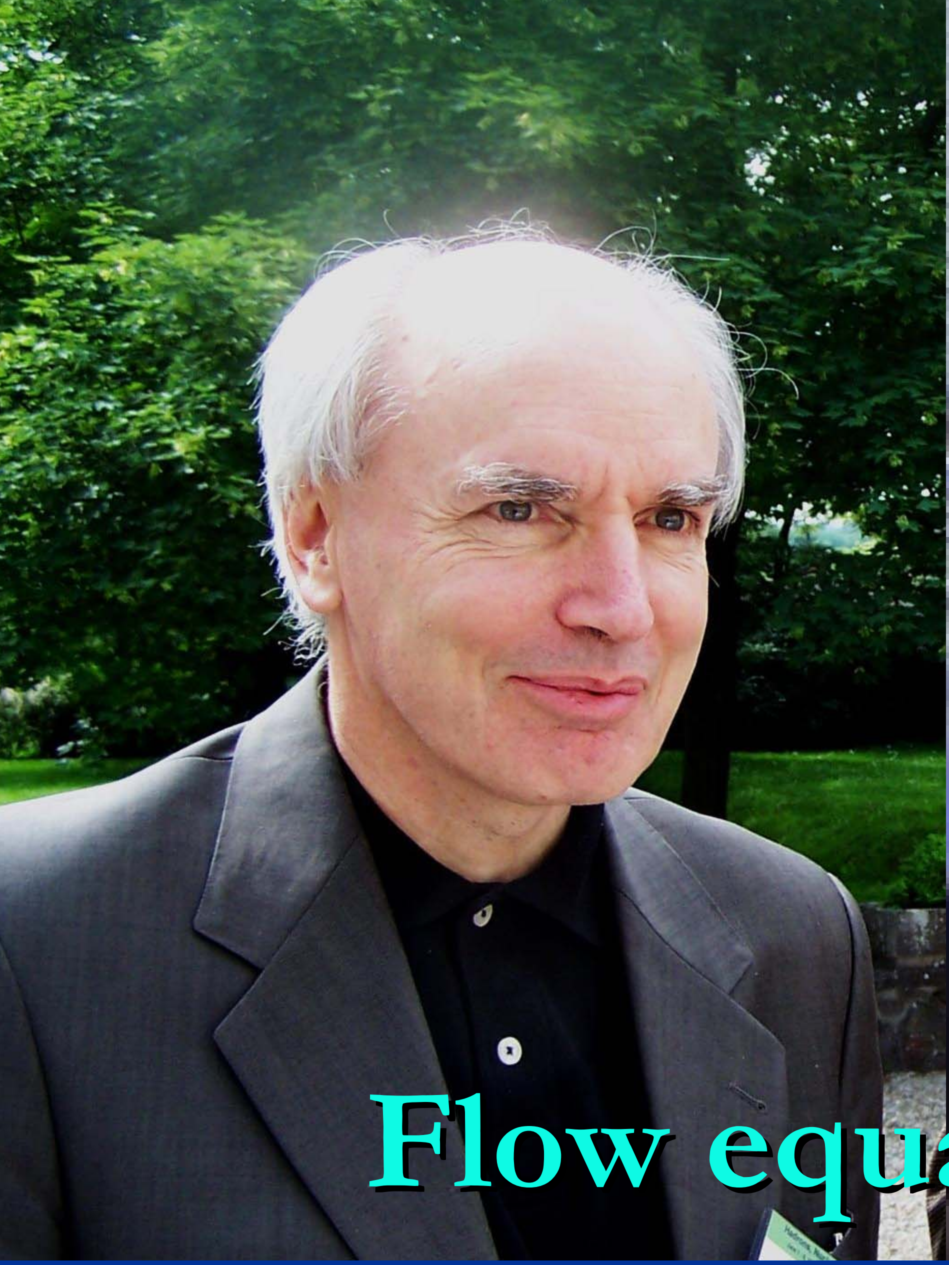



Flow equations and phase transitions

phase transitions

QCD – phase transition

Quark –gluon plasma

- Gluons : 8 x 2 = 16
- Quarks : 9 x 7/2 =12.5
- Dof : 28.5

Chiral symmetry

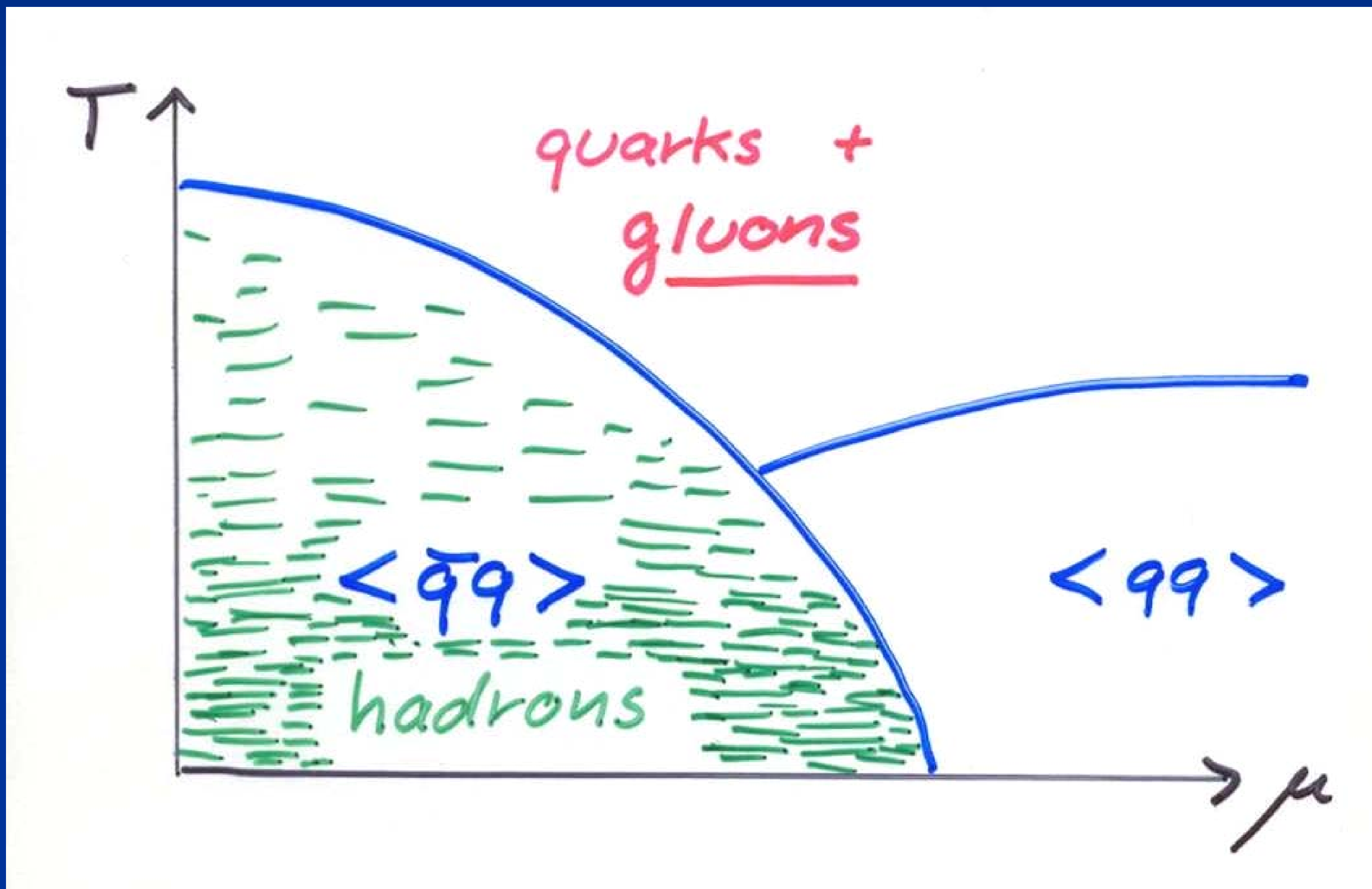
Hadron gas

- Light mesons : 8
- (pions : 3)
- Dof : 8

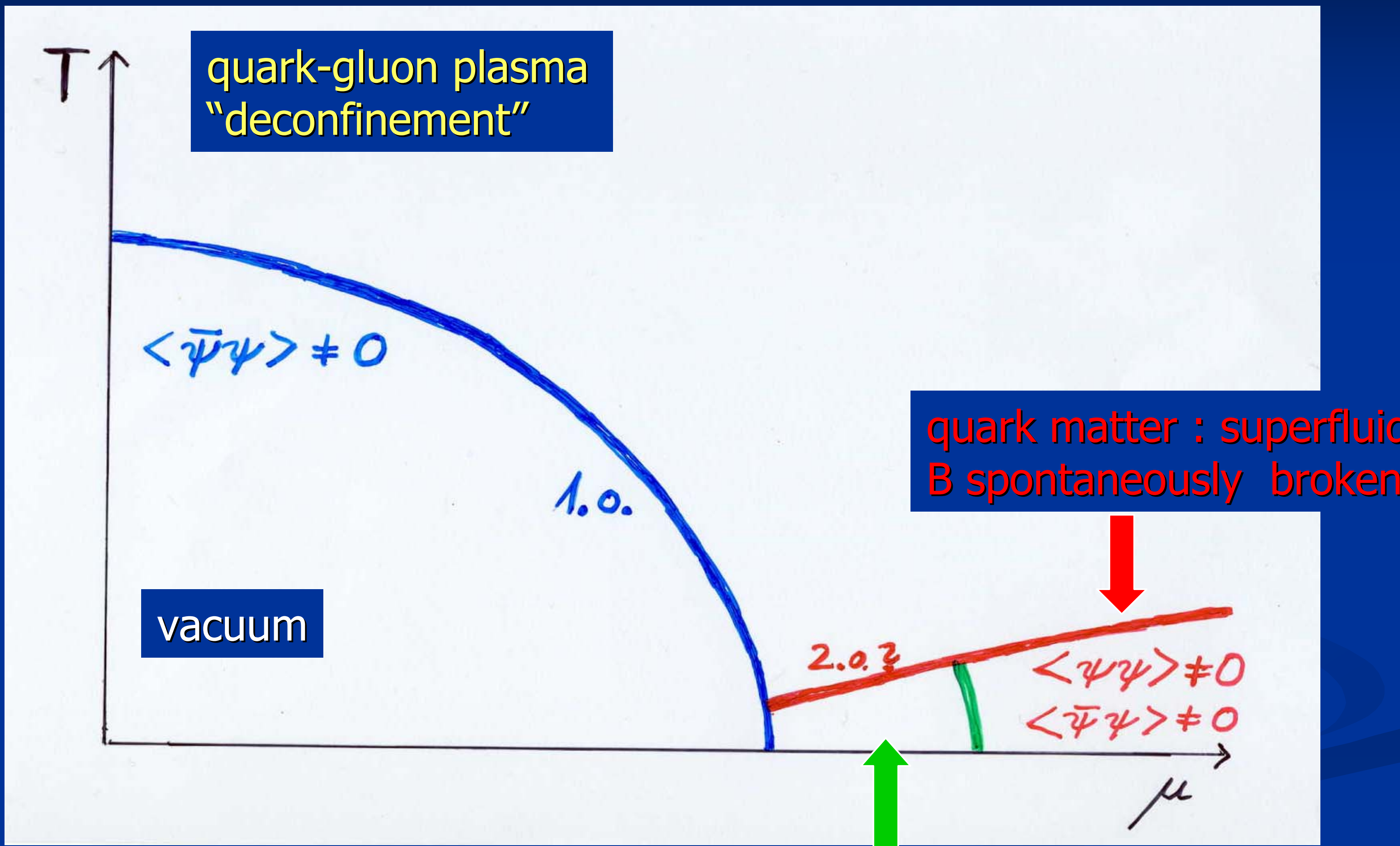
Chiral sym. broken

Large difference in number of degrees of freedom !
Strong increase of density and energy density at T_c !

