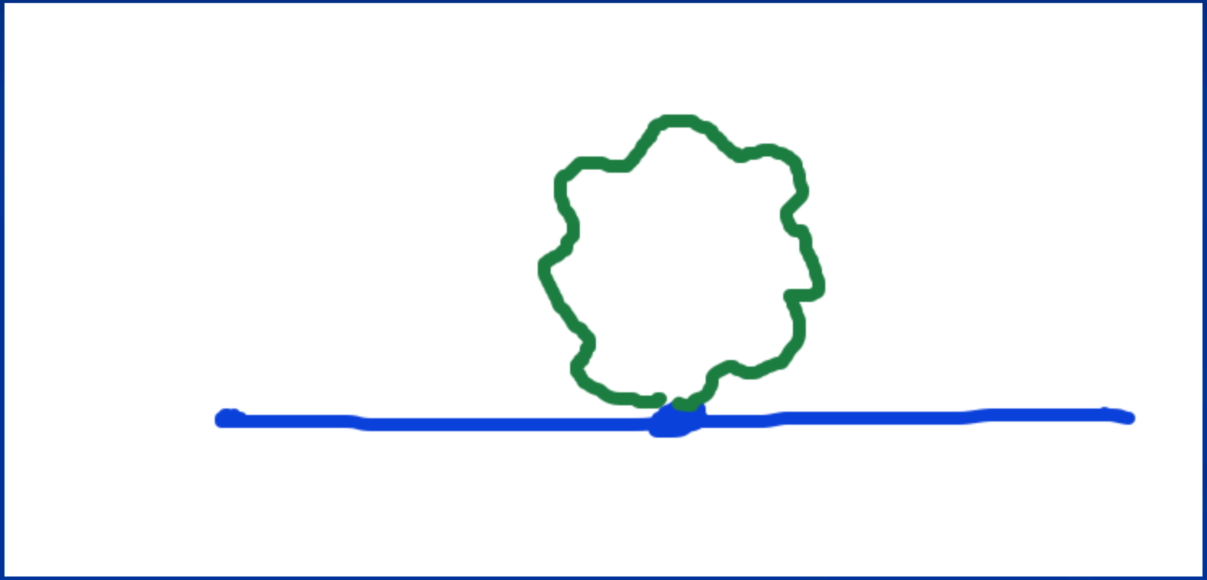
Quantum gravity prediction for dynamical dark energy

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral

Metric fluctuations matter



Quantum gravity needs method to take them into account

Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors
- Difference: Einstein gravity is not perturbatively renormalizable
- no small dimensionless coupling constant, effective coupling q^2/M^2

Quantum gravity

Quantum gravity is

non-perturbatively renormalizable

Asymptotic safety : non-perturbative renormalizabilty

Weinberg, Reuter, …

Use functional renormalization !

Flowing couplings

Couplings change with renormalization scale k due to (quantum) fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included for (quantum) effective action.

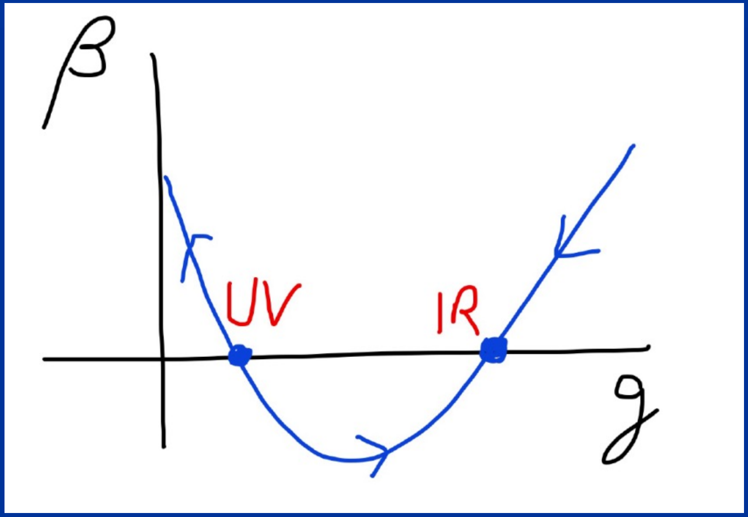
Flow of k to zero : all fluctuations included, IR-limit
Flow of k to infinity : UV-limit

