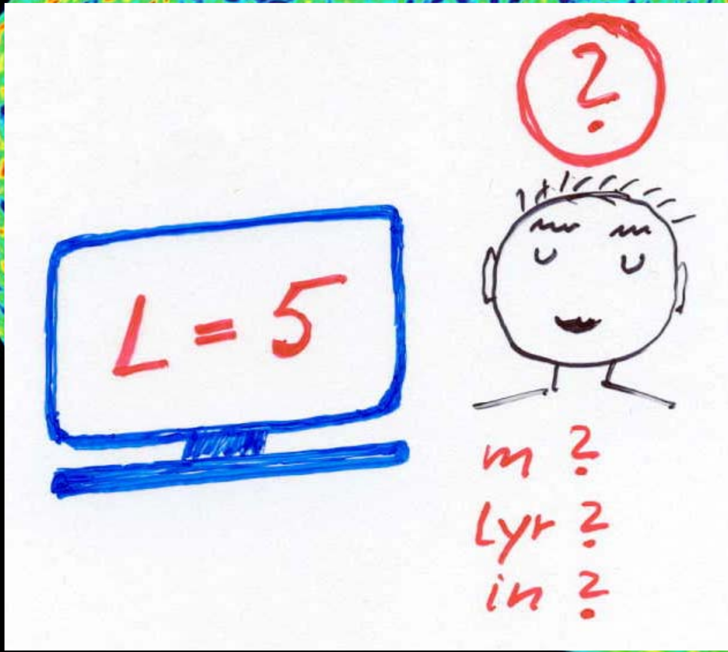
Scale symmetry in quantum gravity

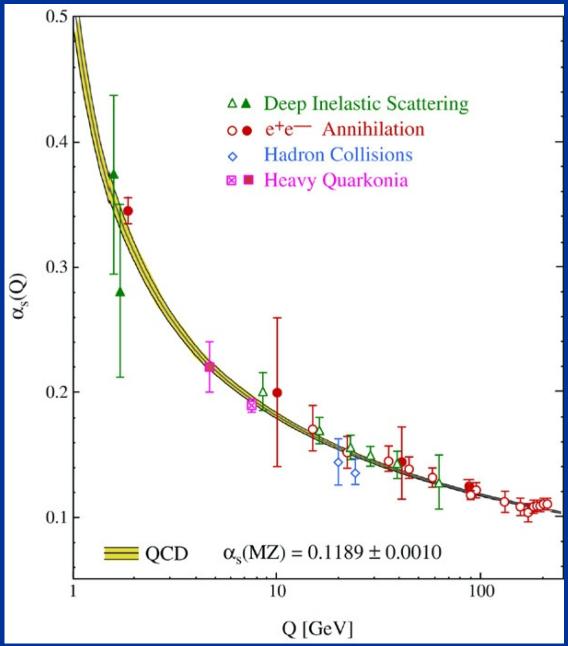


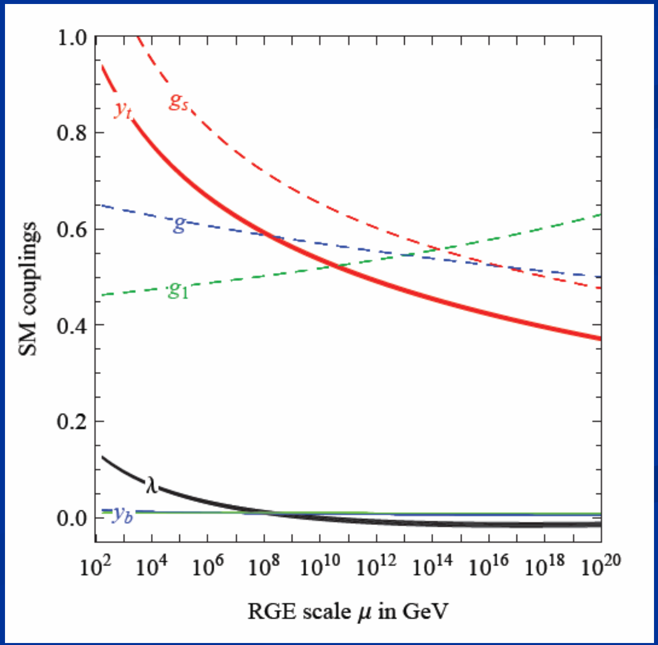
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu \chi \partial_\mu \chi \right\}$$