Understanding the phase diagram



Phase diagram for $m_s > m_{u,d}$



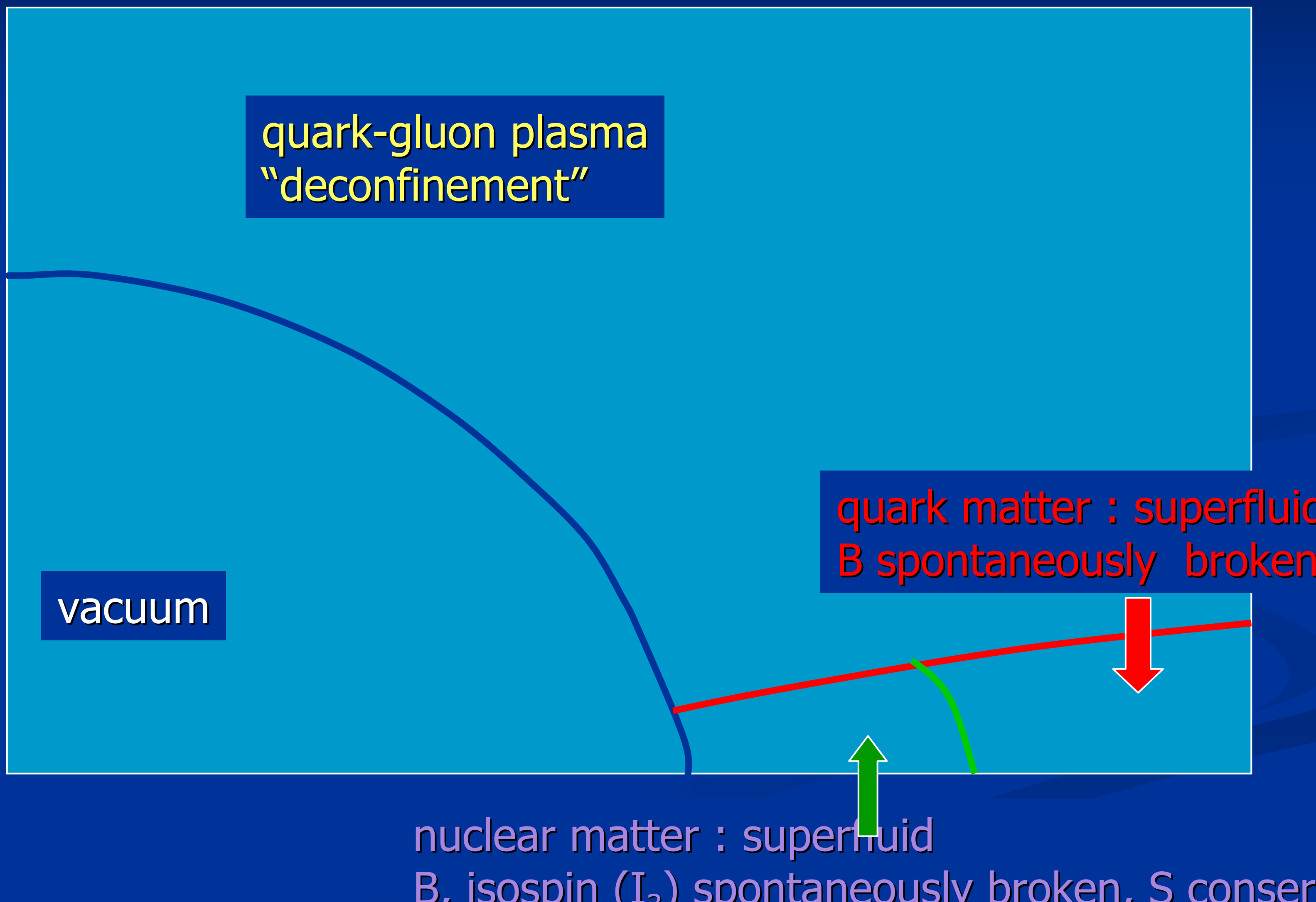
nuclear matter : B,isospin (I_3) spontaneously broken,
S conserved

Order parameters

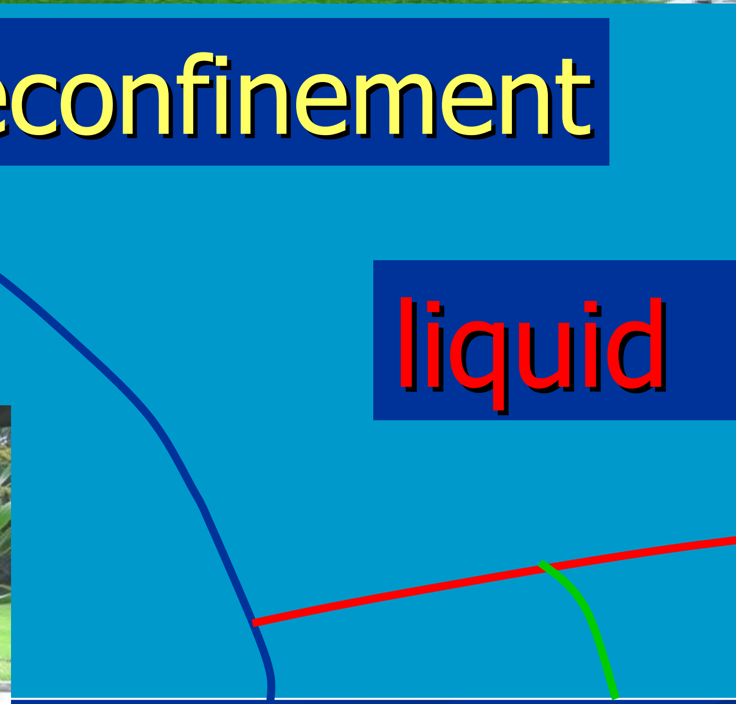
- Nuclear matter and quark matter are separated from other phases by true critical lines
- Different realizations of global symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin I_3