k-dependence can differ from momentum dependence

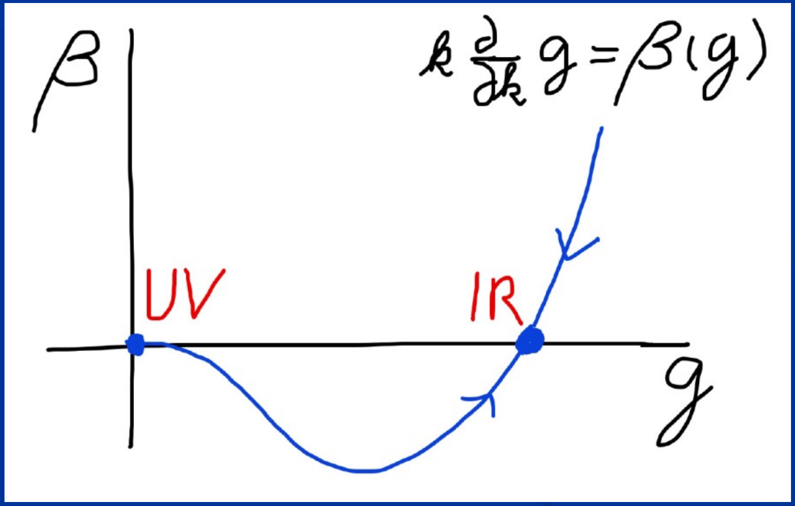
Ultraviolet fixed point

- Flow of couplings stops for k $\rightarrow\infty$
- Self-similarity: no explicit dependence on k
- Theory can be extrapolated to arbitrary short distances
- Completeness
- Renormalizability

Asymptotic safety



Asymptotic freedom



Asymptotically safe gravity

Ultraviolet fixed point exists for

quantum field theory for metric (or vierbein).

Predictivity

- asymptotic safety is predictive
- few relevant parameters govern flow away from UV-fixed point
- translate to renormalizable couplings in standard model or extensions

Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

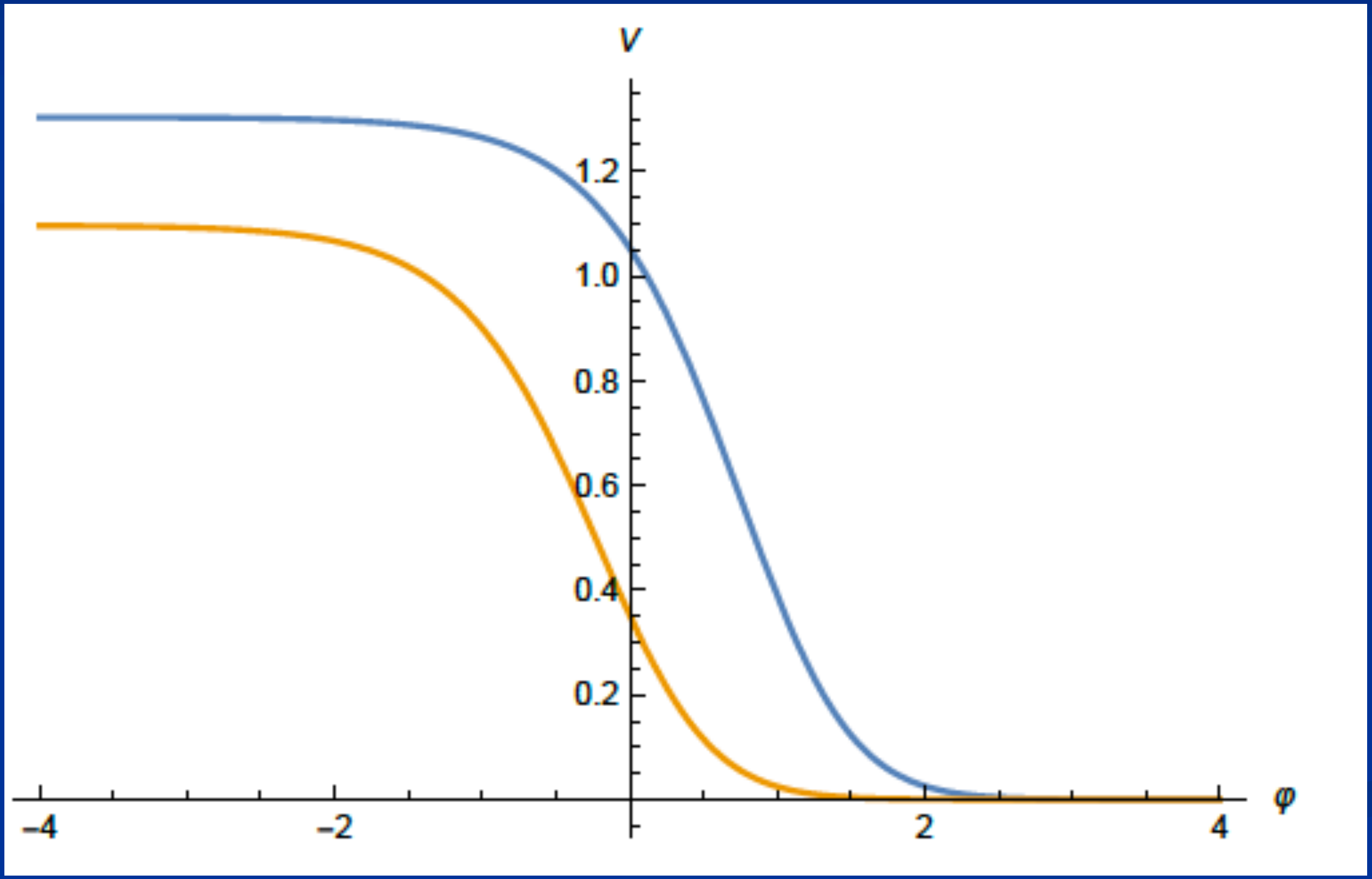
Metric + scalar field

- Inflation :
  add scalar field (inflaton)

- Dynamical dark energy or quintessence:
  add scalar field (cosmon)

Can the scalar potential be predicted by functional renormalization for quantum gravity ?

