Functional renormalization group for the effective average action

wide applications

particle physics

- gauge theories, QCD

  Reuter,…, Marchesini et al, Ellwanger et al, Litim, Pawlowski, Gies ,Freire, Morris et al., many others

- electroweak interactions, gauge hierarchy problem

  Jaeckel,Gies,…

- electroweak phase transition

  Reuter,Tetradis,…Bergerhoff,

wide applications

gravity

- asymptotic safety

  Reuter, Lauscher, Schwindt et al, Percacci et al, Litim, Fischer

wide applications

condensed matter

- unified description for classical bosons

  CW, Tetradis , Aoki, Morikawa, Souma, Sumi, Terao , Morris , Graeter, v.Gersdorff, Litim , Berges, Mouhanna, Delamotte, Canet, Bervilliers,

- Hubbard model Baier,Bick,…, Metzner et al, Salmhofer et al, Honerkamp et al, Krahl,

- disordered systems Tissier, Tarjus ,Delamotte, Canet

wide applications

condensed matter

- equation of state for CO_2 Seide,…
- liquid He^4 Gollisch,… and He^3 Kindermann,…
- frustrated magnets Delamotte, Mouhanna, Tissier
- nucleation and first order phase transitions
  Tetradis, Strumia,…, Berges,…

wide applications

condensed matter

- crossover phenomena Bornholdt, Tetradis,…
- superconductivity (scalar QED_3)
  Bergerhoff, Lola, Litim , Freire,…
- non equilibrium systems
  Delamotte, Tissier, Canet, Pietroni

wide applications

nuclear physics

- effective NJL- type models

  Ellwanger, Jungnickel, Berges, Tetradis,…, Pirner, Schaefer, Wambach, Kunihiro, Schwenk,

- di-neutron condensates

  Birse, Krippa,

- equation of state for nuclear matter

  Berges, Jungnickel …, Birse, Krippa

wide applications

ultracold atoms

- Feshbach resonances Diehl, Gies, Pawlowski ,…, Krippa,
- BEC Blaizot, Wschebor, Dupuis, Sengupta

unified description of scalar models for all d and N

Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

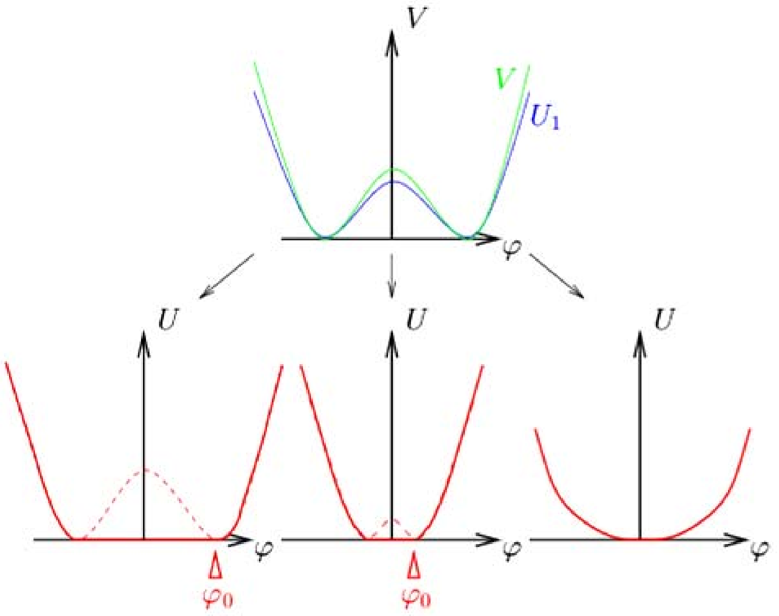
$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

Scalar field theory

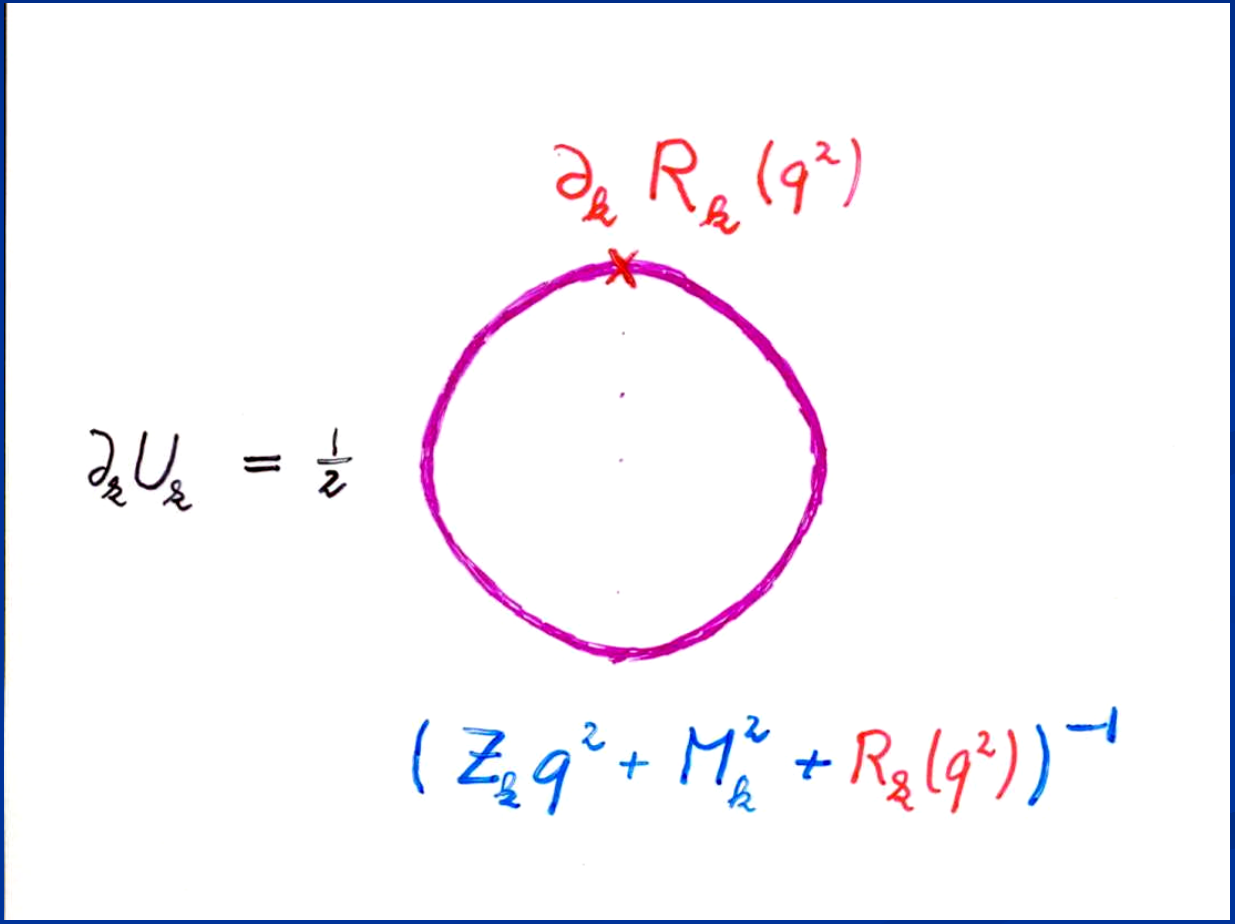
$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Simple one loop structure – nevertheless (almost) exact



Infrared cutoff

R_k : IR-cutoff

e.g $$R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$$

or $$R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad \text{(Litim)}$$

$$\lim_{k\to 0} R_k = 0$$

$$\lim_{k\to\infty} R_k \to \infty$$

Wave function renormalization and anomalous dimension

Z_k: wave function renormalization

$$k\partial_k Z_k = -\eta_k Z_K$$

η_k: anomalous dimension

$$t = \ln(k/\Lambda)$$

$$\partial_t \ln Z = -\eta$$

for $Z_k(\varphi, q^2)$: flow equation is **exact !**

Scaling form of evolution equation

$$u = \frac{U_k}{k^d}$$

$$\tilde{\rho} = Z_k k^{2-d} \rho$$

$$u' = \frac{\partial u}{\partial \tilde{\rho}} \quad \text{etc.}$$

$$\begin{aligned} \partial_t u|_{\tilde{\rho}} = & -du + (d-2+\eta)\tilde{\rho}u' \\ & +2v_d\{l_0^d(u'+2\tilde{\rho}u'';\eta) \\ & +(N-1)\, l_0^d(u';\eta)\} \end{aligned}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

linear cutoff:

$$l_0^d(w;\eta) = \frac{2}{d}\left(1 - \frac{\eta}{d+2}\right)\frac{1}{1+w}$$

On r.h.s. :
neither the scale k
nor the wave function
renormalization Z
appear explicitly.

Scaling solution:
no dependence on t;
corresponds
to second order
phase transition.

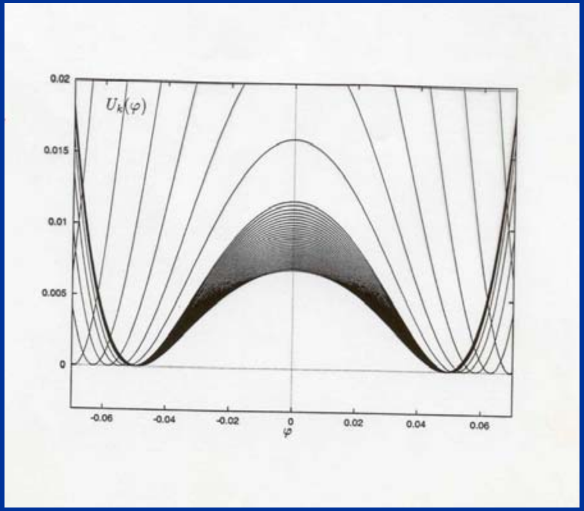
Tetradis …

unified approach

- choose N
- choose d
- choose initial form of potential
- run !