neutron-neutron condensate

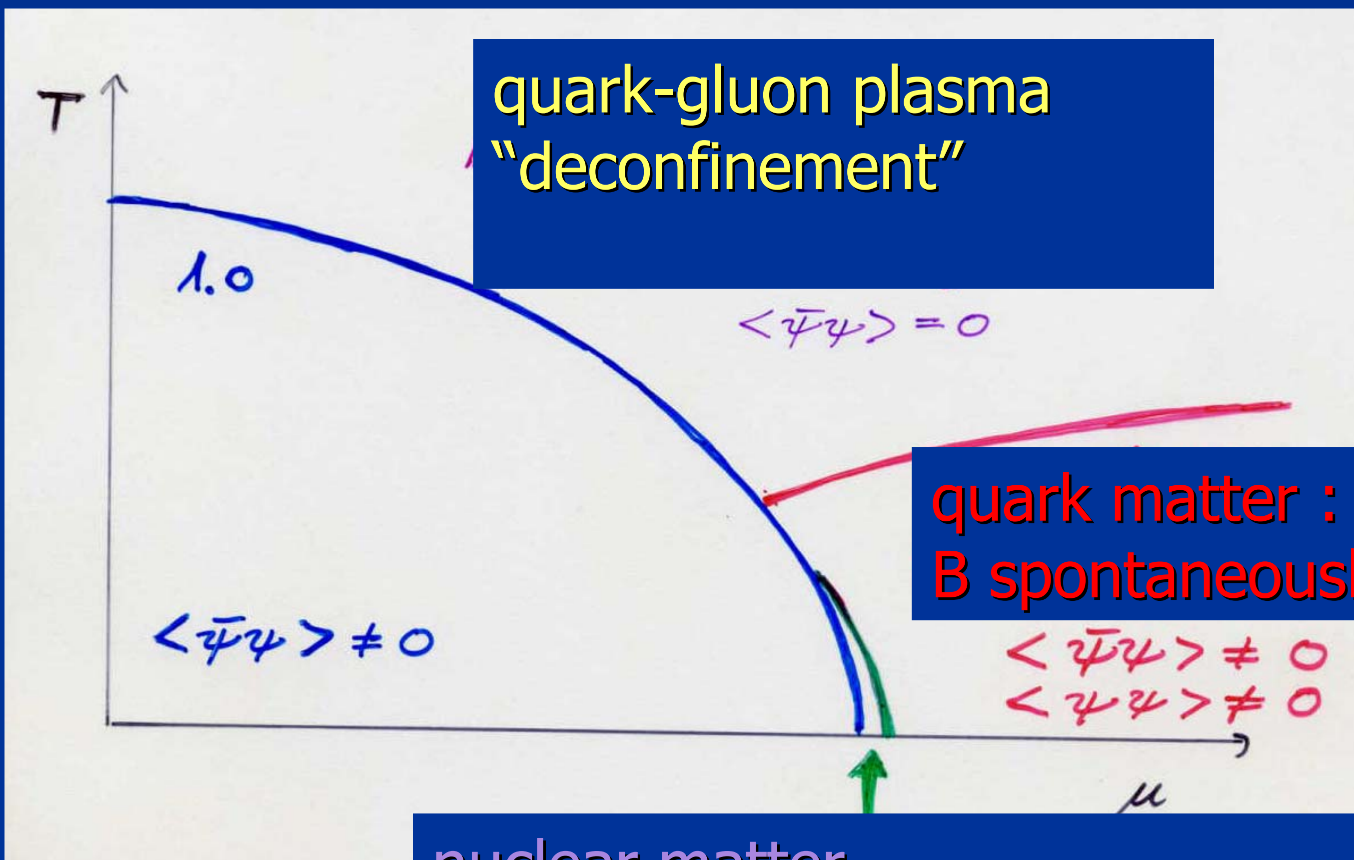
Phase diagram for $m_s > m_{u,d}$



Phase diagram for $m_s > m_{u,d}$



Phase diagram for $m_s > m_{u,d}$

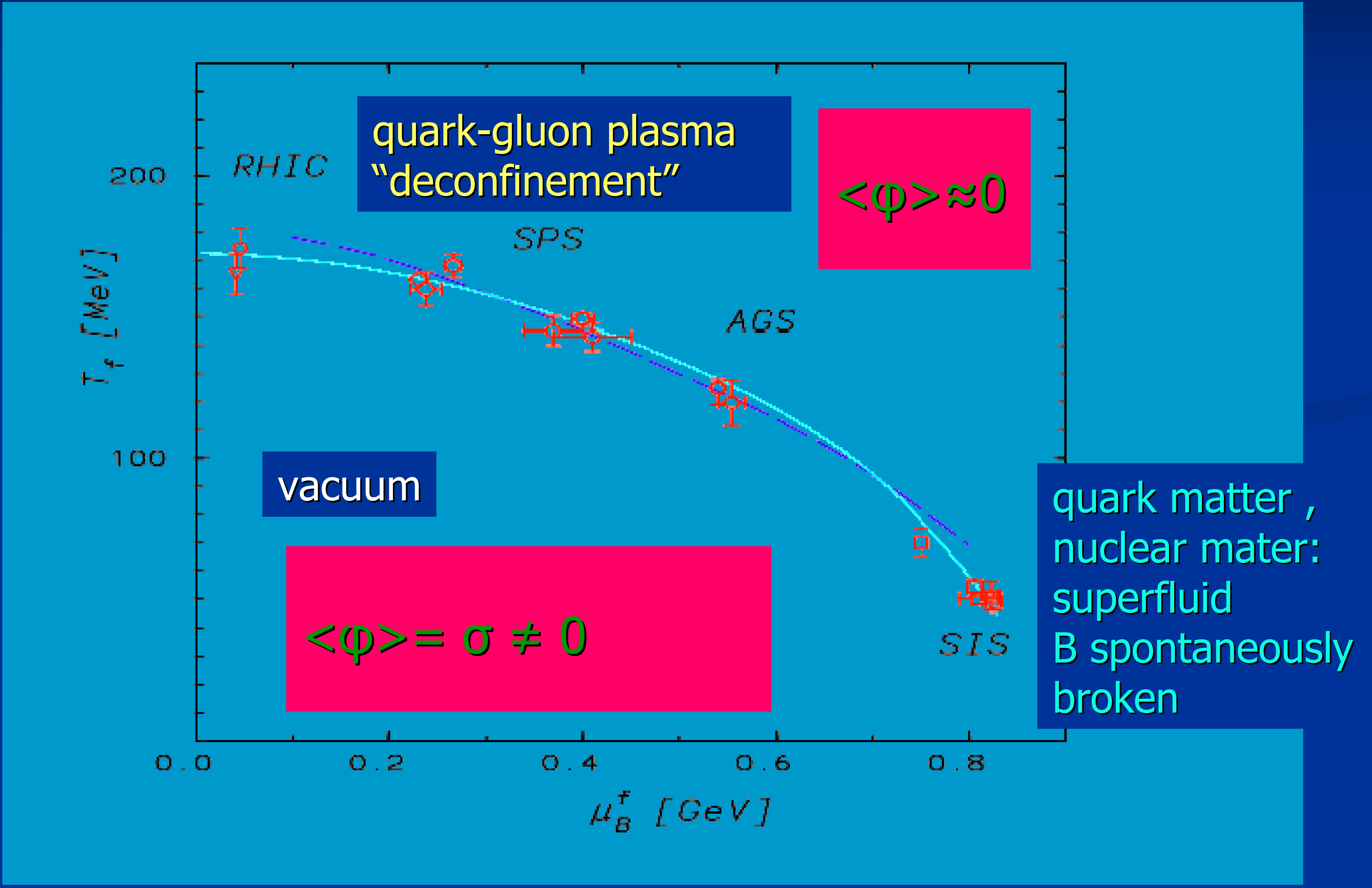


quark-gluon plasma
"deconfinement"

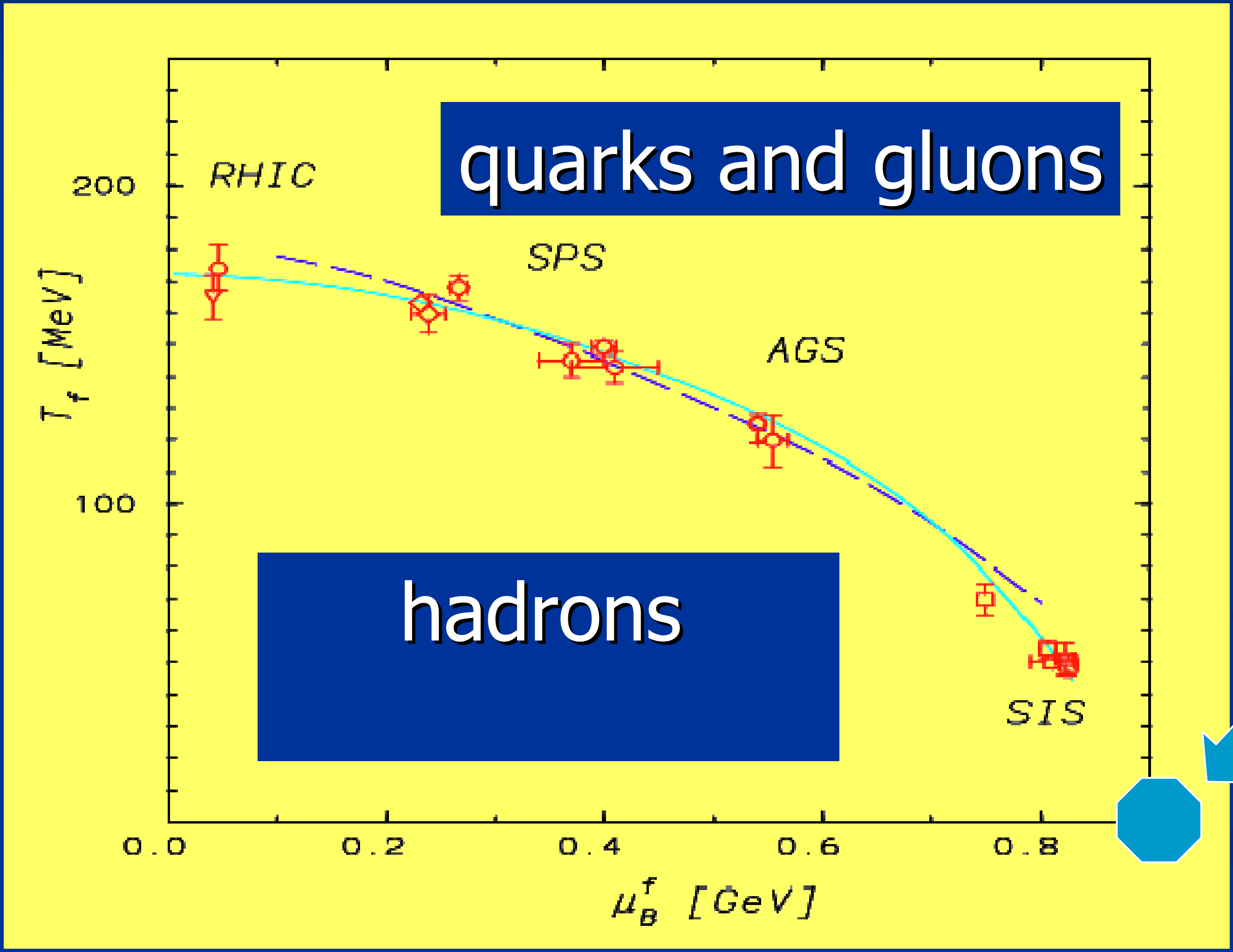
quark matter : superfluid
B spontaneously broken

