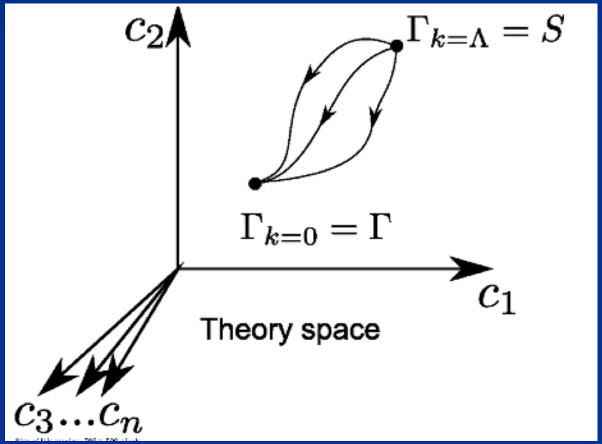
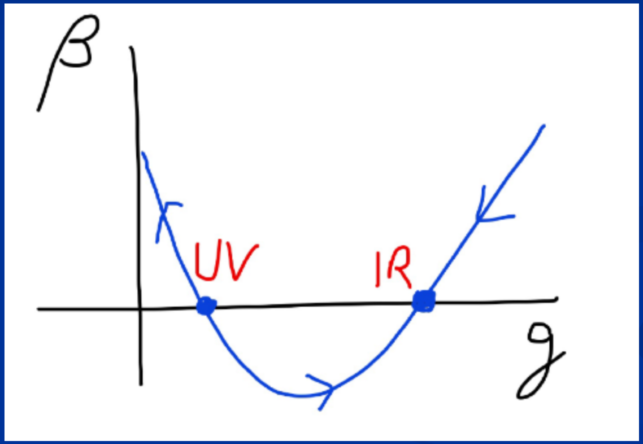
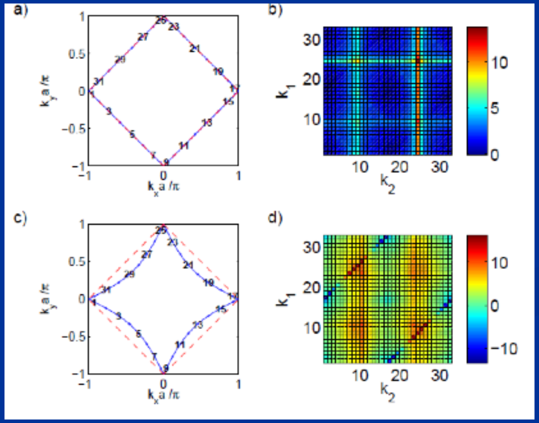
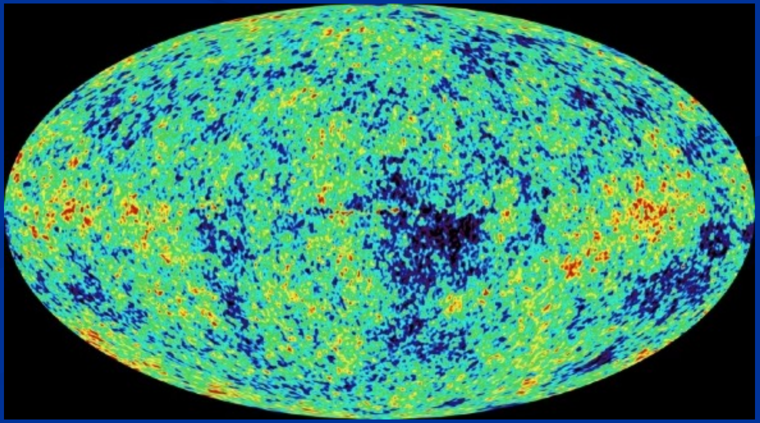
Functional renormalisation: understanding fluctuations from superconductivity to quantum gravity









Functional renormalisation: from microphysical laws to macrophysical complexity

Functional integral

Wide applications:

- partition function in statistical physics
- whenever you deal with a probability distribution
- also more general complex weight distributions

Microphysics

Formulated as partition function or functional integral

Microphysical laws are encoded in classical action S
(microphysical action, related to Hamiltonian)

weight factor in probability distribution e^{-S}

atomic interactions, quantum gravity,
standard model of particle physics, …

Macrophysics

Landau type theories for relevant degrees of freedom

extract properties from variation of effective action :
field equations

superconductors, superfluidity …

Macroscopic understanding does not need all details of underlying microscopic physics

1) motion of planets : m_i

Newtonian mechanics of point particles

probabilistic atoms → deterministic planets

2) thermodynamics : T, μ, Gibbs free energy $J(T,\mu)$

3) antiferromagnetic waves for correlated electrons

$\Gamma[\, s_i(x)]$

How to get from microphysics to macrophysics ?

1) motion of planets : m_i

compute or measure mass of objects

(second order more complicated : tides etc.)

