Unification from Functional Renormalization

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,...
- linear and non-linear sigma models
- vortices and perturbation theory
- bosonic and fermionic models
- relativistic and non-relativistic physics
- classical and quantum statistics
- non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

unification

abstract laws

quantum gravity

grand unification

standard model

electro-magnetism

gravity

Landau theory | universal critical physics | functional renormalization

complexity

unification:
functional integral / flow equation

- simplicity of average action
- explicit presence of scale
- differentiating is easier than integrating…

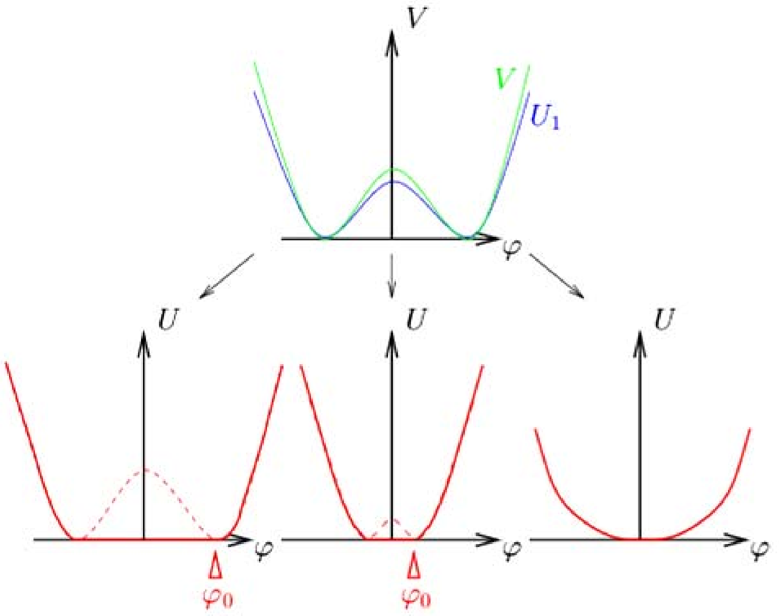
unified description of scalar models for all d and N

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



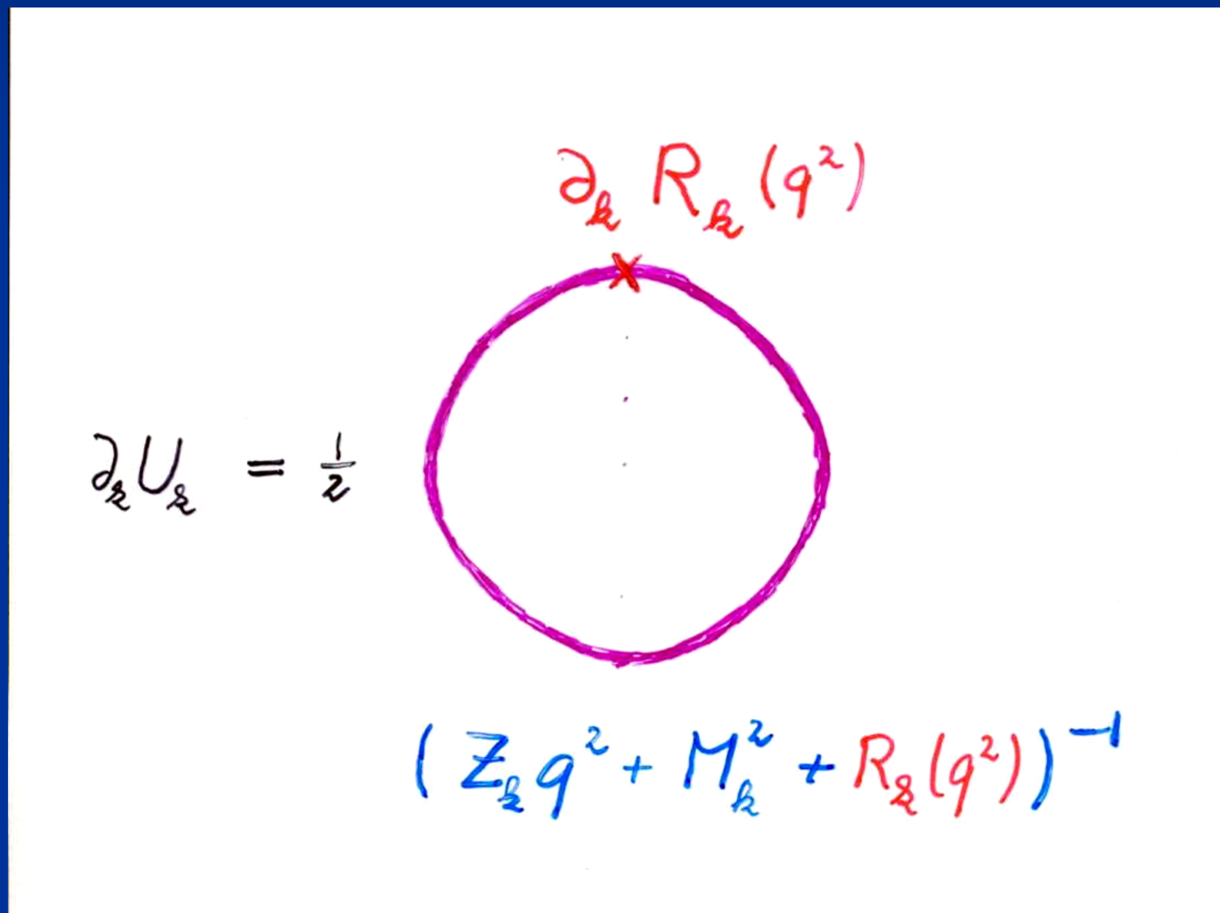
Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

Simple one loop structure – nevertheless (almost) exact



Infrared cutoff

R_k : IR-cutoff

e.g $\quad R_k = \dfrac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $\quad R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad$ (Litim)

$\lim_{k\to 0} R_k = 0$

$\lim_{k\to\infty} R_k \to \infty$

Wave function renormalization and anomalous dimension

Z_k: wave function renormalization

$$k\partial_k Z_k = -\eta_k Z_K$$

η_k: anomalous dimension

$$t = \ln(k/\Lambda)$$

$$\partial_t \ln Z = -\eta$$

for $Z_k\,(\varphi,q^2)$: flow equation is **exact !**

Scaling form of evolution equation

$$u = \frac{U_k}{k^d}$$

$$\tilde{\rho} = Z_k k^{2-d} \rho$$

$$u' = \frac{\partial u}{\partial \tilde{\rho}} \quad \text{etc.}$$

$$\boxed{\begin{aligned} \partial_t u|_{\tilde{\rho}} = & -du + (d-2+\eta)\tilde{\rho}u' \\ & +2v_d\{l_0^d(u' + 2\tilde{\rho}u''; \eta) \\ & \quad +(N-1)\, l_0^d(u'; \eta)\} \end{aligned}}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

linear cutoff:

$$l_0^d(w; \eta) = \frac{2}{d}\left(1 - \frac{\eta}{d+2}\right)\frac{1}{1+w}$$

On r.h.s. :
neither the scale k
nor the wave function
renormalization Z
appear explicitly.

Scaling solution:
no dependence on t;
corresponds
to second order
phase transition.

Tetradis …

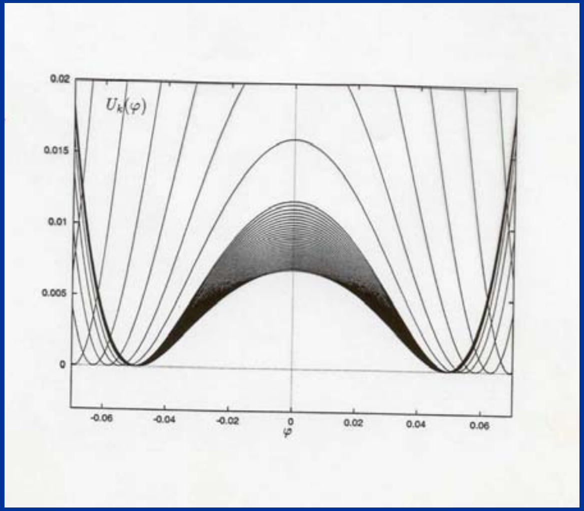
unified approach

- choose N
- choose d
- choose initial form of potential
- run !

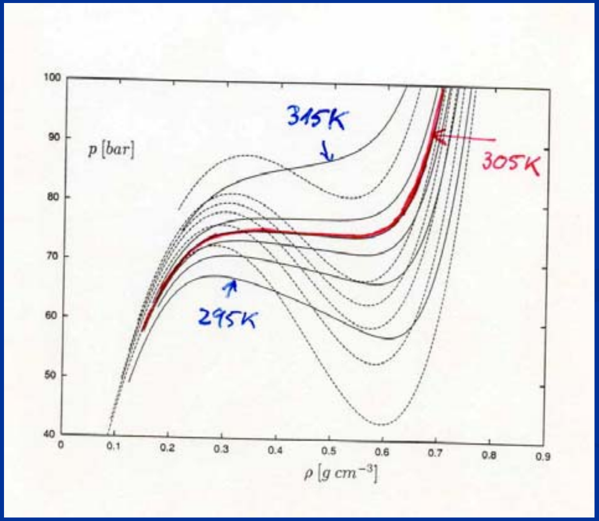
(quantitative results : systematic derivative expansion in second order in derivatives)

Flow of effective potential

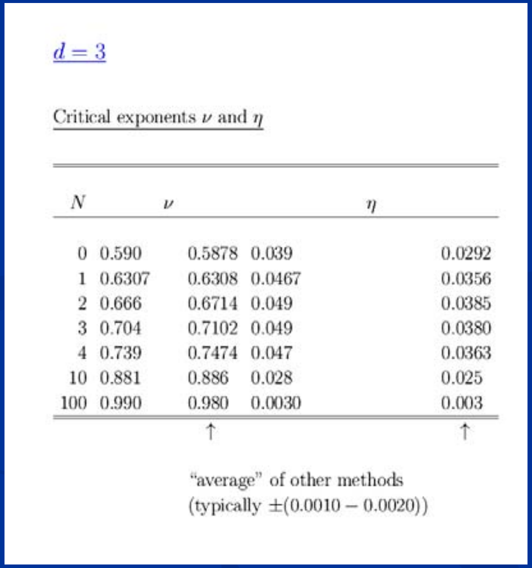
Ising model



CO_2



Critical exponents



$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003
		↑		↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Experiment :

T* =304.15 K
p* =73.8.bar
ρ* = 0.442 g cm-2

S.Seide …

Critical exponents , d=3

N	ν (ERGE)	ν (world)	η (ERGE)	η (world)
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
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ERGE **world** **ERGE** **world**

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

critical exponents , BMW approximation

N	η	η (other)	ν	ν (other)	ω (prelim.)	ω (other)
0	0.033(3)	0.028(3) [1]	0.588	0.588(1) [1]	0.80	
1	0.039(3)	0.0364(2) [2] 0.0368(2) [3] 0.033(3) [1]	0.6298(4)	0.6301(2) [2] 0.6302(1) [3] 0.630(1) [1]	0.78	0.79(1) [1]
2	0.041(3)	0.0381(2) [4] 0.035(3) [1]	0.6719(4)	0.6717(1) [4] 0.670(2) [1]	0.78	0.79(1) [1]
3	0.040(3)	0.0375(5) [5] 0.036(3) [1]	0.709	0.7112(5) [5] 0.707(4) [1]	0.73	
4	0.038(3)	0.035(5)[1] 0.037(1) [6]	0.738	0.741(6) [1] 0.749(2) [6]	0.74	0.77(2) [1]
5	0.035(3)	0.031(3) [8] 0.034(1) [7]	0.768	0.764(4) [8] 0.779(3) [7]	0.73	0.77(2) [1]
10	0.022(2)	0.024 [9]	0.860	0.859 [9]	0.81	
20	0.012(1)	0.014 [9]	0.929	0.930 [9]	0.94	
100	0.0023(2)	0.0027 [10]		0.989 [10]	0.99	

[1] R. Guida and J. Zinn-Justin '98. [2] M. Campostrini, A. Pelissetto, P. Rossi, E. Vicari '02.
[3] Y. Deng and H. W. J. Blote '03. [4] M. Campostrini, M. Hasenbusch, A. Pelissetto, E. Vicari '06.
[5] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, E. Vicari '02. [6] M. Hasenbusch '01.
[7] M. Hasenbusch, A. Pelissetto, E. Vicari '05. [8] A. Butti and F. Parisen Toldin '05.
[9] S. A. Antonenko and A. I. Sokolov '95. [10] M. Moshe and J. Zinn-Justin '03.