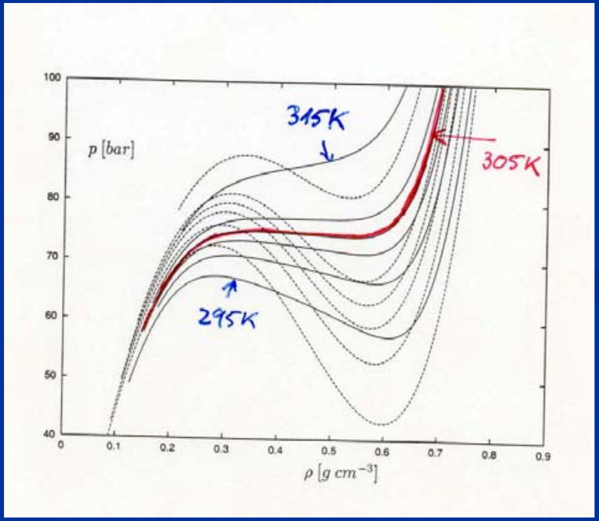
Flow of effective potential

Ising model



S.Seide …

CO_2



Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

Critical exponents

$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003
		↑		↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Critical exponents , d=3

N	ν (ERGE)	ν (world)	η (ERGE)	η (world)
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
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"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

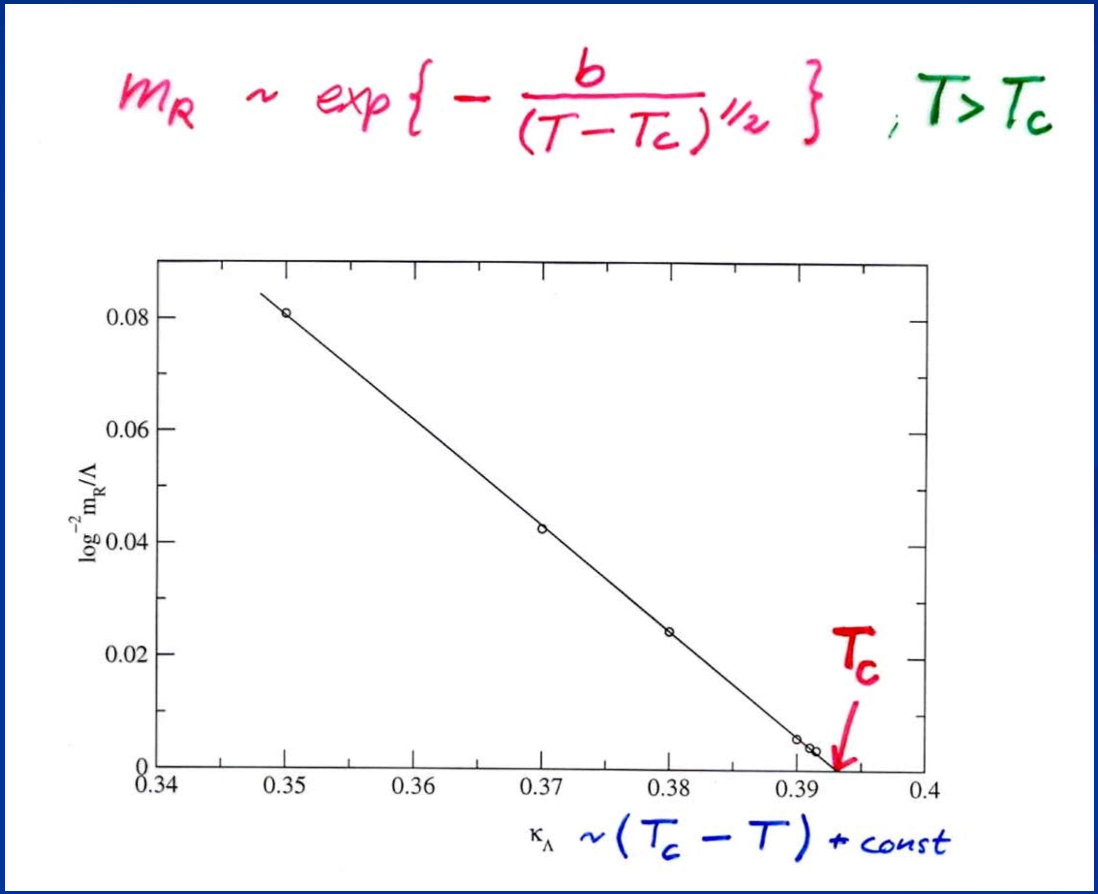
Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:

Kosterlitz-Thouless phase transition

Essential scaling : d=2,N=2



- Flow equation contains correctly the non-perturbative information !
- (essential scaling usually described by vortices)

Von Gersdorff …

Kosterlitz-Thouless phase transition (d=2,N=2)

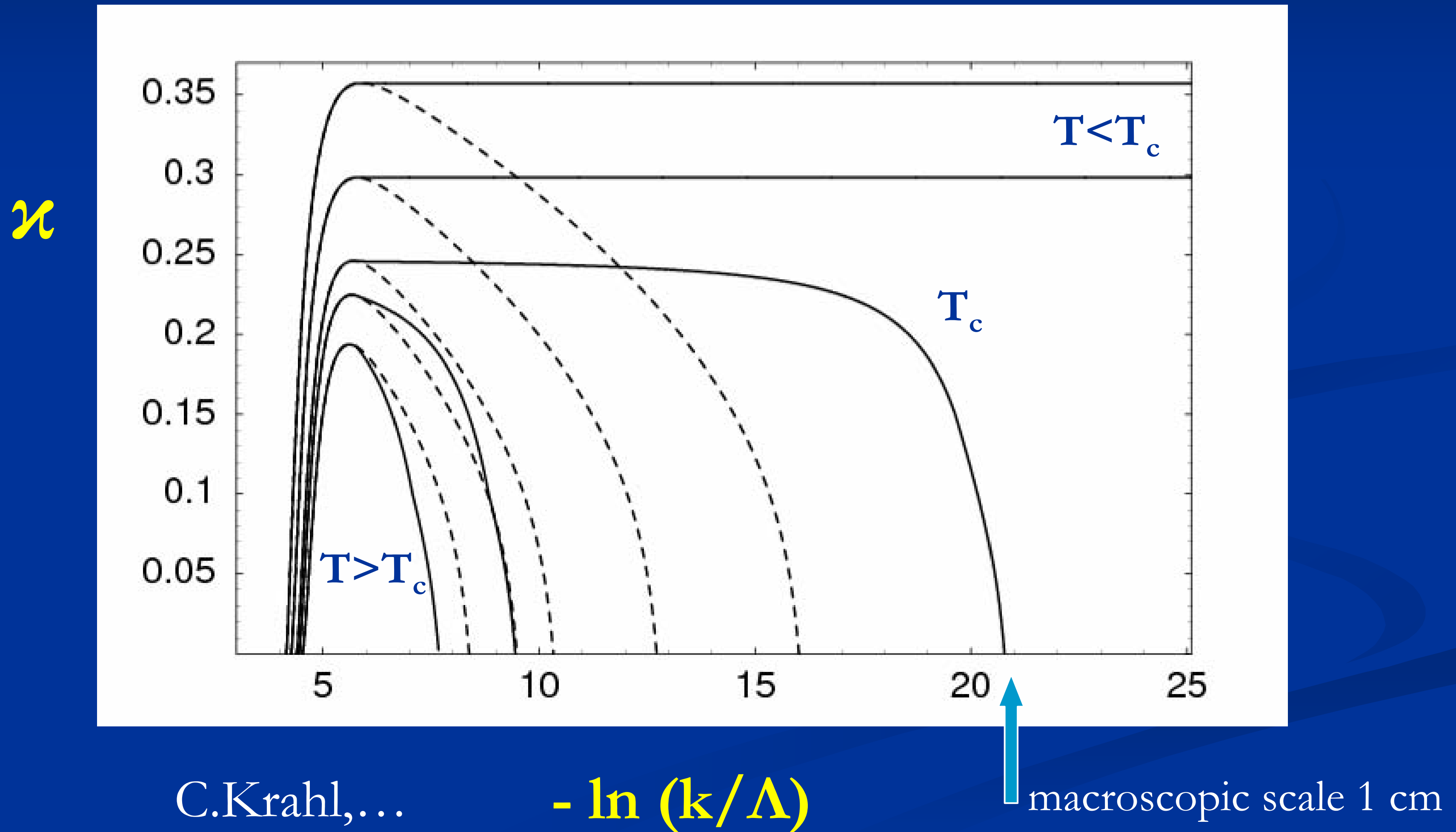
Correct description of phase with

Goldstone boson

(infinite correlation length)