quantum gravity with scalar field – the role of scale symmetry

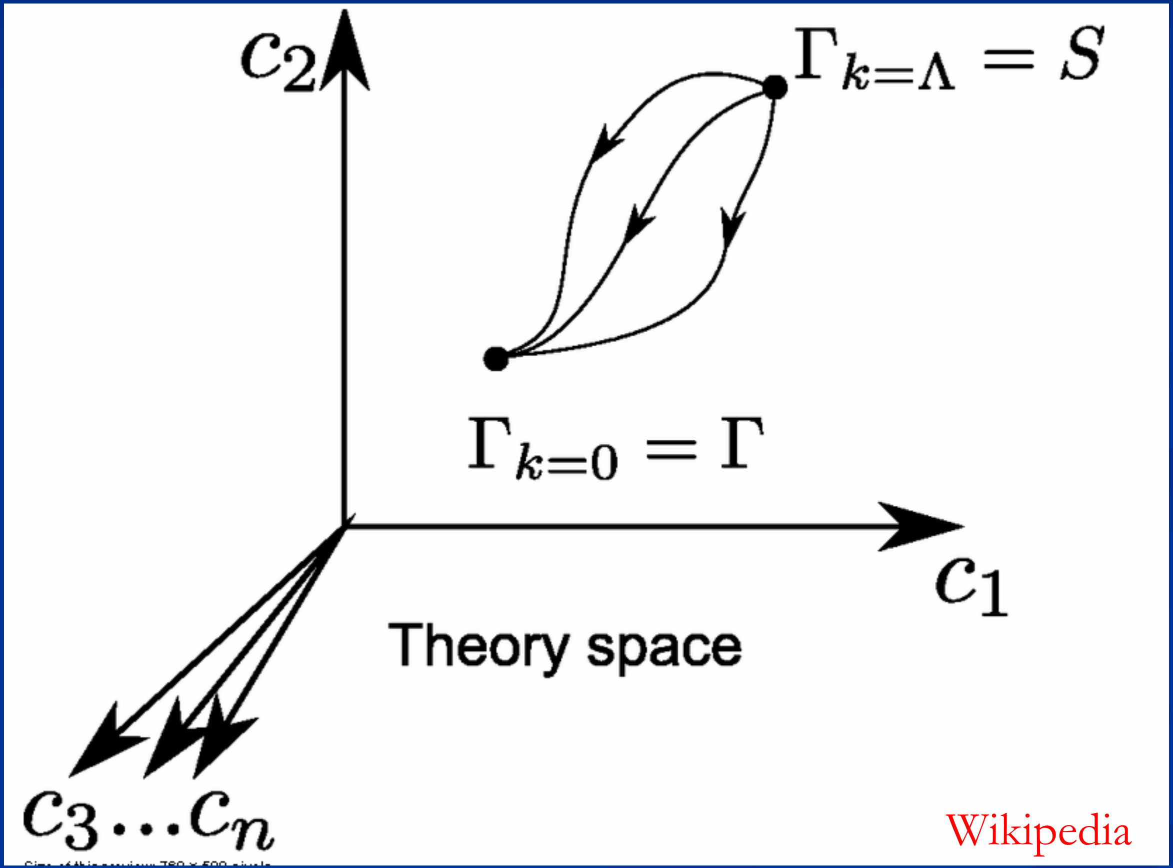
fluctuations induce running couplings

- violation of scale symmetry
- well known in QCD or standard model





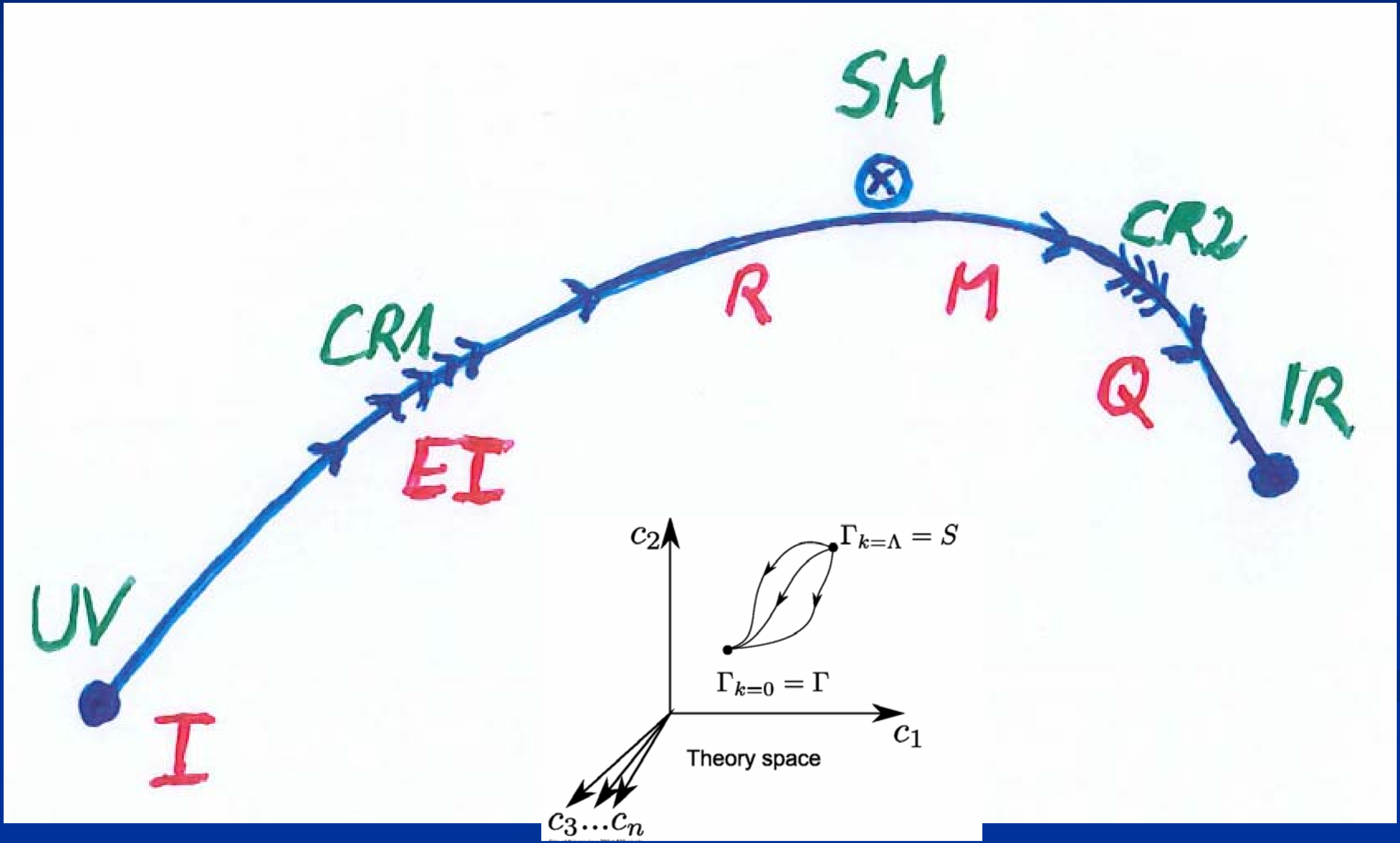
functional renormalization : flowing action



Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

Crossover in quantum gravity



Origin of mass

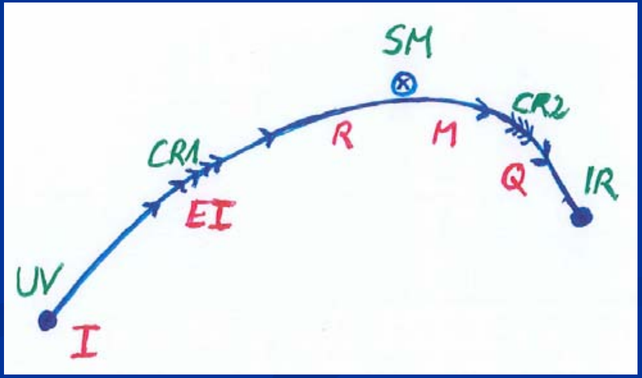
- UV fixed point : scale symmetry unbroken

  all particles are massless

- IR fixed point :

  scale symmetry spontaneously broken,

  massive particles , massless dilaton



- crossover : explicit mass scale μ important

- approximate SM fixed point : approximate scale symmetry spontaneously broken, massive particles , almost massless cosmon, tiny cosmon potential

Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Asymptotic safety

if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

IR fixed point in quantum gravity

Dilaton Quantum Gravity

T. Henz, J. M. Pawlowski, A. Rodigast, and C. Wetterich

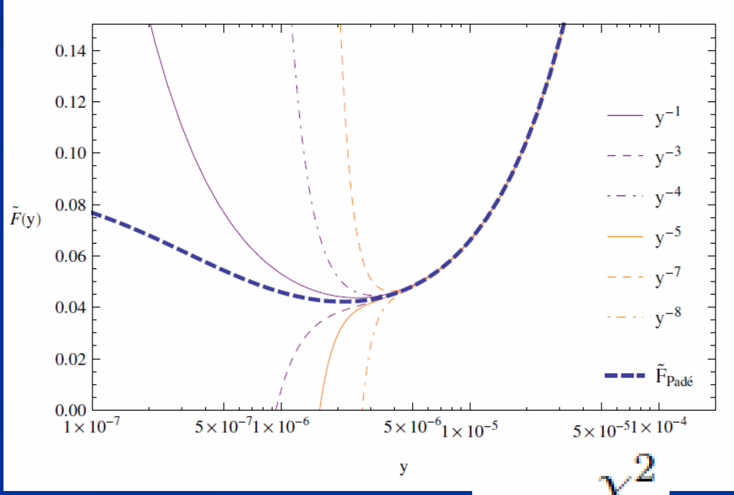
First positive indication from functional renormalization flow with truncation :

$$\Gamma_k = \int d^4x \sqrt{g} \left(V_k(\chi^2) - \frac{1}{2} F_k(\chi^2)\, R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

fixed point effective action :

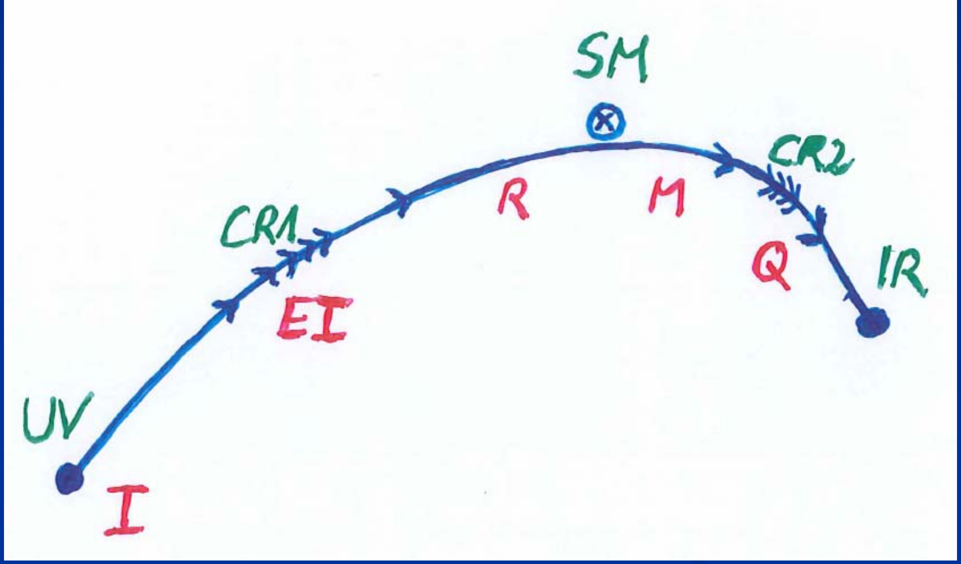
$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi\, \partial_\nu \chi - \frac{1}{2} \xi \chi^2\, R \right)$$

large field behavior of F



$$y = \frac{\chi^2}{k^2}$$

Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy with single scalar field

Variable Gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\right.$ M^2 R $\left.\right\}$

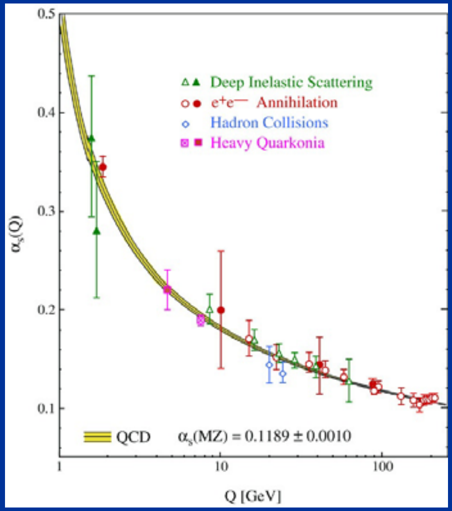
Variable Gravity

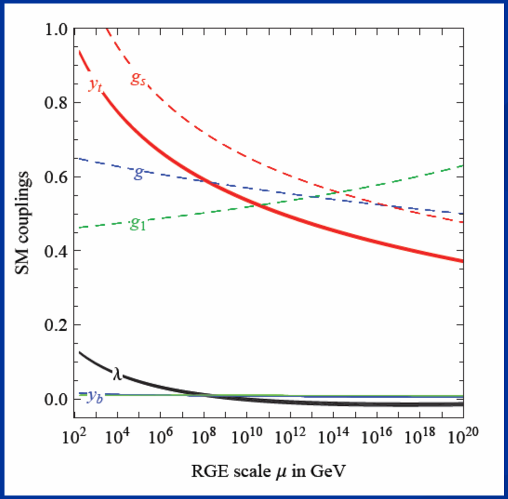
- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

