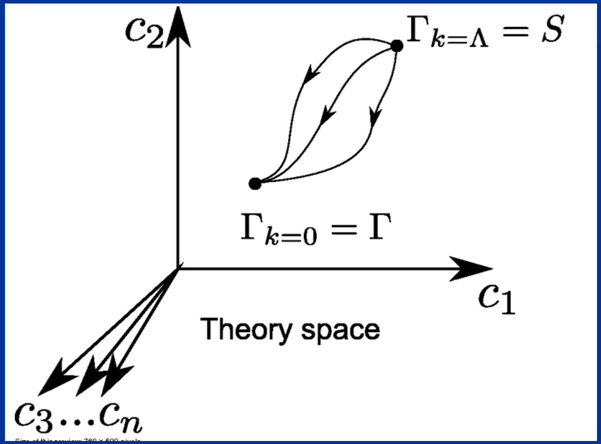
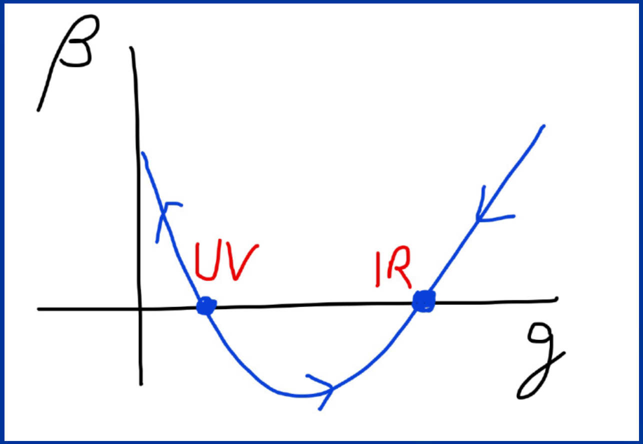
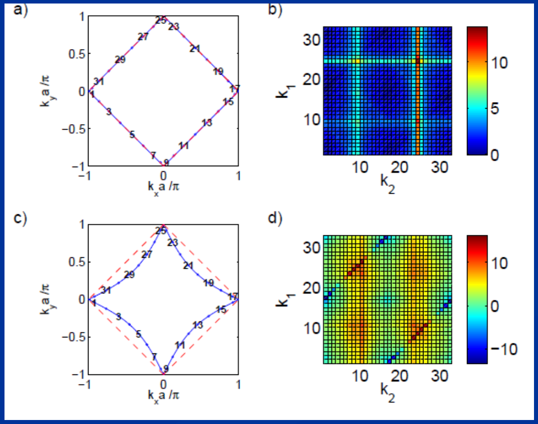
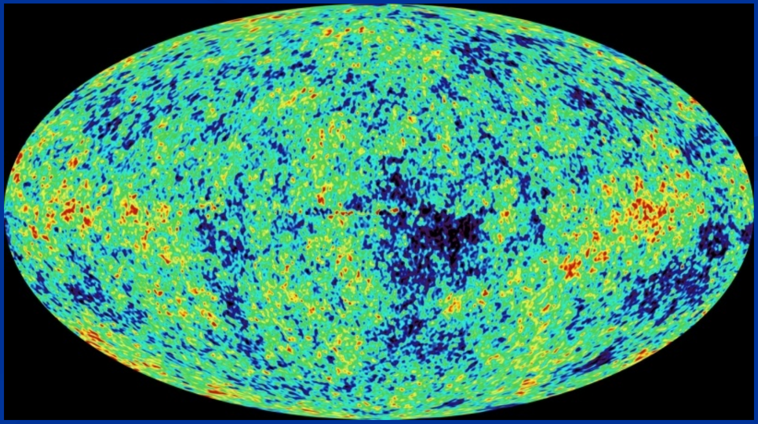
Functional renormalization: from quantum gravity and cosmology to superconducting solids









Functional renormalization: from microphysics to macrophysics

Macrophysics

Landau type theories for relevant degrees of freedom

extract properties from variation : field equations

superconductors, superfluidity …

Microphysics

Formulated as partition function or functional integral

Microphysical laws are encoded in classical action S
(microphysical action, related to Hamiltonian)

weight factor in probability distribution e^{-S}

atomic interactions, quantum gravity,
standard model of particle physics, …

Macroscopic understanding does not need all details of underlying microscopic physics

1) motion of planets : m_i

Newtonian mechanics of point particles

probabilistic atoms → deterministic planets

2) thermodynamics : T, μ, Gibbs free energy $J(T,\mu)$

3) antiferromagnetic waves for correlated electrons

$\Gamma[\, s_i(x)]$

How to get from microphysics to macrophysics ?

1) motion of planets : $\mathbf{m_i}$

compute or measure mass of objects

(second order more complicated : tides etc.)

2) thermodynamics : $\mathbf{J(T, \mu)}$

integrate out degrees of freedom

3) antiferromagnetic waves for correlated electrons

$\mathbf{\Gamma[s_i(x)]}$ change degrees of freedom *central role of fluctuations*

classical and effective action

- classical action : microscopic laws

- quantum effective action : macroscopic laws

  includes all fluctuation effects

  field equations are exact

  Landau type theory

  generates 1PI- correlation functions

Emergence of macroscopic laws
with
Functional Renormalization

Do it stepwise :
functional renormalization

Leo Kadanoff   Kenneth Wilson   Franz Wegner

scale dependent effective action

- average effective action, flowing effective action
- introduces momentum scale k
- all fluctuations with momenta larger k are included
- fluctuations with momenta smaller k are not yet included

effective laws at scale k

From

Microscopic Laws

(Interactions, classical action)

↓

to — Fluctuations!

Macroscopic Observation

(Free energy functional, effective action)