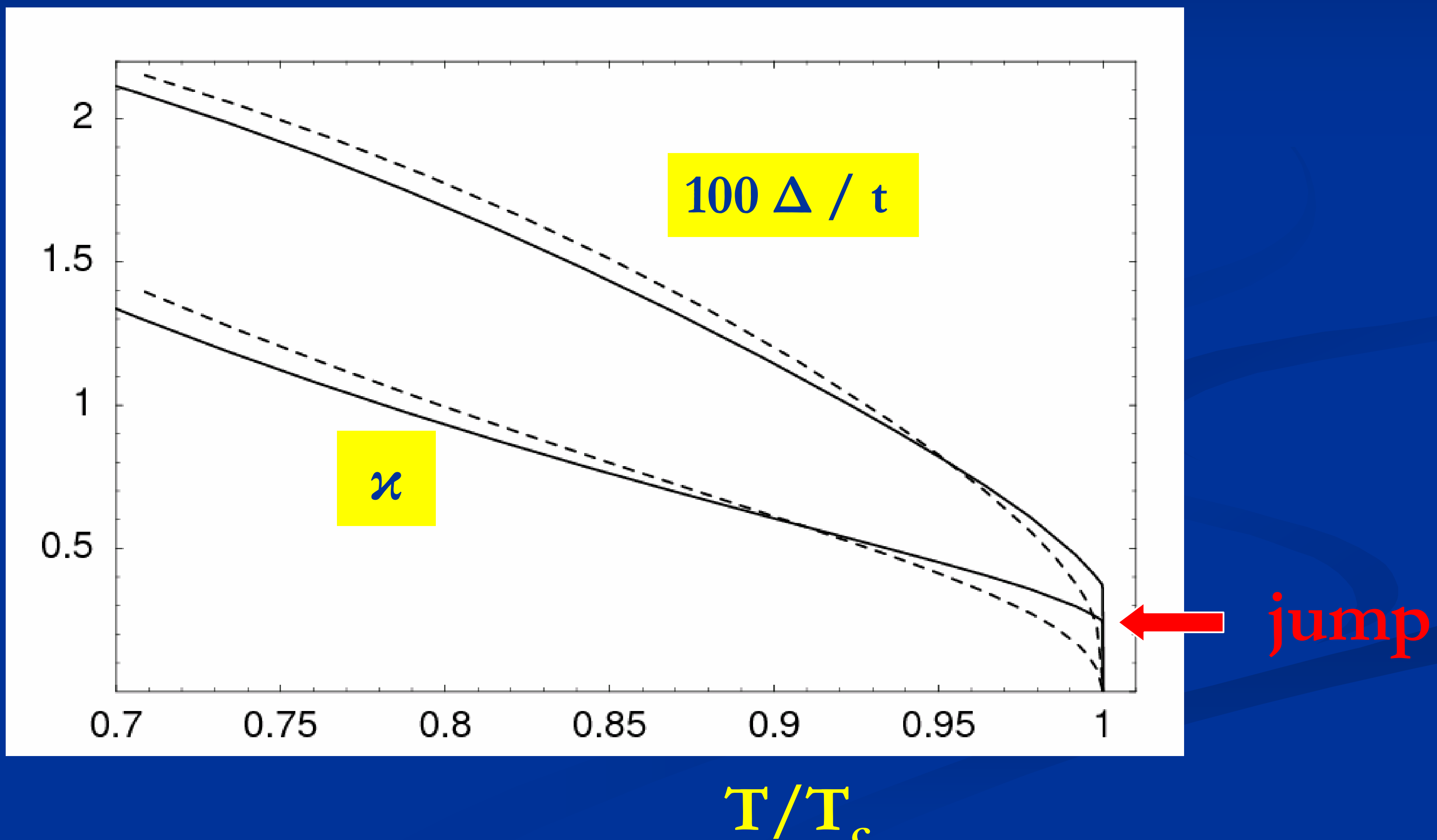
for $T<T_c$

Running renormalized d-wave superconducting order parameter ϰ in doped Hubbard model

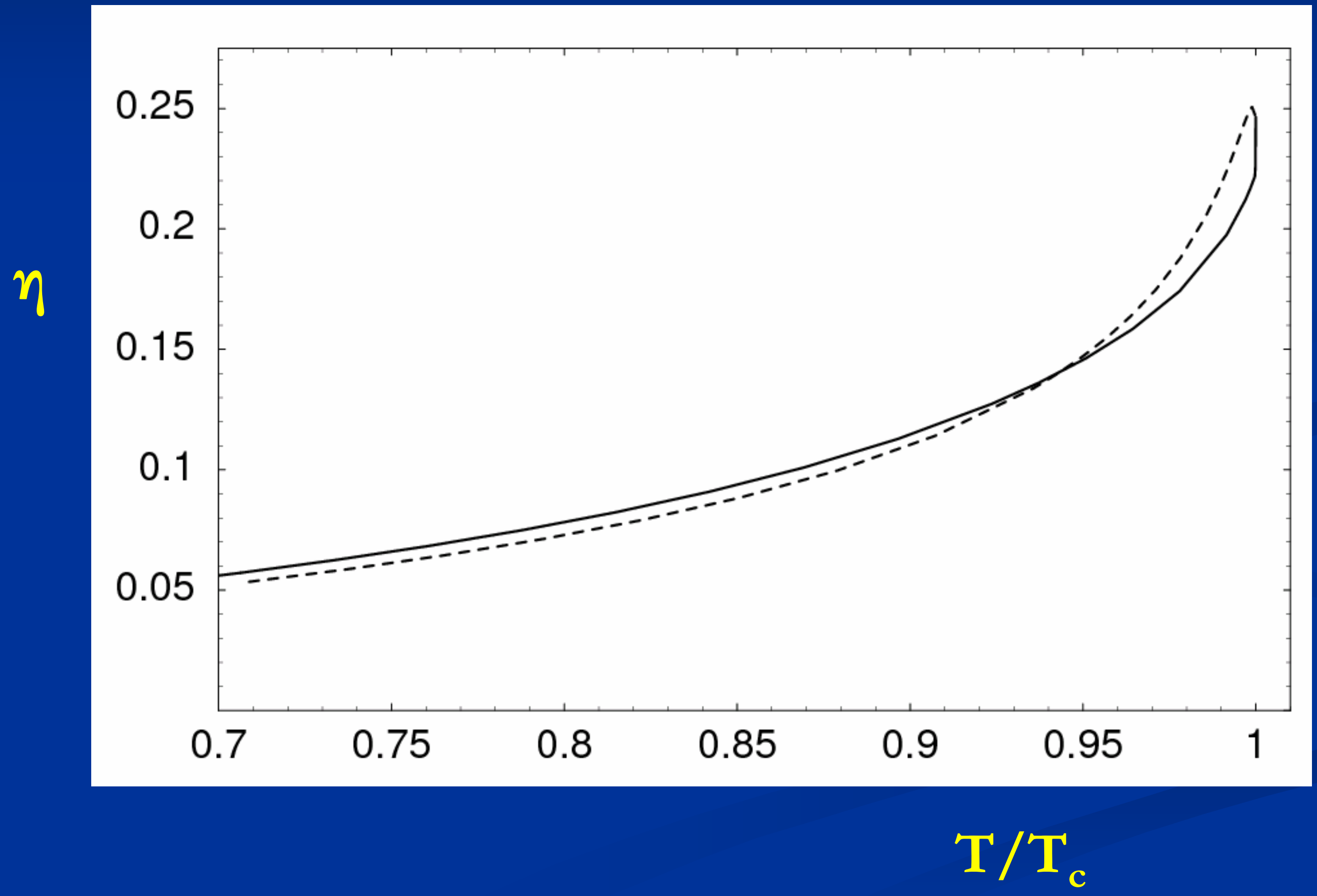


C.Krahl,…

Renormalized order parameter ϰ and gap in electron propagator Δ in doped Hubbard model



Temperature dependent anomalous dimension η



convergence and errors

- for precise results: systematic derivative expansion in second order in derivatives
- includes field dependent wave function renormalization $Z(\rho)$
- fourth order : similar results
- apparent fast convergence : no series resummation
- rough error estimate by different cutoffs and truncations

including fermions :

no particular problem !

changing degrees of freedom

Antiferromagnetic order in the Hubbard model

A functional renormalization group study

T.Baier, E.Bick, …

Hubbard model

Functional integral formulation

$$Z[\eta] = \int_{\hat\psi(\beta)=-\hat\psi(0),\hat\psi^*(\beta)=-\hat\psi^*(0)} \mathcal{D}(\hat\psi^*(\tau),\hat\psi(\tau))$$
$$\exp\Big(-\int_0^\beta d\tau\Big(\sum_{\mathbf{x}}\hat\psi^\dagger_{\mathbf{x}}(\tau)\big(\frac{\partial}{\partial\tau}-\mu\big)\hat\psi_{\mathbf{x}}(\tau)$$
$$+\sum_{\mathbf{xy}}\hat\psi^\dagger_x(\tau)\mathcal{T}_{\mathbf{xy}}\hat\psi_{\mathbf{y}}(\tau)$$
$$+\frac{1}{2}U\sum_{\mathbf{x}}(\hat\psi^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau))^2$$
$$-\sum_{\mathbf{x}}(\eta^\dagger_{\mathbf{x}}(\tau)\hat\psi_{\mathbf{x}}(\tau)+\eta^T_{\mathbf{x}}(\tau)\hat\psi^*_{\mathbf{x}}(\tau))\Big)\Big)$$

next neighbor interaction

$$\mathcal{T}_{xy} = \begin{cases} -t & \text{, if } \boldsymbol{x} \text{ and } \boldsymbol{y} \text{ are nearest neighbors} \\ 0 & \text{, else} \end{cases}$$

U > 0 :
repulsive local interaction

External parameters
T : temperature
μ : chemical potential
(doping)

Fermion bilinears

$$\tilde{\rho}(X) = \hat{\psi}^\dagger(X)\hat{\psi}(X),$$
$$\tilde{\vec{m}}(X) = \hat{\psi}^\dagger(X)\vec{\sigma}\hat{\psi}(X)$$

Introduce sources for bilinears

$$S_F = S_{F,\text{kin}} + \frac{1}{2}U(\hat{\psi}^\dagger\hat{\psi})^2 - J_\rho\tilde{\rho} - \vec{J}_m\tilde{\vec{m}}$$

Functional variation with respect to sources J yields expectation values and correlation functions

$$Z = \int \mathcal{D}(\psi^*,\psi)\exp\left(-(S_F + S_\eta)\right)$$
$$S_\eta = -\eta^\dagger\psi - \eta^T\psi^*$$