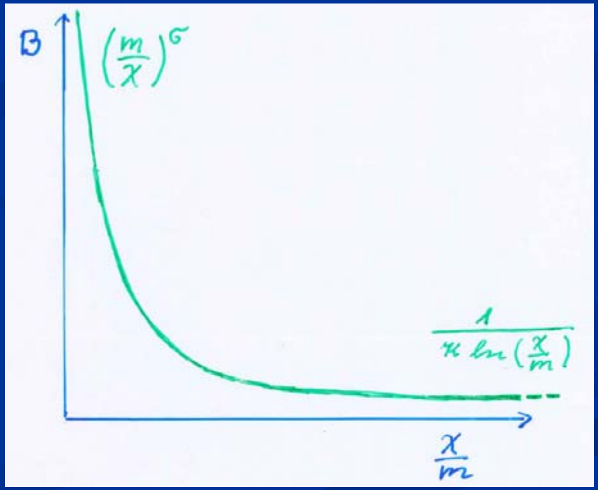
$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi\right\}$$

running coupling B

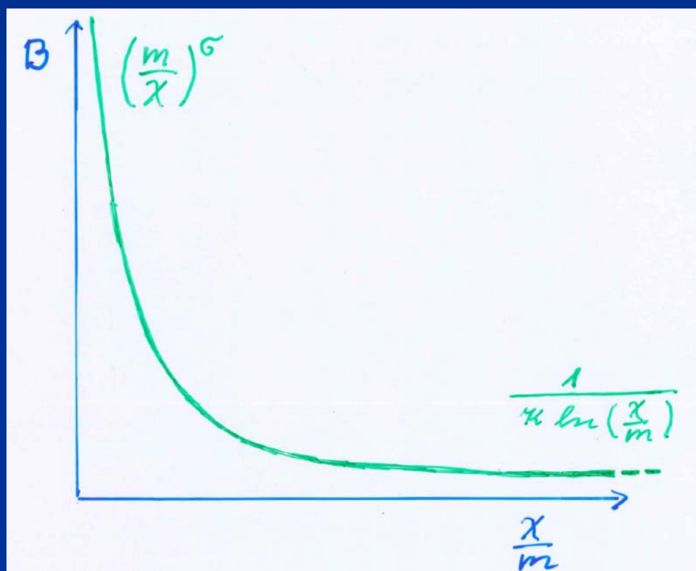
- B varies if intrinsic scale μ changes
- similar to QCD or standard model

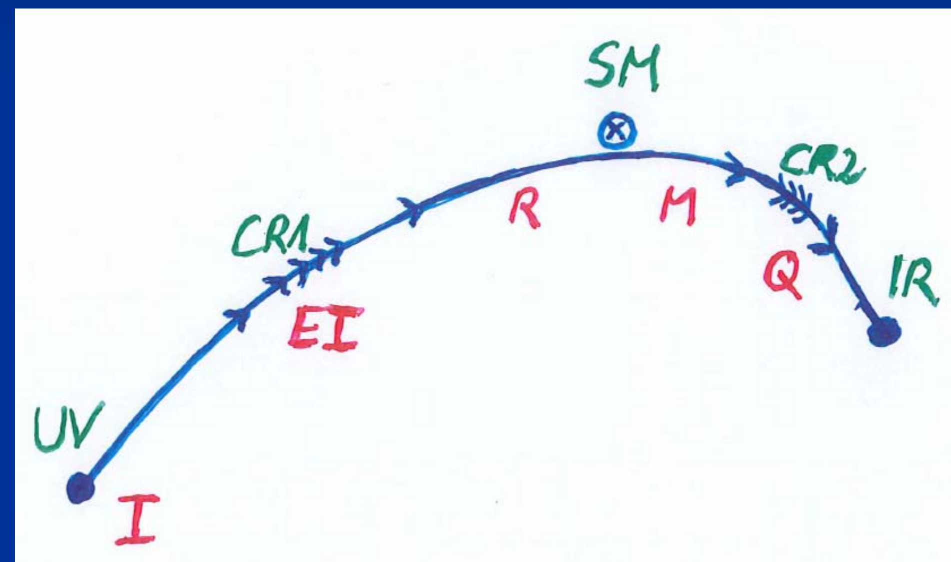




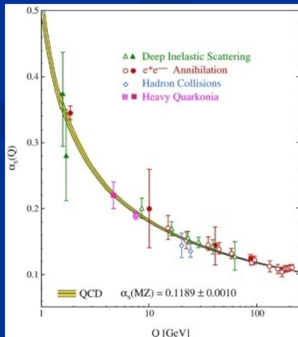


Kinetial B :
Crossover between two fixed points

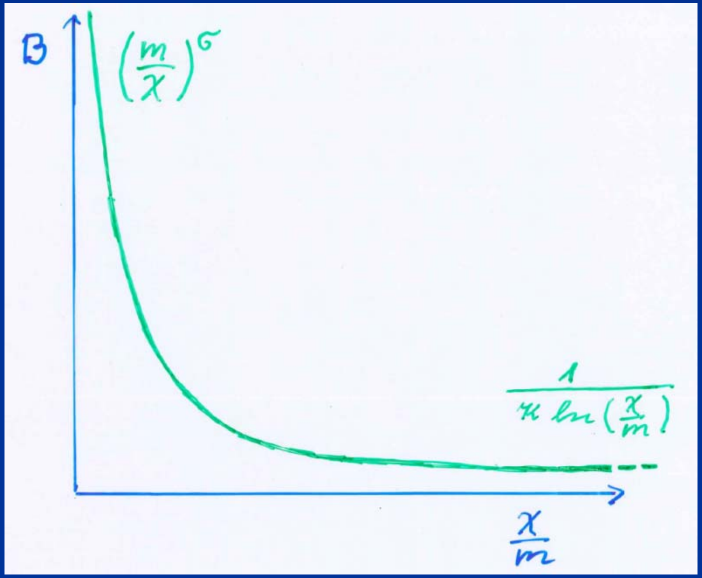




$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$



Kinetial B :
Crossover between two fixed points



running
coupling obeys
flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover
can be exponentially larger than intrinsic scale μ

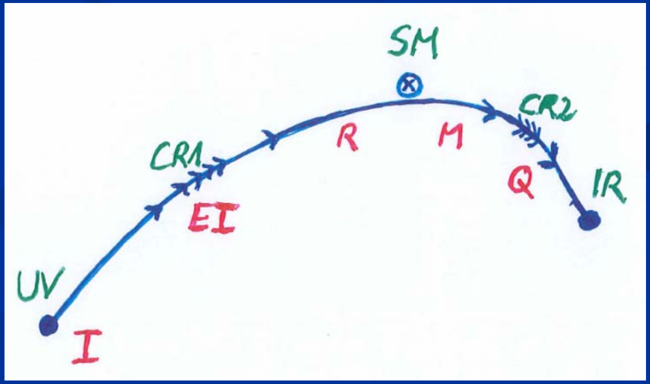
Infrared fixed point

- $\mu \to 0$
- $B \to 0$

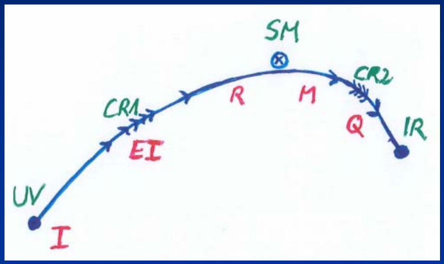
$$\mu\partial_\mu B = \kappa B^2 \qquad \text{for} \qquad B \to 0$$

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

- no intrinsic mass scale
- scale symmetry



Ultraviolet fixed point



- μ→ ∞

- kinetial diverges

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

- scale symmetry with anomalous dimension σ

$$g_{\mu\nu} \to \alpha^2 g_{\mu\nu}\ ,\ \chi \to \alpha^{-\frac{2}{2-\sigma}}\chi$$

Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$1 < \sigma$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left(\mu^2 - \frac{R}{2} \right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