Blaizot, Benitez , … , Wschebor

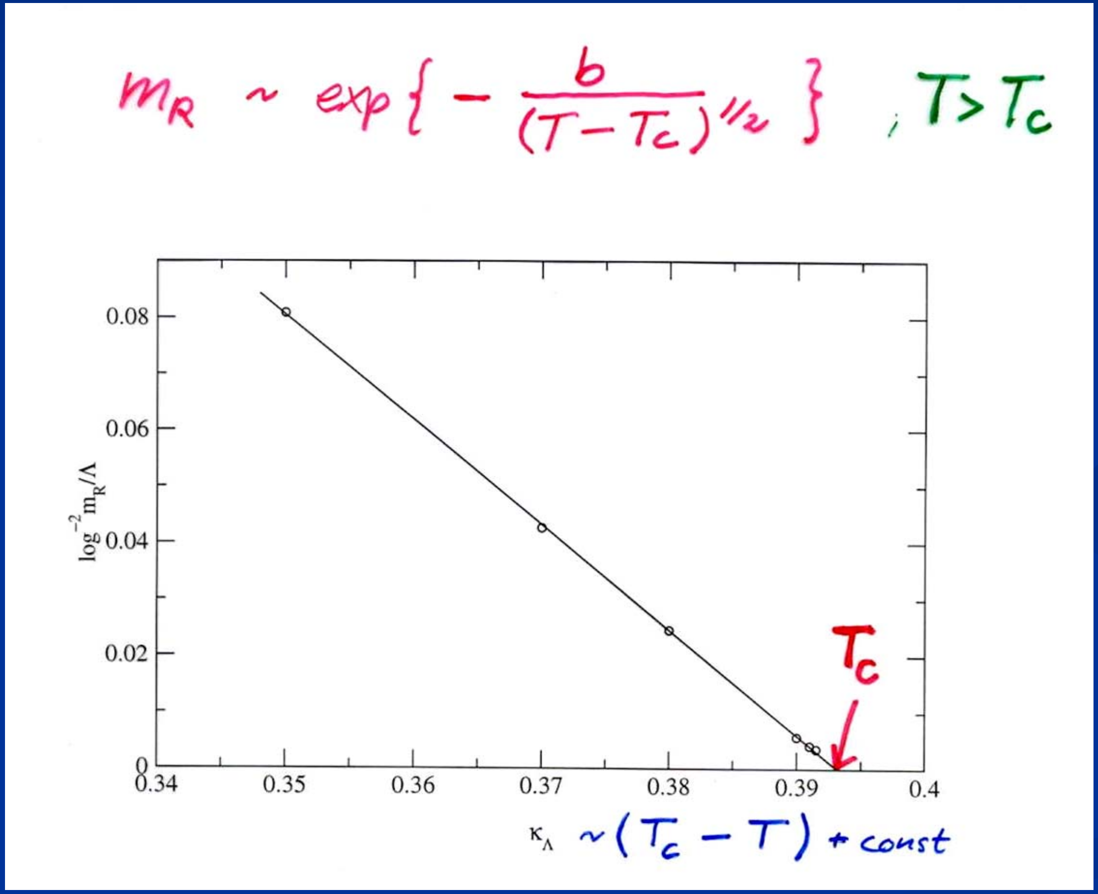
Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:

Kosterlitz-Thouless phase transition

Essential scaling : d=2,N=2



- Flow equation contains correctly the non-perturbative information !
- (essential scaling usually described by vortices)

Von Gersdorff …

Kosterlitz-Thouless phase transition (d=2,N=2)

Correct description of phase with

Goldstone boson

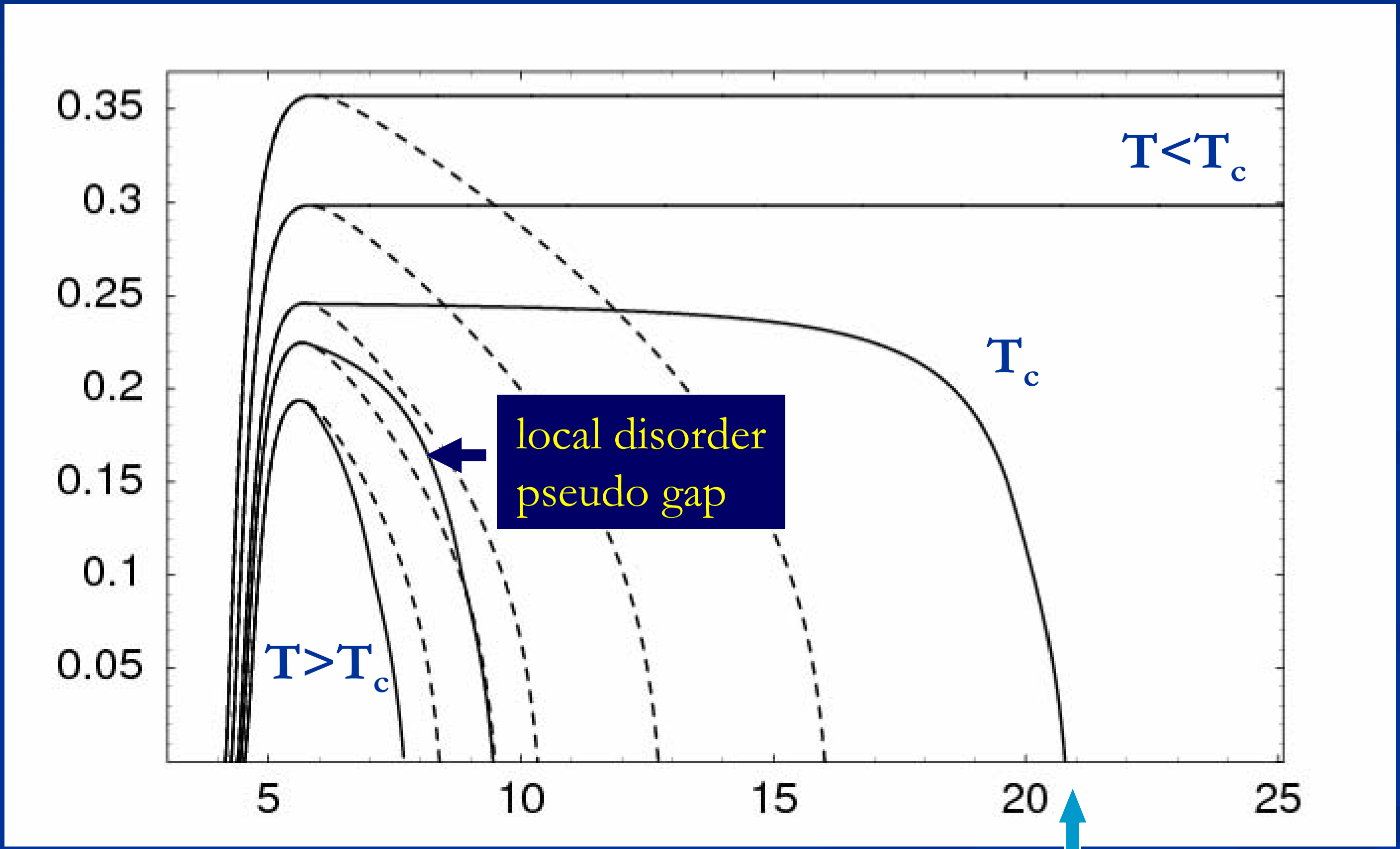
(infinite correlation length)

for $T<T_c$

Running renormalized d-wave superconducting order parameter $\varkappa$ in doped Hubbard (-type) model

$\varkappa$

location of minimum of u

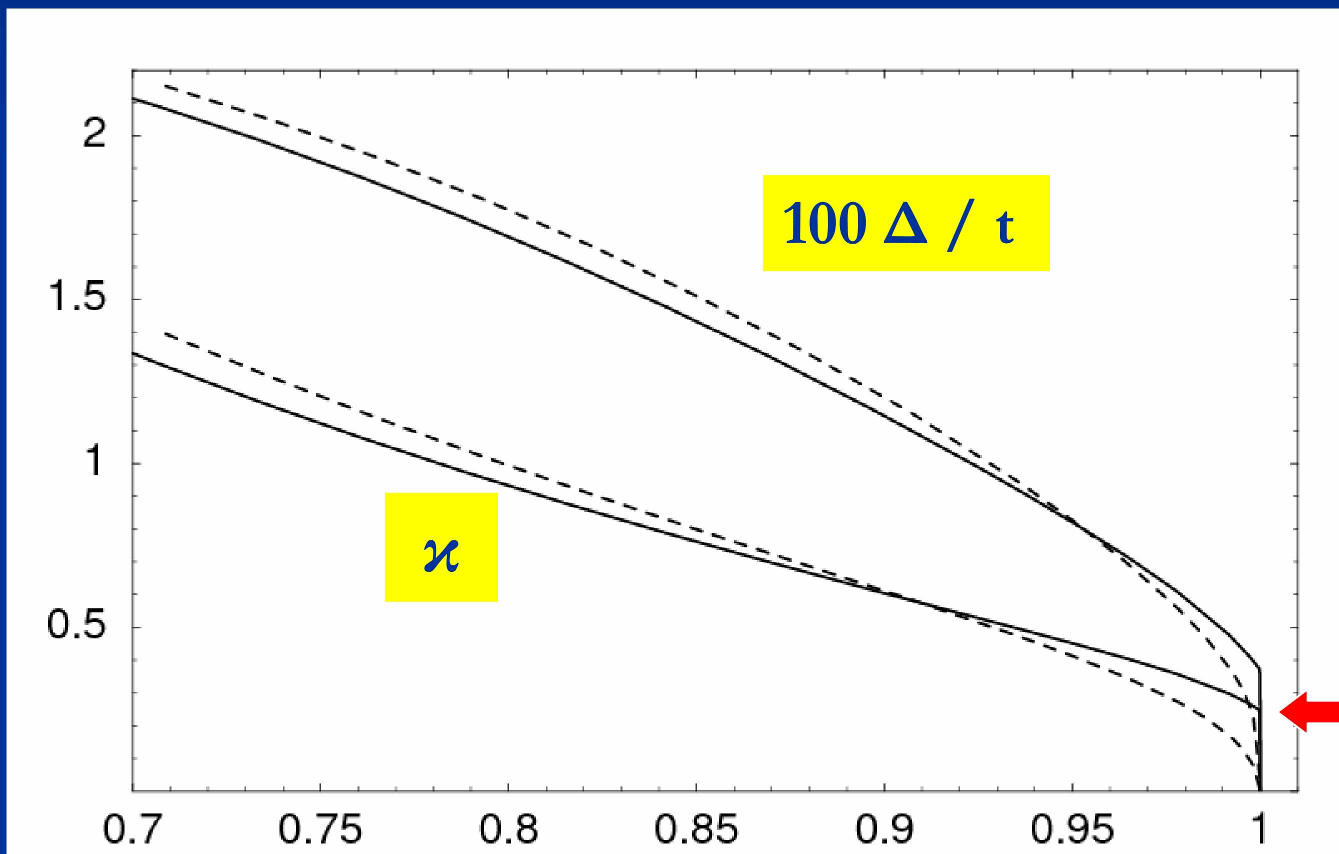


C.Krahl,…

- ln (k/Λ)