Partial Bosonisation

- collective bosonic variables for fermion bilinears
- insert identity in functional integral
  (Hubbard-Stratonovich transformation)
- replace four fermion interaction by equivalent bosonic interaction (e.g. mass and Yukawa terms)
- problem : decomposition of fermion interaction into bilinears not unique (Grassmann variables)

$$(\hat{\psi}^\dagger(X)\hat{\psi}(X))^2 = \tilde{\rho}(X)^2 = -\frac{1}{3}\tilde{\vec{m}}(X)^2$$

Partially bosonised functional integral

$$Z[\eta,\eta^*,J_\rho,\vec{J}_m] = \int \mathcal{D}(\psi^*,\psi,\hat{\rho},\hat{\vec{m}}) \exp\big(-(S+S_\eta+S_J)\big)$$

$$S = S_{F,\text{kin}} + \frac{1}{2}U_\rho\hat{\rho}^2 + \frac{1}{2}U_m\hat{\vec{m}}^2 - U_\rho\hat{\rho}\tilde{\rho} - U_m\hat{\vec{m}}\tilde{\vec{m}},$$
$$S_J = -\,J_\rho\hat{\rho} - \vec{J}_m\hat{\vec{m}}$$

equivalent to
fermionic functional integral

if $U = -U_\rho + 3U_m$

Bosonic integration is Gaussian

or:

solve bosonic field equation as functional of fermion fields and reinsert into action

$$\hat{\rho} = \tilde{\rho} + \frac{J_\rho}{U_\rho}, \qquad \hat{\vec{m}} = \tilde{\vec{m}} + \frac{\vec{J}_m}{U_m}$$

fermion – boson action

$$S = S_{F,\mathrm{kin}} + S_B + S_Y + S_J,$$

fermion kinetic term

$$S_{F,\mathrm{kin}} = \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q),$$

boson quadratic term ("classical propagator")

$$S_B = \frac{1}{2}\sum_Q \left(U_\rho \hat{\rho}(Q)\hat{\rho}(-Q) + U_m \hat{\vec{m}}(Q)\hat{\vec{m}}(-Q)\right),$$

Yukawa coupling

$$S_Y = -\sum_{QQ'Q''} \delta(Q - Q' + Q'') \times (U_\rho \hat{\rho}(Q)\hat{\psi}^\dagger(Q')\hat{\psi}(Q'') + U_m \hat{\vec{m}}(Q)\hat{\psi}^\dagger(Q')\vec{\sigma}\hat{\psi}(Q'')),$$

source term

$$S_J = -\sum_Q \left(J_\rho(-Q)\hat{\rho}(Q) + \vec{J}_m(-Q)\hat{\vec{m}}(Q) \right)$$

is now linear in the bosonic fields

Mean Field Theory (MFT)

Evaluate Gaussian fermionic integral
in background of bosonic field , e.g.

$$\begin{aligned} \hat{\rho}(Q) &\rightarrow \rho\delta(Q) \\ \hat{\vec{m}}(Q) &\rightarrow \vec{a}\delta(Q-\Pi) \end{aligned}$$

$$\begin{aligned} Z_{\mathrm{MF}} &= \int \mathcal{D}(\hat{\psi}^*, \hat{\psi}) \exp(-S_{\mathrm{MF}}), \\ S_{\mathrm{MF}} &= \sum_Q \hat{\psi}^\dagger(Q)(i\omega_F - \mu - 2t(\cos q_1 + \cos q_2))\hat{\psi}(Q) \\ &\quad - \sum_Q (U_\rho \rho \hat{\psi}^\dagger(Q)\hat{\psi}(Q) + U_m \vec{a}\hat{\psi}^\dagger(Q+\Pi)\vec{\sigma}\hat{\psi}(Q)) \\ &\quad + \frac{V_2}{2T}(U_\rho \rho^2 + U_m \vec{a}^2) - J_\rho(0)\rho - \vec{J}_m(-\Pi)\vec{a} \end{aligned}$$

$$\Gamma_{\mathrm{MF}} = -\ln Z_{\mathrm{MF}} + J_\rho(0)\rho + \vec{J}_m(-\Pi)\vec{a}$$

Effective potential in mean field theory

$$U(\rho,\vec{a}) = \frac{T\Gamma}{V_2} = \frac{1}{2}(U_\rho \rho^2 + U_m \vec{a}^2) + \Delta U(\rho,\vec{a})$$

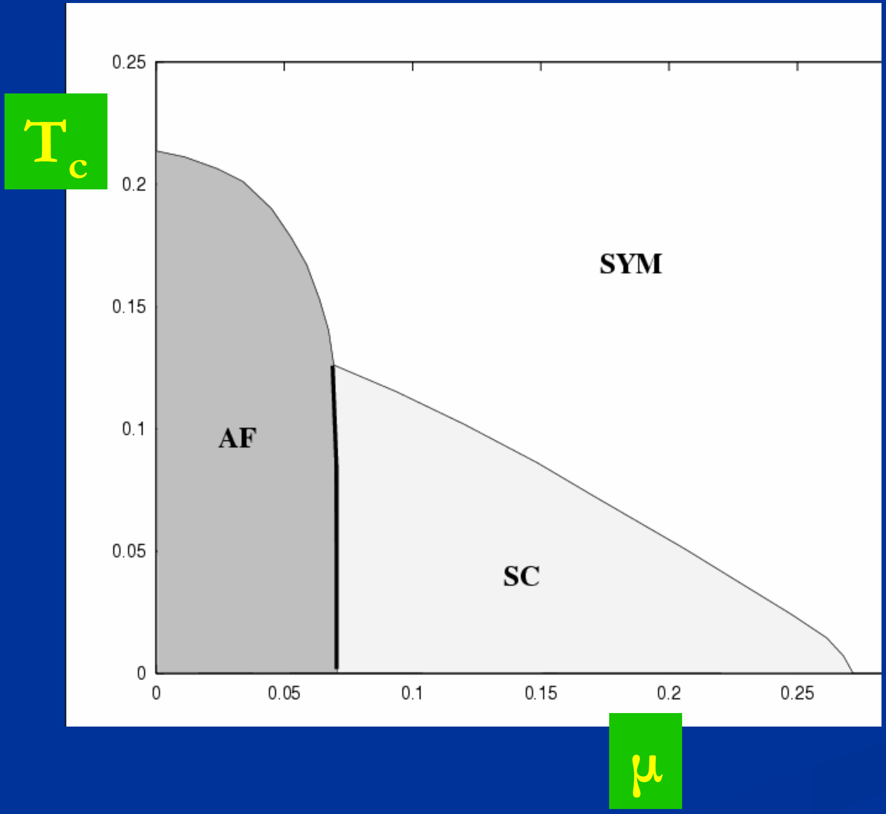
$$\Delta U(\rho,\vec{a}) = -\frac{T}{V_2}\ln\int \mathcal{D}(\hat{\psi}^*,\hat{\psi})\exp(-S_\Delta),$$

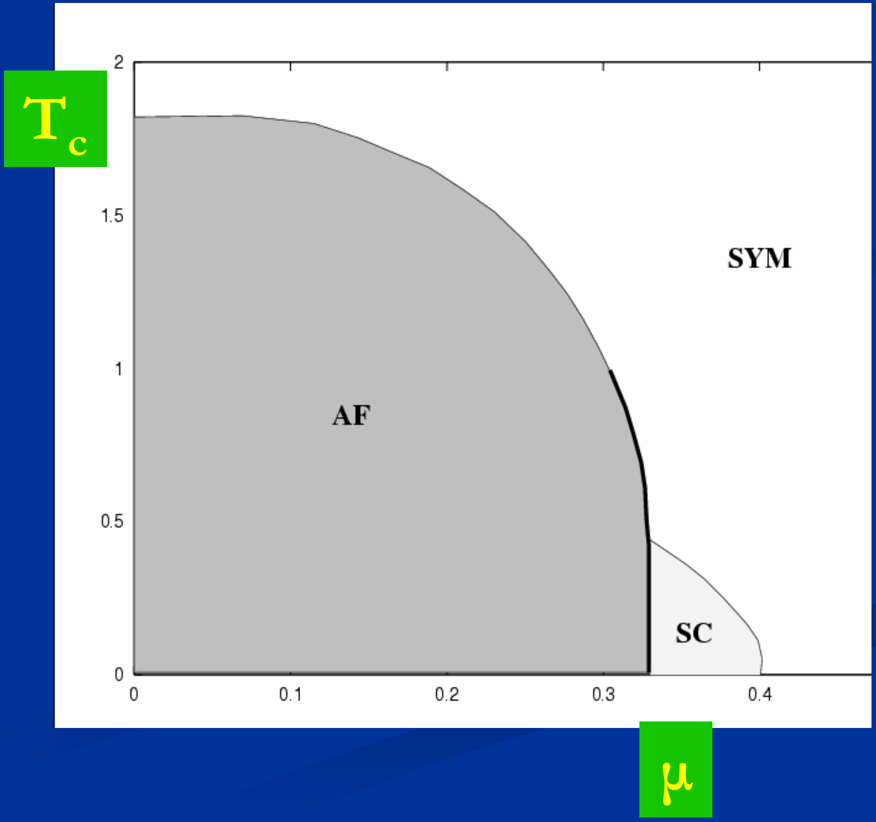
$$S_\Delta = \sum_Q \left(\hat{\psi}^\dagger(Q)P(Q)\hat{\psi}(Q) - U_m\vec{a}\hat{\psi}^\dagger(Q+\Pi)\vec{\sigma}\hat{\psi}(Q)\right)$$

$$\begin{aligned} P(Q) &= i\omega_F - \mu_{\text{eff}} - 2t(\cos q_1 + \cos q_2), \\ \mu_{\text{eff}} &= \mu + U_\rho \rho. \end{aligned}$$