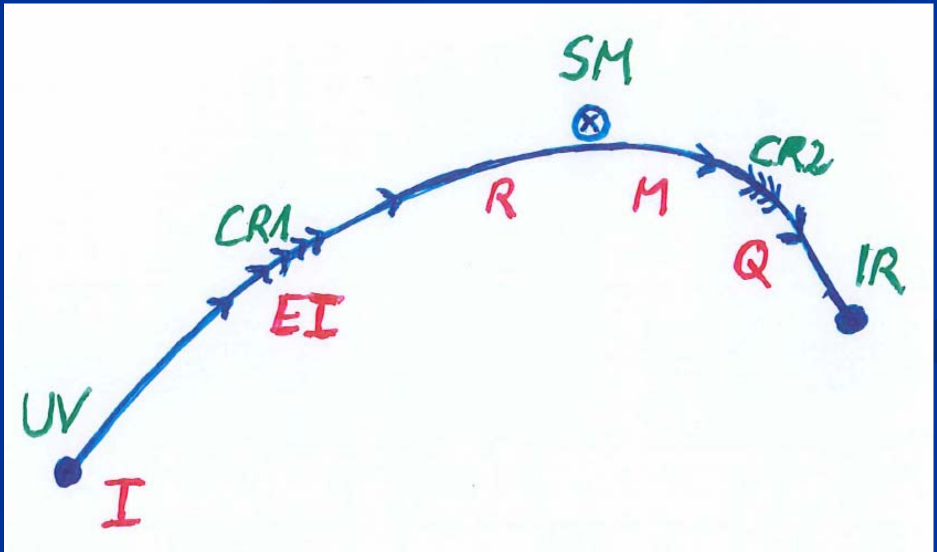
$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

deviation from fixed point vanishes for $\mu \to \infty$

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \to 0$, $\chi \to \infty$
- UV: $\mu \to \infty$, $\chi \to 0$

Cosmology makes crossover between fixed points by variation of χ .



Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

No tiny dimensionless parameters
(except gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33}\ \mathrm{eV}$
- one time scale $\mu^{-1} = 10^{10}\ \mathrm{yr}$
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Particle masses change with time

At SM fixed point :

- All particle masses (except for neutrinos) are proportional to scalar field χ .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

Four-parameter model

- model has four dimensionless parameters
- three in kinetial B :

  $\sigma \sim 2.5$

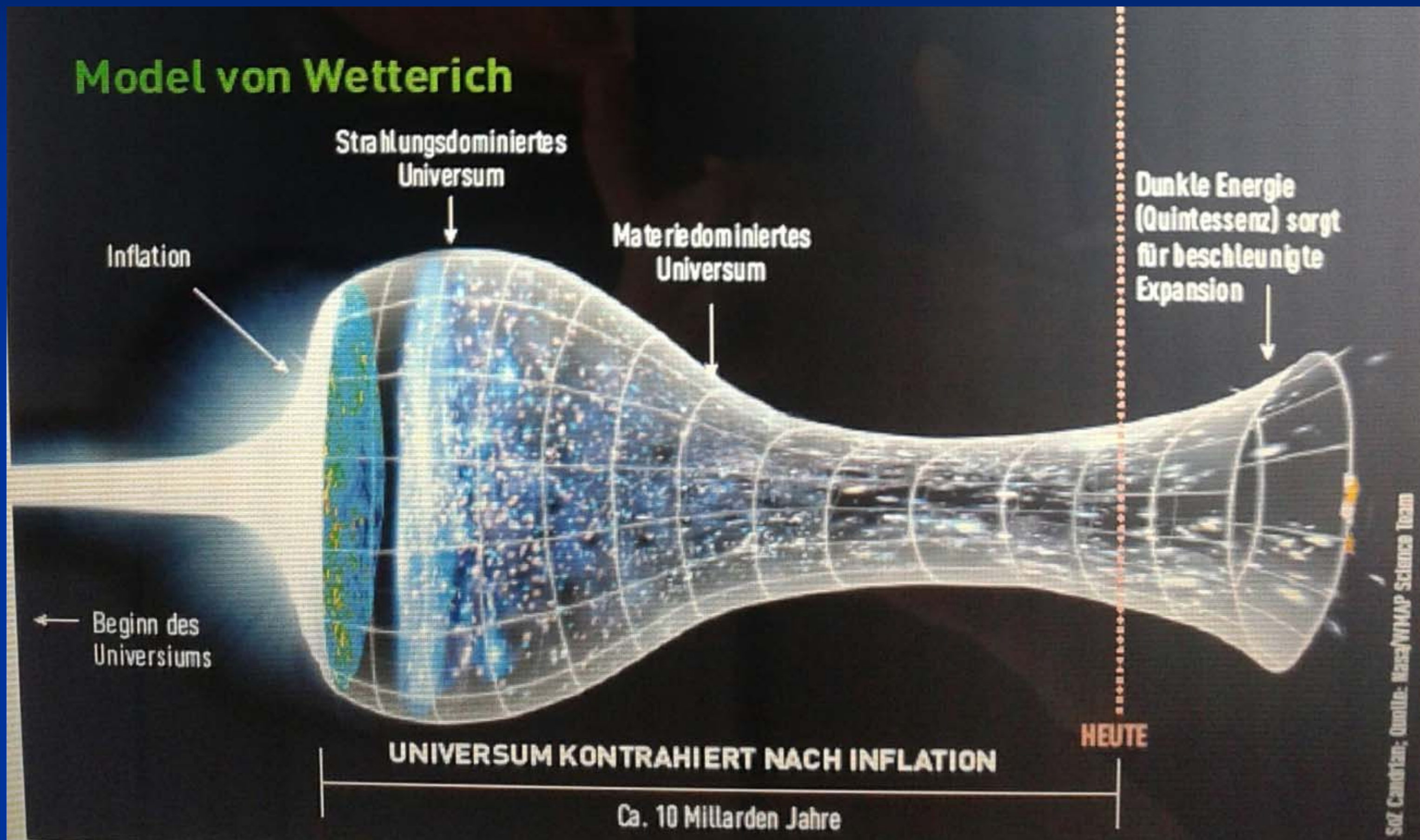
  $\kappa \sim 0.5$

  $c_t \sim 14$ (or m/μ)
- one parameter for present growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than ΛCDM

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Slow Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \;, \; \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33}\ \text{eV}$$

Expansion or shrinking always slow , characteristic time scale of the order of the age of the Universe : $t_{ch} \sim \mu^{-1} \sim$ 10 billion years !

Hubble parameter of the order of present Hubble parameter for all times , including inflation and big bang !

Slow increase of particle masses !

Model is compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations

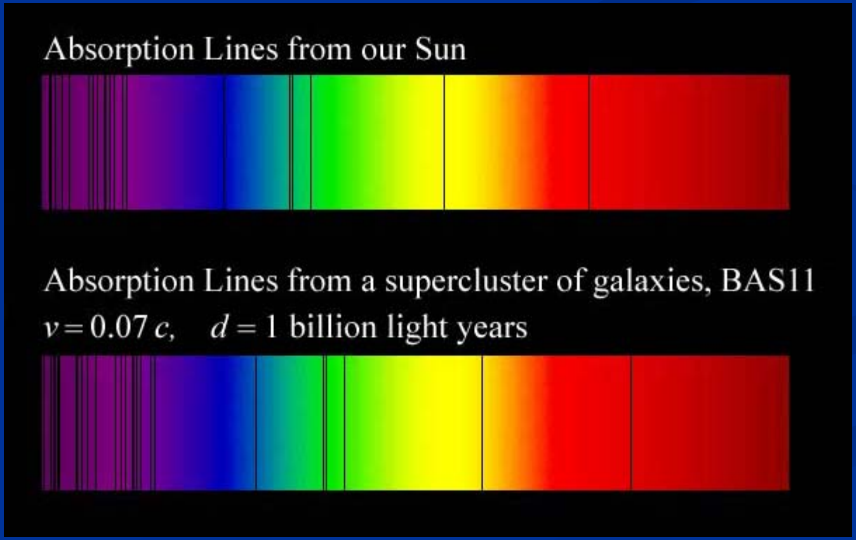
$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa\ln\left(\frac{\chi}{m}\right)$$

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past, because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

Hot plasma ?

- Temperature in radiation dominated Universe : $T \sim \chi^{1/2}$ smaller than today
- Ratio temperature / particle mass : $T / m_p \sim \chi^{-1/2}$ larger than today
- T/m_p counts ! This ratio decreases with time.

- Nucleosynthesis , CMB emission as in standard cosmology !