macroscopic scale 1 cm

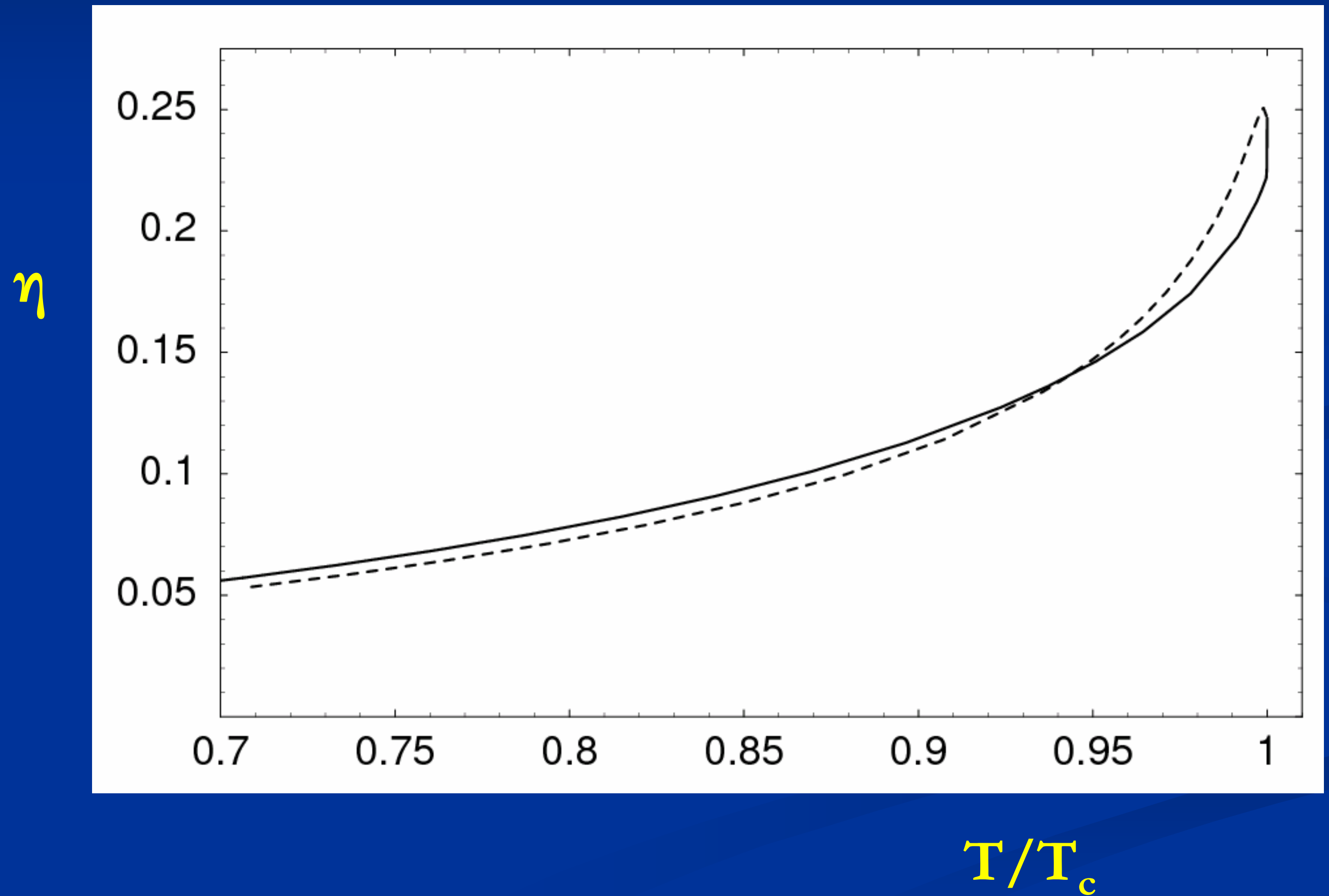
Renormalized order parameter $\varkappa$ and gap in electron propagator Δ in doped Hubbard model



jump

T/T_c

Temperature dependent anomalous dimension η



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- homogenous systems and local disorder
- equilibrium and out of equilibrium

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\boxed{\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}}$$

$$\left(\Gamma_k^{(2)} \right)_{ab} (q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)

15 years

getting adult…

some history … (the parents)

- **exact RG equations :**

  Symanzik eq. , Wilson eq. , Wegner-Houghton eq. , Polchinski eq. ,

  mathematical physics

- **1PI :** RG for 1PI-four-point function and hierarchy

  Weinberg

  formal Legendre transform of Wilson eq.

  Nicoll, Chang

- **non-perturbative flow :**

  d=3 : sharp cutoff ,

  no wave function renormalization or momentum dependence

  Hasenfratz^2

qualitative changes that make non-perturbative physics accessible :

(1) basic object is simple

average action ~ classical action

~ generalized Landau theory

direct connection to thermodynamics

(coarse grained free energy)

qualitative changes that make non-perturbative physics accessible :

(2) Infrared scale k

instead of Ultraviolet cutoff Λ

short distance memory not lost

no modes are integrated out , but only part of the fluctuations is included

simple one-loop form of flow

simple comparison with perturbation theory

infrared cutoff k

cutoff on momentum resolution

or frequency resolution

e.g. distance from pure anti-ferromagnetic momentum or from Fermi surface

intuitive interpretation of k by association with physical IR-cutoff , i.e. finite size of system :

arbitrarily small momentum differences cannot be resolved !

qualitative changes that make non-perturbative physics accessible :

(3) only physics in small momentum range around k matters for the flow

ERGE regularization

simple implementation on lattice

artificial non-analyticities can be avoided

qualitative changes that make non-perturbative physics accessible :

(4) flexibility

change of fields

microscopic or composite variables

simple description of collective degrees of freedom and bound states

many possible choices of "cutoffs"

Proof of
exact flow equation

$$\partial_k \Gamma|_\phi = -\partial_k W|_j - \partial_k \Delta_k S[\varphi]$$
$$= \frac{1}{2}\mathrm{Tr}\,\{\partial_k R_k(\langle\phi\phi\rangle - \langle\phi\rangle\langle\phi\rangle)\}$$
$$= \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k W_k^{(2)}\right\}$$

$$W_k^{(2)}(\Gamma_k^{(2)} + R_k) = \mathbb{1}$$
$$(\Delta_k S^{(2)} \equiv R_k)$$

$$\Longrightarrow$$

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k(\Gamma_k^{(2)} + R_k)^{-1}\right\}$$

sources j can multiply arbitrary operators

φ : associated fields

Truncations

Functional differential equation –

cannot be solved exactly

Approximative solution by truncation of

most general form of effective action

derivative expansion

Tetradis,...; Morris

$O(N)$-model:

$$\Gamma_k = \int d^dx \{U_k(\rho) + \frac{1}{2} Z_k(\rho) \partial_\mu \varphi_a \partial_\mu \varphi_a + \frac{1}{4} Y_k(\rho) \partial_\mu \rho \partial_\mu \rho + \cdots \}$$

$$(N = 1 : \quad Y_k \equiv 0)$$

field expansion (flow eq. for 1PI vertices)

Weinberg; Ellwanger,...

$$\Gamma_k = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=0}^{n} d^d x_j \, \Gamma_k^{(n)}(x_1, x_2, \ldots, x_n) \prod_{j=0}^{n} (\phi(x_j) - \phi_0)$$

Expansion in canonical dimension of couplings

Lowest order:

$$d = 4: \quad \rho_0, \bar{\lambda}, Z$$

$$d = 3: \quad \rho_0, \bar{\lambda}, \bar{\gamma}, Z$$

$$U = \frac{1}{2}\bar{\lambda}(\rho - \rho_0)^2 + \frac{1}{6}\bar{\gamma}(\rho - \rho_0)^3$$

works well for $O(N)$ models

Tetradis,...; Tsypin

polynomial expansion of potential converges if expanded around ρ_0

Tetradis,...; Aoki et al.

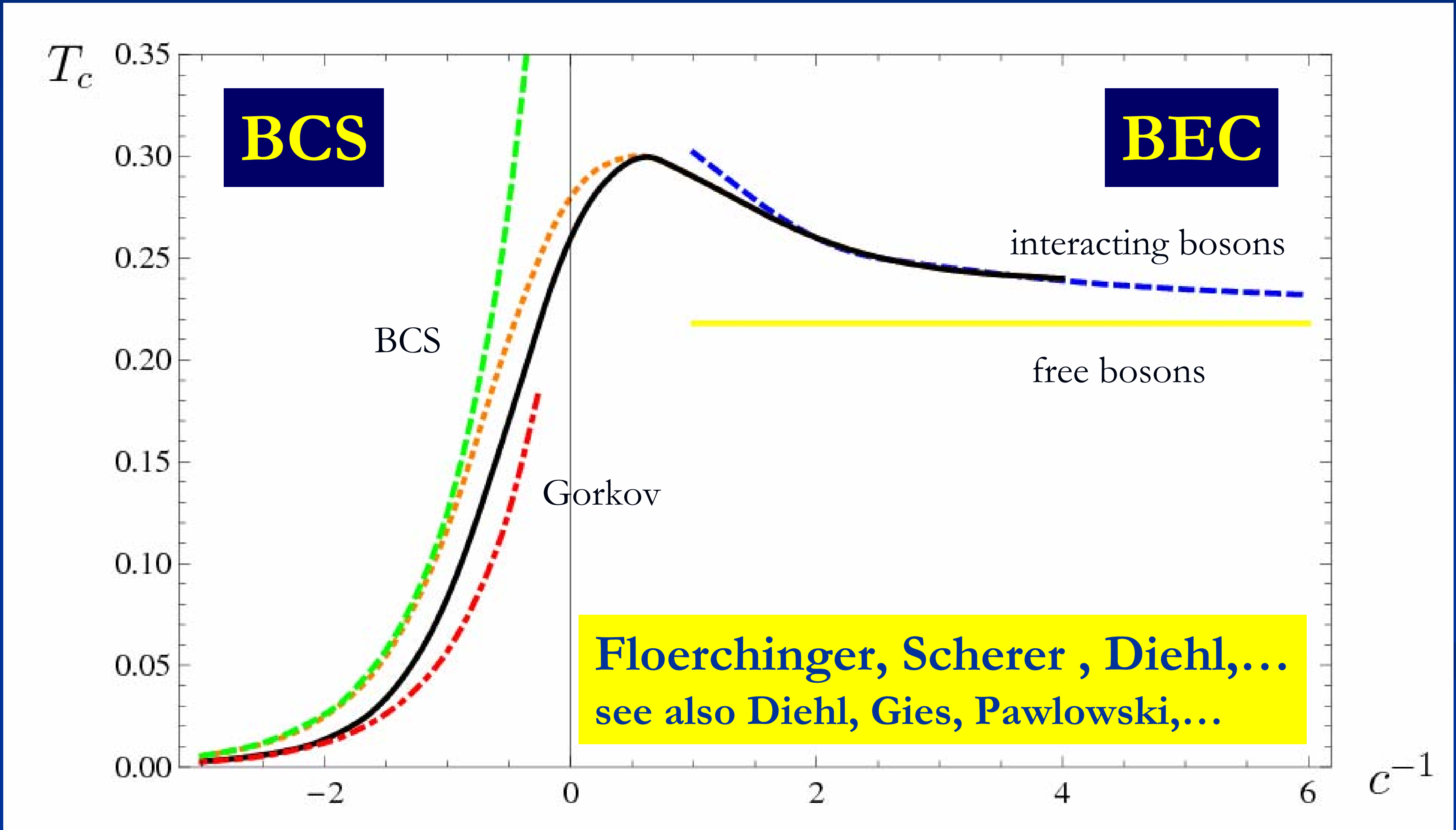
convergence and errors

- apparent fast convergence : no series resummation
- rough error estimate by different cutoffs and truncations , Fierz ambiguity etc.
- in general : understanding of physics crucial
- no standardized procedure

including fermions :

no particular problem !