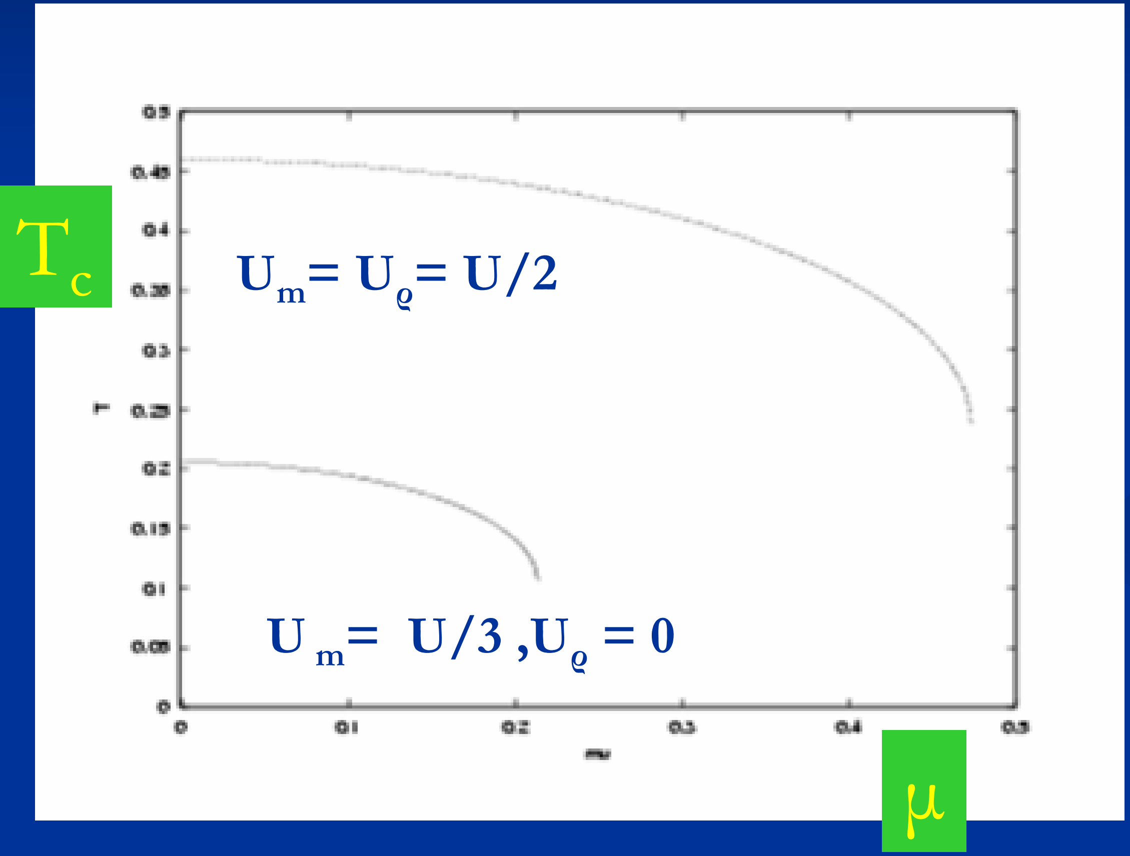
Mean field phase diagram

for two different choices of couplings – same U !





Mean field ambiguity



Artefact of approximation …

cured by inclusion of bosonic fluctuations

J.Jaeckel,…

mean field phase diagram

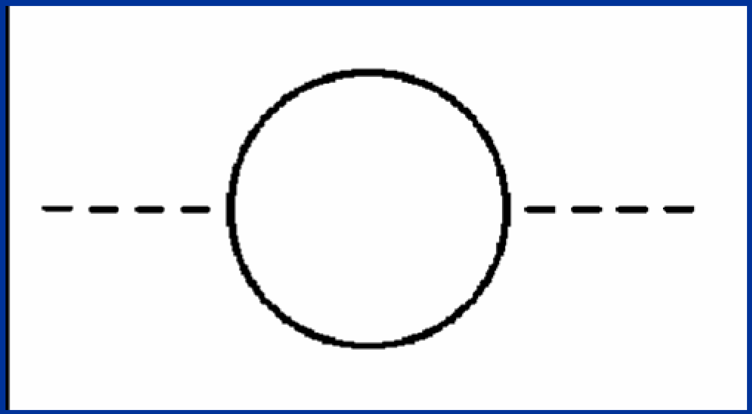
$$U = -U_\rho + 3U_m$$

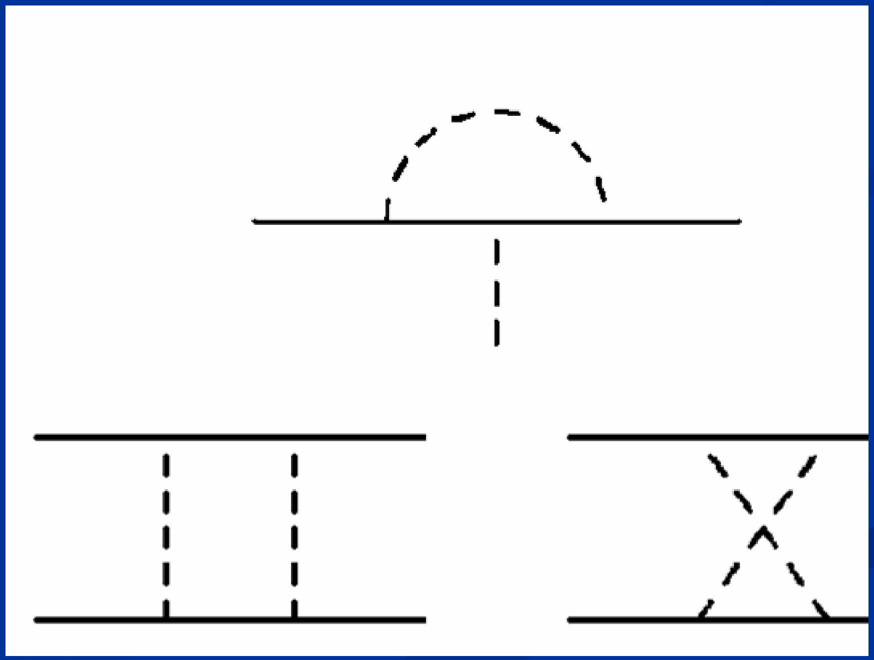
Rebosonization and the mean field ambiguity

Bosonic fluctuations

fermion loops

boson loops

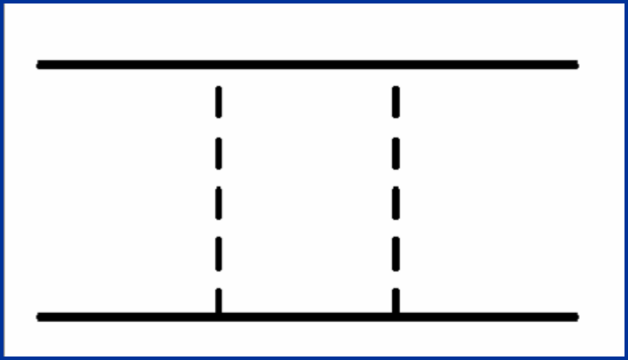




mean field theory

Rebosonization

- adapt bosonization to every scale k such that



is translated to bosonic interaction

H.Gies , …

$$\Gamma_k[\psi,\psi^*,\phi] = \sum_Q \psi^*(Q)P_{\psi,k}\psi(Q) + \frac{1}{2}\sum_Q \phi(-Q)P_{\phi,k}(Q)\phi(Q) - \sum_Q h_k(Q)\phi(Q)\tilde{\phi}(-Q) + \sum_Q \lambda_{\psi,k}(Q)\tilde{\phi}(Q)\tilde{\phi}(-Q)$$

k-dependent field redefinition

$$\phi_k(Q) = \phi_{\bar{k}}(Q) + \Delta\alpha_k(Q)\tilde{\phi}(Q)$$

$$\partial_k\phi_k(Q) = -\,\partial_k\alpha_k(Q)\tilde{\phi}(Q)$$

absorbs four-fermion coupling

Modification of evolution of couplings …

Evolution with k-dependent field variables

$$\partial_k \Gamma_k[\psi,\psi^*,\phi_k] = \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \left(\frac{\delta}{\delta\phi_k}\Gamma[\psi,\psi^*,\phi_k]\right)\partial_k\phi_k$$
$$= \partial_k \Gamma_k[\psi,\psi^*,\phi_k]|_{\phi_k} + \sum_Q \big(-\partial_k\alpha_k(Q)P_{\phi,k}(Q)\phi_k(Q)\tilde{\phi}(-Q) + h_k(Q)\partial_k\alpha_k(Q)\tilde{\phi}(Q)\tilde{\phi}(-Q)\big)$$