nuclear matter
B,isospin (I_3) spontaneously broken, S conserved

Phase diagram

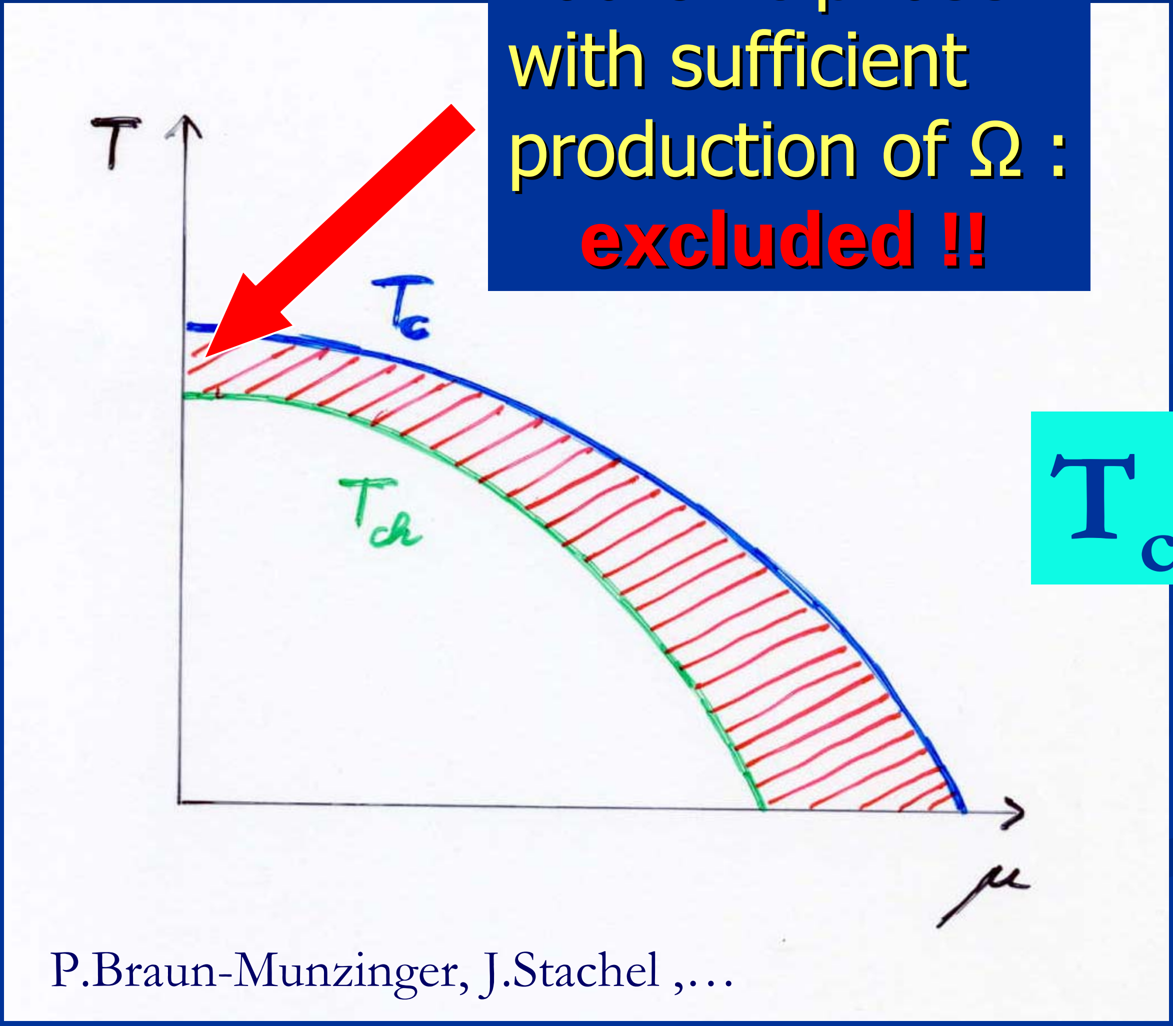


First order phase transition line



μ=923MeV transition to nuclear matter

Exclusion argument

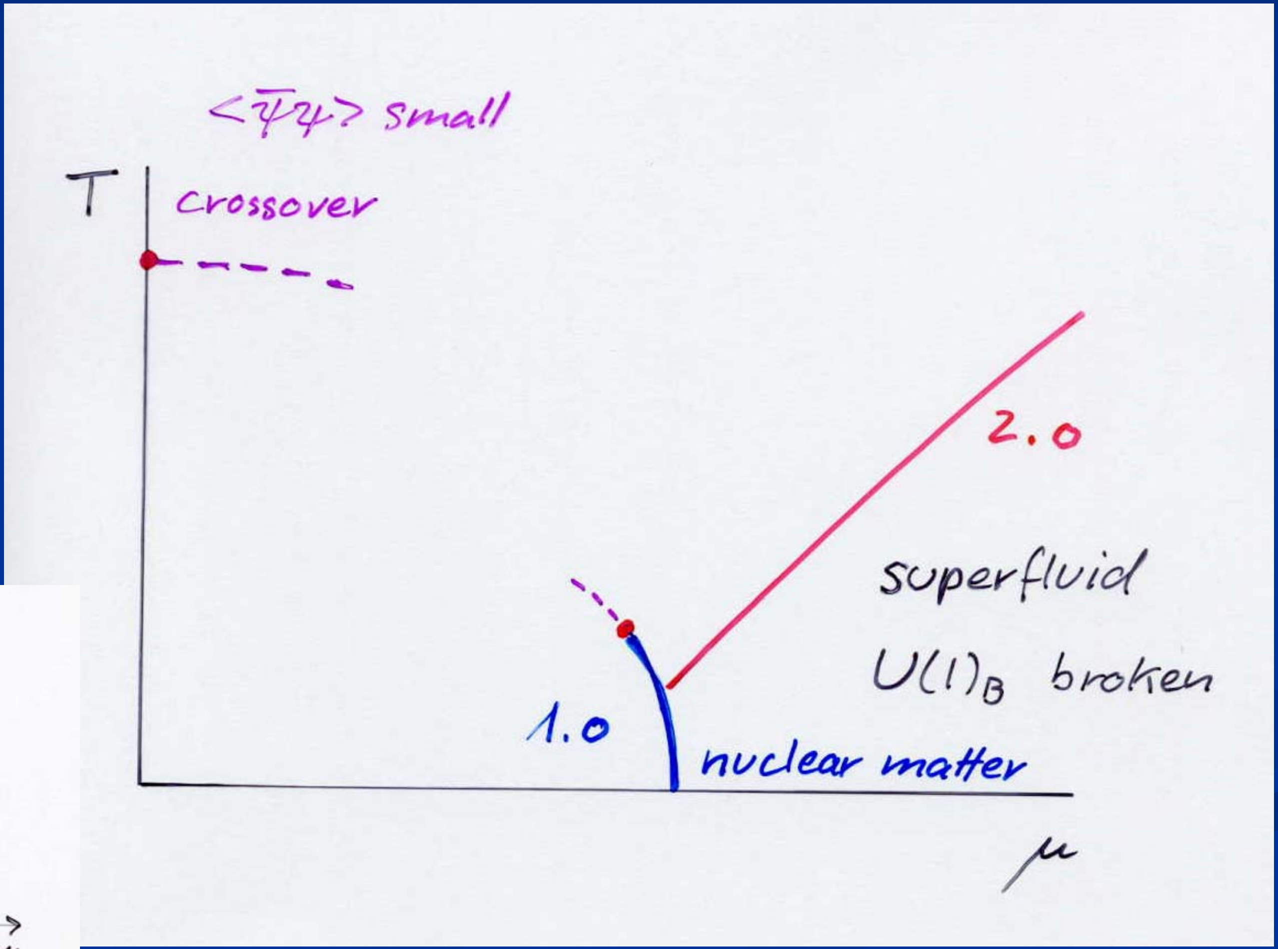


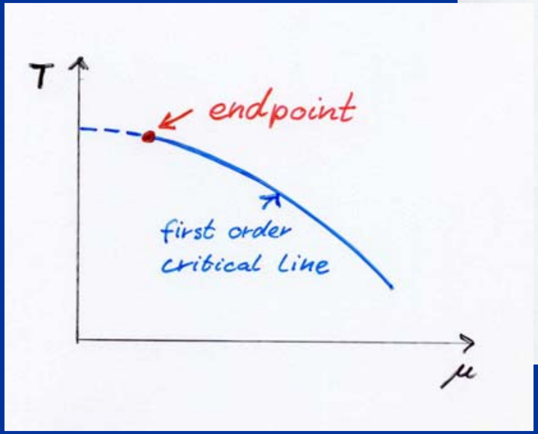
hadronic phase with sufficient production of Ω :
excluded !!

→ $T_c \approx T_{ch}$

“minimal” phase diagram

for equal nonzero quark masses





strong interactions in high T phase

SU(2) gauge theory $(d=3)$

$k \ll 2\pi T$ $\quad$ k: IR-Regulator, z.B. $\sim M_W$

$g^2 \frac{T}{k} \ll 1$ $\quad$ g: Eichkopplung

$$k \frac{\partial}{\partial k} g^2(k) = -\frac{23}{24\pi} g^4(k) \frac{T}{k}$$

$$(\bar{g}_3^{\,2} = g^2 T)$$

solution

$$\frac{1}{\bar{g}_3^{\,2}(k)} = \frac{1}{\bar{g}_3^{\,2}(2\pi T)} + \frac{23}{24\pi}\left(\frac{1}{2\pi T} - \frac{1}{k}\right)$$

High temperature „confinement scale" Λ_3:

where gauge coupling diverges or gets very large

$$\Lambda_3 = \left(\frac{1}{2\pi T} + \frac{24\pi}{23\, \bar{g}_3^{\,2}(2\pi T)}\right)^{-1}$$

$$\Lambda_3 \approx \frac{23}{24\pi} g^2 T = \frac{23}{6} \alpha_w T$$

$$\approx 0.13\, T$$

(similar for QCD !)

Running gauge coupling in three dimensions and the electroweak phase transition

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(Revised 4 June 1993)
Accepted for publication 22 June 1993

We present a method which allows us to deal with the strong infrared effects in three-dimensional gauge theories. In particular, we compute the three-dimensional running of the gauge coupling. Applying these results to the electroweak phase transition in the standard model, we conclude that the transition cannot be of second order. It is either a first-order transition or a smooth cross-over.

Consequences

Typical nonperturbative scales :

(condensates, glue ball …

a consequence of three-dimensional confinement one expects at very high temperatures that the quark degrees of freedom are described by mesons and hadrons as far as the space-like correlations are concerned. (The picture of weakly interacting quark degrees of freedom applies here to momenta much higher than the tempera-

$$\Lambda^{(3)}_{\mathrm{conf}} \approx 5\alpha_s(5T)T,$$

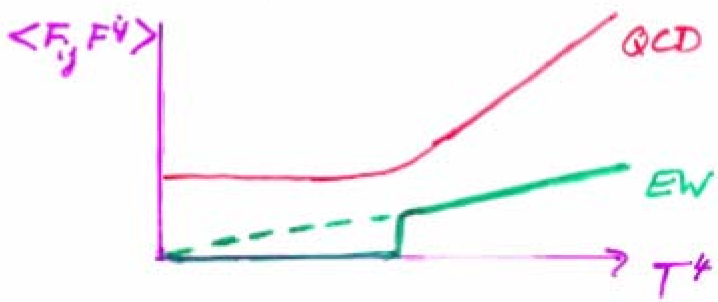
The relevant degrees of freedom in the momentum range $q^2 < T^2$ are not weakly interacting gluons but rather the glueballs of a three-dimensional confining theory. Their mass scale is set by the temperature.

…s emerges: As long as the temperature is smal… …ne four-dimensional (zero temperature) confineme… T/θ, the three-dimensional running of the gauge coupl… temperatures, however, three-dimensional confinement … confinement and $\Lambda^{(3)}_{\mathrm{conf}}$ sets the relevant mass scale for the…

…ch that the value of … is larger than … …tive. For higher … …r-dimensional

$\sim T$

e.g. magnetic condensate in QCD, EW



(Reuter, … Shaposhnikov)

Reliability of perturbation theory for electroweak phase transition ?

presence of strong interactions:

crossover or **first order** phase transition

if no global order parameter distinguishes phases

(second order only at critical endpoint)

→

use flow non-perturbative equations

Electroweak phase transition ?

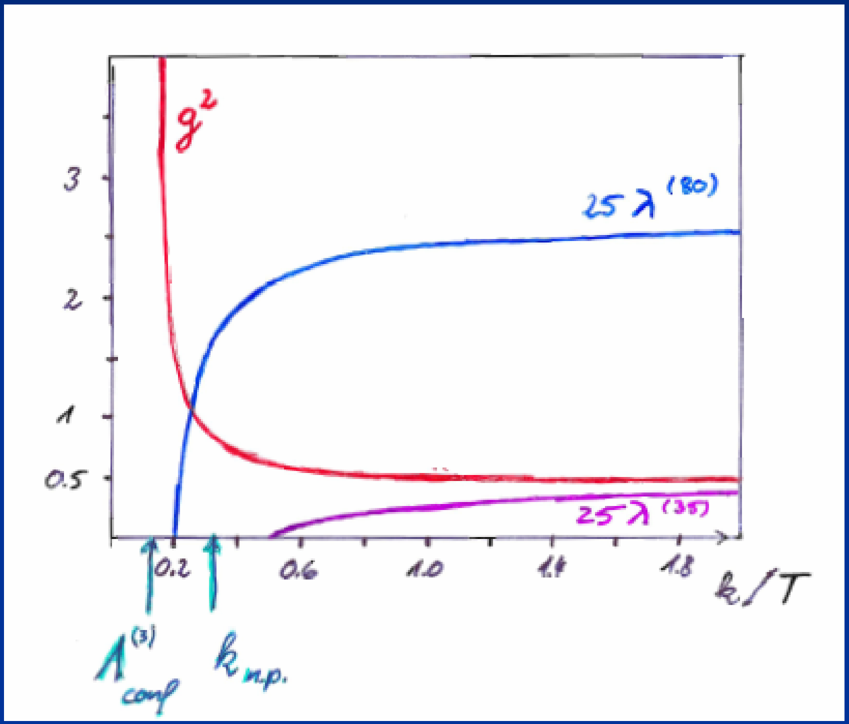
- 10^{-12} s after big bang
- fermions and W-,Z-bosons get mass
- standard model : crossover Reuter,Wetterich '93
- baryogenesis if first order