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

'92

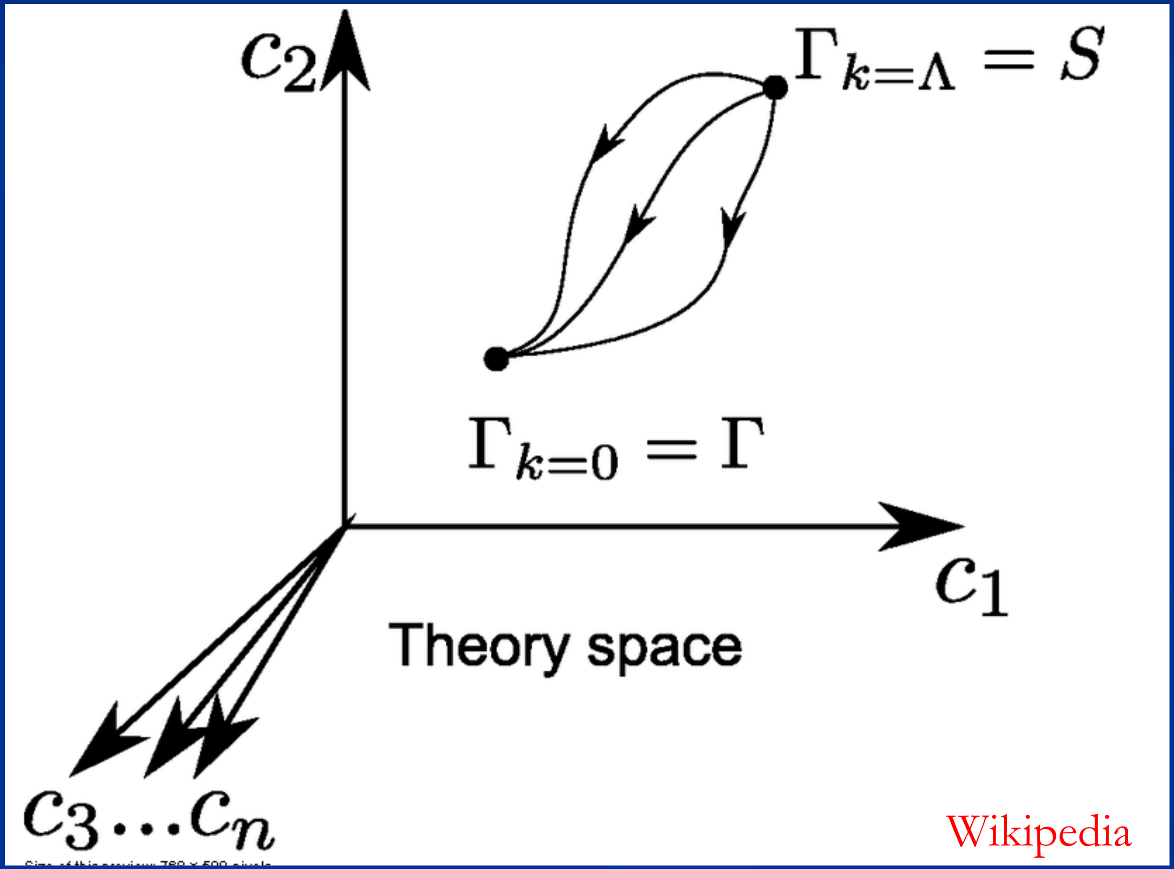
$$\left(\Gamma_k^{(2)}\right)_{ab}(q,q') = \frac{\delta^2\Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$\mathrm{Tr} : \sum_a \int \frac{d^dq}{(2\pi)^d}$

(fermions : STr)

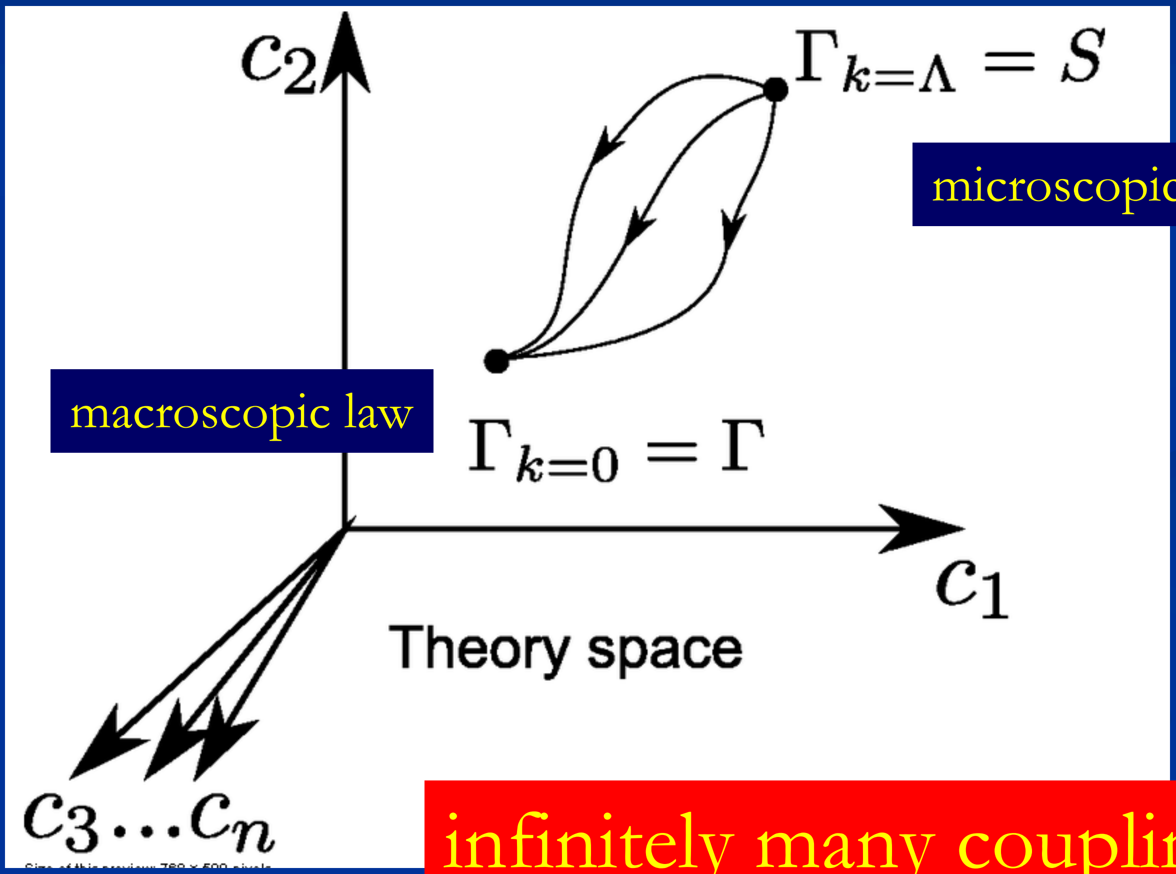
R_k : cutoff function
does not affect
high momentum fluctuations
cuts off
"infrared fluctuations"

flowing action



Wikipedia

flowing action



microscopic law

macroscopic law

infinitely many couplings

Effective potential

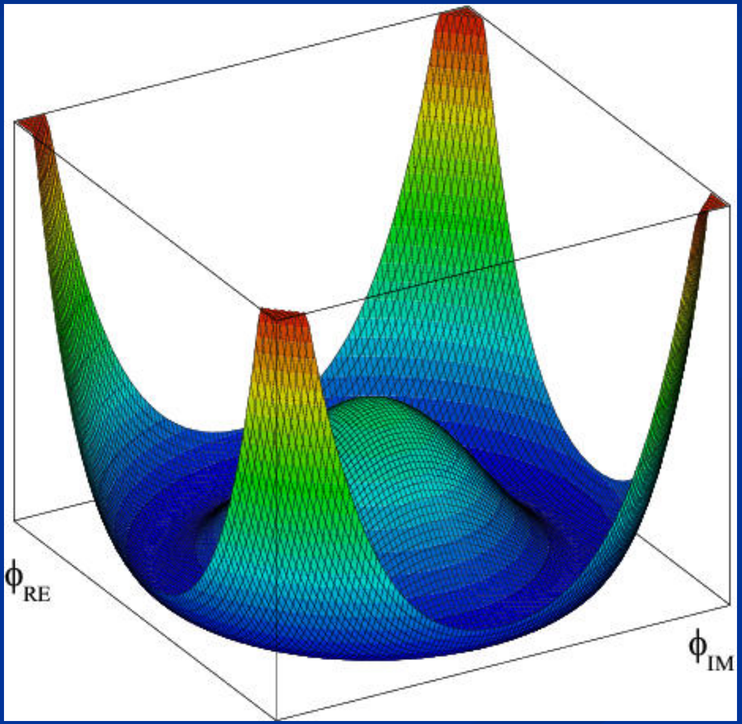
Effective potential

=

non – derivative

part of

effective action



Effective potential includes **all** fluctuations

Average potential U_k

$\equiv$ **scale dependent effective potential**
$\equiv$ **coarse grained free energy**