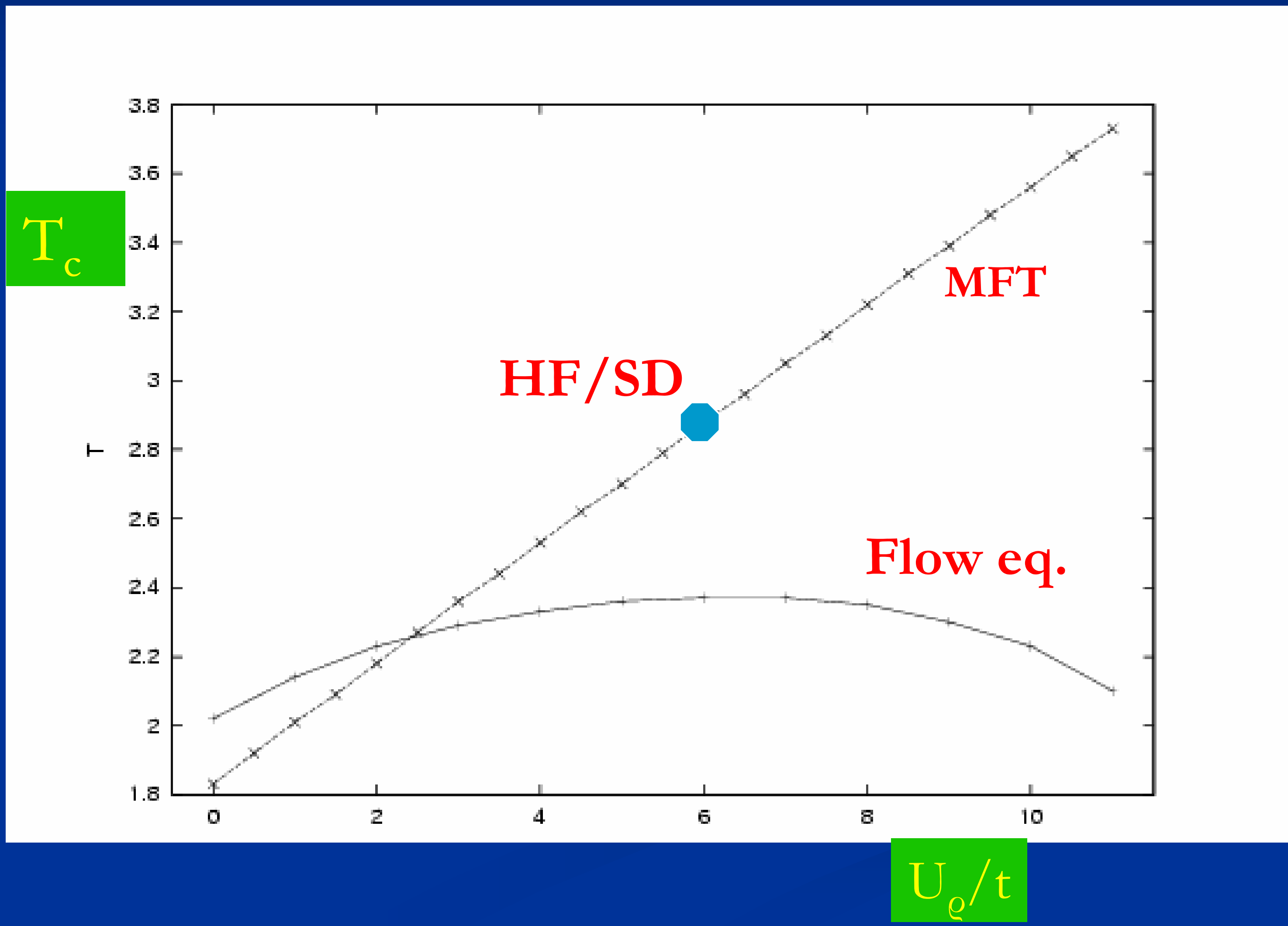
Rebosonisation

$$\partial_k h_k(Q) = \partial_k h_k(Q)|_{\phi_k} + \partial_k\alpha_k(Q)P_{\phi,k}(Q),$$
$$\partial_k \lambda_{\psi,k}(Q) = \partial_k \lambda_{\psi,k}(Q)|_{\phi_k} + h_k(Q)\,\partial_k\alpha_k(Q).$$

Choose α_k such that no four fermion coupling is generated ➡

$$\partial_k h_k(Q) = \partial_k h_k(Q)|_{\phi_k} - \frac{P_{\phi,k}(Q)}{h_k(Q)}\partial_k\lambda_{\psi,k}(Q)|_{\phi_k}$$

…cures mean field ambiguity



Flow equation for the Hubbard model

T.Baier , E.Bick , …,C.Krahl

Truncation

Concentrate on antiferromagnetism

$$\vec{a}(Q) = \vec{m}(Q + \Pi)$$

Potential U depends only on $\alpha = a^2$

$$\Gamma_{\psi,k}[\psi, \psi^*] = \sum_Q \psi^\dagger(Q) P_F(Q) \psi(Q),$$

$$P_F(Q) = i\omega_F + \epsilon - \mu, \quad \epsilon(\boldsymbol{q}) = -2t(\cos q_x + \cos q_y),$$

$$\Gamma_{Y,k}[\psi, \psi^*, \vec{a}] = -\bar{h}_{a,k} \sum_{KQQ'} \vec{a}(K) \psi^*(Q) \vec{\sigma} \psi(Q') \times \delta(K - Q + Q' + \Pi)$$

$$\Gamma_{a,k}[\vec{a}] = \frac{1}{2} \sum_Q \vec{a}(-Q) P_a(Q) \vec{a}(Q) + \sum_X U[\vec{a}(X)]$$

$$\begin{aligned}
\mathrm{SYM}: \sum_X U[\vec{a}] &= \sum_K \bar{m}_a^2 \alpha(-K, K) + \\
&\quad + \frac{1}{2} \sum_{K_1 \ldots K_4} \bar{\lambda}_a \delta(K_1 + K_2 + K_3 + K_4) \\
&\quad \times \alpha(K_1, K_2) \alpha(K_3, K_4), \\
\mathrm{SSB}: \sum_X U[\vec{a}] &= \frac{1}{2} \sum_{K_1 \ldots K_4} \bar{\lambda}_a \delta(K_1 + K_2 + K_3 + K_4) \\
&\quad \times (\alpha(K_1, K_2) - \alpha_0 \delta(K_1) \delta(K_2)) \\
&\quad \times (\alpha(K_3, K_4) - \alpha_0 \delta(K_3) \delta(K_4))
\end{aligned}$$

$$\alpha(K, K') = \frac{1}{2} \vec{a}(K) \vec{a}(K')$$

scale evolution of effective potential for antiferromagnetic order parameter

$$\begin{aligned}\partial_k U\ \ (\alpha) &= \partial_k U^B(\alpha) + \partial_k U^F(\alpha) \\ &= \frac{1}{2}\sum_{Q,i} \tilde{\partial}_k \ln[P_a(Q) + \hat{M}_i^2(\alpha) + R_k^a(Q)] \\ &\quad -2T\int_{-\pi}^{\pi} \frac{d^2q}{(2\pi)^2}\tilde{\partial}_k \ln\cosh y(\alpha).\end{aligned}$$

boson contribution

fermion contribution

$$\hat{M}^2_{1,2,3}(\alpha) = \\ = \begin{cases} (\bar{m}_a^2 + 3\bar{\lambda}_a\alpha, \bar{m}_a^2 + \bar{\lambda}_a\alpha, \bar{m}_a^2 + \bar{\lambda}_a\alpha) & \text{SYM} \\ (\bar{\lambda}_a(3\alpha - \alpha_0), \bar{\lambda}_a(\alpha - \alpha_0), \bar{\lambda}_a(\alpha - \alpha_0)) & \text{SSB} \end{cases}$$

$$y(\alpha) = \frac{1}{2T_k}\sqrt{\epsilon^2(\boldsymbol{q}) + 2\bar{h}_a^2\alpha}.$$

effective masses depend on α !

gap for fermions ~α

running couplings

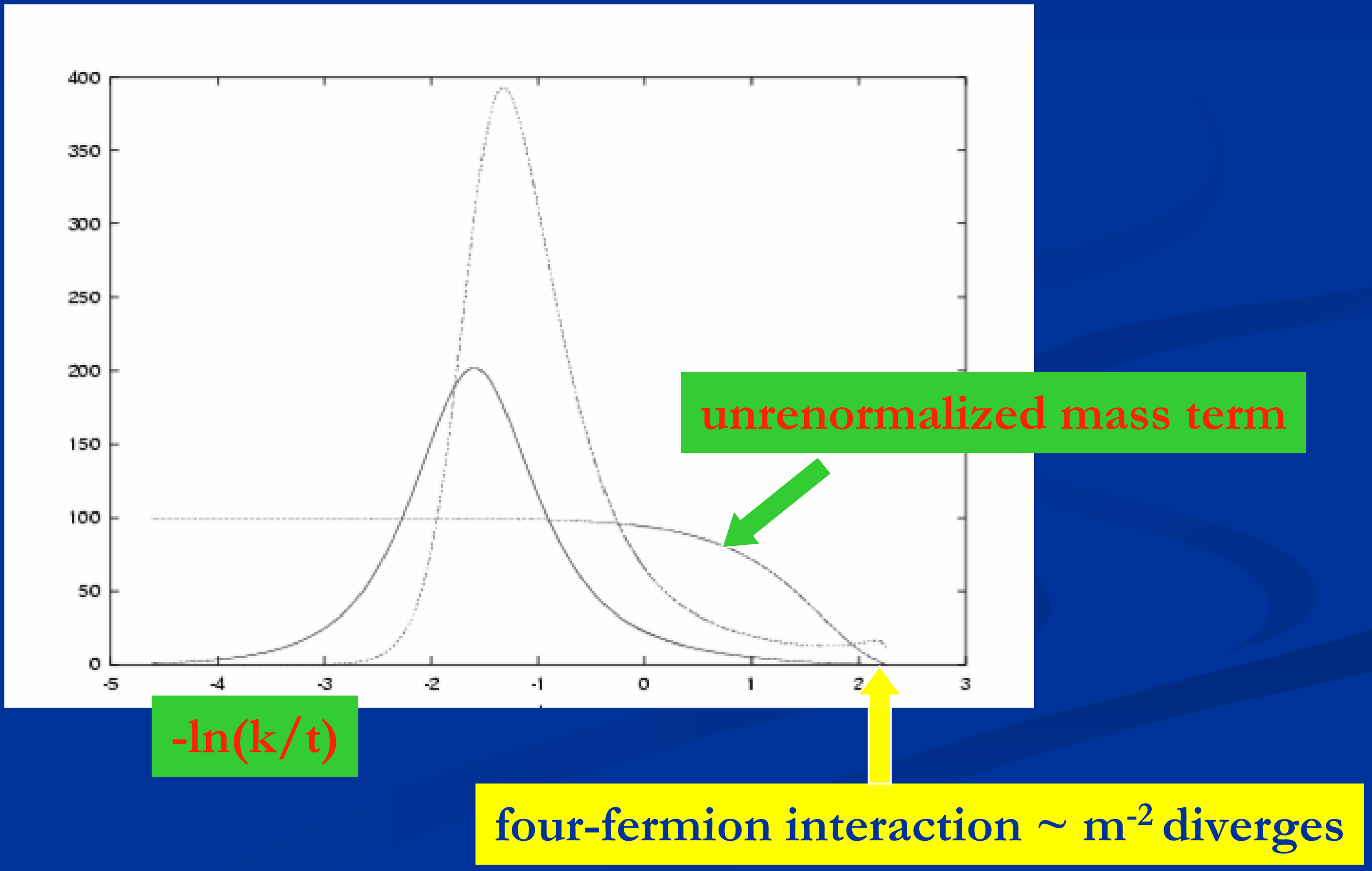
$$\text{SYM:} \quad \partial_k \bar{m}_a^2 = \frac{\partial}{\partial \alpha}(\partial_k U(\alpha))|_{\alpha=0},$$