  (only for some SUSY – models)

  bubble formation of " our vacuum "

Kuzmin,Rubakov,Shaposhnikov '85 , Shaposhnikov '87

strong electroweak interactions responsible for crossover



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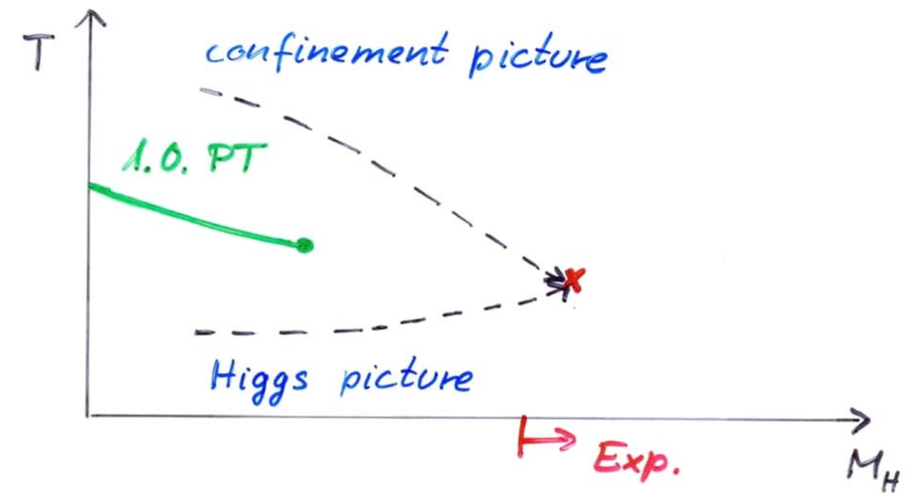
Electroweak phase diagram



Reuter, W.

Philipsen, Buchmüller

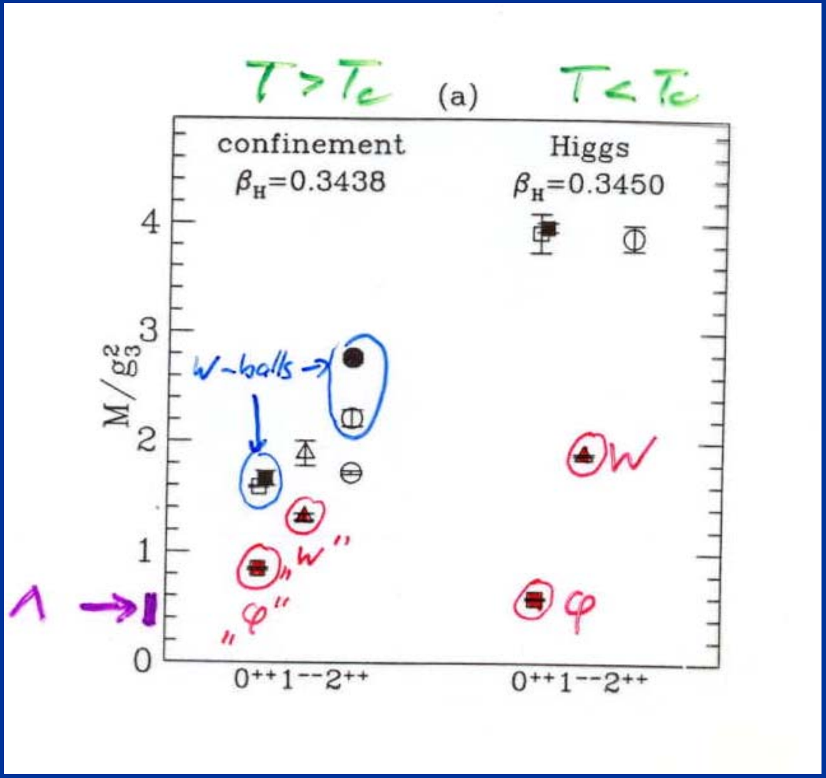
Laine, Kajantie, Rummukainen,
Shaposhnikov

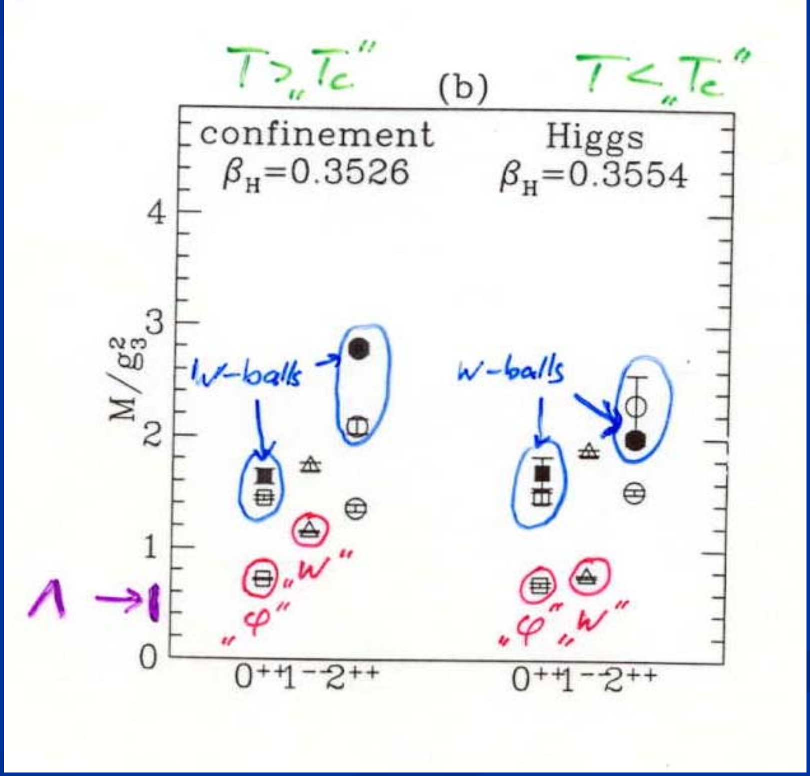
Crossover in the standard model!

M.Reuter,C.Wetterich
Nucl.Phys.B408,91(1993)

Masses of excitations (d=3)

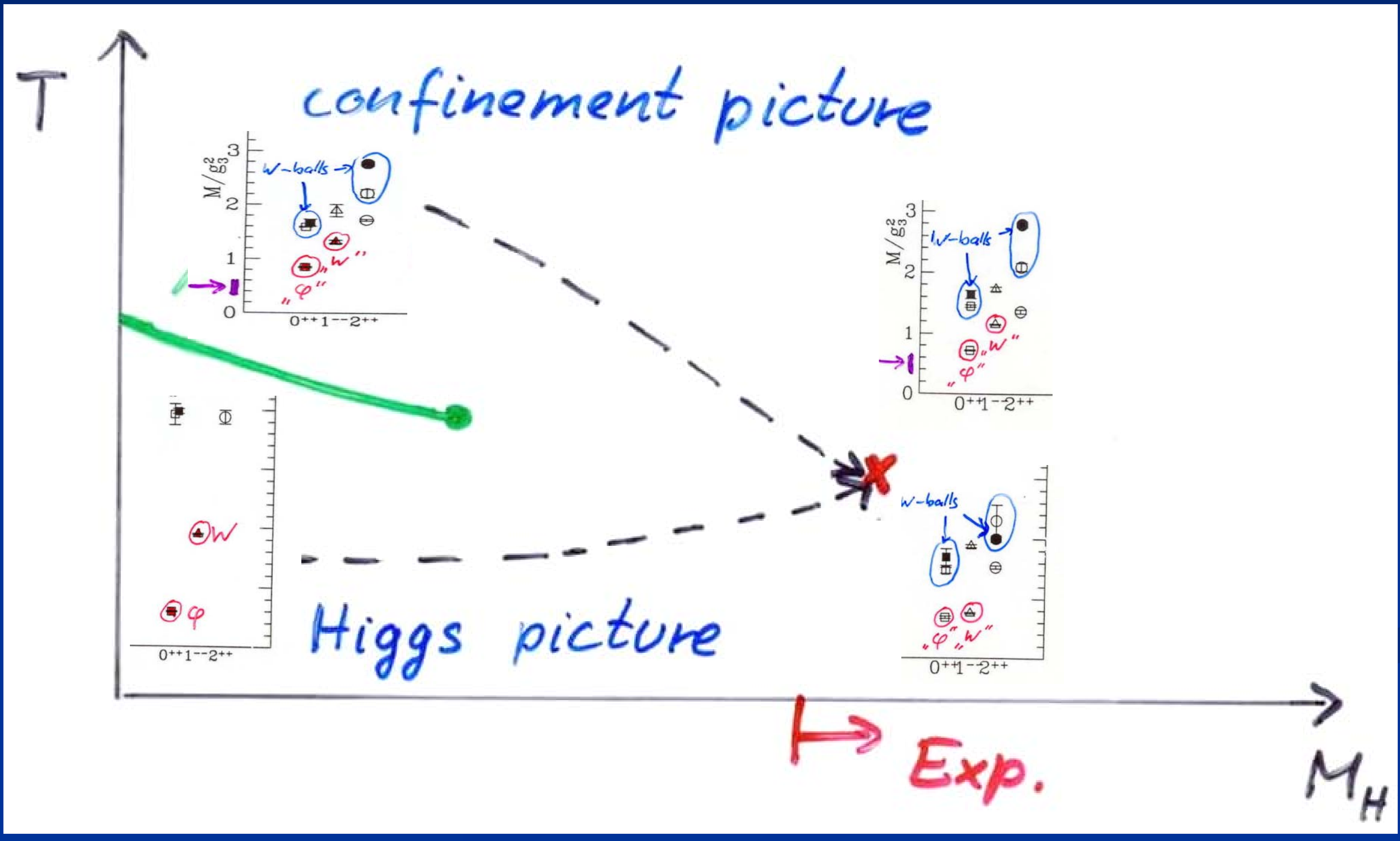
small M_H | large M_H





O.Philipsen,M.Teper,H.Wittig '97

Continuity



T
confinement picture
Higgs picture
W-balls
"W"
"φ"
M/g$_3^2$
0⁺⁺1⁻⁻2⁺⁺
Exp.
M_H

BEC – BCS crossover

Bound molecules of two atoms
on microscopic scale:

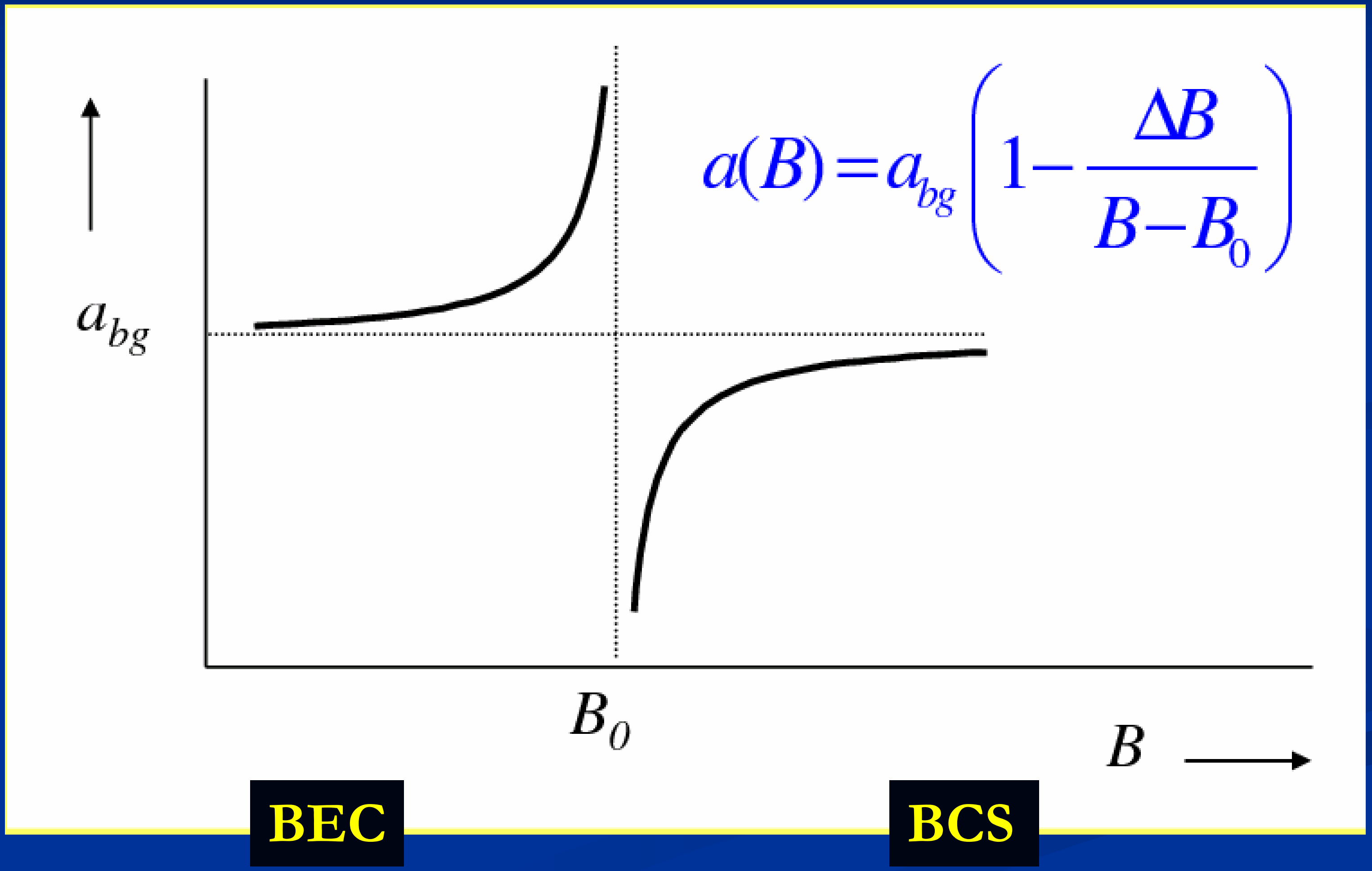
Bose-Einstein condensate (BEC) for low T

Fermions with attractive interactions
(molecules play no role) :

BCS – superfluidity at low T
by condensation of Cooper pairs

Crossover by Feshbach resonance
as a transition in terms of external magnetic field

scattering length

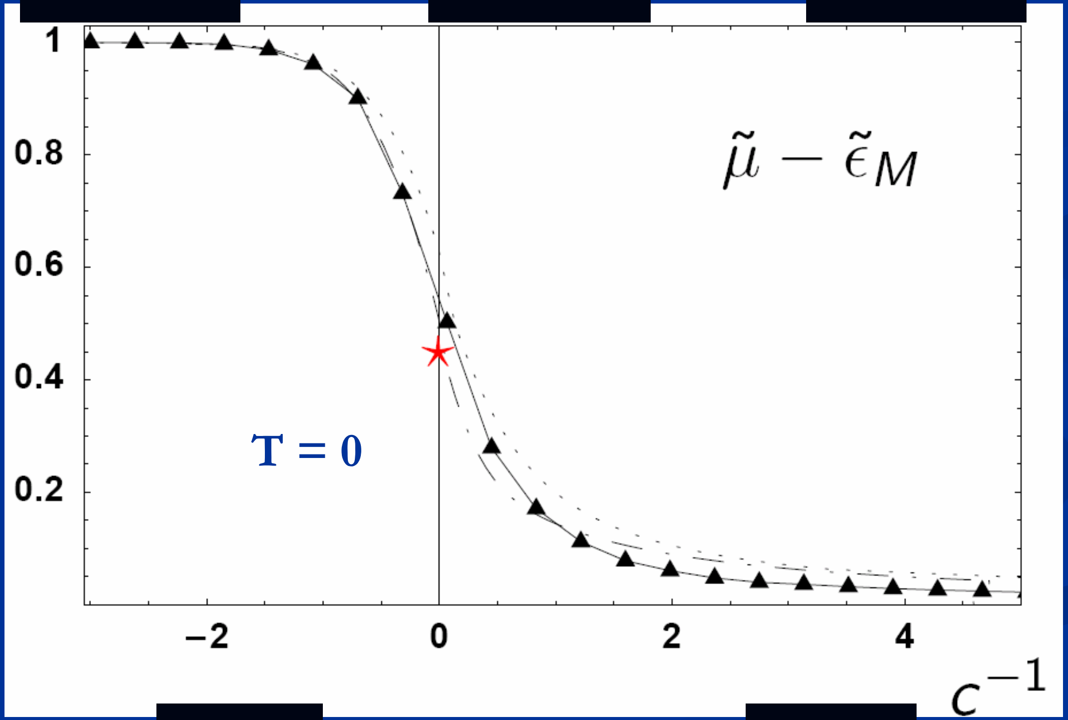


concentration

- $c = a\, k_F$, **a(B)** : scattering length
- needs computation of density $n = k_F^3/(3\pi^2)$

