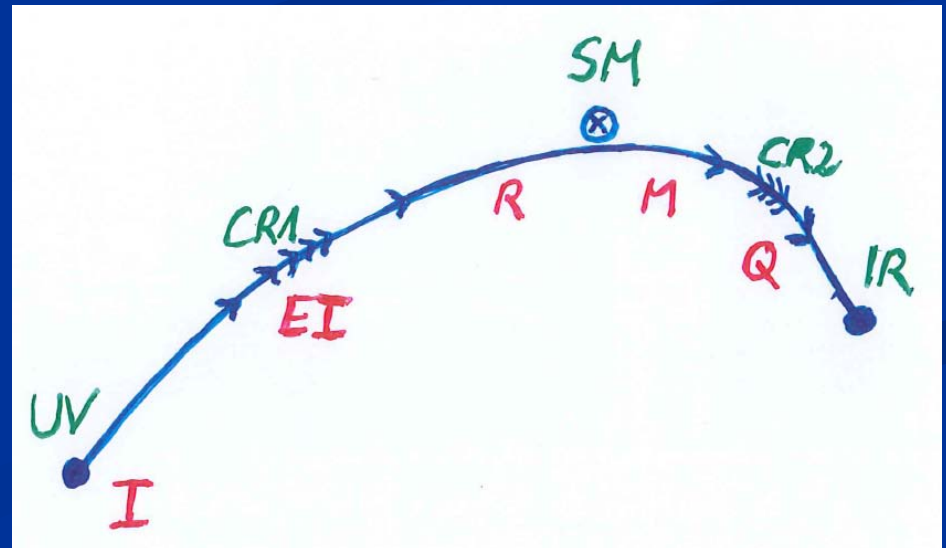
$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

$$\mu \rightarrow \infty$$

# Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as  $B$  depend only on ratio  $\mu/\chi$ .
- IR:  $\mu \rightarrow 0$  ,  $\chi \rightarrow \infty$
- UV:  $\mu \rightarrow \infty$  ,  $\chi \rightarrow 0$

**Cosmology makes  
crossover between  
fixed points by  
variation of  $\chi$ .**



# Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field  $\chi$  vanishes in the infinite past
- scalar field  $\chi$  diverges in the infinite future

# No tiny dimensionless parameters ( except gauge hierarchy )

- one mass scale  $\mu = 2 \cdot 10^{-33} \text{ eV}$
- one time scale  $\mu^{-1} = 10^{10} \text{ yr}$
- Planck mass does not appear as parameter
- Planck mass grows large dynamically



# Particle masses change with time

At SM fixed point :

- All particle masses ( except for neutrinos ) are proportional to scalar field  $\chi$  .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of  $\chi$  and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of  $\chi$  .