$$\partial_k \bar{\lambda}_a = \frac{\partial^2}{\partial \alpha^2}(\partial_k U(\alpha))|_{\alpha=0},$$

$$\text{SSB:} \quad \partial_k \alpha_0 = -\frac{1}{\bar{\lambda}_a}\frac{\partial}{\partial \alpha}(\partial_k U(\alpha))|_{\alpha=\alpha_0},$$

$$\partial_k \bar{\lambda}_a = \frac{\partial^2}{\partial \alpha^2}(\partial_k U(\alpha))|_{\alpha=\alpha_0}.$$

Running mass term



dimensionless quantities

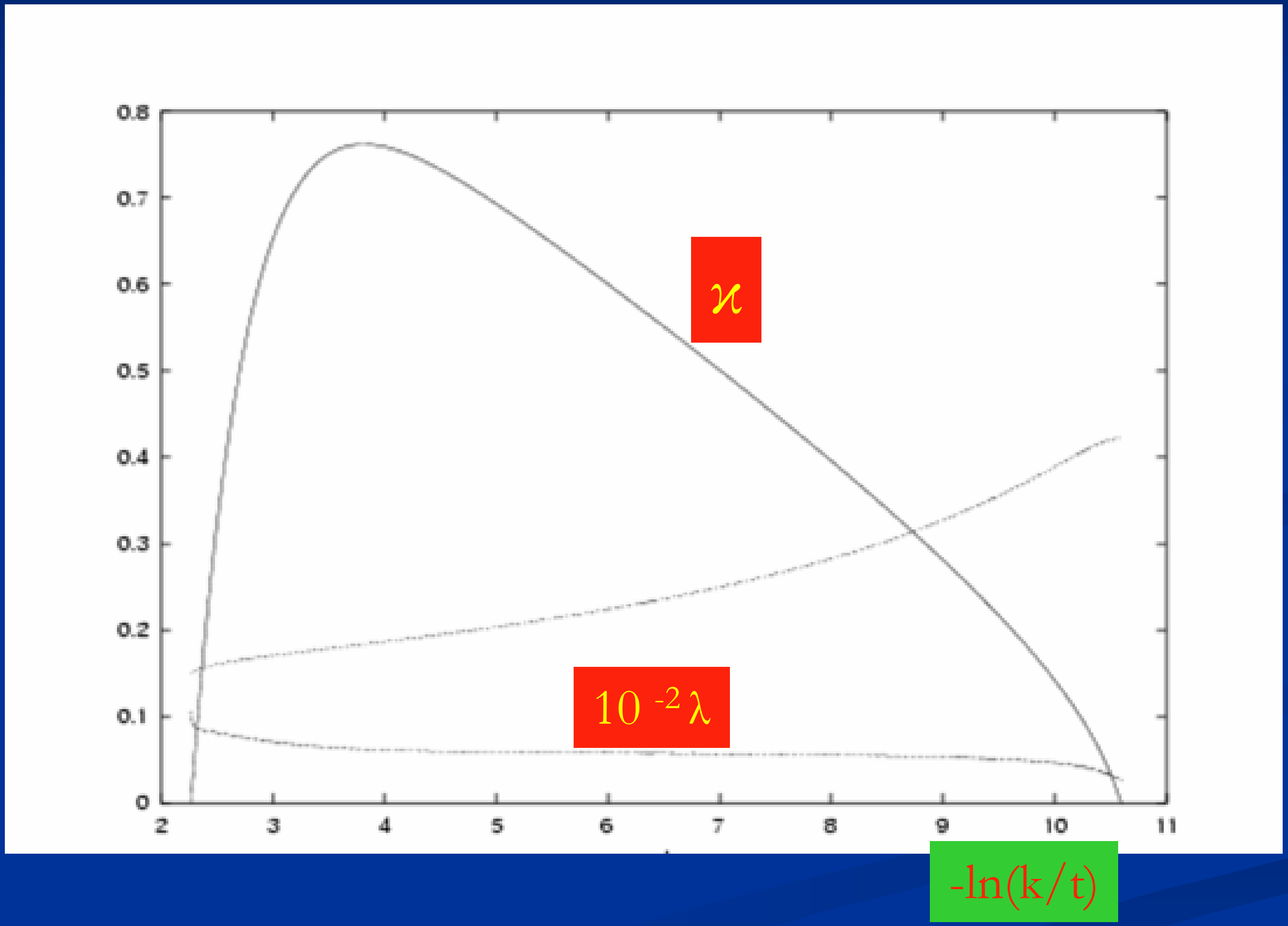
$$u = \frac{Ut^2}{Tk^2}, \quad \tilde{\alpha} = \frac{Z_a t^2 \alpha}{T}$$

$$m_a^2 = \frac{\bar{m}_a^2}{Z_a k^2} = \frac{\partial u}{\partial \tilde{\alpha}}, \quad \kappa_a = \frac{Z_a t^2}{T}\alpha_0,$$

$$\lambda_a = \frac{T}{Z_a^2 t^2 k^2}\bar{\lambda}_a = \frac{\partial^2 u}{\partial \tilde{\alpha}^2}, \quad h_a^2 = \frac{T}{Z_a t^4}\bar{h}_a^2$$