BCS – BEC crossover



changing degrees of freedom

Anti-ferromagnetic order in the Hubbard model

A functional renormalization group study

T.Baier, E.Bick, …
C.Krahl

Hubbard model

Functional integral formulation

$$
\begin{aligned}
Z[\eta] \; = & \int_{\hat\psi(\beta)=-\hat\psi(0),\hat\psi^*(\beta)=-\hat\psi^*(0)} \mathcal{D}(\hat\psi^*(\tau),\hat\psi(\tau)) \\
& \exp\Big(-\int_0^\beta d\tau \Big(\sum_{\mathbf{x}} \hat\psi^\dagger_{\mathbf{x}}(\tau)\big(\frac{\partial}{\partial\tau} - \mu\big)\hat\psi_{\mathbf{x}}(\tau) \\
& \quad + \sum_{\mathbf{xy}} \hat\psi^\dagger_x(\tau)\mathcal{T}_{\mathbf{xy}}\hat\psi_{\mathbf{y}}(\tau) \\
& \quad + \frac{1}{2}U\sum_{\mathbf{x}} \big(\hat\psi^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau)\big)^2 \\
& \quad - \sum_{\mathbf{x}} \big(\eta^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau) + \eta^T_{\mathbf{x}}(\tau)\hat\psi^*_{\mathbf{x}}(\tau)\big)\Big)\Big)
\end{aligned}
$$

next neighbor interaction

$$
\mathcal{T}_{xy} = \begin{cases} -t & \text{, if } \boldsymbol{x} \text{ and } \boldsymbol{y} \text{ are nearest neighbors} \\ 0 & \text{, else} \end{cases}
$$

External parameters
T : temperature
μ : chemical potential
(doping)

U > 0 :
repulsive local interaction

Fermion bilinears

$$\tilde{\rho}(X) = \hat{\psi}^\dagger(X)\hat{\psi}(X),$$
$$\tilde{\vec{m}}(X) = \hat{\psi}^\dagger(X)\vec{\sigma}\hat{\psi}(X)$$

Introduce sources for bilinears

$$S_F = S_{F,\mathrm{kin}} + \frac{1}{2}U(\hat{\psi}^\dagger\hat{\psi})^2 - J_\rho\tilde{\rho} - \vec{J}_m\tilde{\vec{m}}$$

Functional variation with respect to sources J yields expectation values and correlation functions

$$Z = \int \mathcal{D}(\psi^*,\psi)\exp\left(-(S_F+S_\eta)\right)$$
$$S_\eta = -\eta^\dagger\psi - \eta^T\psi^*$$

Partial Bosonisation

- collective bosonic variables for fermion bilinears
- insert identity in functional integral
  (Hubbard-Stratonovich transformation)
- replace four fermion interaction by equivalent bosonic interaction (e.g. mass and Yukawa terms)
- problem : decomposition of fermion interaction into bilinears not unique (Grassmann variables)

$$(\hat{\psi}^\dagger(X)\hat{\psi}(X))^2 = \tilde{\rho}(X)^2 = -\frac{1}{3}\tilde{\vec{m}}(X)^2$$

Partially bosonised functional integral

$$Z[\eta,\eta^*,J_\rho,\vec{J}_m] = \int \mathcal{D}(\psi^*,\psi,\hat\rho,\hat{\vec{m}})\exp\left(-(S+S_\eta+S_J)\right)$$

$$S = S_{F,\mathrm{kin}} + \frac{1}{2}U_\rho\hat\rho^2 + \frac{1}{2}U_m\hat{\vec{m}}^2 - U_\rho\hat\rho\tilde\rho - U_m\hat{\vec{m}}\tilde{\vec{m}},$$
$$S_J = -\,J_\rho\hat\rho - \vec{J}_m\hat{\vec{m}}$$

equivalent to

fermionic functional integral

if $U = -U_\rho + 3U_m$

Bosonic integration is Gaussian

or:

solve bosonic field equation as functional of fermion fields and reinsert into action

$$\hat\rho = \tilde\rho + \frac{J_\rho}{U_\rho}, \qquad \hat{\vec{m}} = \tilde{\vec{m}} + \frac{\vec{J}_m}{U_m}$$

more bosons …

additional fields may be added formally :

only mass term + source term : decoupled boson

introduction of boson fields not linked to Hubbard-Stratonovich transformation

fermion – boson action

$$S = S_{F,\mathrm{kin}} + S_B + S_Y + S_J,$$

fermion kinetic term

$$S_{F,\mathrm{kin}} = \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q),$$

boson quadratic term (“classical propagator”)

$$S_B = \frac{1}{2}\sum_Q \left(U_\rho \hat{\rho}(Q)\hat{\rho}(-Q) + U_m \hat{\vec{m}}(Q)\hat{\vec{m}}(-Q)\right),$$

Yukawa coupling

$$S_Y = -\sum_{QQ'Q''} \delta(Q - Q' + Q'') \times$$
$$(U_\rho \hat{\rho}(Q)\hat{\psi}^\dagger(Q')\hat{\psi}(Q'') + U_m \hat{\vec{m}}(Q)\hat{\psi}^\dagger(Q')\vec{\sigma}\hat{\psi}(Q'')),$$

source term

$$S_J = -\sum_Q \left(J_\rho(-Q)\hat{\rho}(Q) + \vec{J}_m(-Q)\hat{\vec{m}}(Q) \right)$$

is now linear in the bosonic fields

Mean Field Theory (MFT)

Evaluate Gaussian fermionic integral in background of bosonic field , e.g.

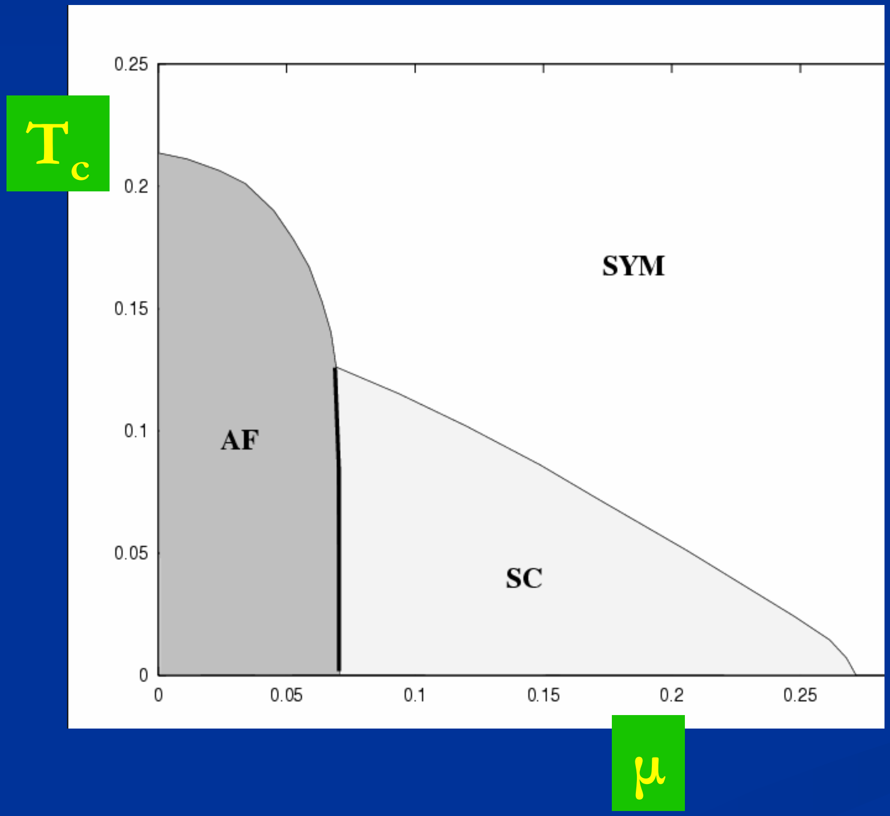
$$\hat{\rho}(Q) \rightarrow \rho\delta(Q)$$
$$\hat{\vec{m}}(Q) \rightarrow \vec{a}\delta(Q-\Pi)$$

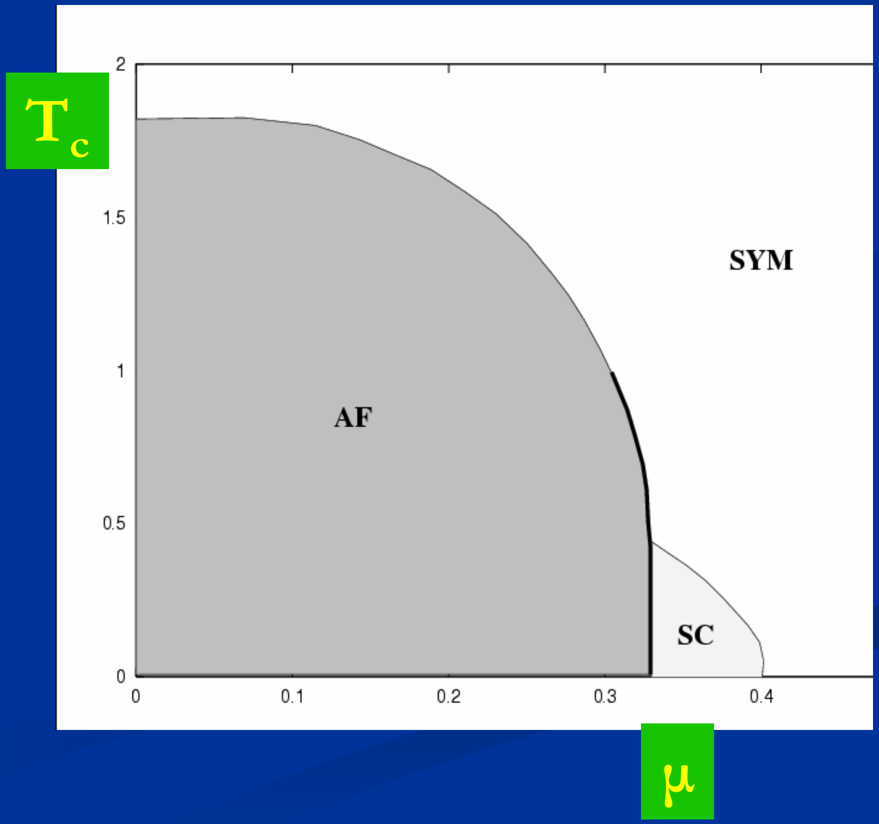
$$Z_{\mathrm{MF}} = \int \mathcal{D}(\hat{\psi}^*, \hat{\psi}) \exp(-S_{\mathrm{MF}}),$$
$$S_{\mathrm{MF}} = \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q)$$
$$-\sum_Q (U_\rho \rho \hat{\psi}^\dagger(Q)\hat{\psi}(Q) + U_m \vec{a}\hat{\psi}^\dagger(Q+\Pi)\vec{\sigma}\hat{\psi}(Q))$$
$$+\frac{V_2}{2T}(U_\rho \rho^2 + U_m \vec{a}^2) - J_\rho(0)\rho - \vec{J}_m(-\Pi)\vec{a}$$

$$\Gamma_{\mathrm{MF}} = -\ln Z_{\mathrm{MF}} + J_\rho(0)\rho + \vec{J}_m(-\Pi)\vec{a}$$