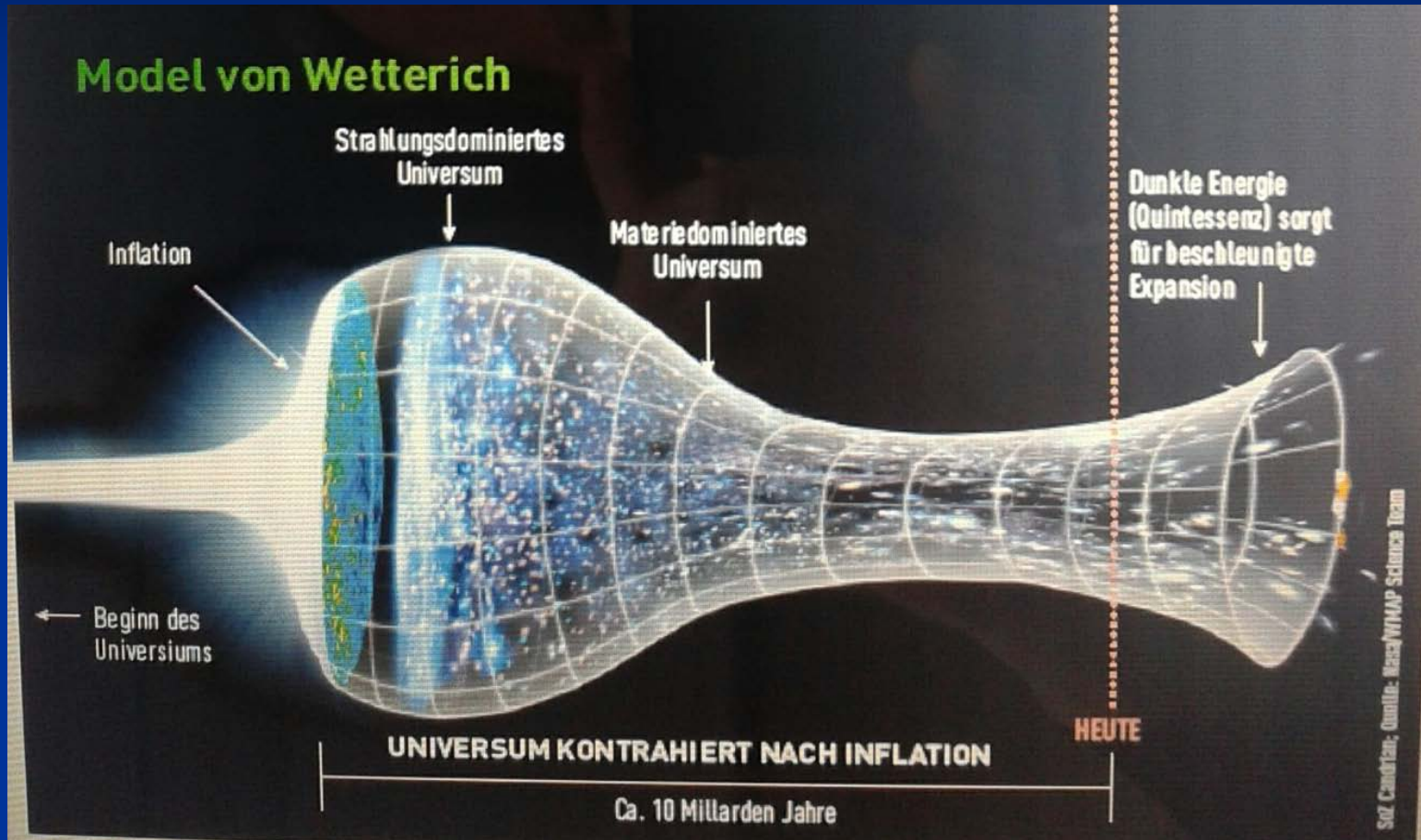
# Four-parameter model

- model has four dimensionless parameters
- three in kinetic B :
  - $\sigma \sim 2.5$
  - $\kappa \sim 0.5$
  - $c_t \sim 14$  ( or  $m/\mu$  )
- one parameter for present growth rate of neutrino mass over electron mass :  $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than  $\Lambda$ CDM

# Cosmological solution

- scalar field  $\chi$  vanishes in the infinite past
- scalar field  $\chi$  diverges in the infinite future

# Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

# Slow Universe

Asymptotic solution in  
freeze frame :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33} \text{ eV}$$

Expansion or shrinking always slow ,  
characteristic time scale of the order of the age of the  
Universe :  $t_{\text{ch}} \sim \mu^{-1} \sim 10 \text{ billion years} !$

Hubble parameter of the order of **present** Hubble  
parameter for all times , including inflation and big bang !  
Slow increase of particle masses !

# Model is compatible with present observations

Together with variation of neutrino mass over  
electron mass in present cosmological epoch :  
model is compatible with all present observations

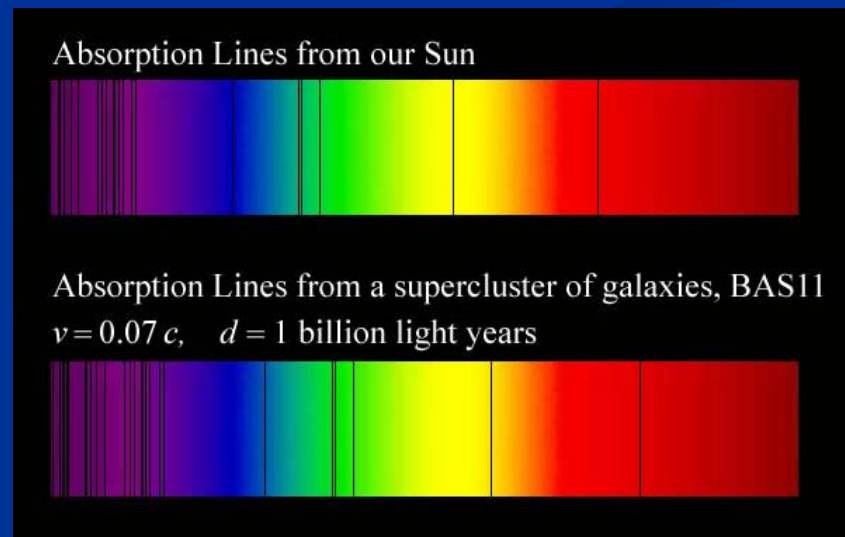
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[ \ln \left( \frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left( \frac{\chi}{m} \right)$$

# Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past,  
because electron mass was smaller !