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

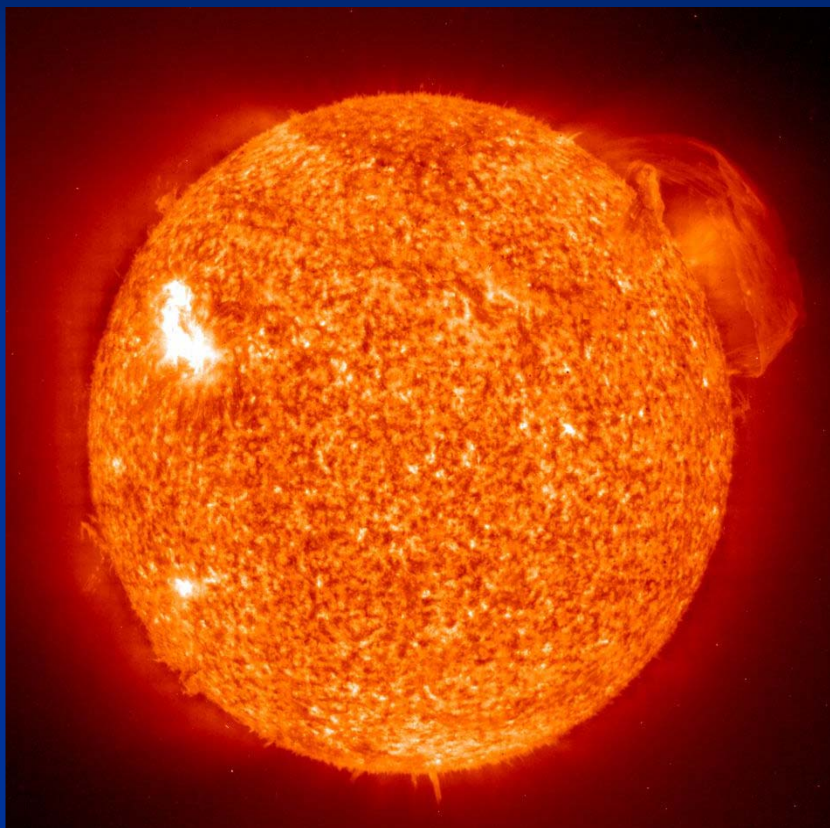
$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Field relativity : different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions ,
  e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Big bang or freeze ?



Big bang or freeze ?

just two ways of looking at same physics

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

small dimensionless number ?

- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V: ratio vanishes asymptotically !

Einstein frame

Weyl scaling :

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$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Infinite past : slow inflation

σ = 2 : field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2}\frac{\dot{\chi}}{\chi}\right)\dot{\chi} = \frac{2\mu^2\chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

solution

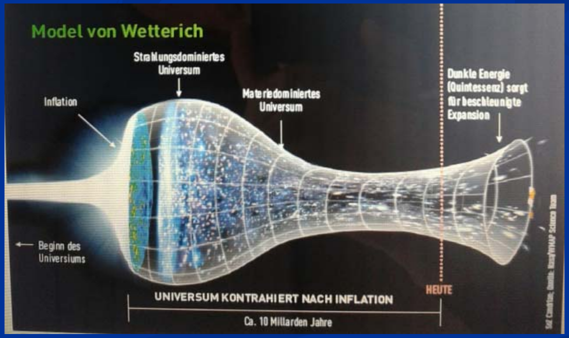
$$H = \frac{\mu}{\sqrt{3}}\ ,\ \ \chi = \frac{3^{\frac{1}{4}}m}{2\sqrt{\mu}}(t_c - t)^{-\frac{1}{2}}$$

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}} \; , \; \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity



- physical time to infinite past is infinite

Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \qquad \left\{\partial_\eta^2 + k^2 + a^2\left(m^2 - \frac{R}{6}\right)\right\}\tilde{\varphi}_k = 0$$

determine physical time by counting number of oscillations

$$\tilde{t}_p = n_k \qquad n_k = \frac{k\eta}{\pi}$$

(m=0)

Big bang singularity in Einstein frame is field singularity !

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

Inflation

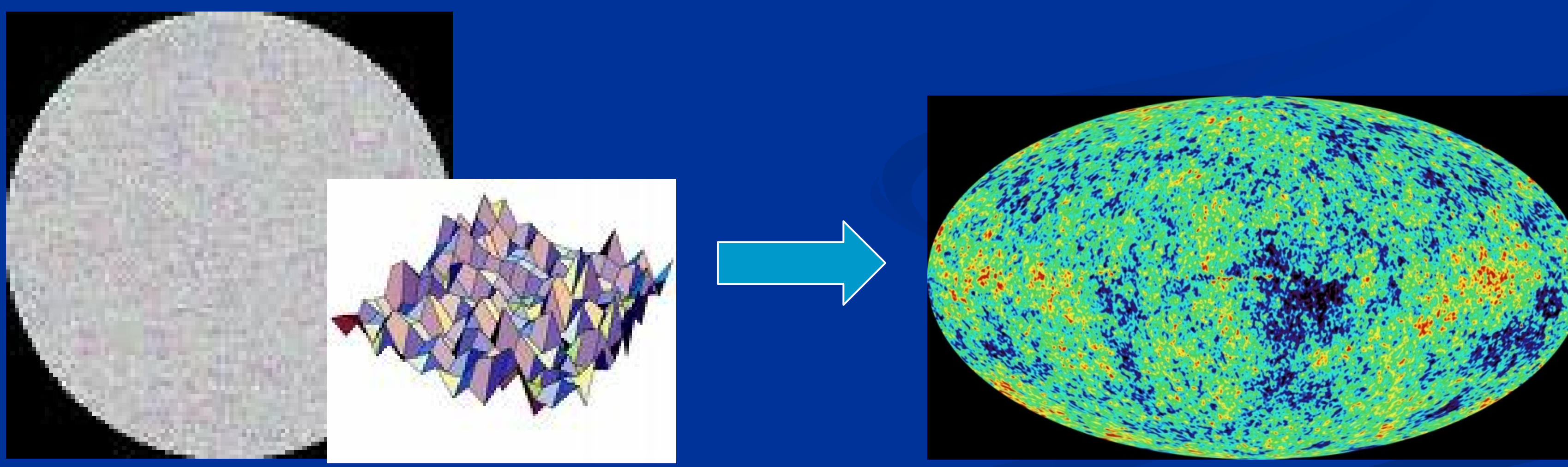
solution for small χ : inflationary epoch

kinetial characterized by anomalous dimension σ

$$B = b\left(\frac{\mu}{\chi}\right)^{\sigma} = \left(\frac{m}{\chi}\right)^{\sigma}$$

Primordial fluctuations

- inflaton field : χ
- primordial fluctuations of inflaton become observable in cosmic microwave background



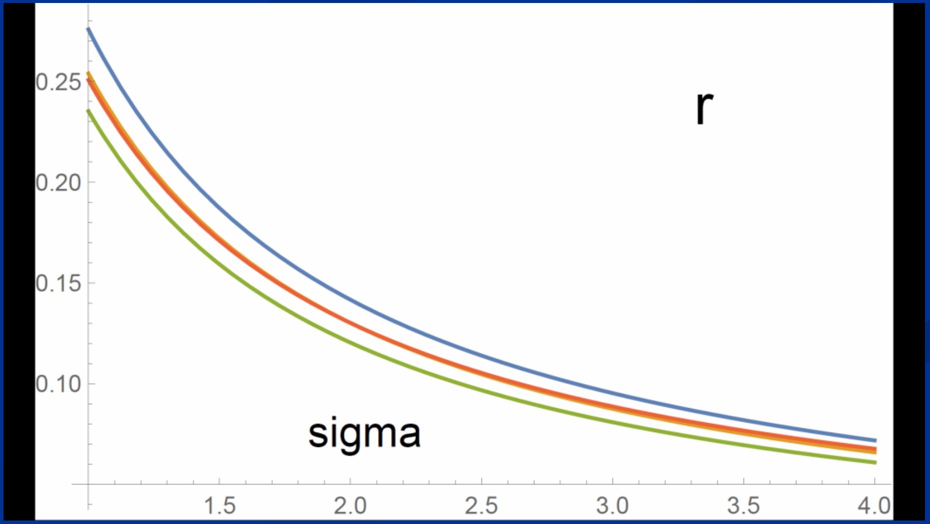
Anomalous dimension determines spectrum of primordial fluctuations

$$r = \frac{0.26}{\sigma}$$

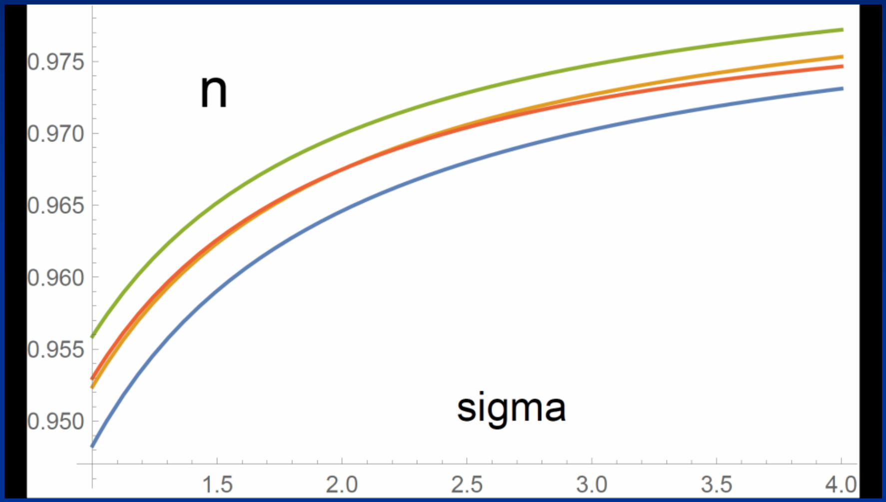
$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$