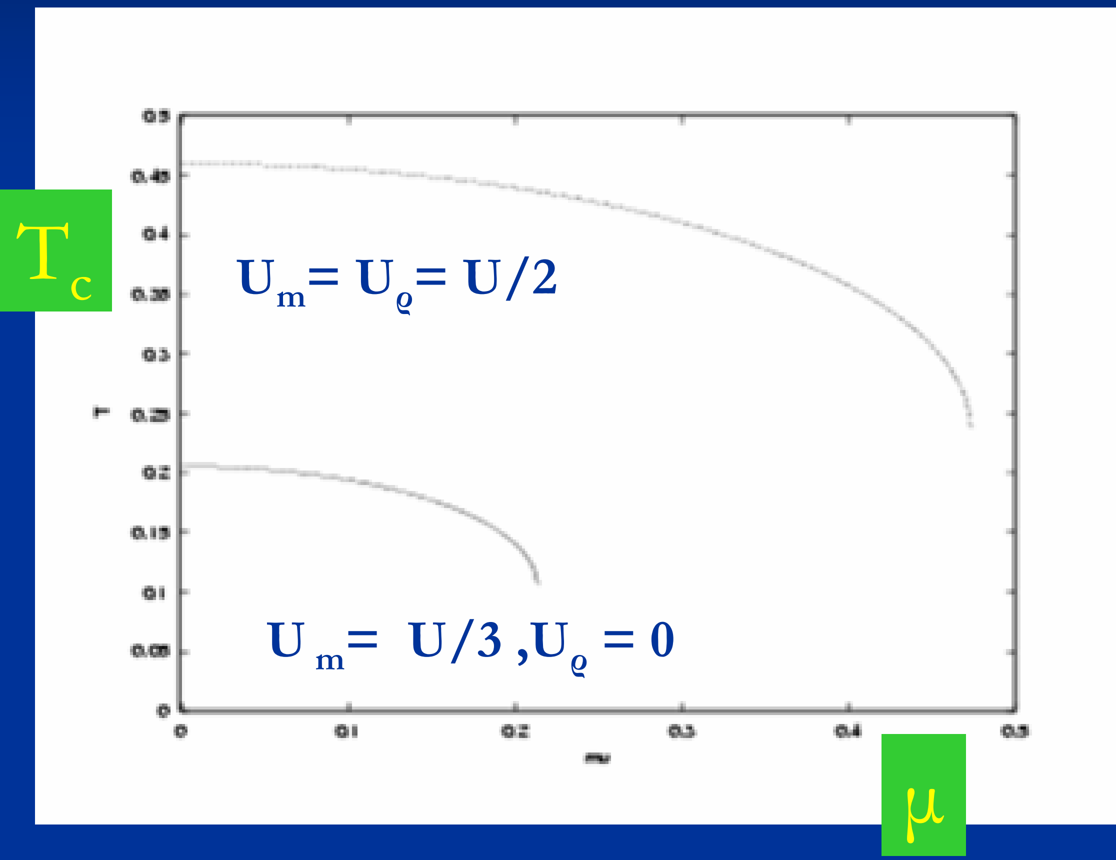
Mean field phase diagram

for two different choices of couplings – same U !





Mean field ambiguity



mean field phase diagram

Artefact of approximation …

cured by inclusion of bosonic fluctuations

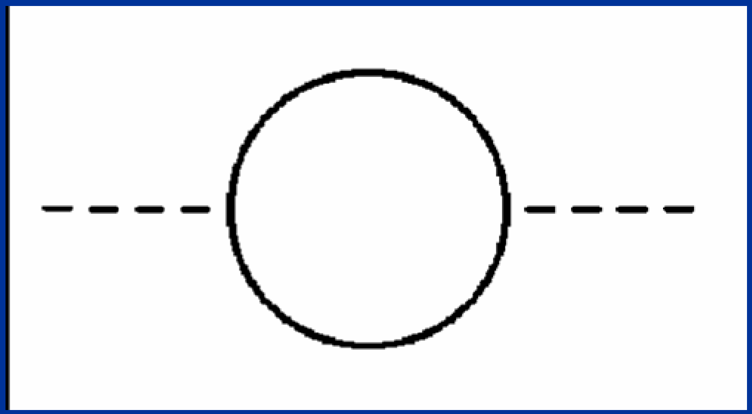
J.Jaeckel,…

$$U = -U_\rho + 3U_m$$

Bosonisation and the mean field ambiguity

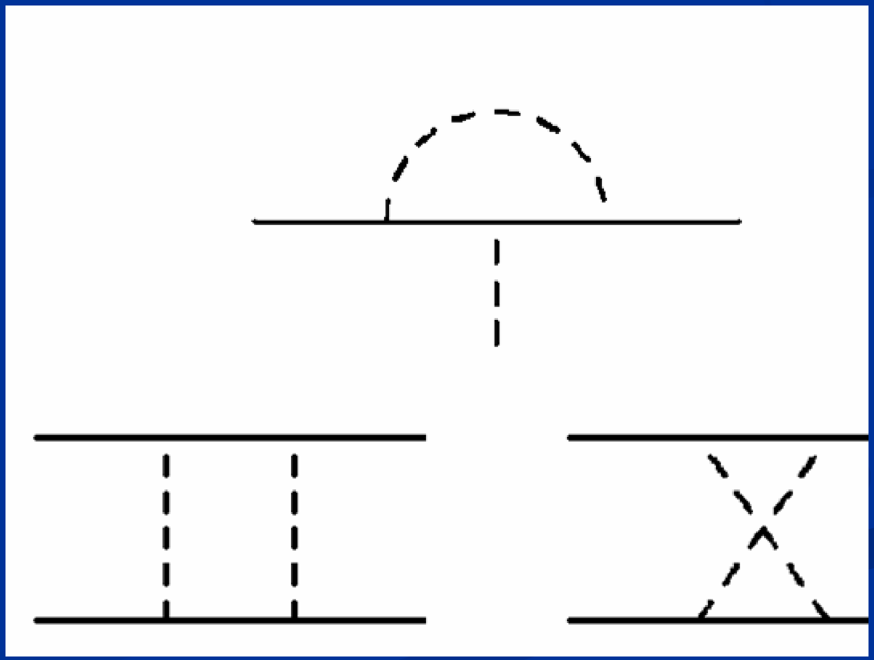
Bosonic fluctuations

fermion loops



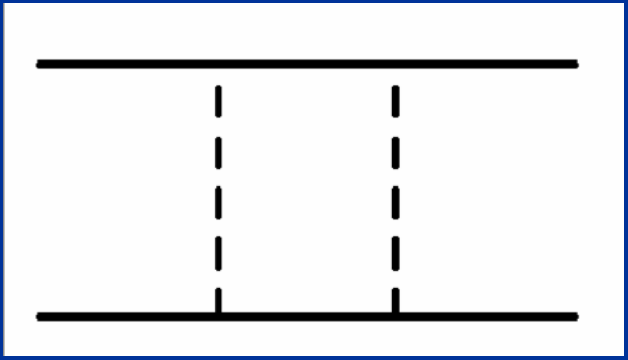
mean field theory

boson loops



Bosonisation

- adapt bosonisation to every scale k such that



is translated to bosonic interaction

H.Gies , …

$$
\begin{aligned}
\Gamma_k[\psi,\psi^*,\phi] = & \sum_Q \psi^*(Q) P_{\psi,k} \psi(Q) \\
& + \frac{1}{2}\sum_Q \phi(-Q) P_{\phi,k}(Q)\phi(Q) \\
& - \sum_Q h_k(Q)\phi(Q)\tilde{\phi}(-Q) \\
& + \sum_Q \lambda_{\psi,k}(Q)\tilde{\phi}(Q)\tilde{\phi}(-Q)
\end{aligned}
$$

k-dependent field redefinition

$$\phi_k(Q) = \phi_{\bar{k}}(Q) + \Delta\alpha_k(Q)\tilde{\phi}(Q)$$

$$\partial_k\phi_k(Q) = -\,\partial_k\alpha_k(Q)\tilde{\phi}(Q)$$

absorbs four-fermion coupling

Modification of evolution of couplings …

Evolution with k-dependent field variables

$$\partial_k \Gamma_k[\psi,\psi^*,\phi_k] = \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \left(\frac{\delta}{\delta\phi_k}\Gamma[\psi,\psi^*,\phi_k]\right)\partial_k\phi_k$$
$$= \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \big(-\partial_k\alpha_k(Q)P_{\phi,k}(Q)\phi_k(Q)\tilde{\phi}(-Q) + h_k(Q)\partial_k\alpha_k(Q)\tilde{\phi}(Q)\tilde{\phi}(-Q)\big)$$

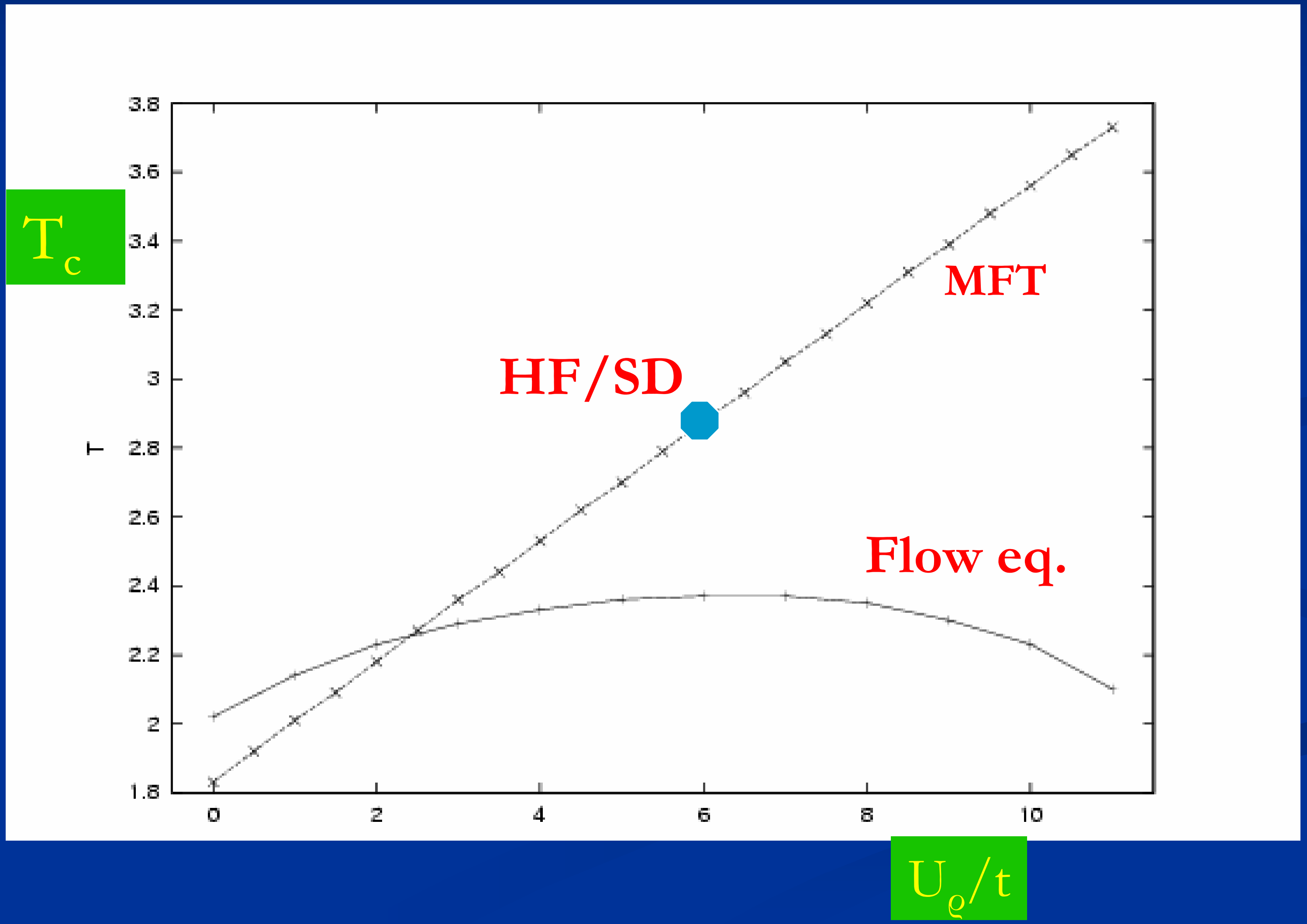
Bosonisation

$$\begin{aligned}\partial_k h_k(Q) &= \partial_k h_k(Q)|_{\phi_k} + \partial_k\alpha_k(Q)P_{\phi,k}(Q),\\ \partial_k \lambda_{\psi,k}(Q) &= \partial_k\lambda_{\psi,k}(Q)|_{\phi_k} + h_k(Q)\,\partial_k\alpha_k(Q).\end{aligned}$$