# What is increasing ?

Ratio of distance between galaxies  
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

# Hot plasma ?

- Temperature in radiation dominated Universe :  
 $T \sim \chi^{1/2}$  **smaller** than today
- Ratio temperature / particle mass :  
 $T / m_p \sim \chi^{-1/2}$  **larger** than today
- $T/m_p$  counts ! This ratio decreases with time.
- Nucleosynthesis , CMB emission as in standard cosmology !

# Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

# Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left( \frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp \left( -\frac{\alpha \varphi}{M} \right)$$

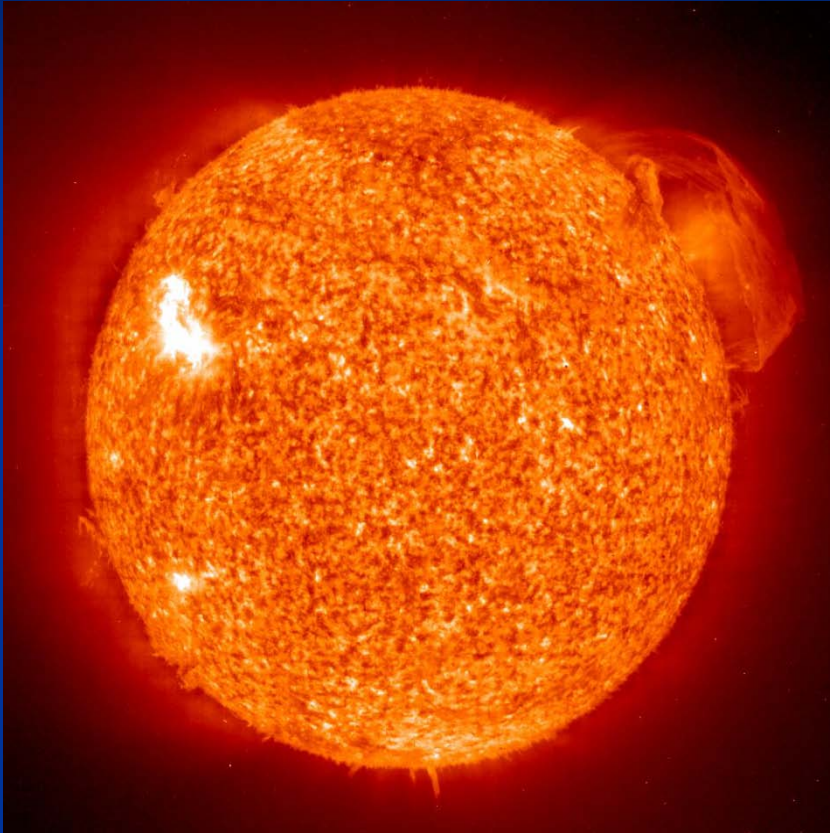
$$k^2 = \frac{\alpha^2 B}{4}$$

# Field relativity :

## different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions ,  
e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

# Big bang or freeze ?





A deep-field astronomical image showing a vast field of galaxies and distant stars against a black background. The galaxies are of various shapes and sizes, some appearing as bright, diffuse clouds and others as more compact, point-like sources. The stars are scattered throughout the field, with some showing prominent diffraction patterns.

# Big bang or freeze ?

just two ways of looking at same physics



# asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for  $\chi \rightarrow \infty$  !

# small dimensionless number ?

- needs two intrinsic mass scales
- $V$  and  $M$  ( cosmological constant and Planck mass )
- variable Planck mass moving to infinity , with fixed  $V$ : **ratio vanishes asymptotically !**

# Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left( \frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp \left( -\frac{\alpha \varphi}{M} \right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