Only fluctuations with momenta $q^2 > k^2$ included

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

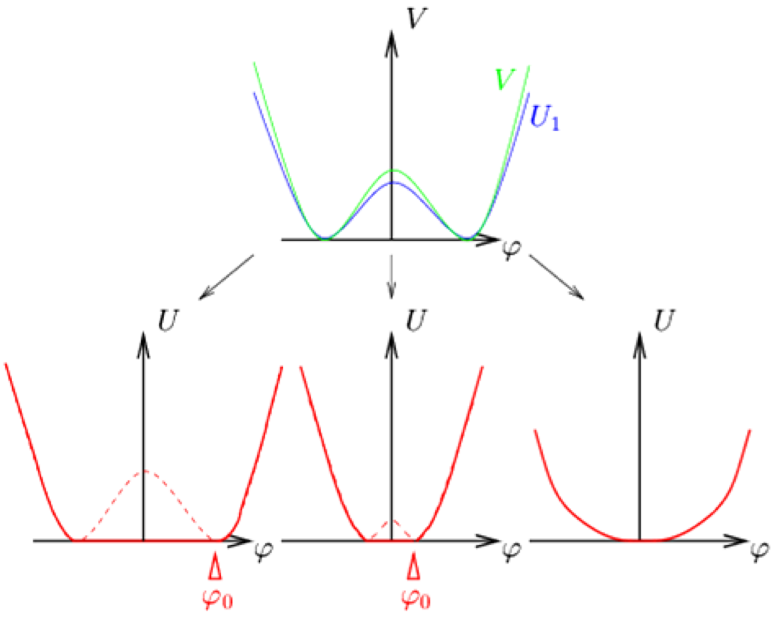
$$U_\Lambda \approx S \to U_0 \equiv U$$

Scalar field theory

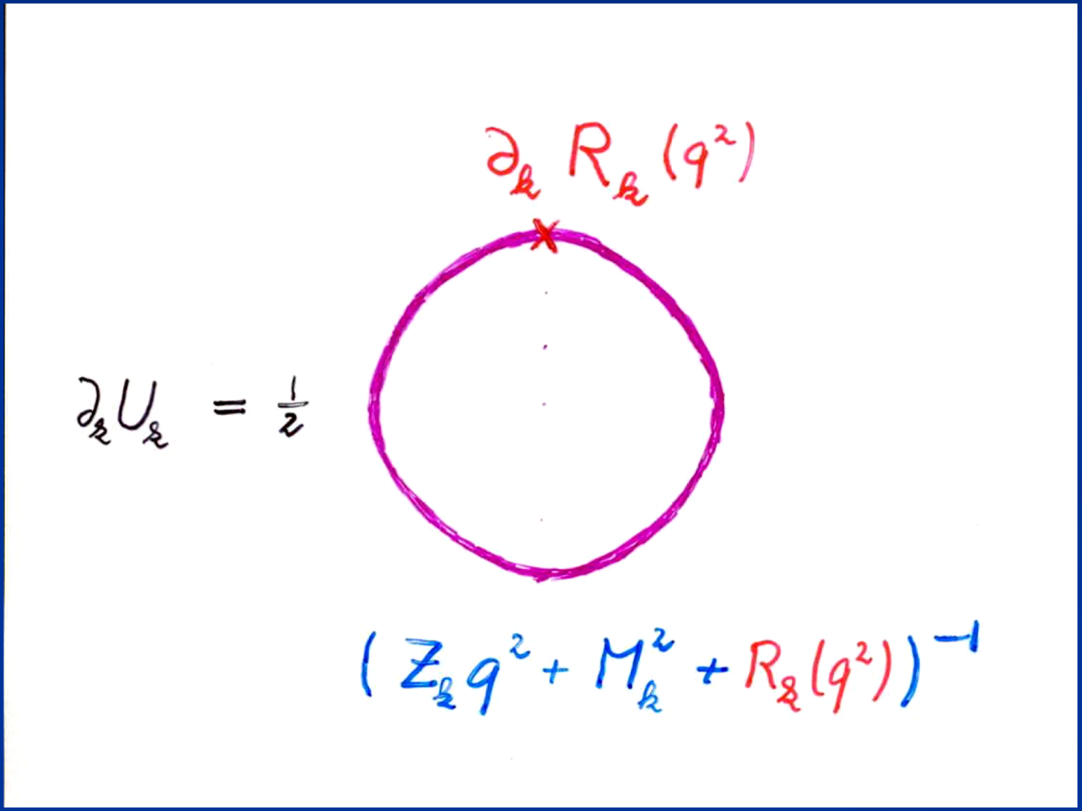
$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Simple one loop structure – nevertheless (almost) exact



$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}_{k,i}^2(\varphi)}$$

Simple differential equation for O(N) – models , dimension d

$$\partial_t u|_{\tilde\rho} = -du + (d-2+\eta)\tilde\rho u' + 2v_d\{l_0^d(u'+2\tilde\rho u'';\eta) + (N-1)\, l_0^d(u';\eta)\}$$

$$u = \frac{U_k}{k^d}$$
$$\tilde\rho = Z_k k^{2-d}\rho$$
$$u' = \frac{\partial u}{\partial\tilde\rho} \quad \text{etc.}$$

linear cutoff:

$$l_0^d(w;\eta) = \frac{2}{d}\left(1-\frac{\eta}{d+2}\right)\frac{1}{1+w}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

$t = \ln(k)$

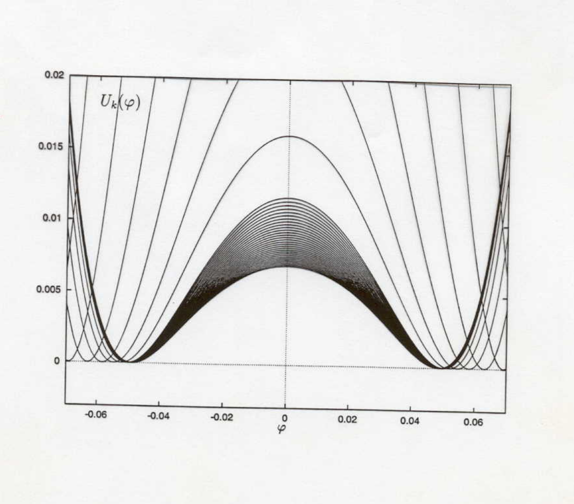
unified approach

- choose N
- choose d
- choose initial form of potential
- run !