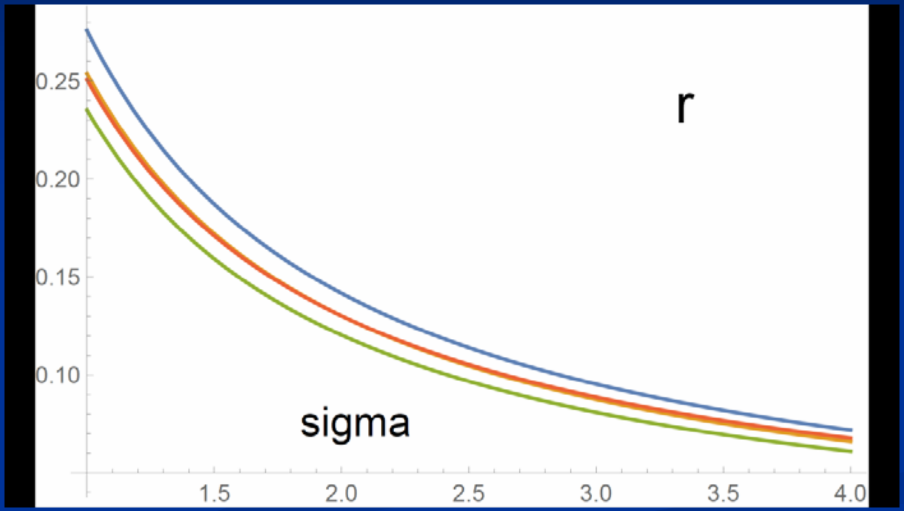
spectral index n

tensor amplitude r



relation between n and r





$$r = 8.19\,(1 - n) - 0.1365$$

Amplitude of density fluctuations

small because of logarithmic running near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln\left(\frac{m}{\mu}\right) = 14.1.$$

σ=1

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60}\right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

N : number of e – foldings at horizon crossing

Cosmon inflation

Unified picture of inflation and dynamical dark energy

Cosmon and inflaton are the same scalar field

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

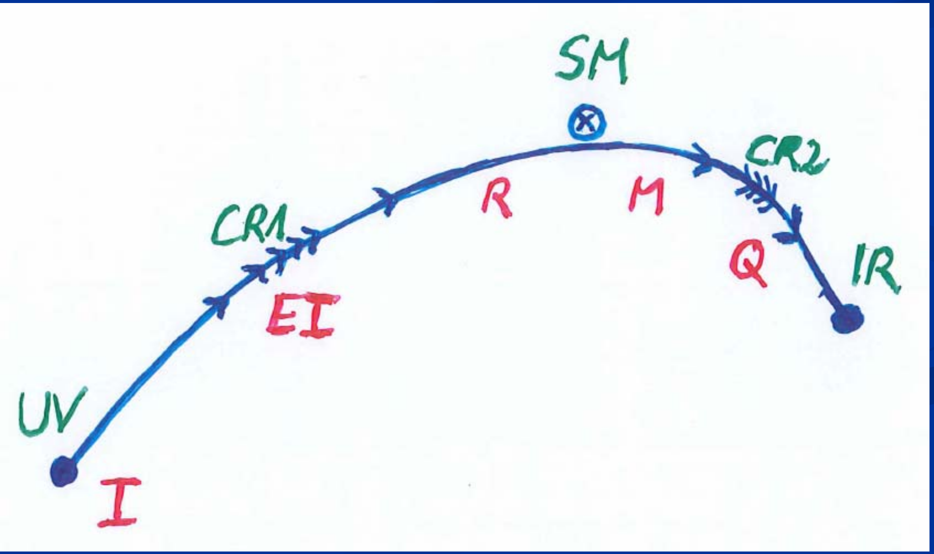
homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications
(different growth of neutrino mass)

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that **ratio neutrino mass over electron mass grows .**

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

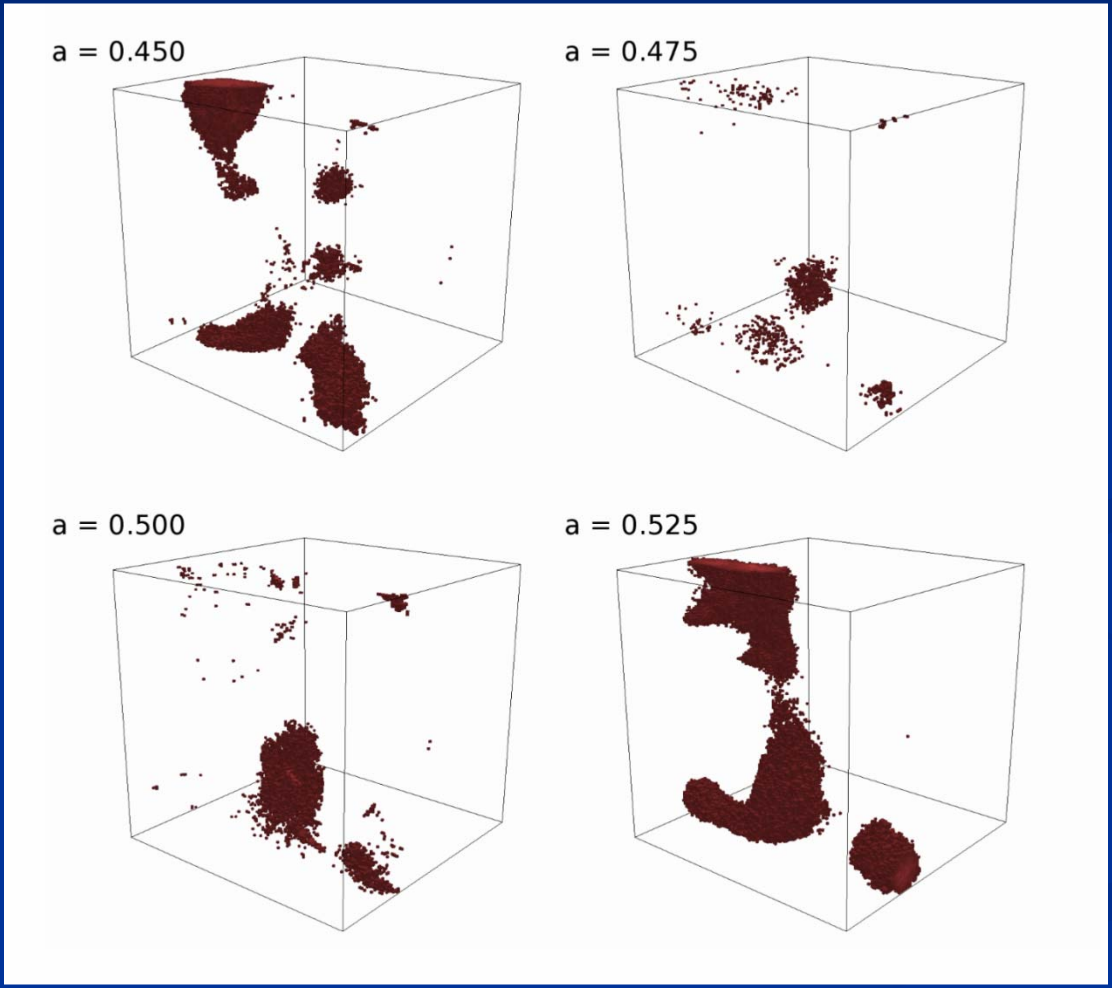
L.Amendola, M.Baldi, …

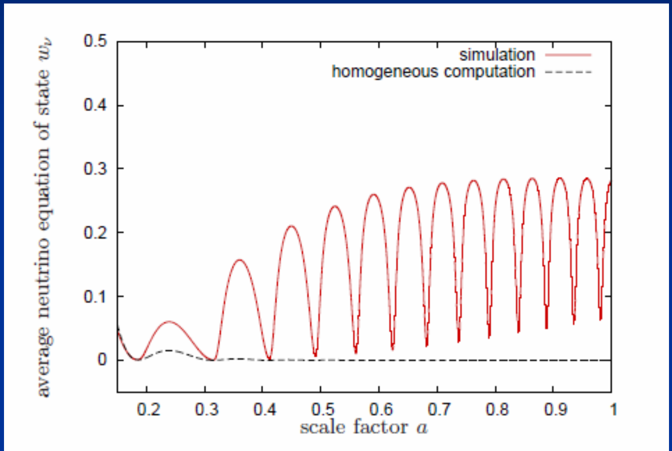
present dark energy density given by neutrino mass