Choose α_k in order to absorb the four fermion coupling in corresponding channel ⇒

$$\partial_k h_k(Q) = \partial_k h_k(Q)|_{\phi_k} - \frac{P_{\phi,k}(Q)}{h_k(Q)}\partial_k\lambda_{\psi,k}(Q)|_{\phi_k}$$

Bosonisation cures mean field ambiguity



Flow equation for the Hubbard model

T.Baier , E.Bick , …,C.Krahl

Truncation

Concentrate on antiferromagnetism

$$\vec{a}(Q) = \vec{m}(Q+\Pi)$$

Potential U depends only on $\alpha = a^2$

$$\Gamma_{\psi,k}[\psi,\psi^*] = \sum_Q \psi^\dagger(Q) P_F(Q)\psi(Q),$$

$$P_F(Q) = i\omega_F + \epsilon - \mu, \quad \epsilon(\boldsymbol{q}) = -2t(\cos q_x + \cos q_y),$$

$$\Gamma_{Y,k}[\psi,\psi^*,\vec{a}] = -\bar{h}_{a,k} \sum_{KQQ'} \vec{a}(K)\psi^*(Q)\vec{\sigma}\psi(Q') \times \delta(K - Q + Q' + \Pi)$$

$$\Gamma_{a,k}[\vec{a}] = \frac{1}{2}\sum_Q \vec{a}(-Q)P_a(Q)\vec{a}(Q) + \sum_X U[\vec{a}(X)]$$

$$\begin{aligned}
\mathrm{SYM}: \sum_X U[\vec{a}] &= \sum_K \bar{m}_a^2 \alpha(-K,K) + \\
&+ \frac{1}{2}\sum_{K_1 \ldots K_4} \bar{\lambda}_a \delta(K_1+K_2+K_3+K_4) \\
&\quad \times \alpha(K_1,K_2)\alpha(K_3,K_4), \\
\mathrm{SSB}: \sum_X U[\vec{a}] &= \frac{1}{2}\sum_{K_1 \ldots K_4} \bar{\lambda}_a \delta(K_1+K_2+K_3+K_4) \\
&\quad \times (\alpha(K_1,K_2) - \alpha_0\delta(K_1)\delta(K_2)) \\
&\quad \times (\alpha(K_3,K_4) - \alpha_0\delta(K_3)\delta(K_4))
\end{aligned}$$

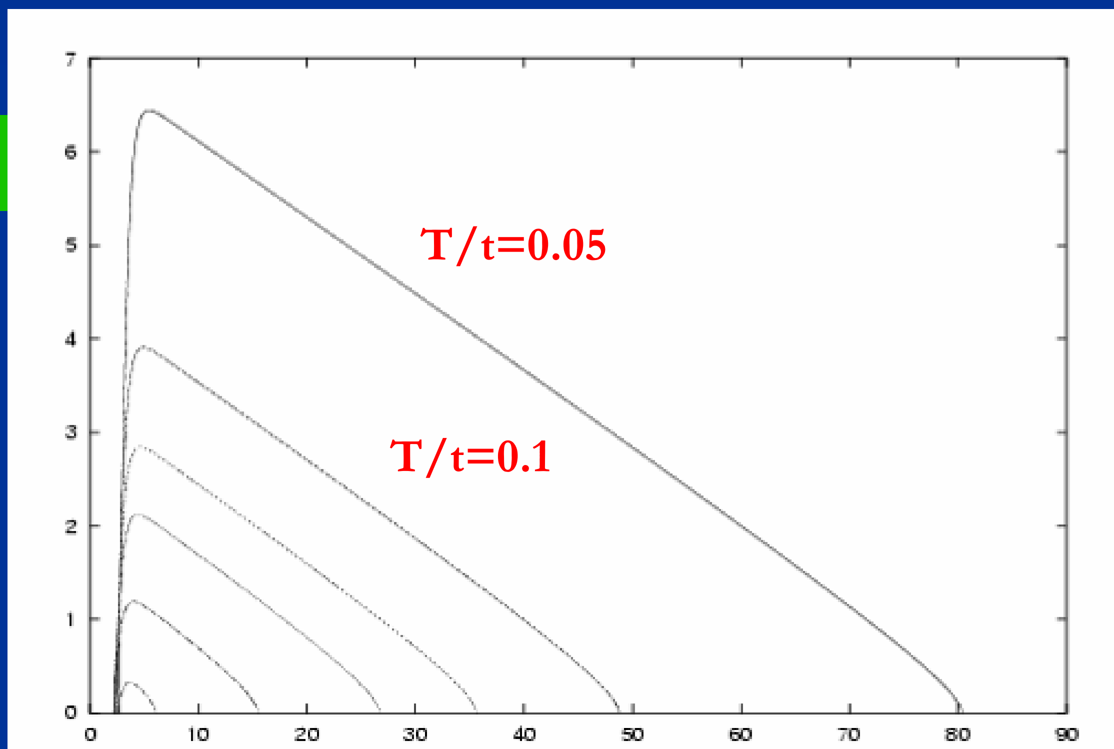
$$\alpha(K,K') = \frac{1}{2}\vec{a}(K)\vec{a}(K')$$

Critical temperature

For $T<T_c$: $\varkappa$ remains positive for $k/t > 10^{-9}$

size of probe > 1 cm

$\varkappa$



$$\kappa_a = \frac{Z_a t^2}{T} \alpha_0$$

local disorder
pseudo gap

SSB

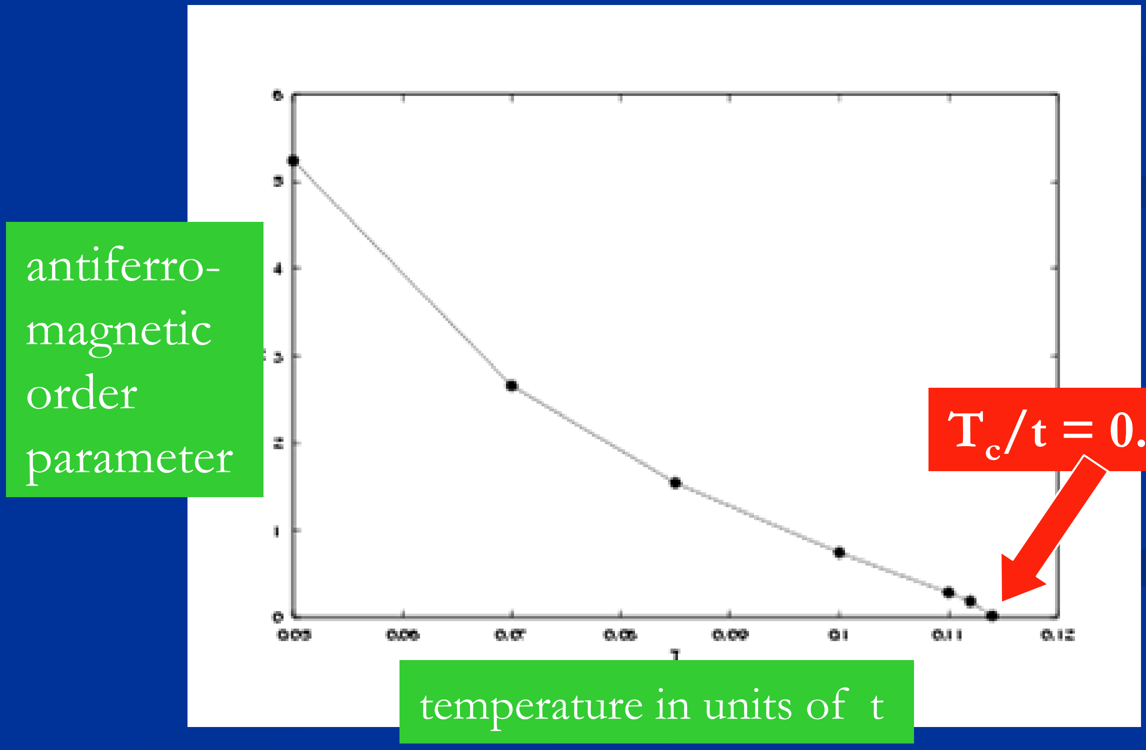
-ln(k/t)

T_c=0.115

Below the critical temperature :

Infinite-volume-correlation-length becomes larger than sample size

finite sample ≈ finite k : order remains effectively



Pseudo-critical temperature T_{pc}

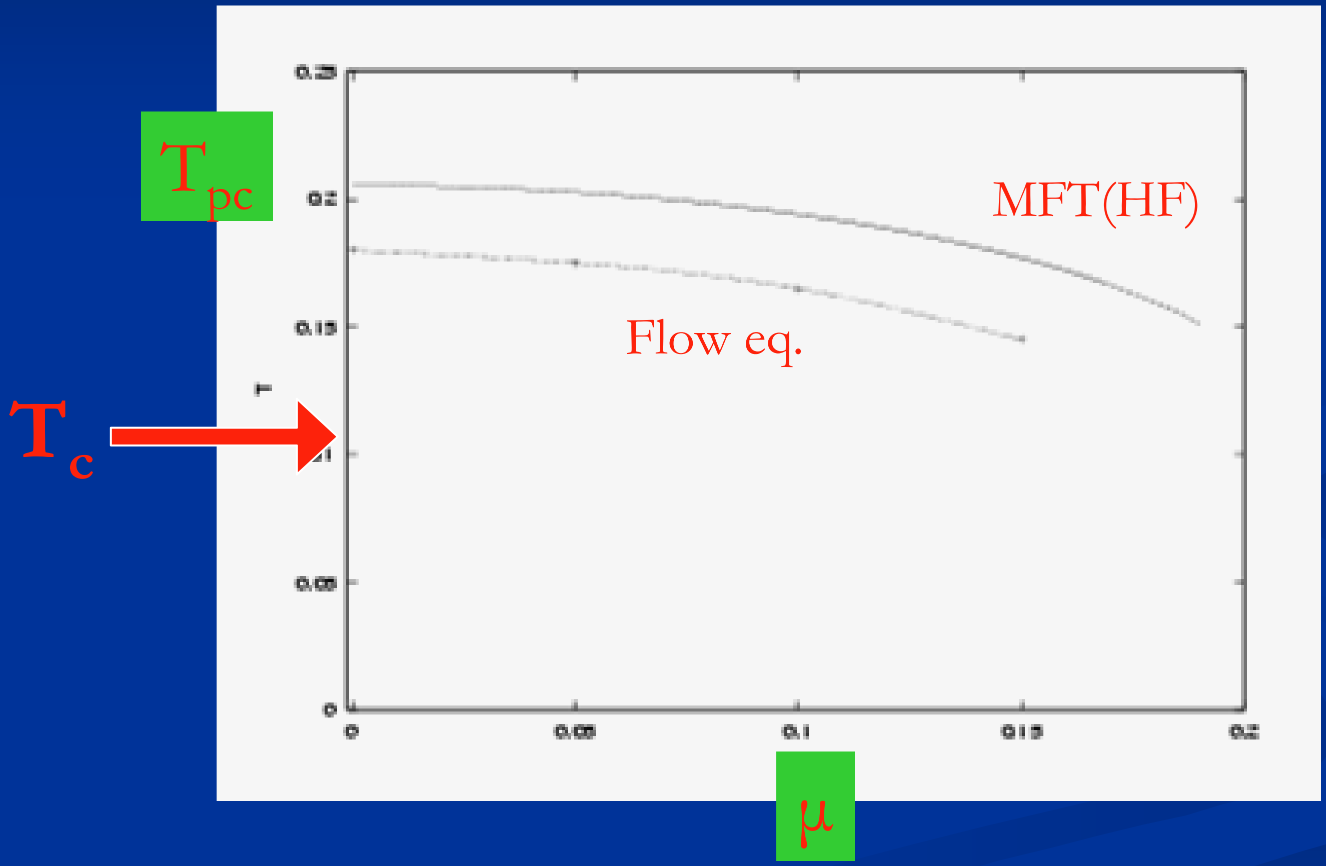
Limiting temperature at which bosonic mass term vanishes ($\varkappa$ becomes nonvanishing)

It corresponds to a diverging four-fermion coupling

This is the "critical temperature" computed in MFT !