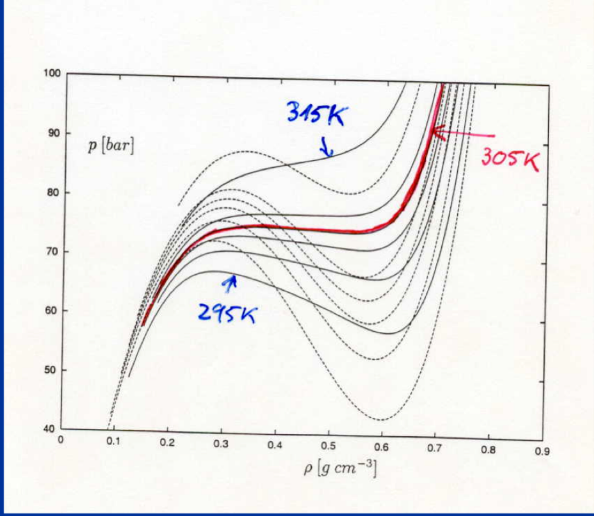
unified description of scalar models for all d and N

Flow of effective potential

Ising model



CO_2



Critical exponents

$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003
		↑		↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

S.Seide …

Critical exponents , d=3

N	ν		η	
	ERGE	world	ERGE	world
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
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"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:
Kosterlitz-Thouless phase transition

Kosterlitz-Thouless phase transition (d=2,N=2)

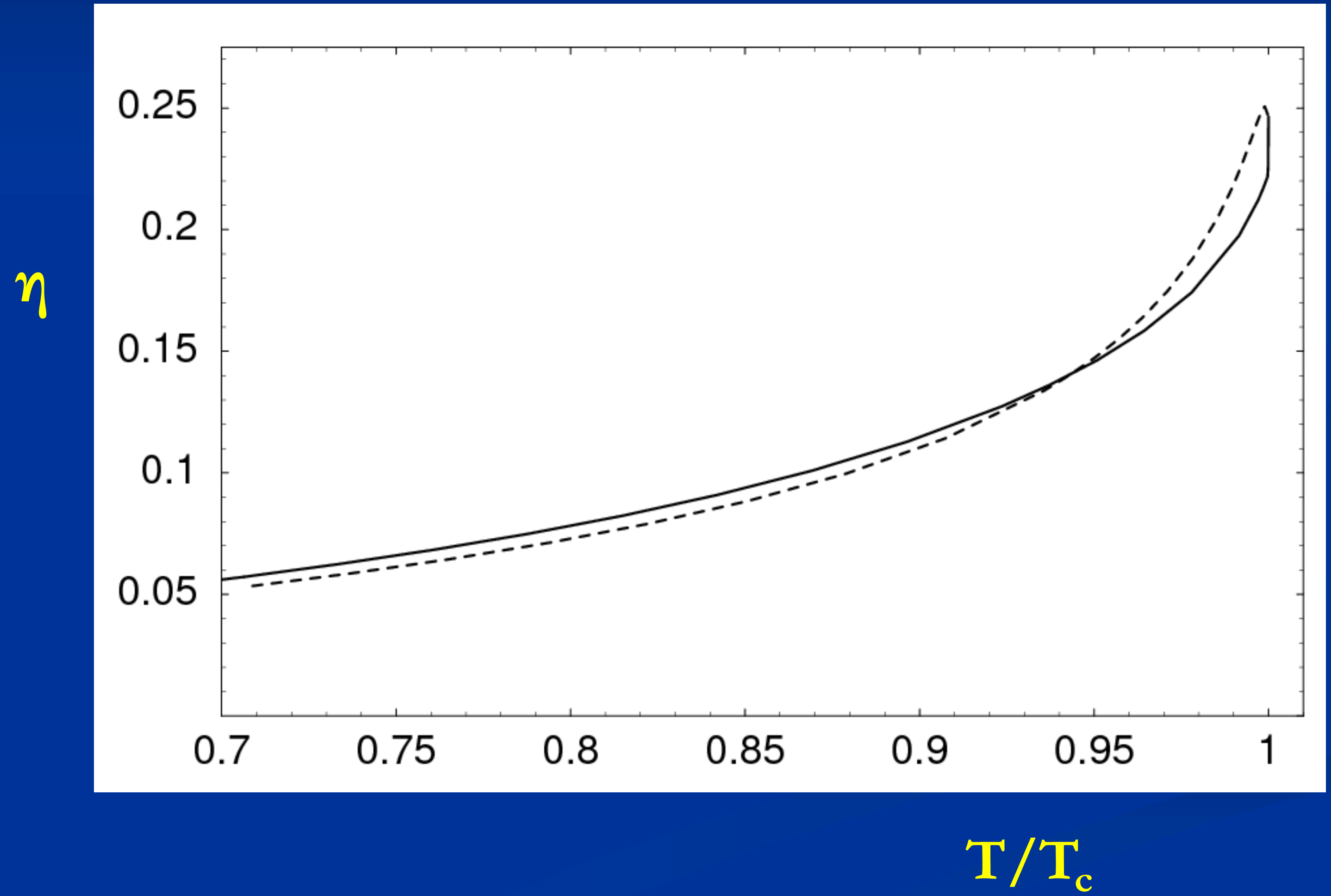
Correct description of phase with

Goldstone boson

(infinite correlation length)