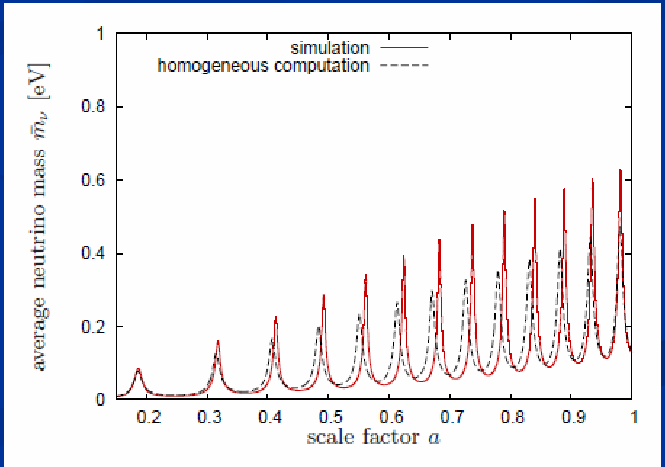
present equation of state given by neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$

Oscillating neutrino lumps



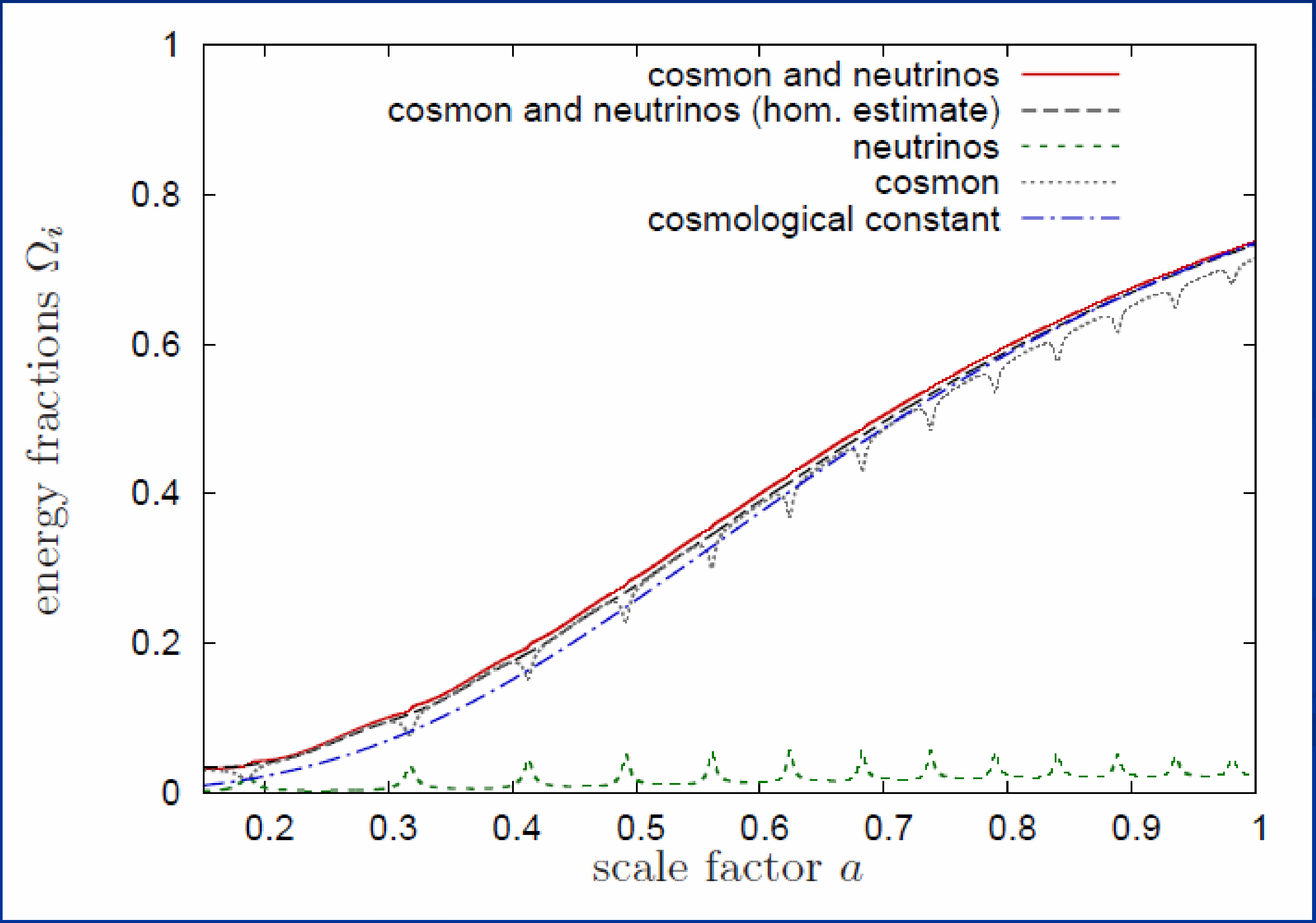




Y.Ayaita, M.Weber,…

Ayaita, Baldi, Fuehrer, Puchwein,…

Evolution of dark energy similar to ΛCDM



Compatibility with observations and possible tests

- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as ΛCDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

conclusions

Quantum gravity may be observable in
dynamics of present Universe

Fixed points and scale symmetry crucial

Big bang singularity is artefact
of inappropriate choice of field variables –
no physical singularity

conclusions (2)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than ΛCDM : tests possible

conclusions (3)

- Variable gravity cosmologies can give a simple and realistic description of Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Cosmon dependence of ratio neutrino mass/ electron mass can explain why Universe makes a transition to Dark Energy domination now
- characteristic signal : neutrino lumps

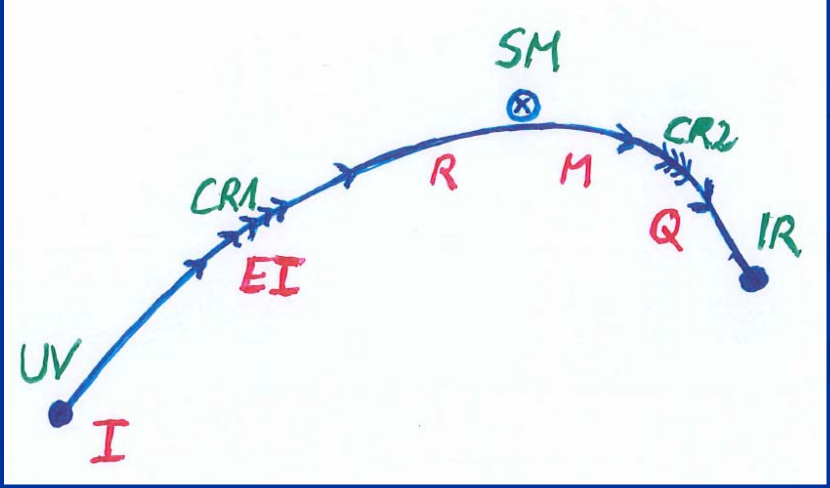
end

First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t \right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

- after crossover B changes only very slowly



Scaling solutions near SM fixed point

(approximation for constant B)

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Different scaling solutions for radiation domination and matter domination

Radiation domination

$$c = \frac{2}{\sqrt{K+6}}$$

$$b = -\frac{c}{2}.$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2.$$

$$\bar{\rho}_r = -3\frac{K+5}{K+6}.$$

K = B - 6

solution exists for B < 1 or K< -5

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t).$$

Varying particle masses near SM fixed point

- All particle masses are proportional to χ .
  (scale symmetry)
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.

Scaling of particle masses

mass of electron or nucleon is proportional to variable Planck mass χ !

effective potential for Higgs doublet h

$$\tilde{V}_h = \frac{1}{2}\lambda_h(\tilde{h}^\dagger\tilde{h} - \epsilon_h\chi^2)^2$$

cosmon coupling to matter

$$K(\ddot{\chi} + 3H\dot{\chi}) + \frac{1}{2}\frac{\partial K}{\partial \chi}\dot{\chi}^2 = -\frac{\partial V}{\partial \chi} + \frac{1}{2}\frac{\partial F}{\partial \chi}R + q_\chi$$

$$q_\chi = -(\rho - 3p)/\chi$$

$$F = \chi^2$$

Matter domination

$$c = \sqrt{\frac{2}{K+6}},$$

$$b = -\frac{1}{3}\sqrt{\frac{2}{K+6}} = -\frac{1}{3}c,$$

Universe shrinks !