# Infinite past : slow inflation

$\sigma = 2$  : field equations

$$\ddot{\chi} + \left( 3H + \frac{1}{2} \frac{\dot{\chi}}{\chi} \right) \dot{\chi} = \frac{2\mu^2 \chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

solution

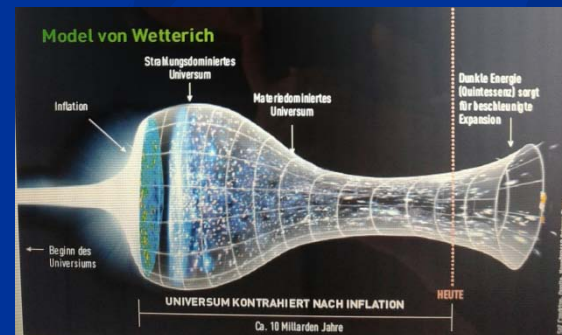
$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

# Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite



# Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \quad \left\{ \partial_\eta^2 + k^2 + a^2 \left( m^2 - \frac{R}{6} \right) \right\} \tilde{\varphi}_k = 0$$

determine **physical time** by counting number of oscillations

$$\tilde{t}_p = n_k$$

$$n_k = \frac{k\eta}{\pi}$$

( m=0 )

*Big bang singularity  
in Einstein frame is  
field singularity !*

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left( \frac{\chi}{\mu} \right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !



# Inflation

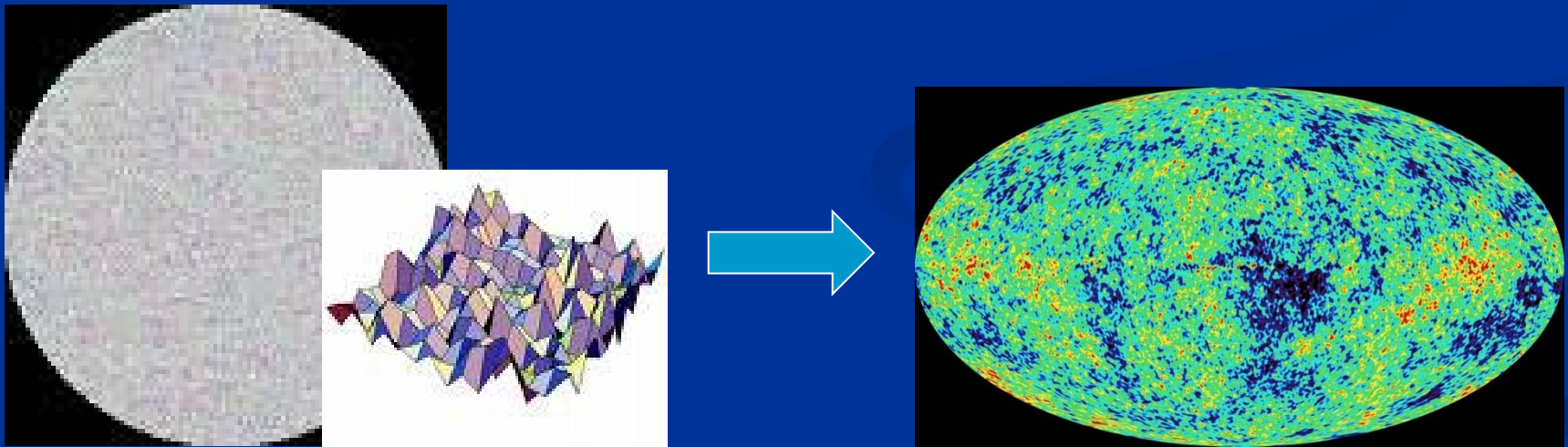
solution for small  $\chi$  : inflationary epoch

kinetial characterized by  
anomalous dimension  $\sigma$

$$B = b \left( \frac{\mu}{\chi} \right)^{\sigma} = \left( \frac{m}{\chi} \right)^{\sigma}$$

# Primordial fluctuations

- inflaton field :  $\chi$
- primordial fluctuations of inflaton become observable in cosmic microwave background