for $T<T_c$

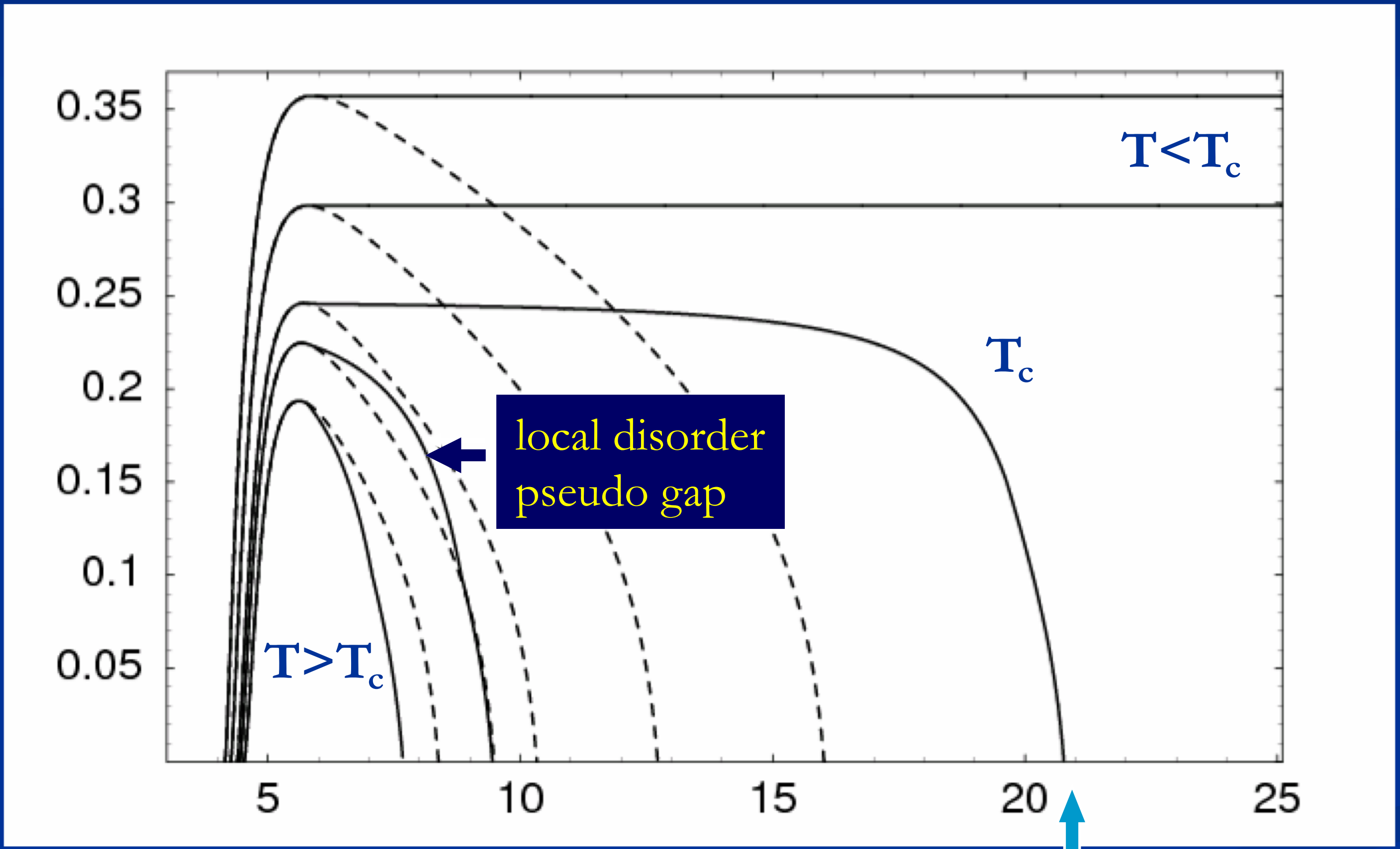
Temperature dependent anomalous dimension η



Running renormalized d-wave superconducting order parameter κ in doped Hubbard (-type) model