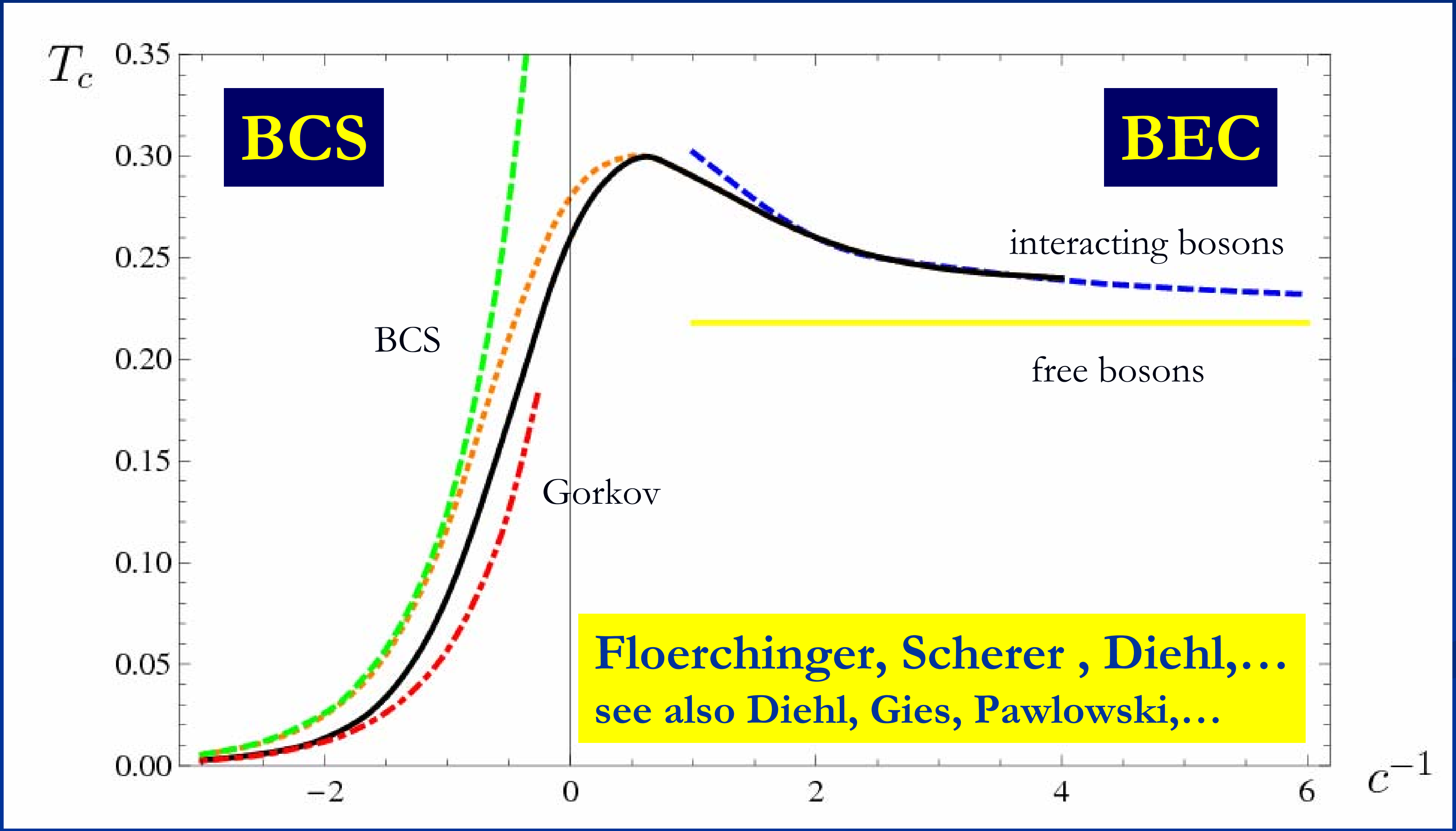
dilute | dense | dilute

non-interacting Fermi gas ← → non-interacting Bose gas



BCS | BEC

BCS – BEC crossover

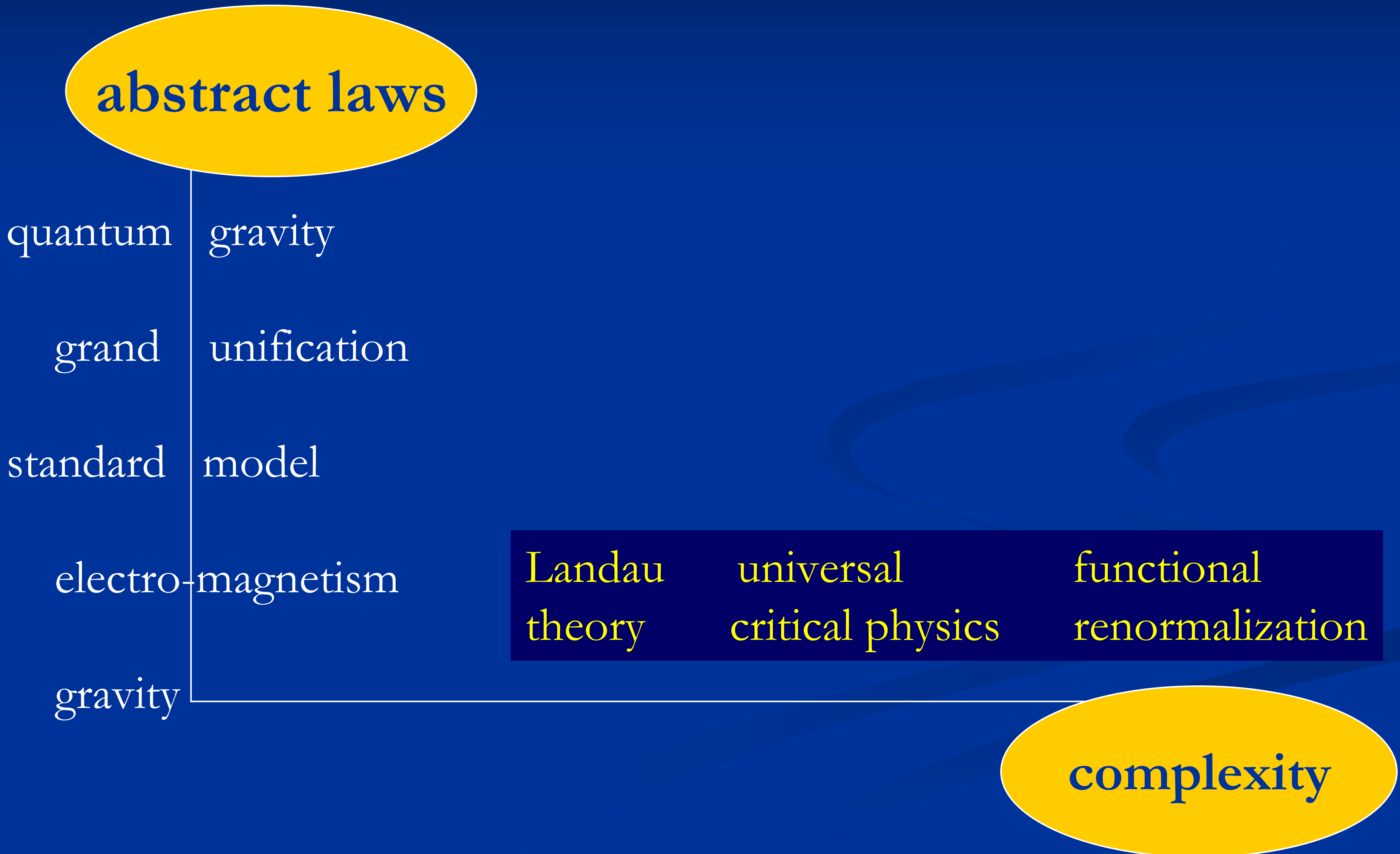


Flow equations

Unification from Functional Renormalization

- fluctuations in d=0,1,2,3,...
- linear and non-linear sigma models
- vortices and perturbation theory
- bosonic and fermionic models
- relativistic and non-relativistic physics
- classical and quantum statistics
- non-universal and universal aspects
- homogenous systems and local disorder
- equilibrium and out of equilibrium

unification



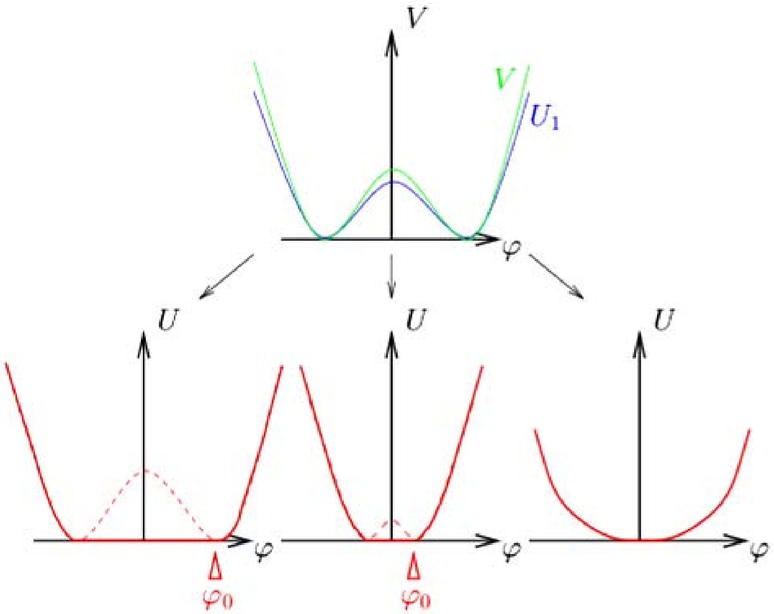
unified description of scalar models for all d and N

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^dx \left\{ \frac{1}{2}\partial_\mu\varphi_a\partial_\mu\varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2}\varphi_a\varphi_a$$



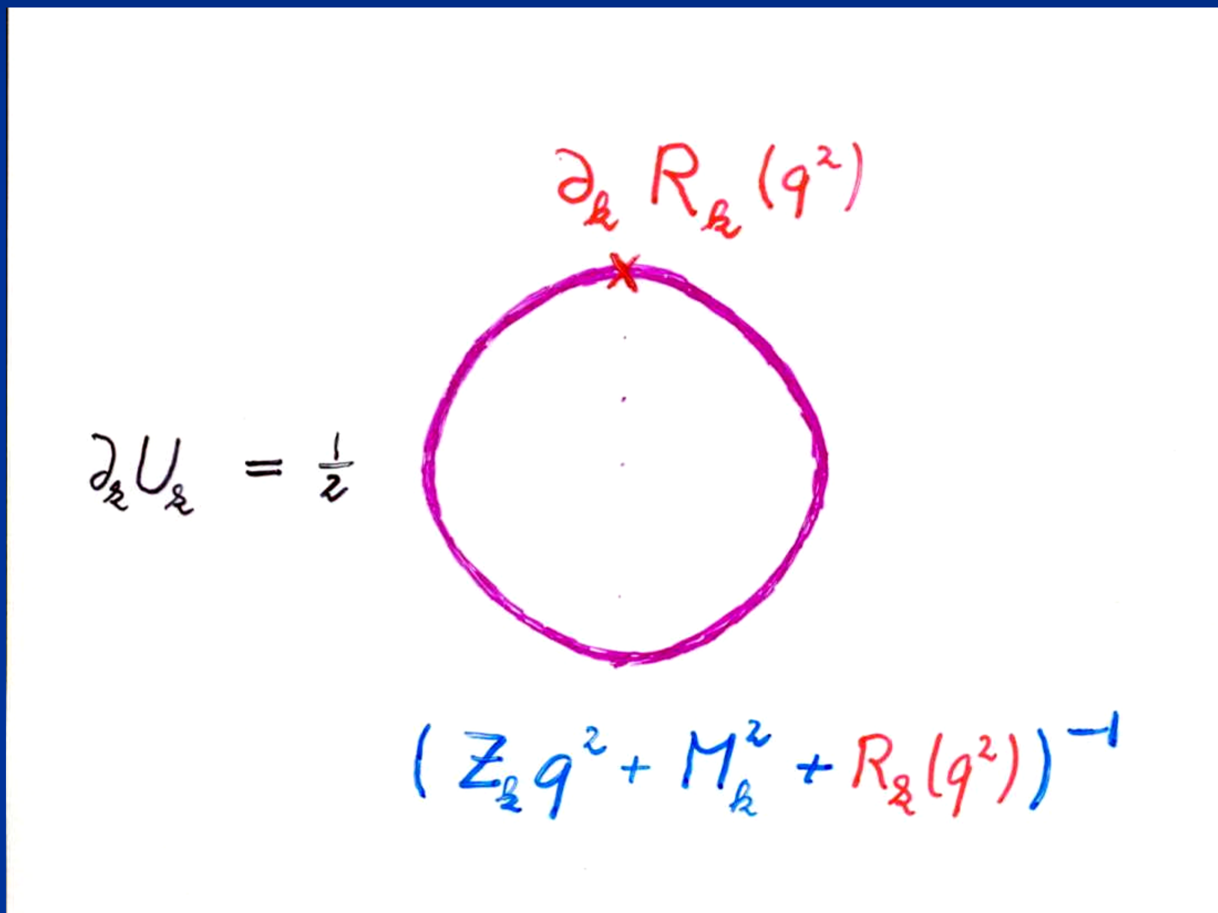
Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

Simple one loop structure – nevertheless (almost) exact



Infrared cutoff

R_k : IR-cutoff

e.g $$R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$$

or $$R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \qquad \text{(Litim)}$$

$$\lim_{k\to 0} R_k = 0$$

$$\lim_{k\to\infty} R_k \to \infty$$

Wave function renormalization and anomalous dimension

Z_k: wave function renormalization

$$k\partial_k Z_k = -\eta_k Z_K$$

η_k: anomalous dimension

$$t = \ln(k/\Lambda)$$

$$\partial_t \ln Z = -\eta$$

for $Z_k(\varphi, q^2)$: flow equation is **exact !**

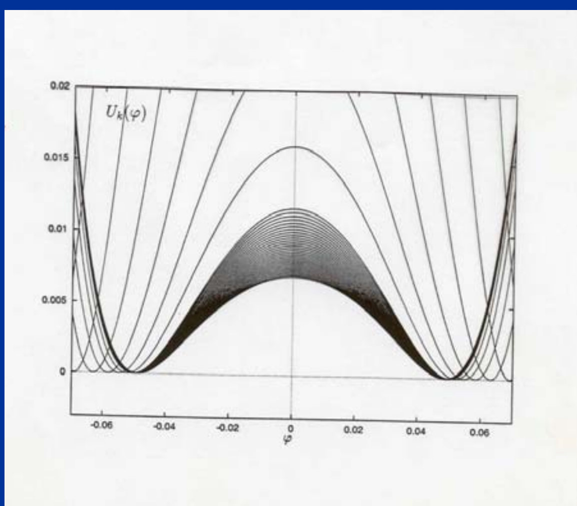
unified approach

- choose N
- choose d
- choose initial form of potential
- run !