Quantum gravity prediction for generic form of potential for singlet scalar



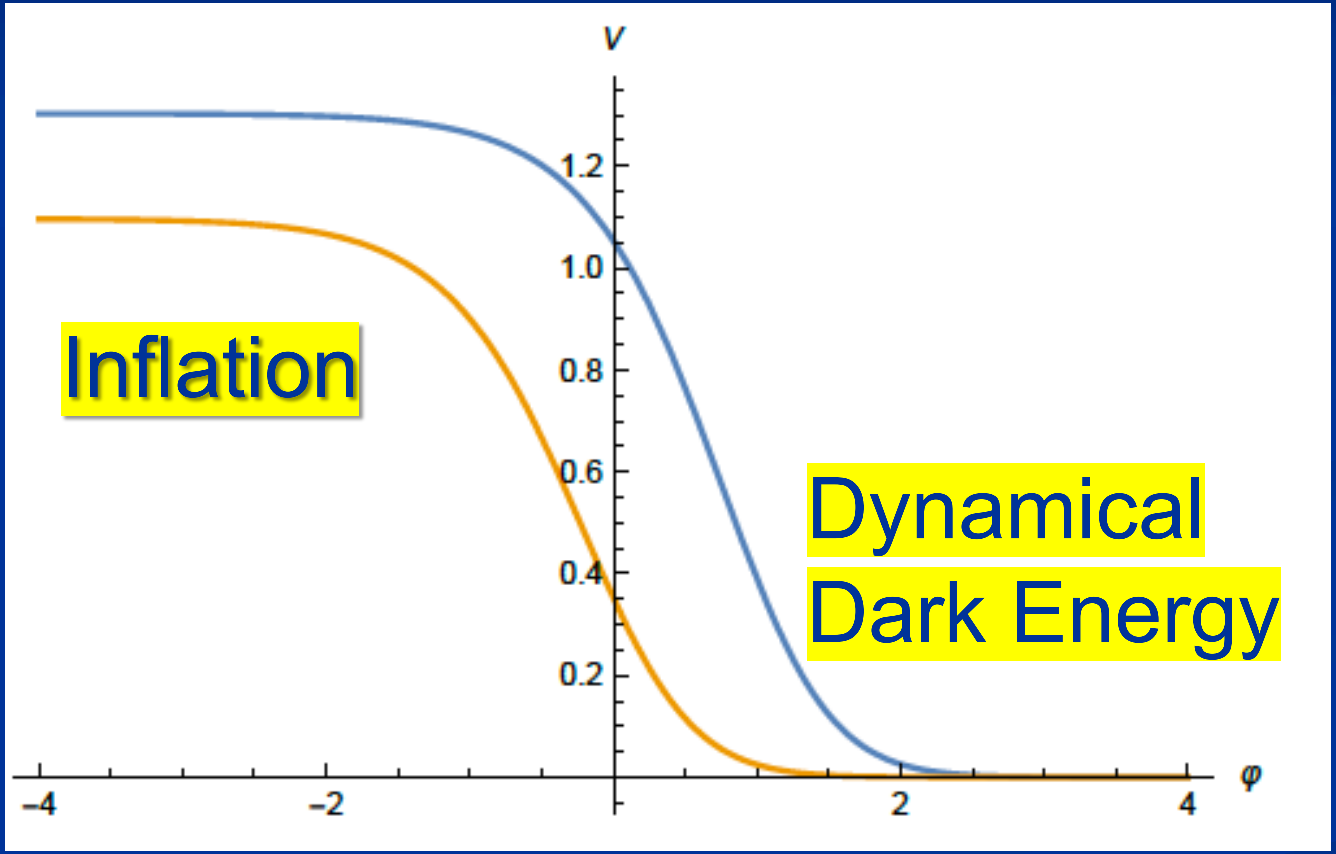
prediction of (approximate) quantum scale symmetry:

dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich, Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles, B.Ratra, ApJ.Lett.325(1988)L17, 20.10.87

Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini,
Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

Scaling solution

Scaling solutions

- At fixed point: all (infinitely many) dimensionless couplings take fixed values
- Full momentum dependence of graviton propagator (no polynomial expansion)
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Scaling solutions are restrictive

- scaling solutions are solutions of non-linear differential equations
- scaling potential needs to extend over whole range of scalar field
- predictivity !

in presence of gravitational fluctuations:

scalar effective potential no longer approximated by polynomial

Relevant parameters and relevant functions

- flow away from scaling solution is dominated by relevant functions
- these should not be singular for finite values of scalar field, or diverge exponentially for increasing scalar field

- typically only a few, associated to relevant parameters
- Predictivity !