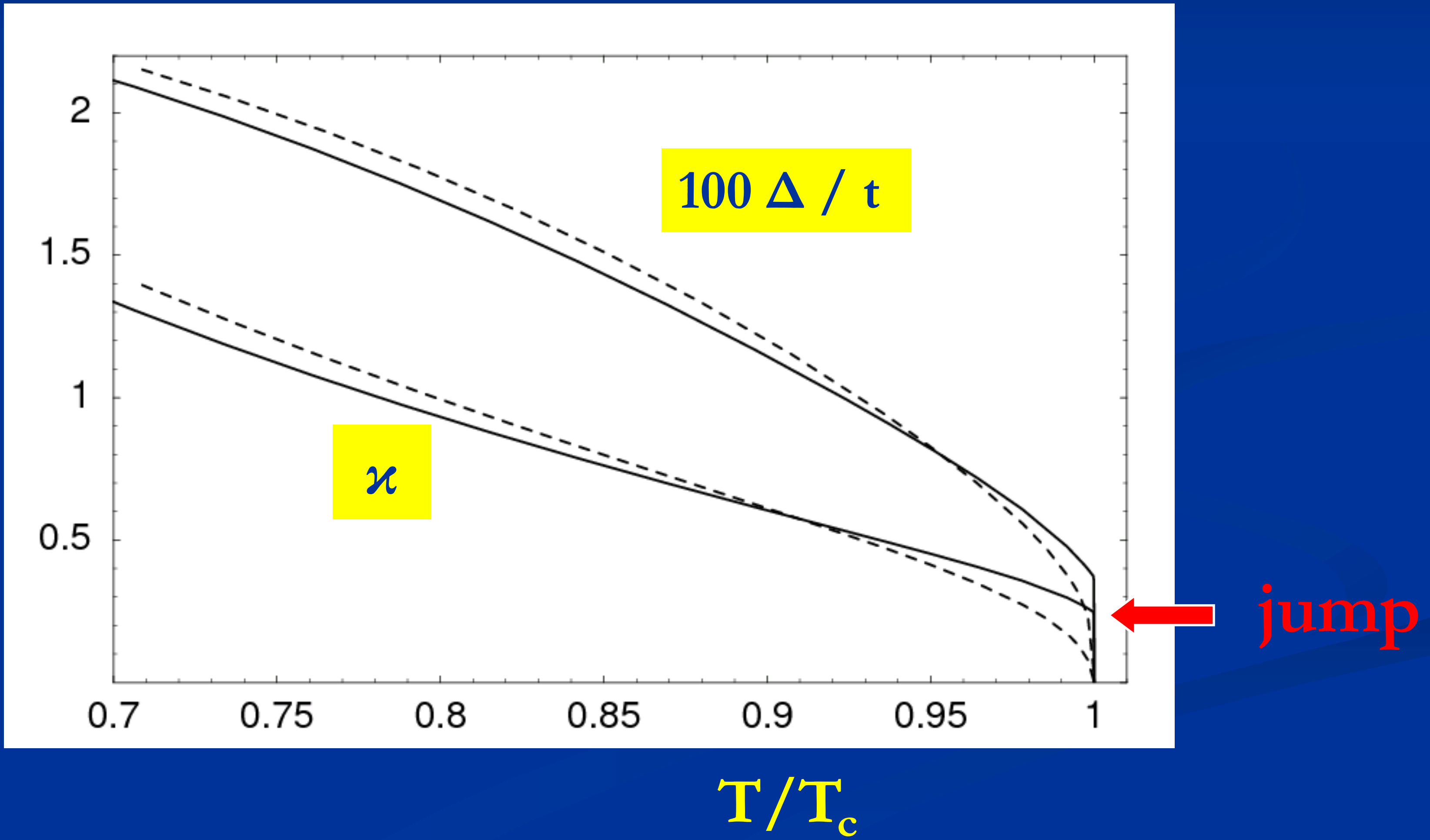
κ

location of minimum of u

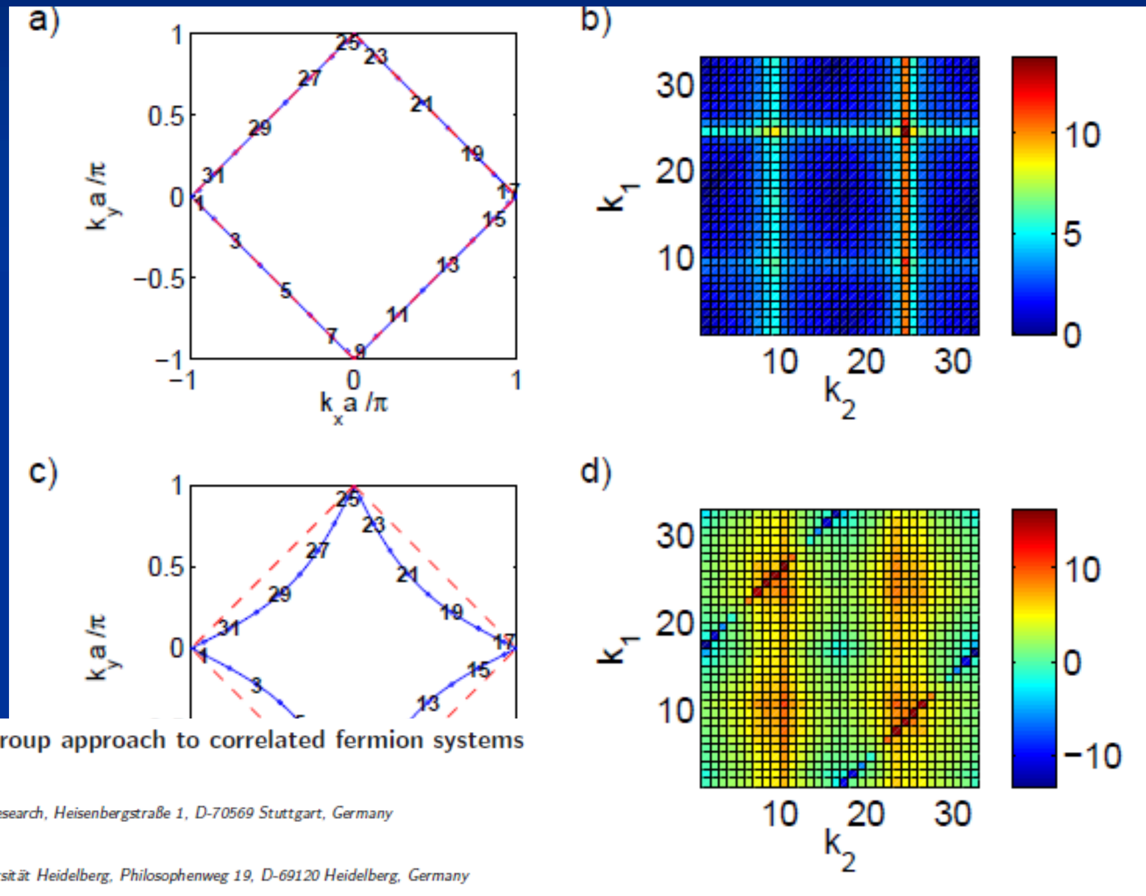


C.Krahl,… **- ln (k/Λ)** macroscopic scale 1 cm

Renormalized order parameter ϰ and gap in electron propagator Δ in doped Hubbard model



Flow of four point function
Hubbard model



Functional renormalization group approach to correlated fermion systems

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Quantum Gravity

Quantum Gravity can be a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

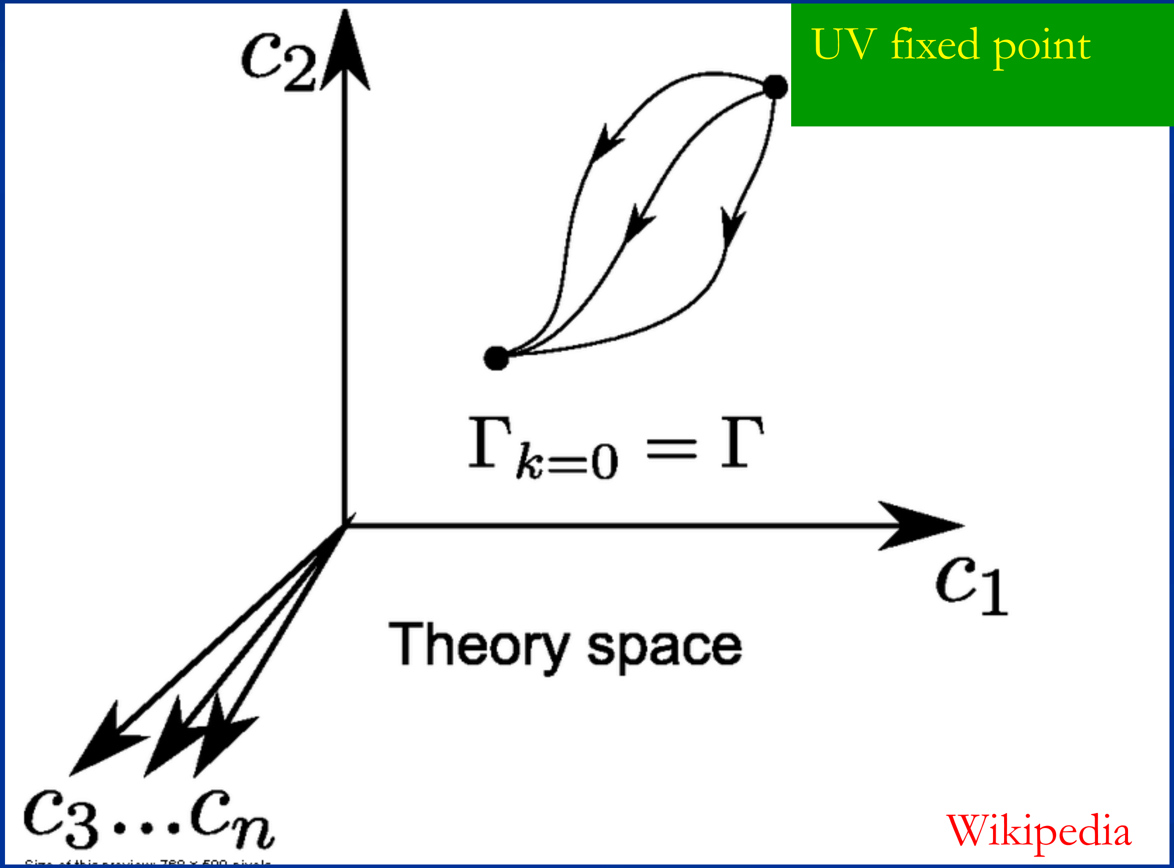
if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