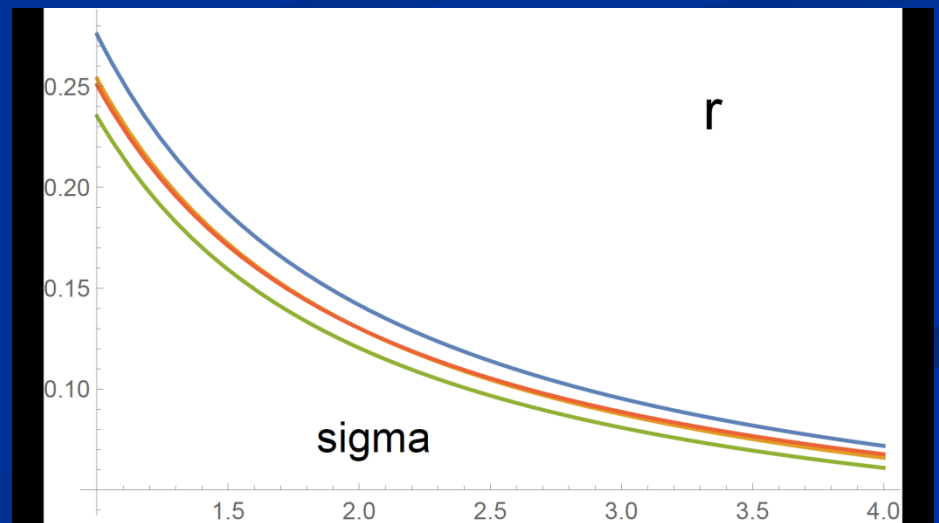
# Anomalous dimension determines spectrum of primordial fluctuations

$$r = \frac{0.26}{\sigma}$$

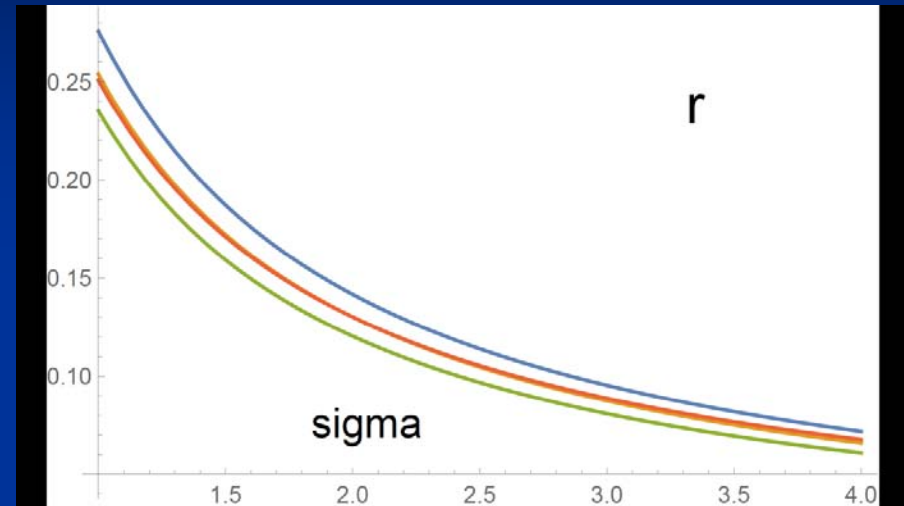
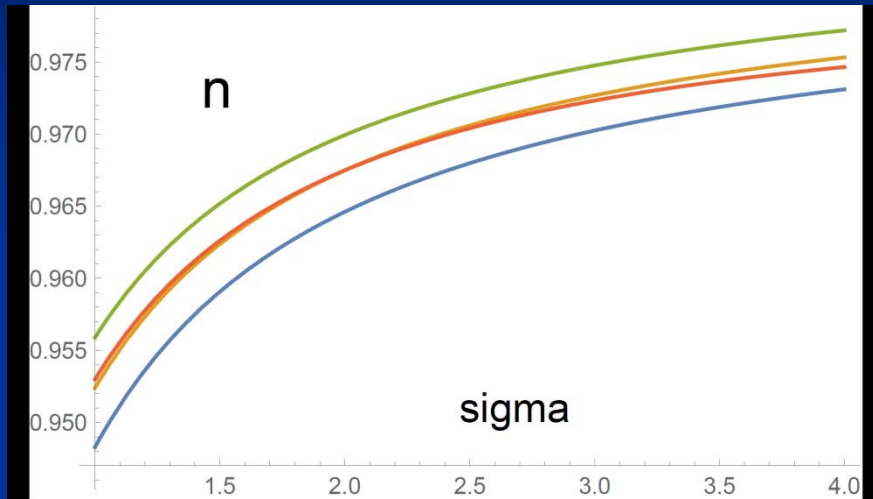
$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$

spectral index  $n$

tensor amplitude  $r$



# relation between n and r



$$r = 8.19 ( 1 - n ) - 0.1365$$

# Amplitude of density fluctuations

small because of logarithmic running  
near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln \left( \frac{m}{\mu} \right) = 14.1 \quad \sigma=1$$