Dominant relevant mass scale

- Flow away from fixed point induces intrinsic mass scales by dimensional transmutation
- Intrinsic violation of quantum scale symmetry
- Can be in different steps (example Planck mass, QCD scale)
- Dominant relevant mass scale is the largest intrinsic scale

Field dependent mass scales

- Scenario: dominant relevant mass scale much below electron mass (say 10^{-3} eV in appropriate units)
- Planck mass and particle masses are given by cosmological (expectation) value of scalar field
- Spontaneous breaking of scale symmetry
- Scaling solution very good approximation for all momentum scales larger than dominant relevant mass scale

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$ $\rightarrow$ $\varphi_0^2 = \epsilon\chi^2$

Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \quad \rightarrow \quad \Lambda_{\text{QCD}} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \qquad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetry CW'87, Shaposhnikov et al

Cosmon

- Spontaneously broken scale symmetry induces a Goldstone boson
- Massless dilaton

- Intrinsic mass scale from dominant relevant mass scale: Pseudo-Goldstone boson acquires a tiny mass
- Cosmon

Naturally very light scalar particle !

quantum gravity prediction
for
scaling potential

Dilaton quantum gravity

functional renormalization for
quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert,
Eichhorn, Pauly, Laporte, Pereira, Saueressig,
Wang, Knorr, …

for low order polynomial expansion of potential :

Percacci, Narain, …

Derivative expansion of effective action

$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

variable gravity

Flow equation for scalar potential

$$u = \frac{U}{k^4}, \quad w = \frac{F}{2k^2}$$

$$v = \frac{2U}{Fk^2} = \frac{u}{w}$$

$$\rho = \chi^2/2$$

$$\tilde{\rho} = \rho/k^2$$

$$\partial_t u = \beta_u = -4u + 2\tilde{\rho}\,\partial_{\tilde{\rho}} u + 4c_{\mathrm{U}}$$

$$c_{\mathrm{U}} = \frac{1}{96\pi^2}\left(\frac{5}{1-v} + \frac{1}{1-v/4}\right) + b_{\mathrm{U}}$$

$$b_{\mathrm{U}} = \frac{N-4}{128\pi^2}$$

$$N = N_{\mathrm{S}} + 2N_{\mathrm{V}} - 2N_F$$

Differential equation for scaling solution

$$2\tilde{\rho}\frac{\partial u}{\partial \tilde{\rho}} = 4u - \frac{1}{24\pi^2}\left(\frac{5}{1-u/w} + \frac{1}{1-u/4w}\right) - 4b_{\mathrm{U}}$$

Differential equation for scaling solution for effective Planck mass

$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

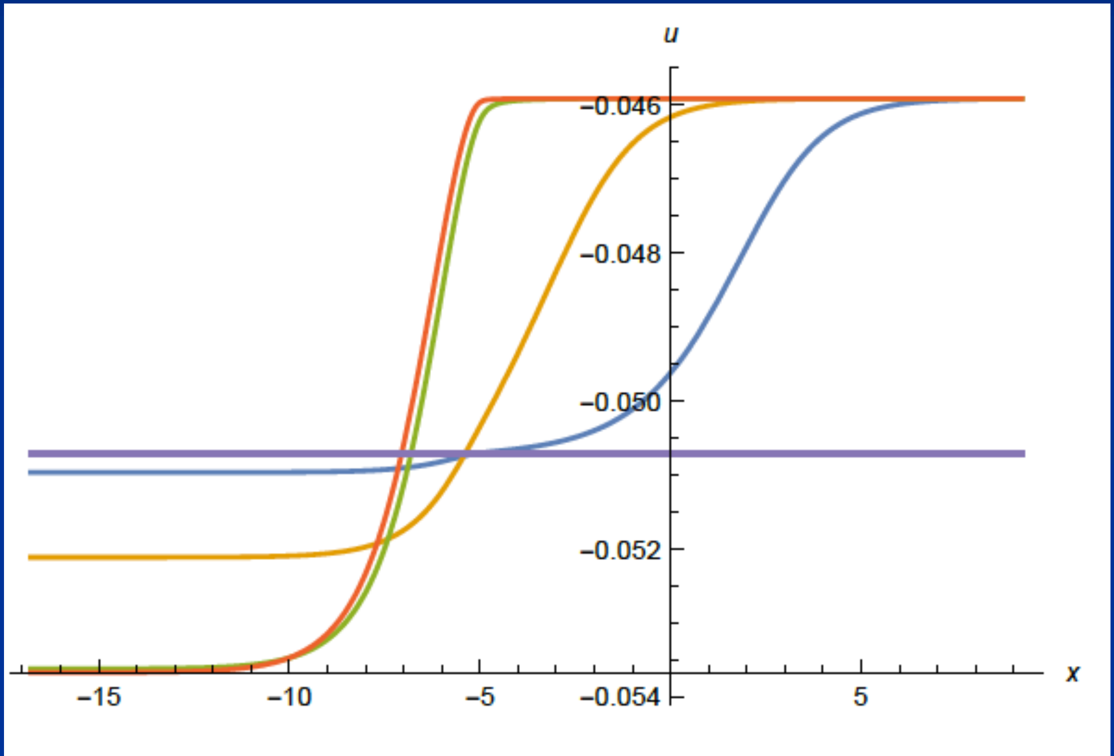
$$u = \frac{U}{k^4}\,, \quad w = \frac{F}{2k^2}$$