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2,$$

$$\bar{\rho}_m = -\frac{2(3K+14)}{3(K+6)}$$

solution exists for

B < 4/3 , $K < -14/3$

K = B - 6

Early Dark Energy

Energy density in radiation increases , proportional to cosmon potential

$$T_{00} = \rho = \bar{\rho}\mu^2\chi^2$$

$$V(\chi) = \mu^2\chi^2$$

fraction in early dark energy $\Omega_h = \dfrac{\rho_h}{\rho_r + \rho_h} = \dfrac{nB(\chi)}{4}$

or m

observation requires B < 0.02 (at CMB emission)

Dark Energy domination

neutrino masses scale
differently from electron mass

$$\frac{\partial \ln m_\nu}{\partial \ln \chi}\Big|_{\text{today}} = 2\tilde{\gamma} + 1$$

$$m_\nu = \bar{c}_\nu \chi^{2\tilde{\gamma}+1}$$

$$\chi q_\chi = -(2\tilde{\gamma} + 1)(\rho_\nu - 3p_\nu)$$

new scaling solution. not yet reached.
at present : transition period

$$\frac{\rho_\nu}{\chi^2} = \bar{\rho}_\nu \mu^2$$

$$b = \frac{1}{3}(2\tilde{\gamma} - 1)c.$$

Dilaton quantum gravity

Dilaton Quantum Gravity

T. Henz, J. M. Pawlowski, A. Rodigast, and C. Wetterich

Functional renormalization flow,
with truncation :

$$\Gamma_k = \int d^4x \sqrt{g} \left(V_k(\chi^2) - \frac{1}{2} F_k(\chi^2)\, R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

Functional renormalization equations

$$V_k = k^4\, y^2\, v_k(y),\ \ F_k = k^2\, y\, f_k(y)$$

$$y = \frac{\chi^2}{k^2}$$

$$\begin{aligned}\partial_t v_k(y) &= 2\, y\, v_k'(y) + \frac{1}{y^2}\,\zeta_V\\ \partial_t f_k(y) &= 2\, y\, f_k'(y) + \frac{1}{y}\,\zeta_F\,.\end{aligned}$$

$$\begin{aligned}\zeta_V &= \frac{1}{192\pi^2}\Bigg\{6 + \frac{30\,\tilde V}{\Sigma_0} + \frac{3(2\Sigma_0 + 24\,y\,\tilde F'\,\Sigma_0' + \tilde F\Sigma_1)}{\Delta}\\ &\quad +\delta_V\Bigg\},\end{aligned}$$

$$\begin{aligned}\zeta_F &= \frac{1}{1152\pi^2}\Bigg\{150 + \frac{30\,\tilde F\,(3\,\tilde F - 2\tilde V)}{\Sigma_0^2} \qquad (10)\\ &\quad -\frac{12}{\Delta}\Big(24\,y\,\tilde F'\,\Sigma_0' + 2\Sigma_0 + \tilde F\Sigma_1\Big) - 6y\,(3\,\tilde F'^2 + 2\Sigma_0'^2)\\ &\quad -\frac{36}{\Delta^2}\Big[2y\,\Sigma_0\,\Sigma_0'\,(7\,\tilde F' - 2\tilde V')\,(\Sigma_1 - 1) + 2\,\Sigma_0^2\,\Sigma_2\\ &\quad +2\,y\Sigma_1\,(7\,\tilde F' - 2\tilde V')\,(2\,\Sigma_0\,\tilde V' - \tilde V\,\Sigma_0')\\ &\quad +24\,y\,\tilde F'\,\Sigma_0\,\Sigma_0'\,\Sigma_2 - 12\,y\,\tilde F\,\Sigma_0'^2\,\Sigma_2\Big] + \delta_F\Bigg\}.\end{aligned}$$

$$\begin{aligned}\tilde V = y^2\, v_k(y)\ ,\ \ \tilde F &= y\, f_k(y),\\ \Sigma_0 = \frac{1}{2}\tilde F - \tilde V\ ,\ \ \Delta &= (12\,y\,\Sigma_0'^2 + \Sigma_0\,\Sigma_1)\\ \Sigma_1 = 1 + 2\,\tilde V' + 4\,y\,\tilde V''\ ,\ \ \Sigma_2 &= \tilde F' + 2\,y\,\tilde F''.\end{aligned}$$

Percacci, Narain

Fixed point for large scalar field

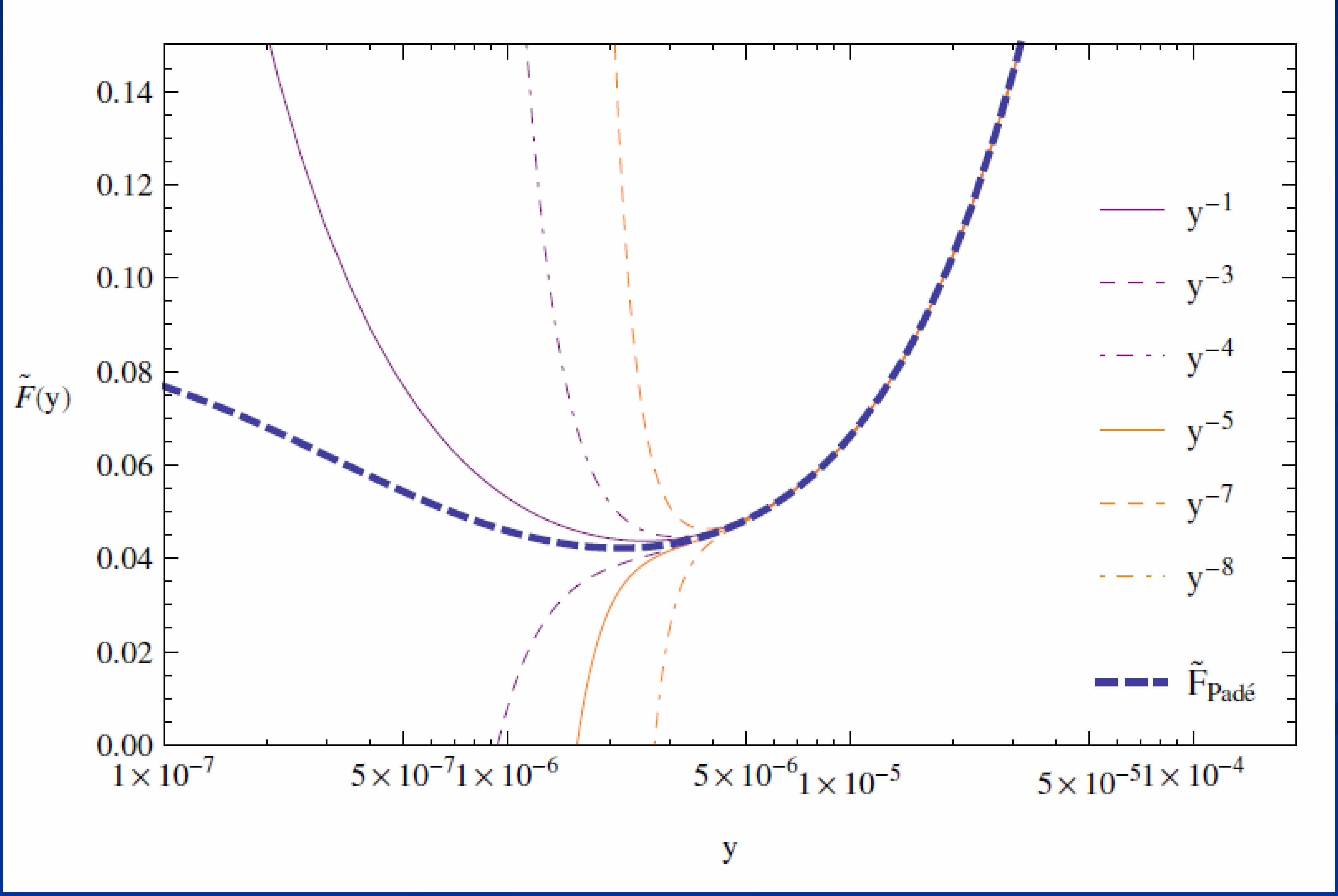
$$\lim_{y \to \infty} f(y) = \xi$$

$$\lim_{y \to \infty} v(y) = 0$$

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - \frac{1}{2} \xi \chi^2 R \right)$$

This fixed point describes already realistic gravity !
Limit k → 0 can be taken !

Fixed point for large scalar field



Vicinity of fixed point

$$\partial_t V = \bar{\zeta}_V k^4 \ , \ \partial_t F = \bar{\zeta}_F k^2$$

$$\bar{\zeta}_V = -\frac{1}{48\pi^2}\left(6 - \frac{\partial_t f_0}{f_0}\right),$$
$$\bar{\zeta}_F = \frac{1}{1728\pi^2}\left(249 - 41\frac{\partial_t f_0}{f_0}\right)$$

solution :

$$V = \frac{\bar{\zeta}_V}{4}k^4 + \bar{V},$$
$$F = \xi\chi^2 + \frac{\bar{\zeta}_F}{2}k^2 + \bar{F}$$

$$\Gamma = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - \frac{1}{2}(\xi\chi^2 + \bar{F})\,R + \bar{V}\right)$$

Cosmology with dynamical dark energy !
Cosmological constant vanishes asymptotically !