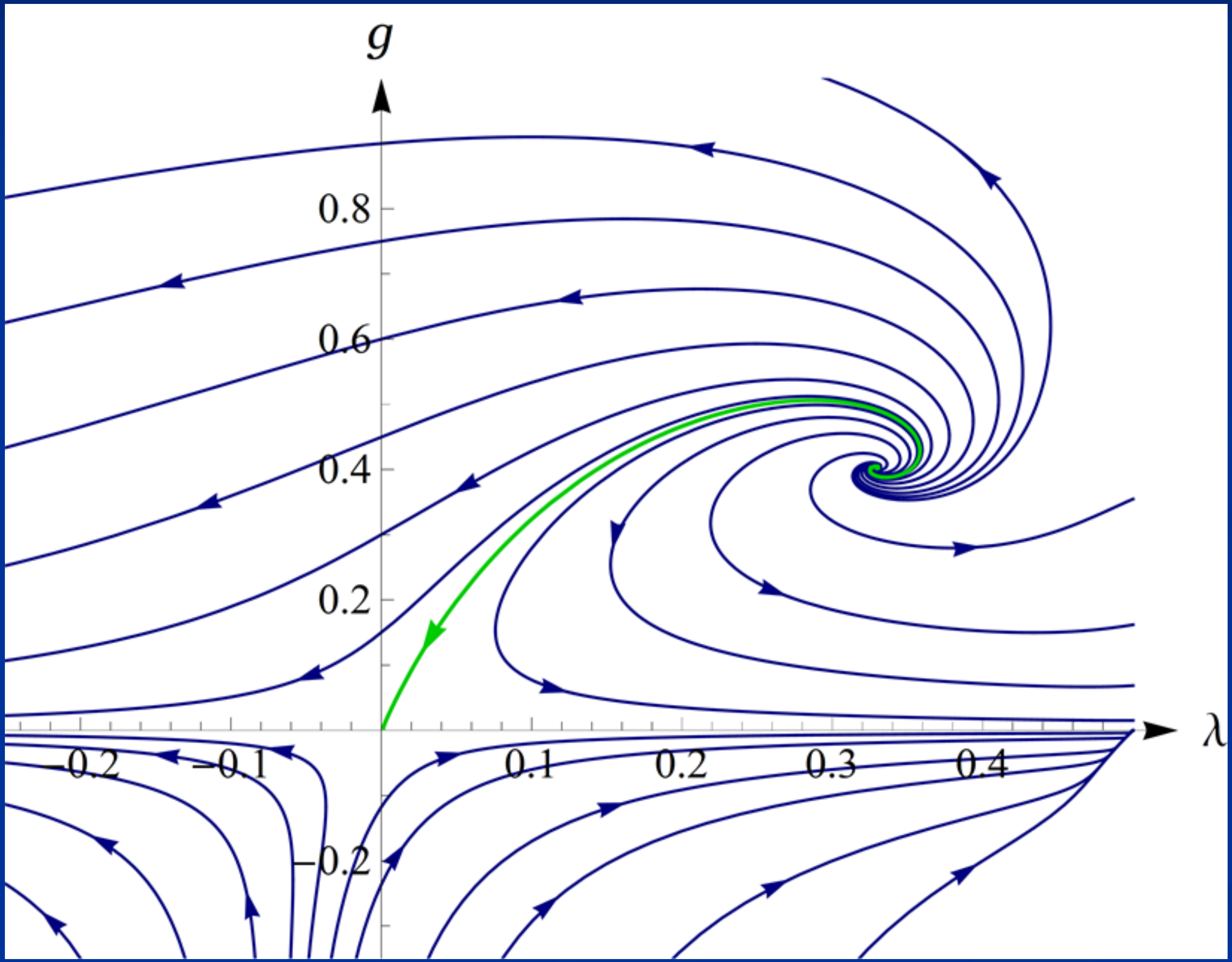
Ultraviolet fixed point



UV fixed point

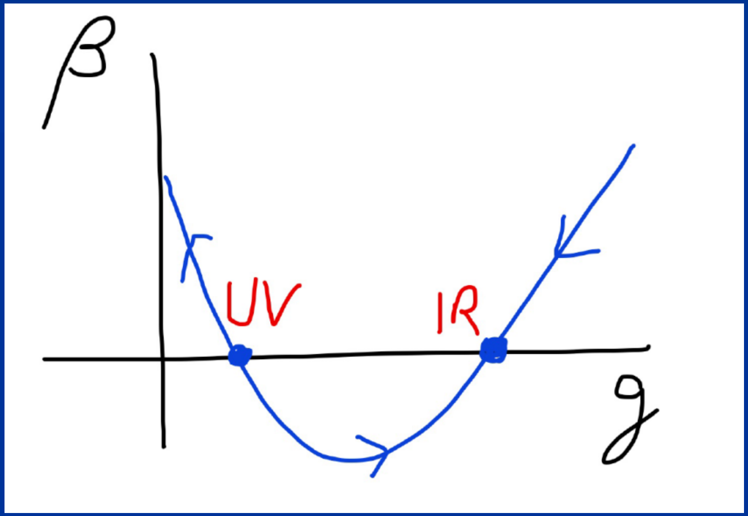
Wikipedia

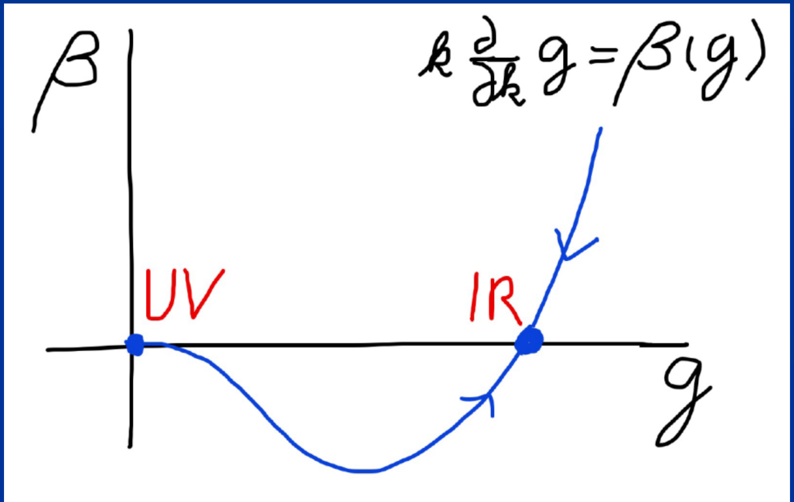
UV- fixed point for quantum gravity



Wikipedia

Asymptotic safety   Asymptotic freedom





Relevant parameters yield undetermined couplings.
Quartic scalar coupling is not relevant and can therefore be predicted.

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

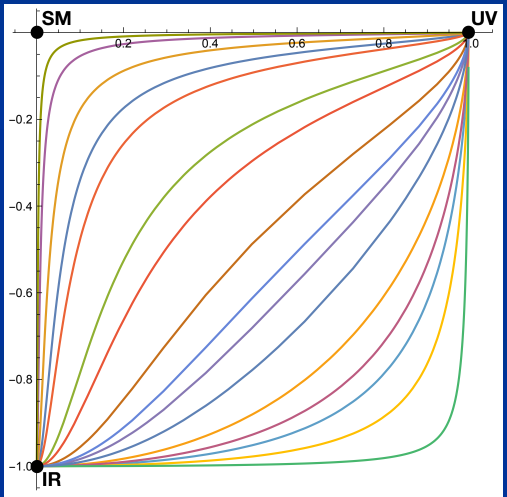
Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\text{min}} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\text{min}} = 126$ GeV, with o

Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.