Pseudo-gap behavior below this temperature

Pseudocritical temperature



Below the pseudocritical temperature

the reign of the goldstone bosons

effective nonlinear O(3) – σ - model

critical behavior

for interval $T_c < T < T_{pc}$
evolution as for classical Heisenberg model

cf. Chakravarty,Halperin,Nelson

$$k\partial_k \kappa = \frac{1}{4\pi} + \frac{1}{16\pi^2 \kappa} + 0(\kappa^{-2})$$

critical correlation length

$$\xi t = c(T)\exp\left\{20.7\beta(T)\frac{T_c}{T}\right\}$$

c,β : slowly varying functions

exponential growth of correlation length
compatible with observation !

at T_c : correlation length reaches sample size !

Mermin-Wagner theorem ?

No spontaneous symmetry breaking of continuous symmetry in $d=2$!

not valid in practice !

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,4,...
- ☺ linear and non-linear sigma models
- vortices and perturbation theory
- ☺ bosonic and fermionic models
- relativistic and non-relativistic physics
- ☺ classical and quantum statistics
- ☺ non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

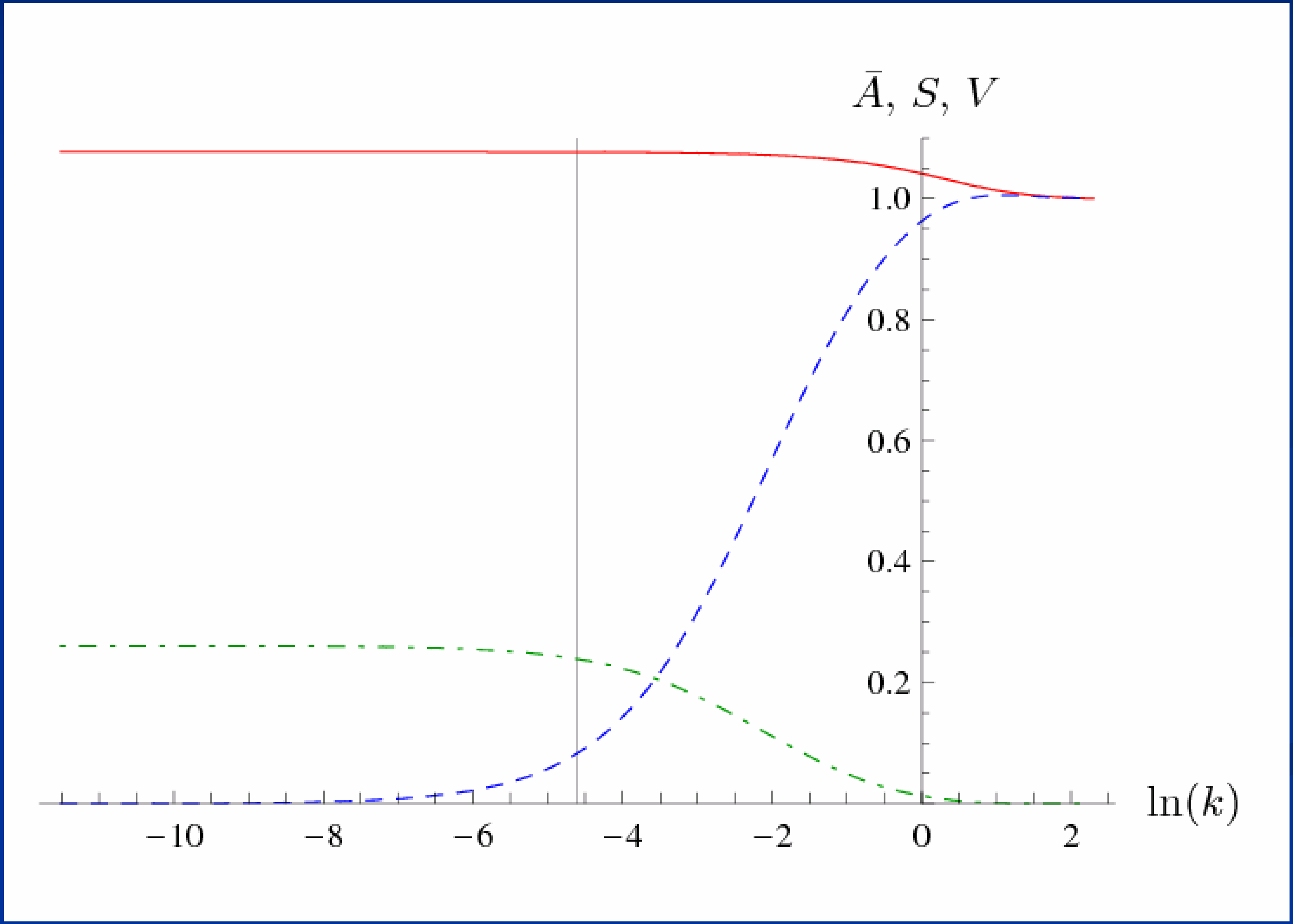
non – relativistic bosons

S. Floerchinger , …
see also N. Dupuis

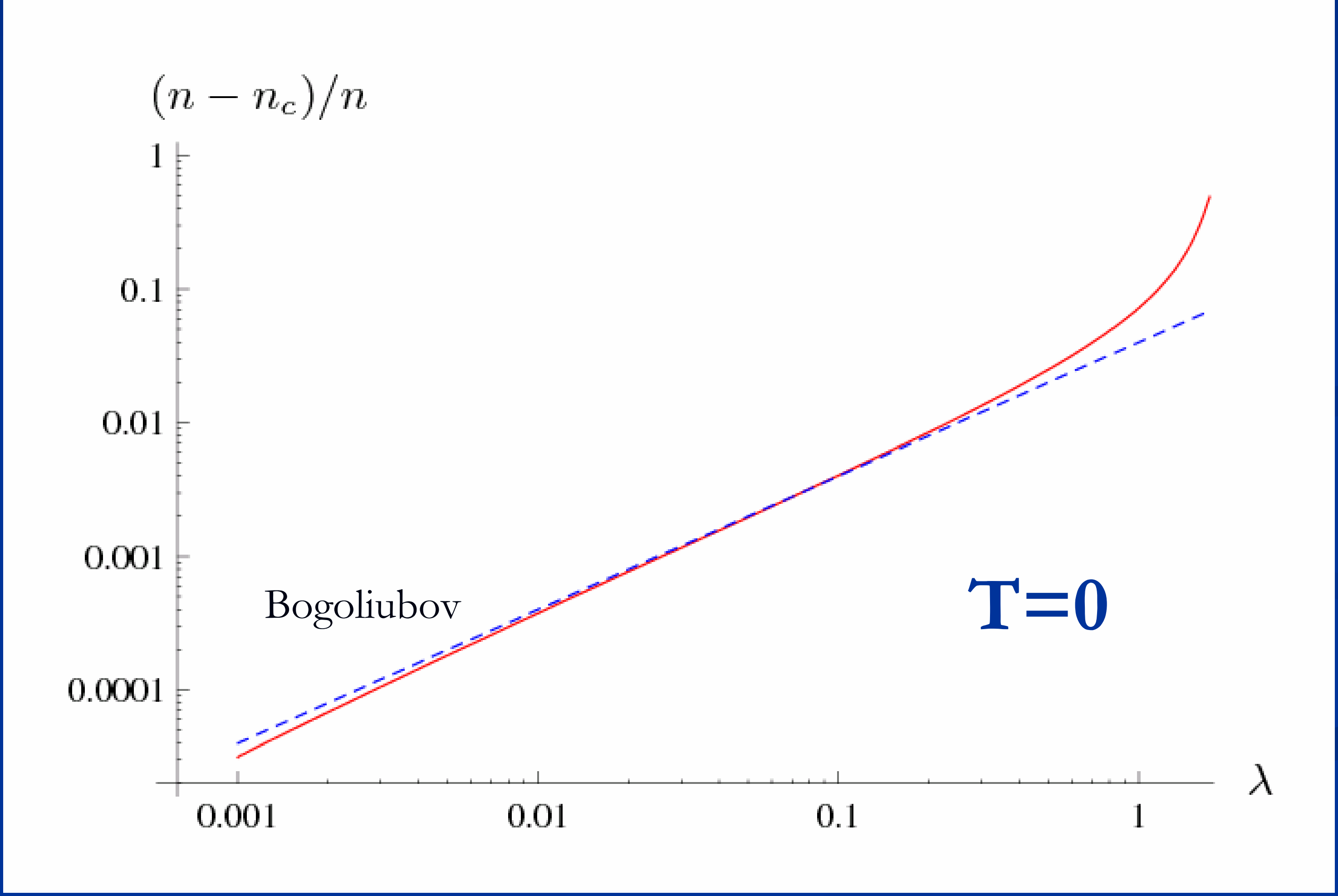
$$\Gamma_k = \int_x \Big\{ \phi^* \left(S\partial_\tau - \Delta - V\partial_\tau^2 \right) \phi$$
$$+2V(\mu - \mu_0)\, \phi^* \left(\partial_\tau - \Delta \right) \phi + U(\rho, \mu) \Big\}$$

arbitrary d , here d=2

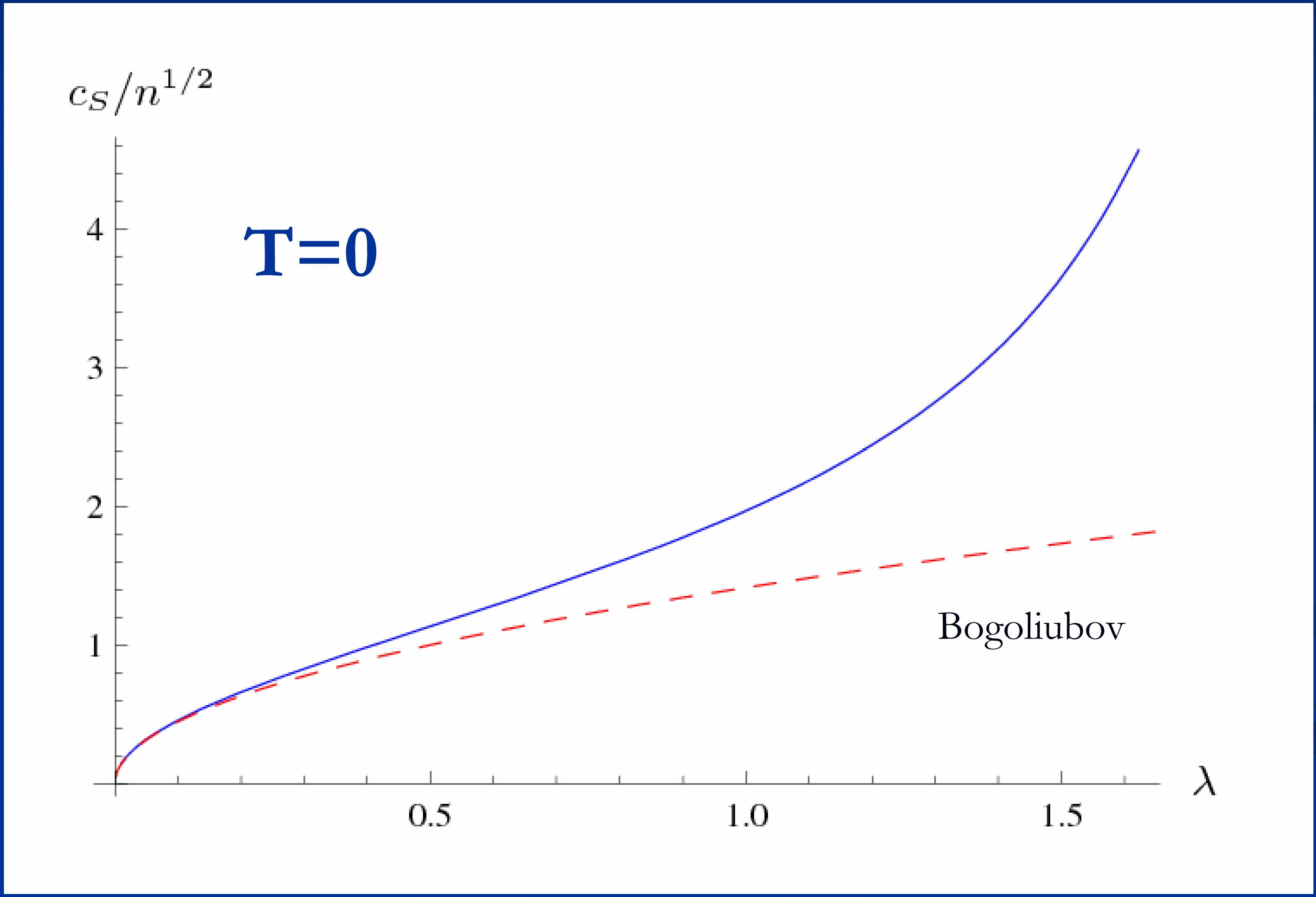
flow of kinetic and gradient coefficients