renormalized antiferromagnetic order parameter $\varkappa$

evolution of potential minimum

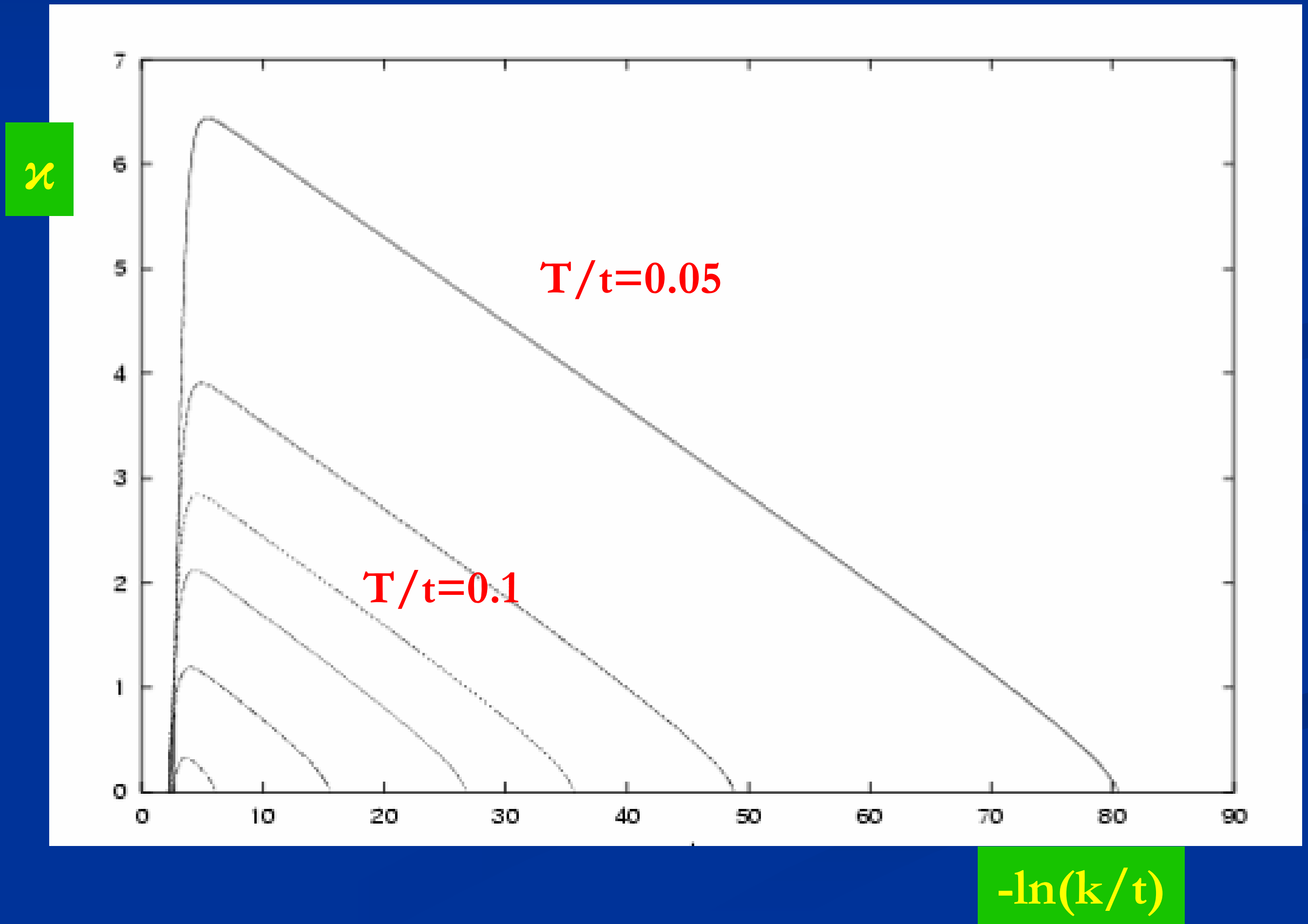


U/t = 3 , T/t = 0.15

Critical temperature

For $T<T_c$: $\varkappa$ remains positive for $k/t > 10^{-9}$

size of probe > 1 cm

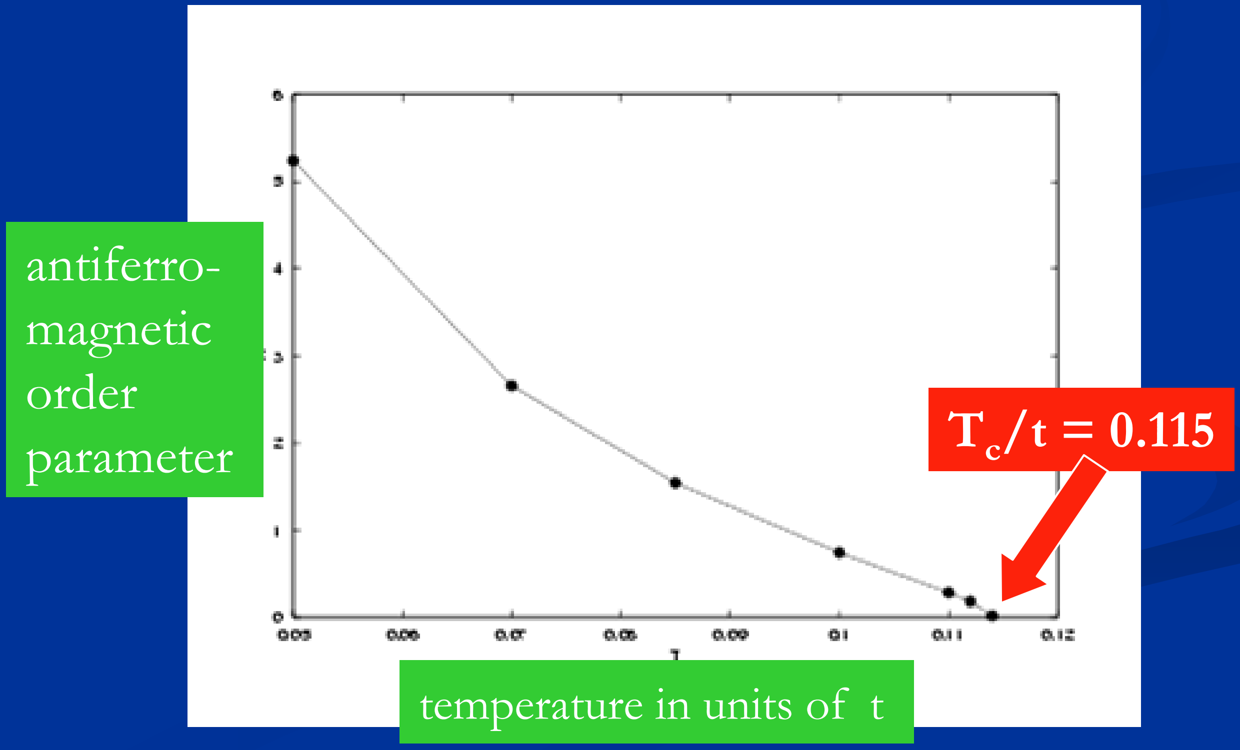


$T_c=0.115$

Below the critical temperature :

Infinite-volume-correlation-length becomes larger than sample size

finite sample ≈ finite k : order remains effectively



Pseudo-critical temperature T_{pc}

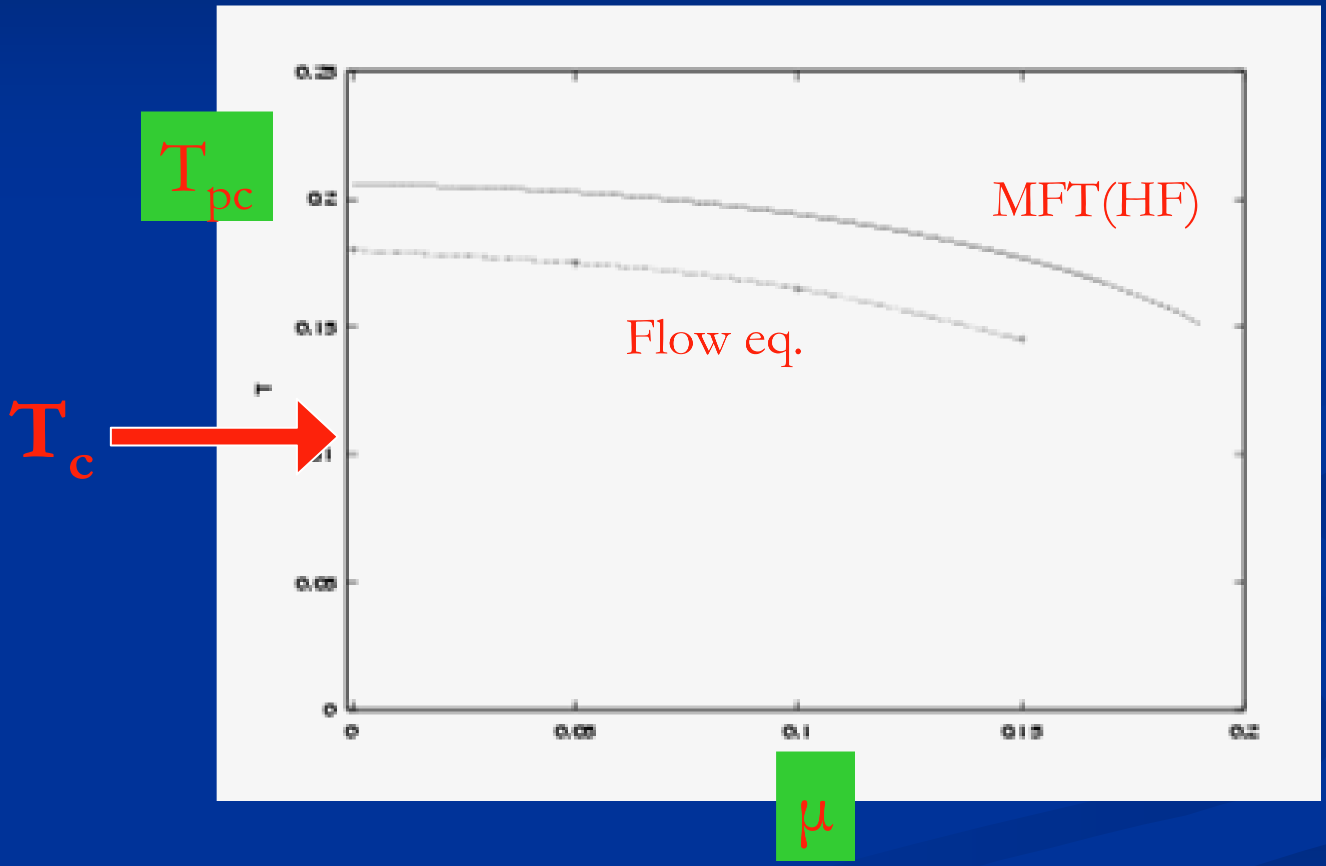
Limiting temperature at which bosonic mass term vanishes ($\varkappa$ becomes nonvanishing)

It corresponds to a diverging four-fermion coupling

This is the "critical temperature" computed in MFT !

Pseudo-gap behavior below this temperature

Pseudocritical temperature



Below the pseudocritical temperature

the reign of the goldstone bosons

effective nonlinear O(3) – σ - model

critical behavior

for interval $T_c < T < T_{pc}$
evolution as for classical Heisenberg model

cf. Chakravarty,Halperin,Nelson

$$k\partial_k\kappa = \frac{1}{4\pi} + \frac{1}{16\pi^2\kappa} + 0(\kappa^{-2})$$

$$\kappa(k) = \kappa_m(T) - \frac{1}{4\pi}\ln\frac{k_m(T)}{k}$$

critical correlation length

$$\xi t = c(T) \exp\left\{20.7\beta(T)\frac{T_c}{T}\right\}$$

c,β : slowly varying functions

exponential growth of correlation length
compatible with observation !

at T_c : correlation length reaches sample size !

$$\beta(T) = \frac{\hat{\alpha}_0(T)\hat{Z}_a(T)}{\hat{\alpha}_0(T_c)\hat{Z}_a(T_c)},$$
$$c(T) = C_{\rm SR}\frac{k_m(T_c)}{k_m(T)}\left(\frac{k_m(T_c)}{t}\right)^{\delta(T)},$$
$$\delta(T) = \beta(T)\frac{T_c}{T} - 1$$

$$\xi = \frac{C_{\rm SR}}{k_m(T)} \exp\left(4\pi\kappa_m(T)\right)$$

$$\xi = \tilde{C} \exp\left(\frac{\gamma}{T}\right)$$

$$\gamma = 4\pi\hat{\alpha}_0(T)\hat{Z}_a(T)t^2.$$

$$T_c(k) = \frac{\gamma(T_c)}{\ln\left(k_m(T_c)/k\right)}$$

critical behavior for order parameter and correlation function

$$\kappa_a(T) = \left(\frac{\gamma(T)T_c}{T} - 1\right)\kappa_m(T_c) + \frac{1}{4\pi}\ln\frac{k_m(T_c)}{k_m(T)}.$$

$$G(q^2) = (Z_a(k = \sqrt{q^2})q^2)^{-1} \sim (q^2)^{-1+\eta_a/2}$$

Mermin-Wagner theorem ?

No spontaneous symmetry breaking of continuous symmetry in d=2 !

end