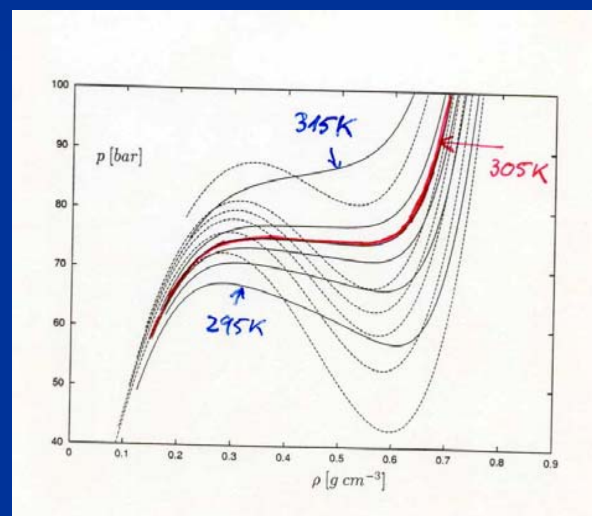
(quantitative results : systematic derivative expansion in second order in derivatives)

Flow of effective potential

Ising model



CO_2



Critical exponents

d = 3

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003

↑ ↑

"average" of other methods
(typically ±(0.0010 − 0.0020))

Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

S.Seide …

critical exponents , BMW approximation

N	η	η (other)	ν	ν (other)	ω (prelim.)	ω (other)
0	0.033(3)	0.028(3) [1]	0.588	0.588(1) [1]	0.80	
1	0.039(3)	0.0364(2) [2]	0.6298(4)	0.6301(2) [2]	0.78	0.79(1) [1]
		0.0368(2) [3]		0.6302(1) [3]		
		0.033(3) [1]		0.630(1) [1]		
2	0.041(3)	0.0381(2) [4]	0.6719(4)	0.6717(1) [4]	0.78	0.79(1) [1]
		0.035(3) [1]		0.670(2) [1]		
3	0.040(3)	0.0375(5) [5]	0.709	0.7112(5) [5]	0.73	
		0.036(3) [1]		0.707(4) [1]		
4	0.038(3)	0.035(5)[1]	0.738	0.741(6) [1]	0.74	0.77(2) [1]
		0.037(1) [6]		0.749(2) [6]		
5	0.035(3)	0.031(3) [8]	0.768	0.764(4) [8]	0.73	0.77(2) [1]
		0.034(1) [7]		0.779(3) [7]		
10	0.022(2)	0.024 [9]	0.860	0.859 [9]	0.81	
20	0.012(1)	0.014 [9]	0.929	0.930 [9]	0.94	
100	0.0023(2)	0.0027 [10]		0.989 [10]	0.99	

[1] R. Guida and J. Zinn-Justin '98. [2] M. Campostrini, A. Pelissetto, P. Rossi, E. Vicari '02.
[3] Y. Deng and H. W. J. Blote '03. [4] M. Campostrini, M. Hasenbusch, A. Pelissetto, E. Vicari '06.
[5] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, E. Vicari '02. [6] M. Hasenbusch '01.
[7] M. Hasenbusch, A. Pelissetto, E. Vicari '05. [8] A. Butti and F. Parisen Toldin '05.
[9] S. A. Antonenko and A. I. Sokolov '95. [10] M. Moshe and J. Zinn-Justin '03.

Blaizot, Benitez , … , Wschebor

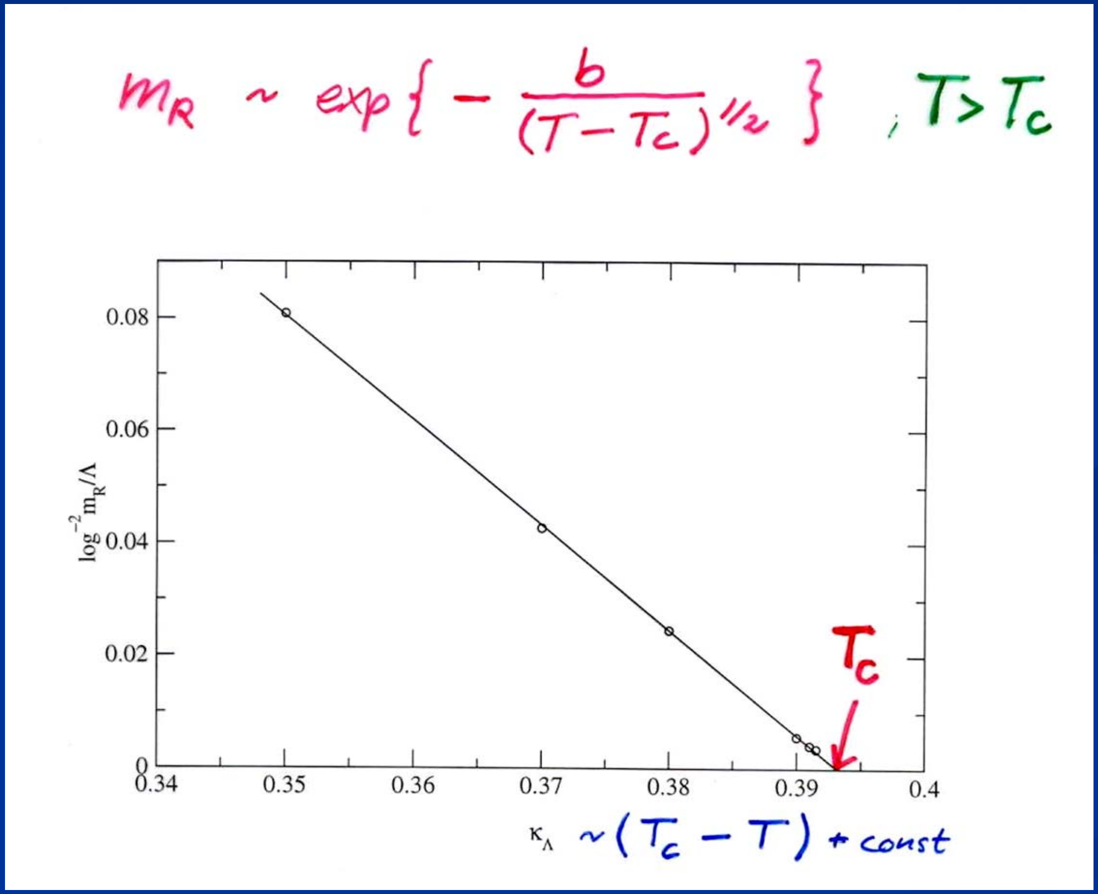
Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:

Kosterlitz-Thouless phase transition

Essential scaling : d=2,N=2



- Flow equation contains correctly the non-perturbative information !
- (essential scaling usually described by vortices)

Von Gersdorff …

Kosterlitz-Thouless phase transition (d=2,N=2)

Correct description of phase with
Goldstone boson
(infinite correlation length)
for $T<T_c$

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

'92

$\left(\Gamma_k^{(2)}\right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$

$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$

(fermions : STr)

wide applications

particle physics

- gauge theories, QCD

  Reuter,…, Marchesini et al, Ellwanger et al, Litim, Pawlowski, Gies ,Freire, Morris et al., Braun , many others

- electroweak interactions, gauge hierarchy problem

  Jaeckel,Gies,…

- electroweak phase transition

  Reuter,Tetradis,…Bergerhoff,

wide applications

gravity

- asymptotic safety

  Reuter, Lauscher, Schwindt et al, Percacci et al, Litim, Fischer, Saueressig

wide applications

condensed matter

- unified description for classical bosons

CW , Tetradis , Aoki , Morikawa , Souma, Sumi , Terao , Morris , Graeter , v.Gersdorff , Litim , Berges , Mouhanna , Delamotte , Canet , Bervilliers , Blaizot , Benitez , Chatie , Mendes-Galain , Wschebor

- Hubbard model

Baier , Bick,…, Metzner et al, Salmhofer et al, Honerkamp et al, Krahl , Kopietz et al, Katanin , Pepin , Tsai , Strack , Husemann , Lauscher

wide applications

condensed matter

- quantum criticality

  Floerchinger , Dupuis , Sengupta , Jakubczyk ,

- sine- Gordon model

  Nagy , Polonyi

- disordered systems

  Tissier , Tarjus , Delamotte , Canet

wide applications

condensed matter

- equation of state for CO_2 Seide,…
- liquid He^4 Gollisch,… and He^3 Kindermann,…
- frustrated magnets Delamotte, Mouhanna, Tissier
- nucleation and first order phase transitions
  Tetradis, Strumia,…, Berges,…

wide applications

condensed matter

- crossover phenomena

  Bornholdt , Tetradis ,…
- superconductivity (scalar QED_3)

  Bergerhoff , Lola , Litim , Freire,…
- non equilibrium systems

  Delamotte , Tissier , Canet , Pietroni , Meden , Schoeller , Gasenzer , Pawlowski , Berges , Pletyukov , Reininghaus

wide applications

nuclear physics

- effective NJL- type models

  Ellwanger , Jungnickel , Berges , Tetradis,…, Pirner , Schaefer , Wambach , Kunihiro , Schwenk

- di-neutron condensates

  Birse, Krippa,

- equation of state for nuclear matter

  Berges, Jungnickel …, Birse, Krippa

- nuclear interactions

  Schwenk

wide applications

ultracold atoms

- Feshbach resonances

  Diehl, Krippa, Birse , Gies, Pawlowski , Floerchinger , Scherer , Krahl ,

- BEC

  Blaizot, Wschebor, Dupuis, Sengupta, Floerchinger

conclusions

- There is still a lot of interesting theory to learn for an understanding of phase transitions
- Perhaps non-perturbative low equations can help

qualitative changes that make non-perturbative physics accessible :

(1) basic object is simple

average action ~ classical action

~ generalized Landau theory

direct connection to thermodynamics

(coarse grained free energy)

qualitative changes that make non-perturbative physics accessible :

(2) Infrared scale k

instead of Ultraviolet cutoff Λ

short distance memory not lost

no modes are integrated out , but only part of the fluctuations is included

simple one-loop form of flow

simple comparison with perturbation theory

infrared cutoff k

cutoff on momentum resolution

or frequency resolution

e.g. distance from pure anti-ferromagnetic momentum or from Fermi surface

intuitive interpretation of k by association with physical IR-cutoff , i.e. finite size of system :

arbitrarily small momentum differences cannot be resolved !

qualitative changes that make non-perturbative physics accessible :

(3) only physics in small momentum range around k matters for the flow

ERGE regularization

simple implementation on lattice

artificial non-analyticities can be avoided

qualitative changes that make non-perturbative physics accessible :

(4) flexibility

- change of fields
- microscopic or composite variables
- simple description of collective degrees of freedom and bound states
- many possible choices of "cutoffs"

some history …

- **exact RG equations :**
  Symanzik eq. , Wilson eq. , Wegner-Houghton eq. , Polchinski eq. ,
  mathematical physics

- **1PI :** RG for 1PI-four-point function and hierarchy
  Weinberg
  formal Legendre transform of Wilson eq.
  Nicoll, Chang

- **non-perturbative flow :**
  d=3 : sharp cutoff ,
  no wave function renormalization or momentum dependence
  Hasenfratz^2