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left( \frac{N}{60} \right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[ \ln \left( \frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left( \frac{\chi}{m} \right)$$

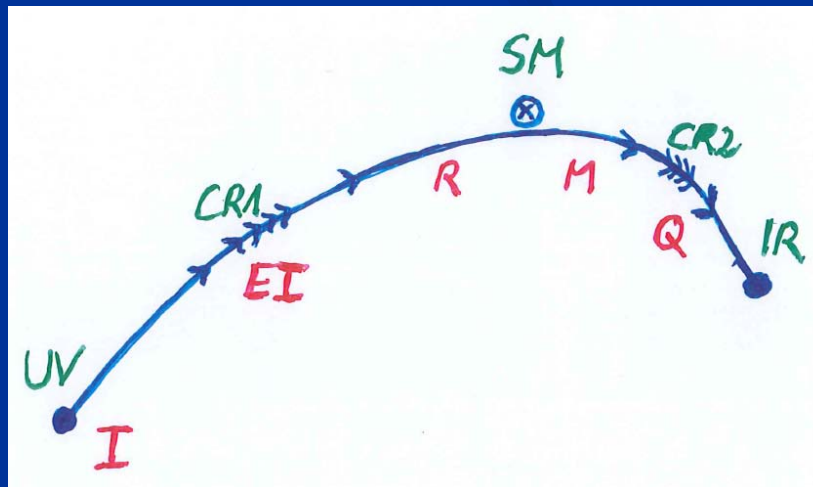
N : number of e – foldings at horizon crossing

# First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[ \ln \left( \frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left( \frac{\chi}{m} \right)$$

- after crossover B changes only very slowly



# Scaling solution

- Heating of the Universe after inflation
- Scaling solution with almost fixed fraction of Early Dark Energy

# Cosmon inflation

Unified picture of inflation and  
dynamical dark energy

Cosmon and inflaton are the same  
scalar field



# Quintessence

Dynamical dark energy ,  
generated by scalar field (cosmon )

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

**Prediction :**

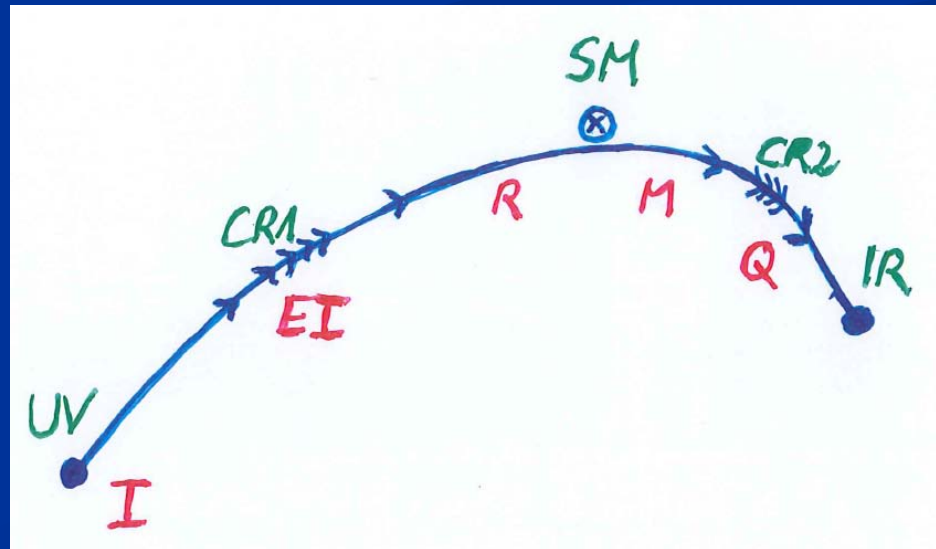
**homogeneous dark energy  
influences recent cosmology**

**- of same order as dark matter -**

Original models do not fit the present observations  
.... modifications  
( different growth of neutrino mass )

# Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first ( seesaw or cascade mechanism )



# Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to  $\chi$ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with  $\chi$ , such that **ratio neutrino mass over electron mass grows**.

# connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left( \frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

L.Amendola,  
M.Baldi, ...

present dark energy density given by neutrino mass

present equation  
of state given by  
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

# Compatibility with observations and possible tests

- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as  $\Lambda$ CDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

# Simplicity

simple description of **all** cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch



# conclusions

Quantum gravity may be observable in  
dynamics of present Universe

Fixed points and scale symmetry crucial

Big bang singularity is artefact  
of inappropriate choice of field variables –  
no physical singularity

## conclusions (2)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than  $\Lambda$ CDM : tests possible

# conclusions (3)

- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Cosmon dependence of ratio neutrino mass/ electron mass can explain why Universe makes a transition to Dark Energy domination **now**
- **characteristic signal : neutrino lumps**

The background is a solid dark blue color. On the right side, there are several overlapping, wavy, light blue lines that create a sense of movement and depth. These lines are curved and flow from the top right towards the bottom right.

end

# Scaling solutions near SM fixed point

( approximation for constant B )

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Different scaling solutions for  
radiation domination and  
matter domination

# Dilaton quantum gravity

## Dilaton Quantum Gravity

T. Henz, J. M. Pawłowski, A. Rodigast, and C. Wetterich

Functional renormalization flow,  
with truncation :

$$\Gamma_k = \int d^4x \sqrt{g} \left( V_k(\chi^2) - \frac{1}{2} F_k(\chi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

# Functional renormalization equations

$$V_k = k^4 y^2 v_k(y), \quad F_k = k^2 y f_k(y)$$

$$y = \frac{\chi^2}{k^2}$$

$$\partial_t v_k(y) = 2 y v'_k(y) + \frac{1}{y^2} \zeta_V$$

$$\partial_t f_k(y) = 2 y f'_k(y) + \frac{1}{y} \zeta_F.$$

$$\zeta_V = \frac{1}{192\pi^2} \left\{ 6 + \frac{30 \tilde{V}}{\Sigma_0} + \frac{3(2\Sigma_0 + 24 y \tilde{F}' \Sigma'_0 + \tilde{F} \Sigma_1)}{\Delta} + \delta_V \right\},$$

$$\begin{aligned} \zeta_F = \frac{1}{1152\pi^2} & \left\{ 150 + \frac{30 \tilde{F} (3 \tilde{F} - 2 \tilde{V})}{\Sigma_0^2} \right. & (10) \\ & - \frac{12}{\Delta} \left( 24 y \tilde{F}' \Sigma'_0 + 2 \Sigma_0 + \tilde{F} \Sigma_1 \right) - 6 y (3 \tilde{F}'^2 + 2 \Sigma_0'^2) \\ & - \frac{36}{\Delta^2} \left[ 2 y \Sigma_0 \Sigma'_0 (7 \tilde{F}' - 2 \tilde{V}') (\Sigma_1 - 1) + 2 \Sigma_0'^2 \Sigma_2 \right. \\ & \left. + 2 y \Sigma_1 (7 \tilde{F}' - 2 \tilde{V}') (2 \Sigma_0 \tilde{V}' - \tilde{V} \Sigma'_0) \right. \\ & \left. \left. + 24 y \tilde{F}' \Sigma_0 \Sigma'_0 \Sigma_2 - 12 y \tilde{F} \Sigma_0'^2 \Sigma_2 \right] + \delta_F \right\}. \end{aligned}$$

$$\tilde{V} = y^2 v_k(y), \quad \tilde{F} = y f_k(y),$$

$$\Sigma_0 = \frac{1}{2} \tilde{F} - \tilde{V}, \quad \Delta = (12 y \Sigma_0'^2 + \Sigma_0 \Sigma_1)$$

$$\Sigma_1 = 1 + 2 \tilde{V}' + 4 y \tilde{V}'', \quad \Sigma_2 = \tilde{F}' + 2 y \tilde{F}''.$$