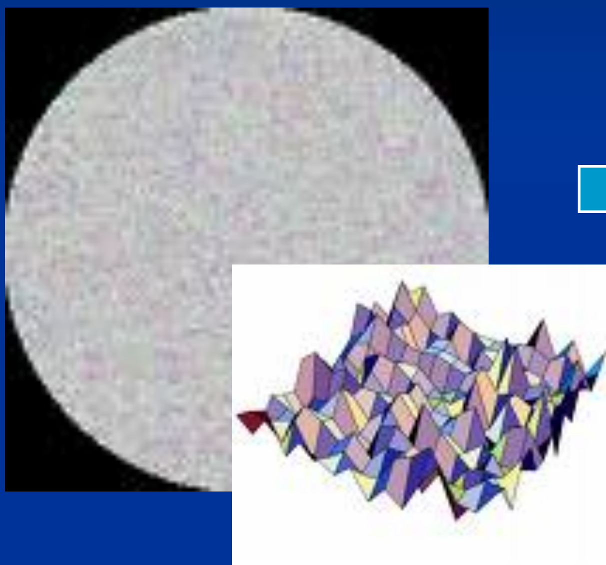
Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

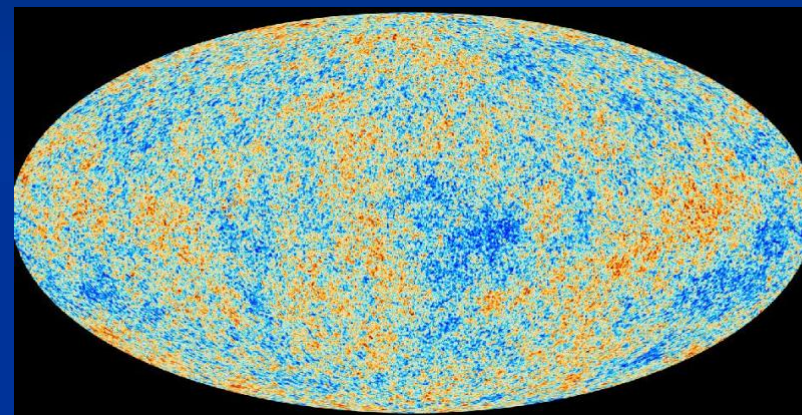
Continuous global symmetry

Approximate scale symmetry near fixed points

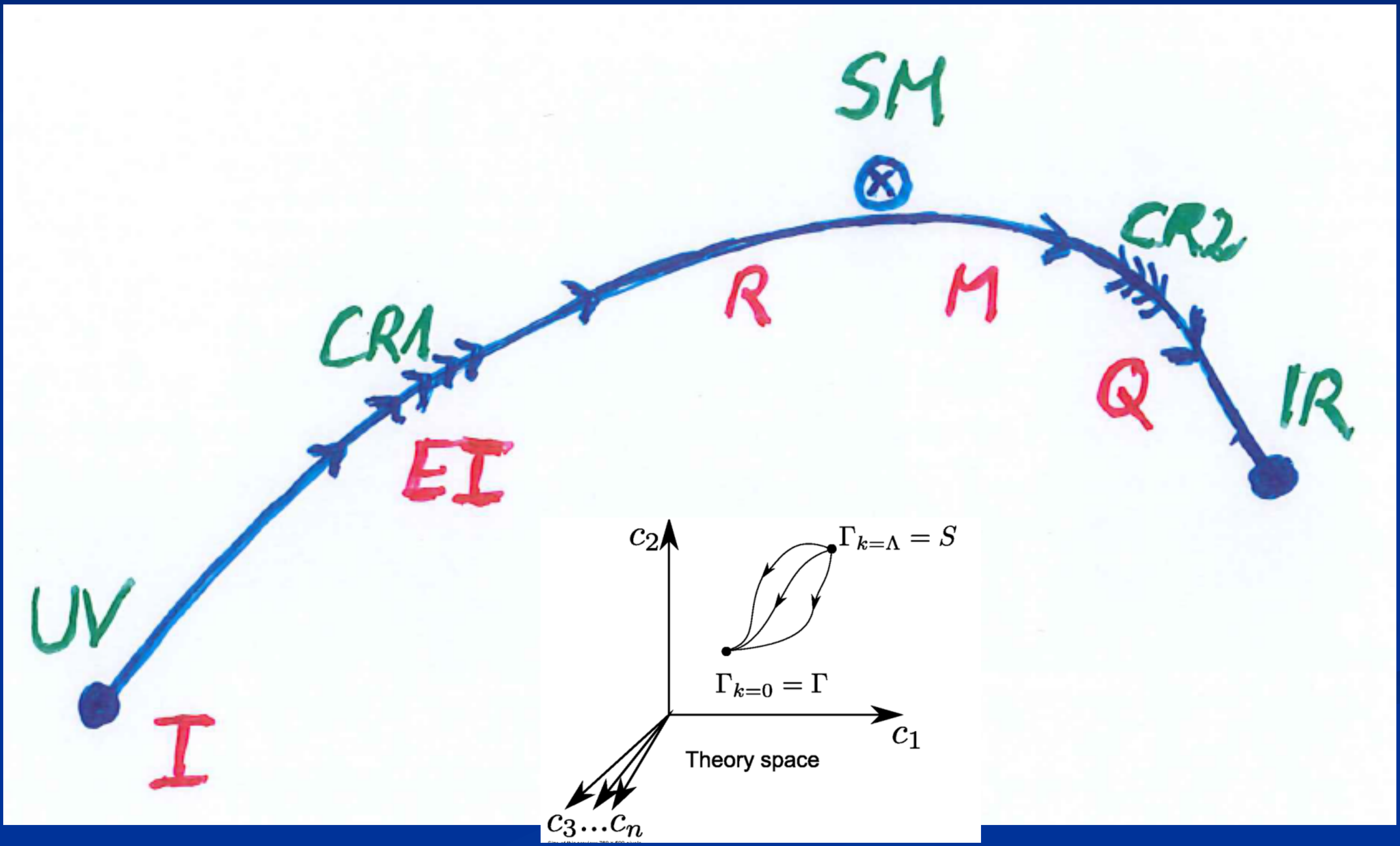
UV : approximate scale invariance of primordial fluctuation spectrum from inflation

Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe





Crossover in quantum gravity



Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

Cosmic scale symmetry and the cosmological constant problem

- IR – fixed point reached for $\chi \rightarrow \infty$
- Impact of intrinsic mass scale disappears

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

asymptotically vanishing cosmological „constant"

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2\chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Conclusions

- Functional renormalization has worked out in many areas of physics,

  even biology and economics…
- try it out !

end