$$2\tilde{\rho}\,\partial_{\tilde{\rho}} w = 2\,(w - c_M)$$

$$\begin{aligned} c_M = & \frac{25}{128\pi^2}\Big(1-\frac{u}{w}\Big)^{-1} \\ & + \frac{1}{192\pi^2}\Big[-N_S\Big(\frac{1}{1+u'} + \frac{3w'}{(1+u')^2}\Big) \\ & + 2N_V\Big(\frac{3}{1+g^2\tilde{\rho}} - 1\Big) - \frac{N_F}{1+y^2\tilde{\rho}} + \frac{43}{6}\Big] \end{aligned}$$

M.Yamada, …

Scaling potential for particles of standard model



u : dimensionless scalar potential

u= U/k^4

x : logarithm of scalar field value

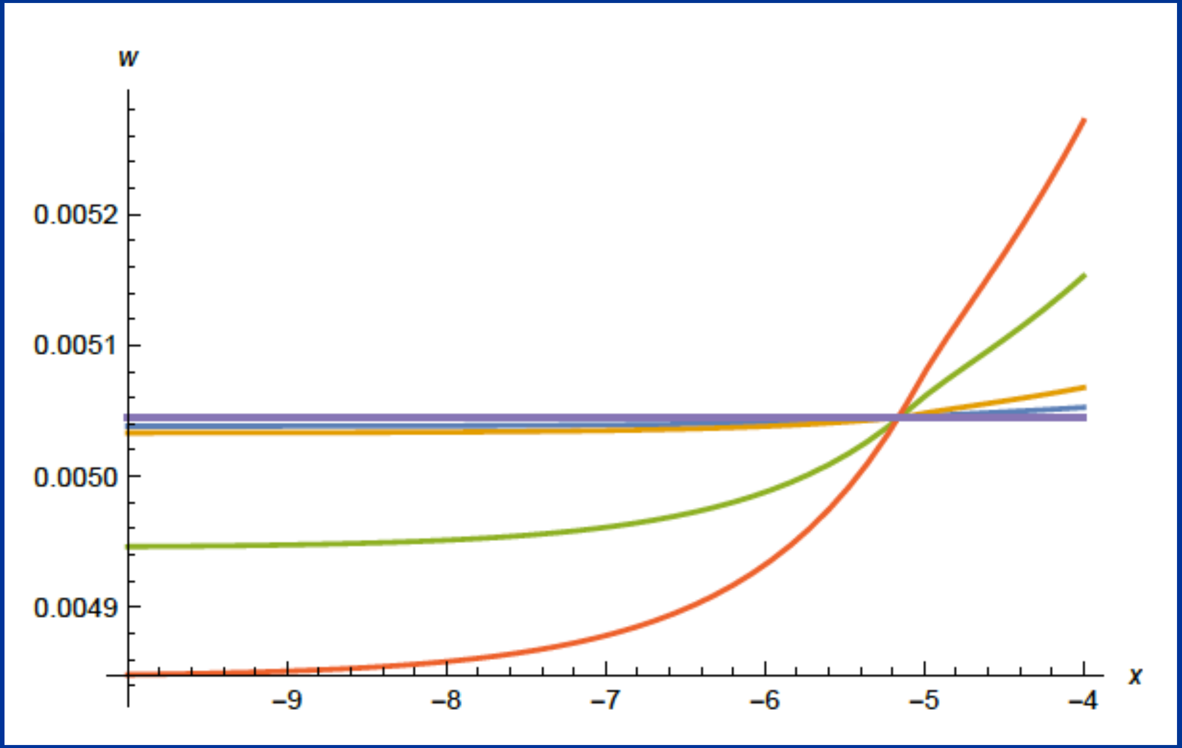
$$2\tilde{\rho}\frac{\partial u}{\partial \tilde{\rho}} = 4u - \frac{1}{24\pi^2}\left(\frac{5}{1-u/w} + \frac{1}{1-u/4w}\right) - 4b_{\mathrm{U}}$$

Generic form of scaling potential

- Interpolates between two plateaus
- Scalar potential =

  field dependent "cosmological constant"
- Effectively massless particles contribute to flow
- Different numbers of massless particles in different regions of field space

Coefficient of curvature scalar in standard model



w : dimensionless field dependent squared Planck mass

$$u = \frac{U}{k^4}, \quad w = \frac{F}{2k^2}$$

non-minimal coupling of scalar field to gravity:

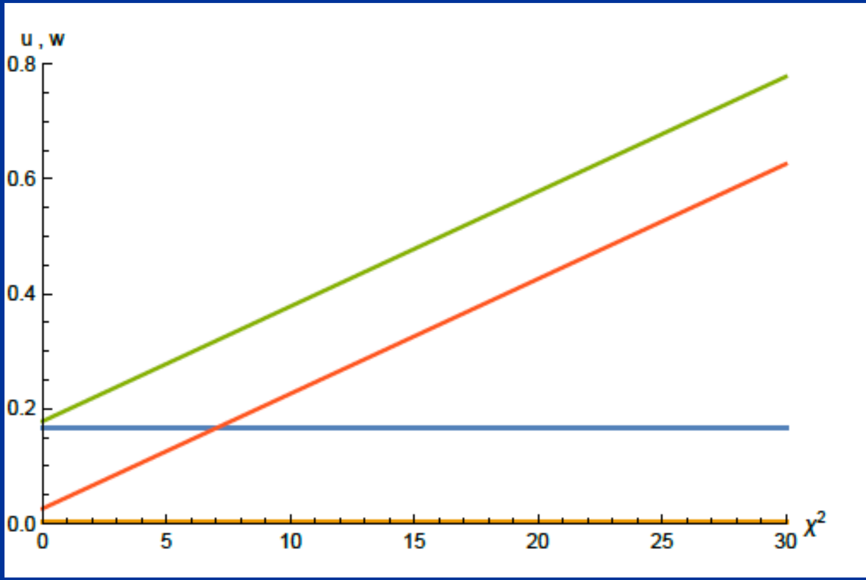
$$F = \xi \chi^2 R$$

x : logarithm of scalar field value

Approximate scaling solution

- flat potential: u constant
- non-minimal scalar- gravity coupling:

  for large scalar field w increases proportional χ^2

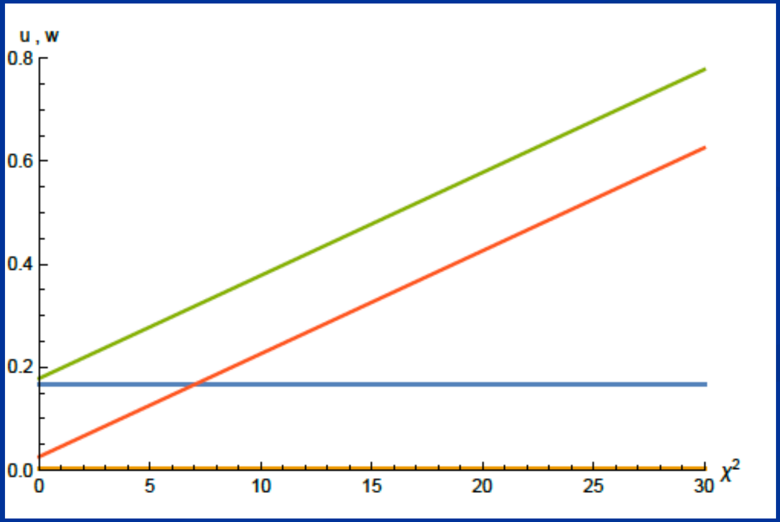


squared scalar field value χ^2

looks natural

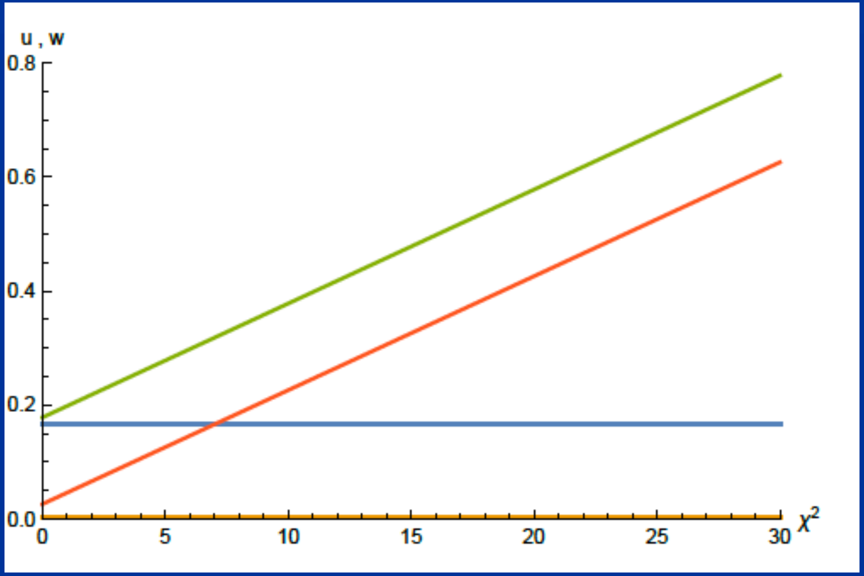
no small parameter

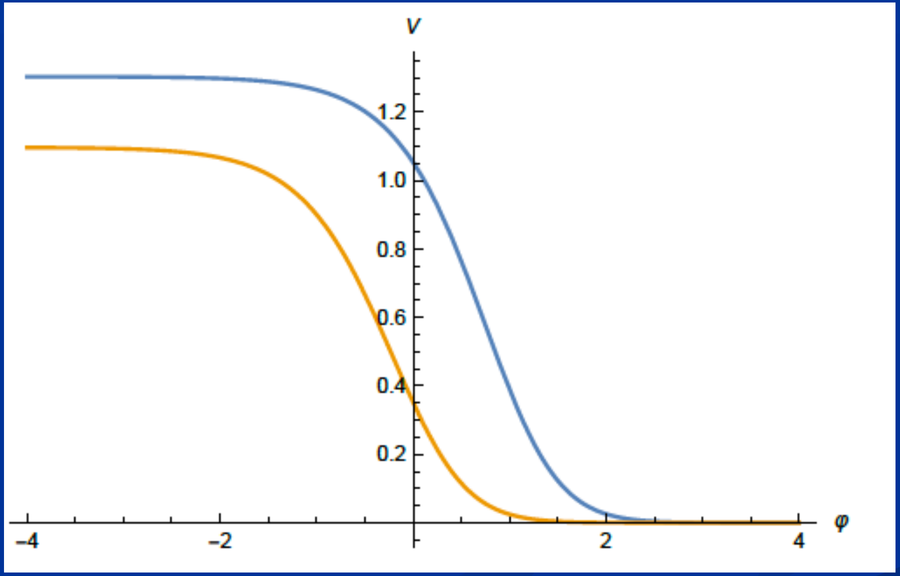
no tuning



Cosmology

Scaling solution in Einstein frame





Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu}$$

$$\varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_\chi \sqrt{g}\left\{-\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\right\}$$

$$\Gamma = \int_\chi \sqrt{g}\left\{-\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\right\}$$

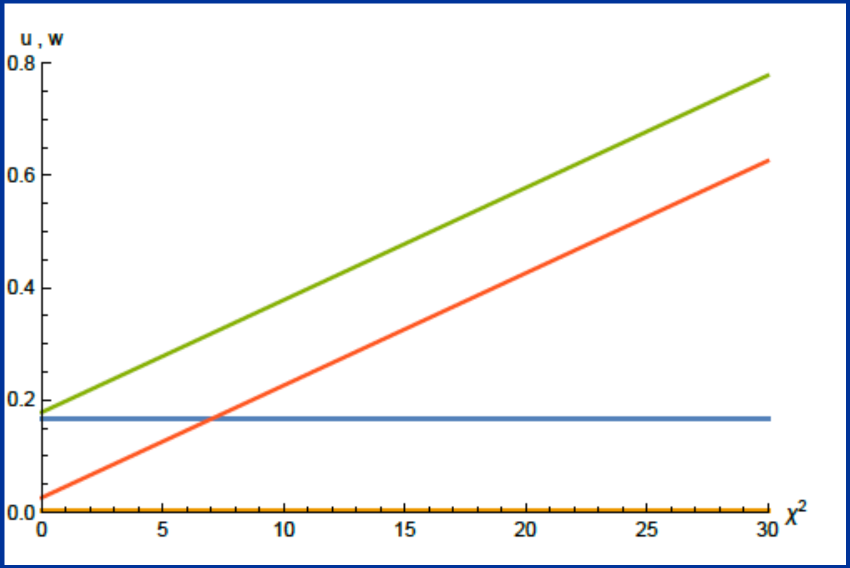
$$V(\varphi) = \frac{UM^4}{F^2}$$

$$Z(\varphi) = \frac{1}{16}\left\{\frac{\chi^2K}{F} + \frac{3}{2}\left(\frac{\partial\ln F}{\partial\ln\chi}\right)^2\right\}$$

Scaling solution

$$U = u_0 k^4$$

$$F = 2w_0 k^2 + \xi \chi^2$$



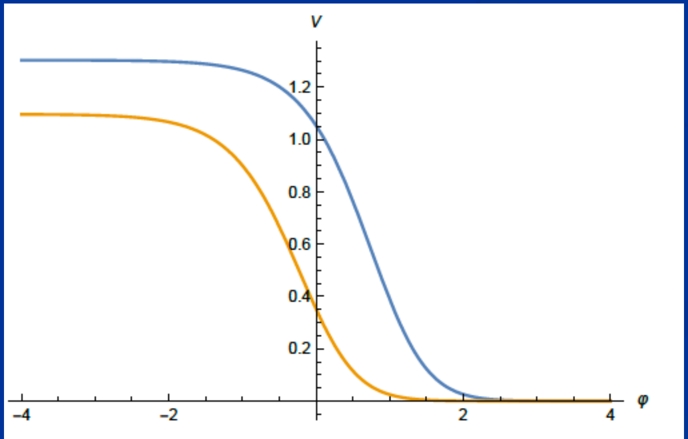
For low energy

standard model :