Percacci, Narain



# Fixed point for large scalar field

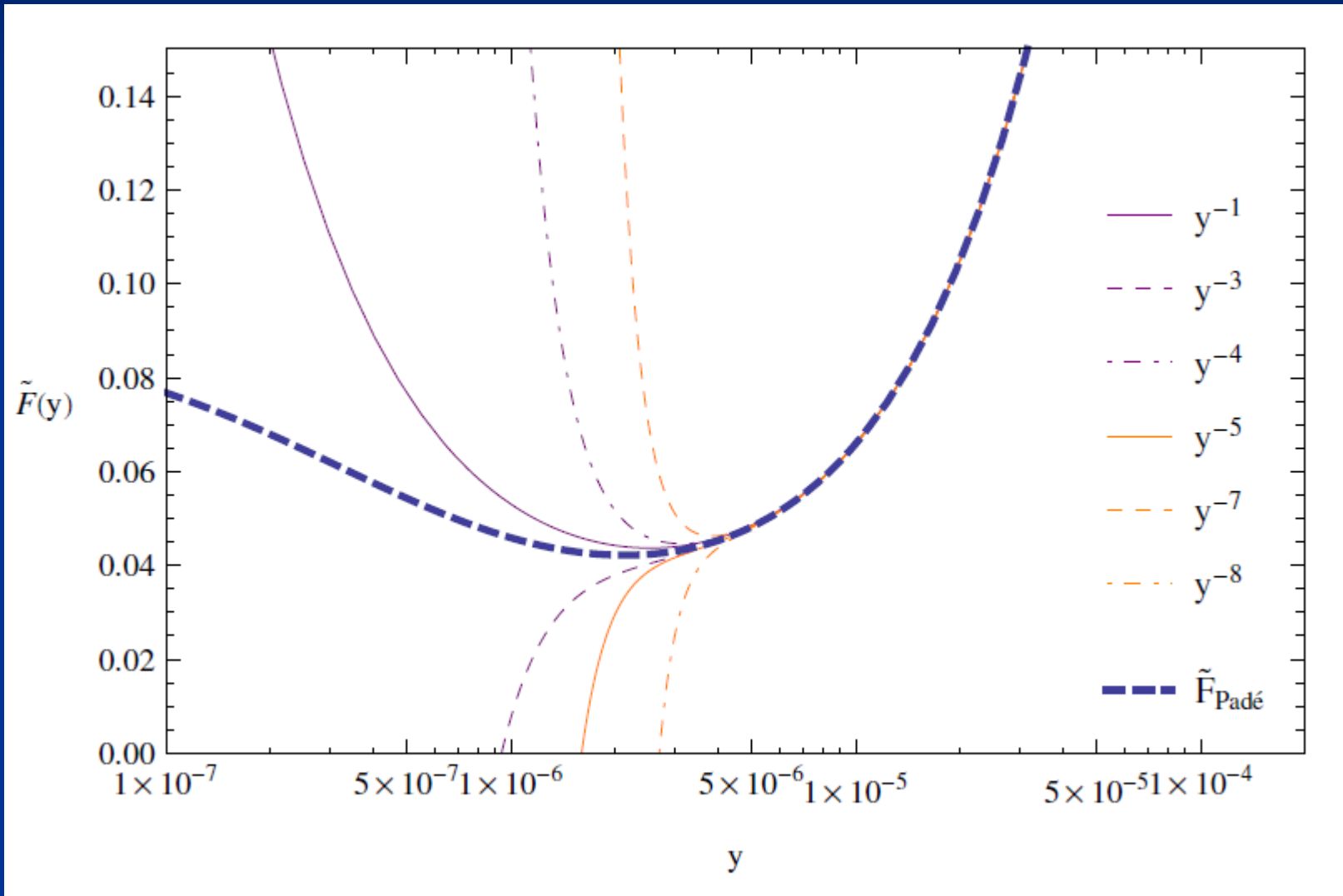
$$\lim_{y \rightarrow \infty} f(y) = \xi$$

$$\lim_{y \rightarrow \infty} v(y) = 0$$

$$\Gamma = \int d^4x \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} \xi \chi^2 R \right)$$

This fixed point describes already realistic gravity !  
Limit  $k \rightarrow 0$  can be taken !

# Fixed point for large scalar field



# Vicinity of fixed point

$$\partial_t V = \bar{\zeta}_V k^4, \quad \partial_t F = \bar{\zeta}_F k^2$$

$$\bar{\zeta}_V = -\frac{1}{48\pi^2} \left( 6 - \frac{\partial_t f_0}{f_0} \right),$$
$$\bar{\zeta}_F = \frac{1}{1728\pi^2} \left( 249 - 41 \frac{\partial_t f_0}{f_0} \right)$$

solution :

$$V = \frac{\bar{\zeta}_V}{4} k^4 + \bar{V},$$

$$F = \xi \chi^2 + \frac{\bar{\zeta}_F}{2} k^2 + \bar{F}$$

$$\Gamma = \int d^4x \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} (\xi \chi^2 + \bar{F}) R + \bar{V} \right)$$

Cosmology with dynamical dark energy !

Cosmological constant vanishes asymptotically !