density depletion

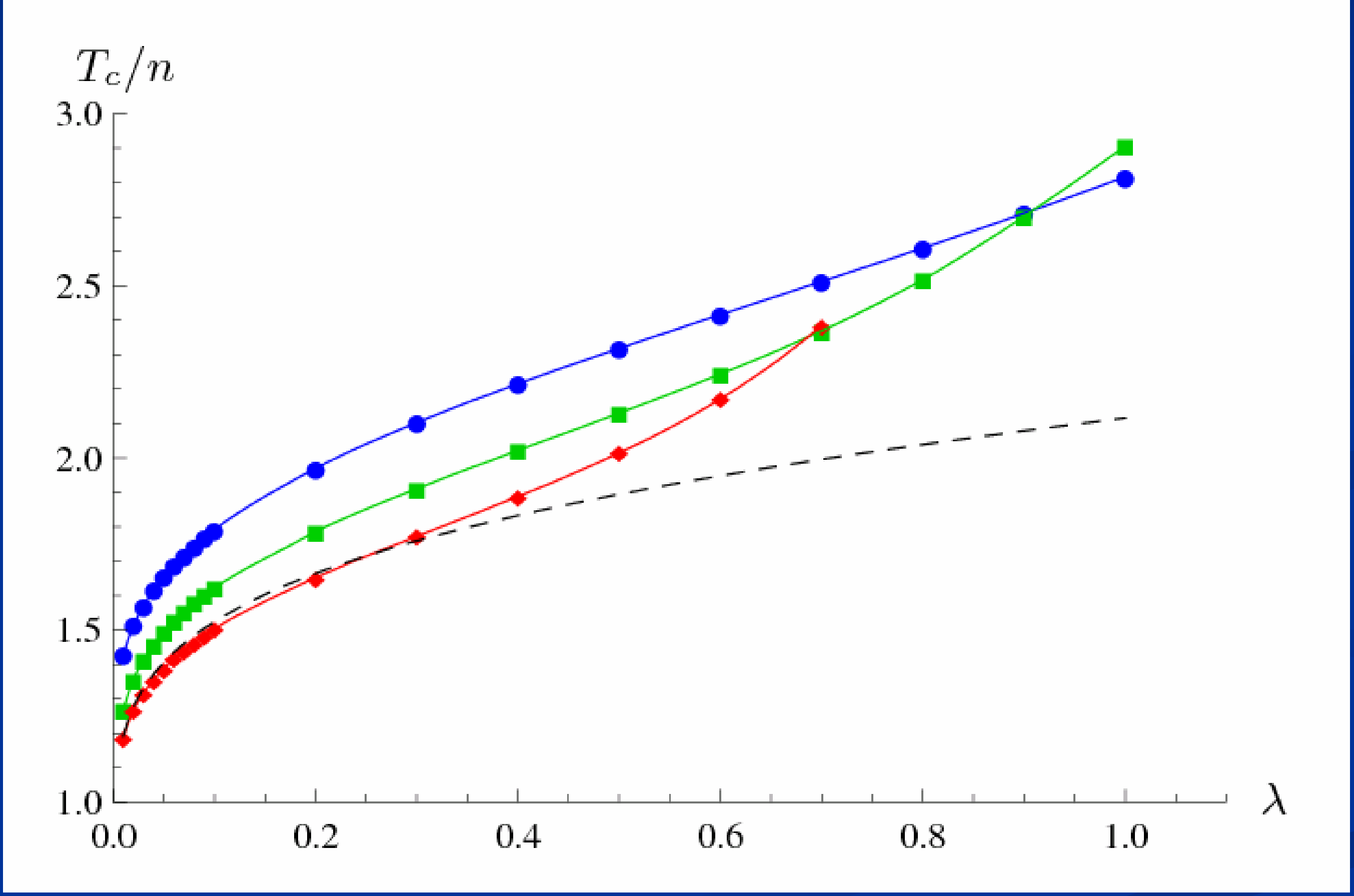


sound velocity



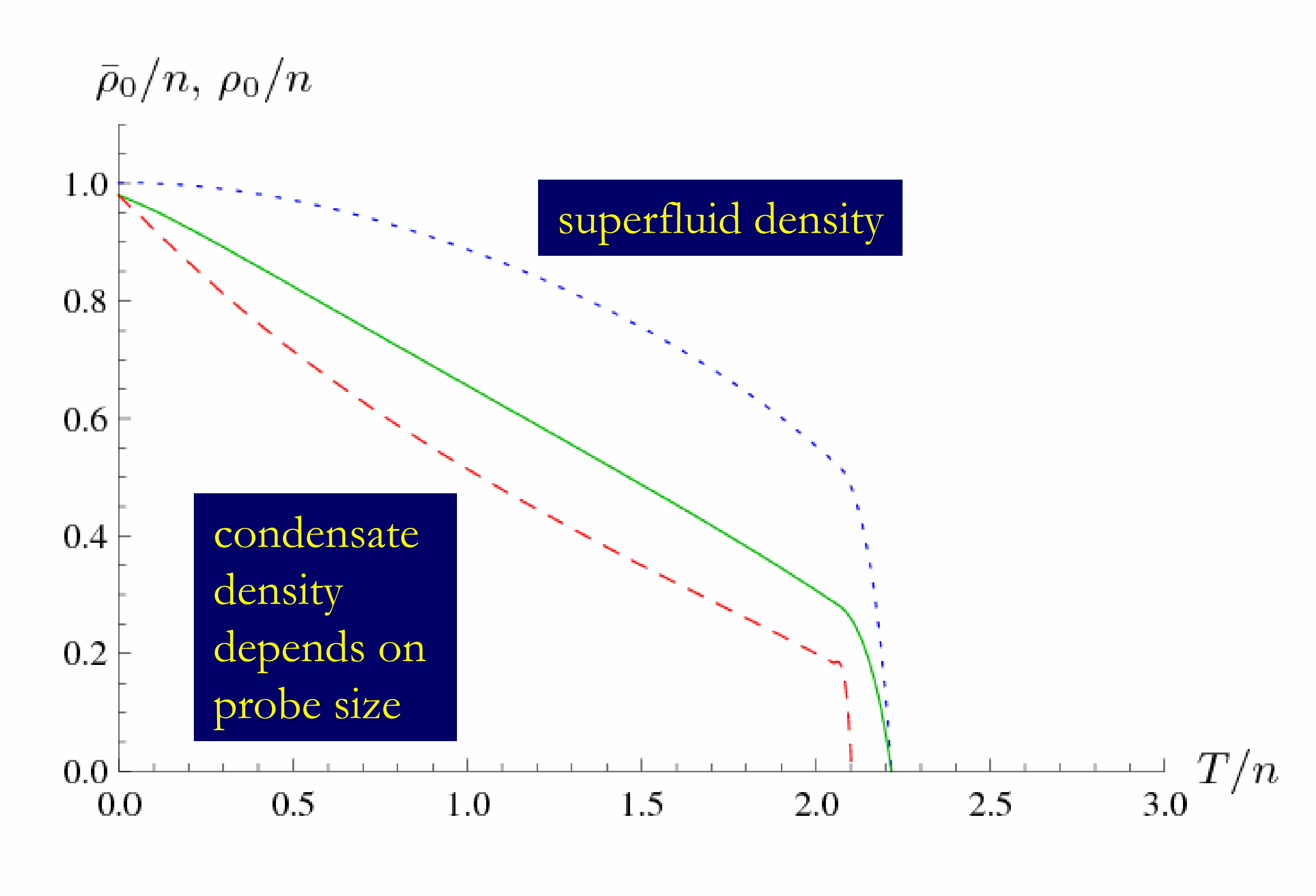
critical temperature

depends on size of probe



T_c vanishes logarithmically for infinite volume

condensate and superfluid density



thermodynamics
for
large finite systems

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,4,...
- linear and non-linear sigma models
- vortices and perturbation theory
- bosonic and fermionic models

☺ relativistic and non-relativistic physics

☺ classical and quantum statistics

☺ non-universal and universal aspects

- homogenous systems and local disorder
- equilibrium and out of equilibrium

wide applications

particle physics

- gauge theories, QCD

  Reuter,…, Marchesini et al, Ellwanger et al, Litim, Pawlowski, Gies ,Freire, Morris et al., Braun , many others

- electroweak interactions, gauge hierarchy problem

  Jaeckel,Gies,…

- electroweak phase transition

  Reuter,Tetradis,…Bergerhoff,

wide applications

gravity

- asymptotic safety

  Reuter, Lauscher, Schwindt et al, Percacci et al, Litim, Fischer, Saueressig

wide applications

condensed matter

- unified description for classical bosons

  CW , Tetradis , Aoki , Morikawa , Souma, Sumi , Terao , Morris , Graeter , v.Gersdorff , Litim , Berges , Mouhanna , Delamotte , Canet , Bervilliers , Blaizot , Benitez , Chatie , Mendes-Galain , Wschebor

- Hubbard model

  Baier , Bick,…, Metzner et al, Salmhofer et al, Honerkamp et al, Krahl , Kopietz et al, Katanin , Pepin , Tsai , Strack , Husemann , Lauscher

wide applications

condensed matter

- quantum criticality

  Floerchinger , Dupuis , Sengupta , Jakubczyk ,

- sine- Gordon model

  Nagy , Polonyi

- disordered systems

  Tissier , Tarjus , Delamotte , Canet

wide applications

condensed matter

- equation of state for CO_2 Seide,…
- liquid He^4 Gollisch,… and He^3 Kindermann,…
- frustrated magnets Delamotte, Mouhanna, Tissier
- nucleation and first order phase transitions
  Tetradis, Strumia,…, Berges,…

wide applications

condensed matter

- crossover phenomena

  Bornholdt , Tetradis ,…
- superconductivity (scalar QED_3)

  Bergerhoff , Lola , Litim , Freire,…
- non equilibrium systems

  Delamotte , Tissier , Canet , Pietroni , Meden , Schoeller , Gasenzer , Pawlowski , Berges , Pletyukov , Reininghaus

wide applications

nuclear physics

- effective NJL- type models

  Ellwanger , Jungnickel , Berges , Tetradis,…, Pirner , Schaefer , Wambach , Kunihiro , Schwenk

- di-neutron condensates

  Birse, Krippa,

- equation of state for nuclear matter

  Berges, Jungnickel …, Birse, Krippa

- nuclear interactions

  Schwenk

wide applications

ultracold atoms

- Feshbach resonances

  Diehl, Krippa, Birse , Gies, Pawlowski , Floerchinger , Scherer , Krahl ,

- BEC

  Blaizot, Wschebor, Dupuis, Sengupta, Floerchinger