$$u_\infty = \frac{7}{256\pi^2}$$

Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$

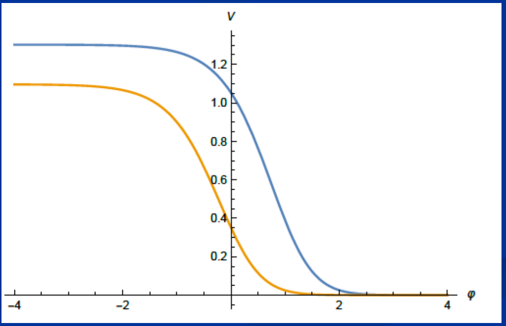


Mass scales in Einstein frame

Renormalization scale k is no longer present

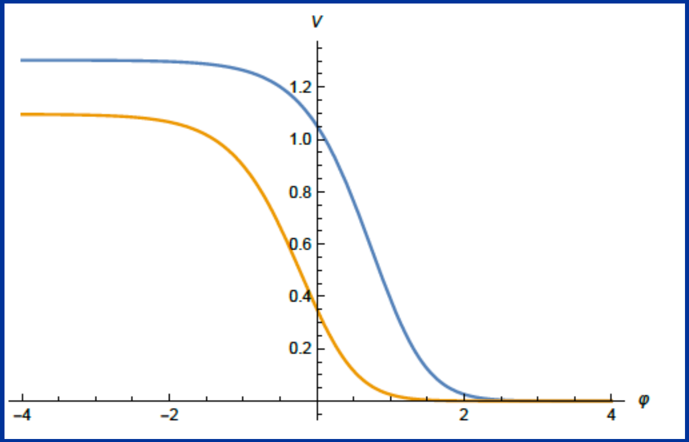
Planck mass M not intrinsic: introduced only by change of variables

$$
\begin{aligned}
V &= \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} \\
&= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)
\end{aligned}
$$



Asymptotic solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !

$$V(\varphi) = \frac{UM^4}{F^2}$$

Predictions for primordial cosmic fluctuations in inflationary cosmology

- Depend on form of kinetial K

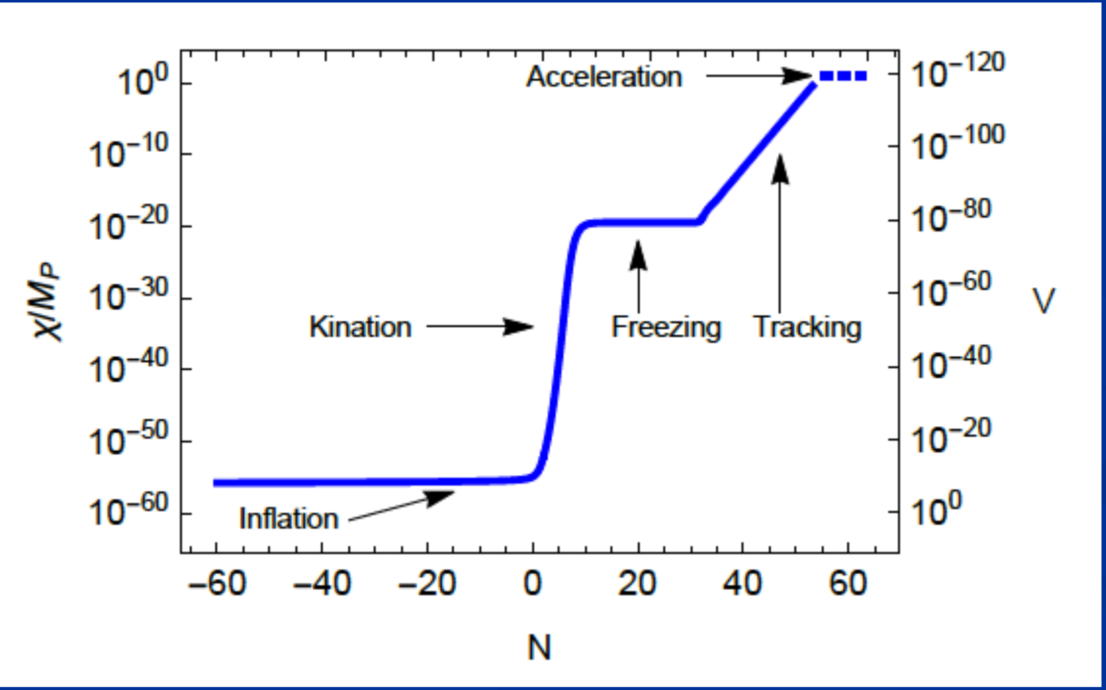
$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

- so far form of K assumed :

  realistic cosmology possible for suitable K
- K needs to be computed!

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Conclusions

- Fluctuations of the metric matter
- They influence the behavior of scalar potentials for all field values
- Quantum gravity relevant for early cosmology and late cosmology
- Quantum scale symmetry is central ingredient for understanding cosmology
- Fundamental scale invariance is highly predictive
- Understanding of quantum gravity fluctuations still in beginning stage

Conclusion

Fixed points of quantum gravity

with associated quantum scale symmetry,

scaling solutions and relevant parameters

are crucial for understanding the

evolution of our Universe

end