2) thermodynamics : $J(T, \mu)$

integrate out degrees of freedom

3) antiferromagnetic waves for correlated electrons

$\Gamma[s_i(x)]$ change degrees of freedom *central role of fluctuations*

Classical and effective action

- classical action : microscopic laws

- quantum effective action : macroscopic laws

  includes all fluctuation effects

  (quantum, thermal, whatsoever…)

  field equations are exact

  Landau type theory

  generates 1PI- correlation functions

Effective action

$$\exp(-\Gamma[\tilde{\varphi}]) = \int D\tilde{\chi} \exp\left\{-S[\tilde{\varphi} + \tilde{\chi}] + \int_x \frac{\partial \Gamma}{\partial \tilde{\varphi}} \tilde{\chi}\right\}$$

Field equations

- The field equations we use for electromagnetism, gravity, or superfluidity are macroscopic equations.
- They obtain by variation of the effective action, not the microscopic action.
- "Classical field theory" is exact, but only with macroscopic field equations

Emergence of macroscopic laws
with
Functional Renormalisation

Do it stepwise : functional renormalisation



Leo Kadanoff   Kenneth Wilson   Franz Wegner

Scale dependent effective action

- average effective action, flowing effective action
- introduces momentum scale k by an

  infrared cutoff
- all fluctuations with momenta larger k are included
- fluctuations with momenta smaller k are not yet included

effective laws at scale k

From

Microscopic Laws

(Interactions, classical action)

to

Fluctuations!

Macroscopic Observation

(Free energy functional, effective action)

Exact renormalisation group equation

Exact flow equation

for scale dependence of average action

$$\boxed{\partial_k \Gamma_k[\varphi] = \tfrac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}}$$

'92

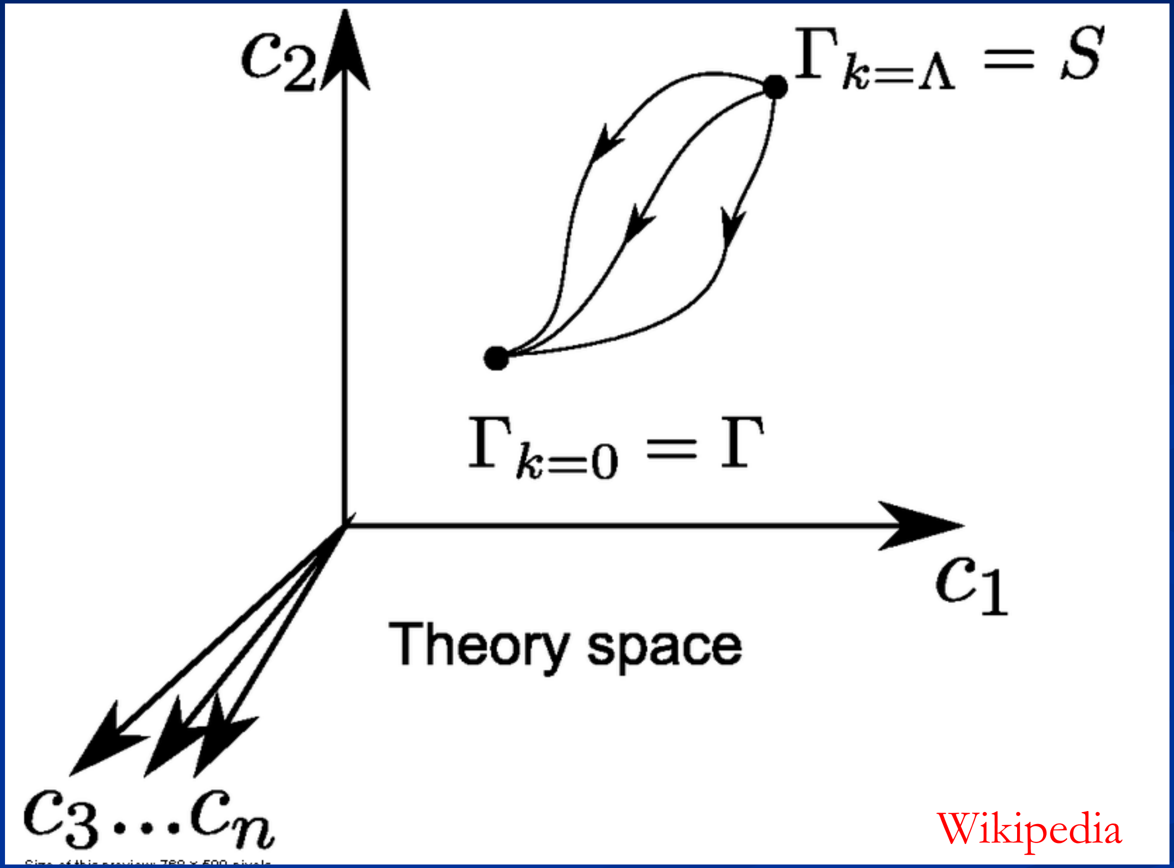
$$\left(\Gamma_k^{(2)}\right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

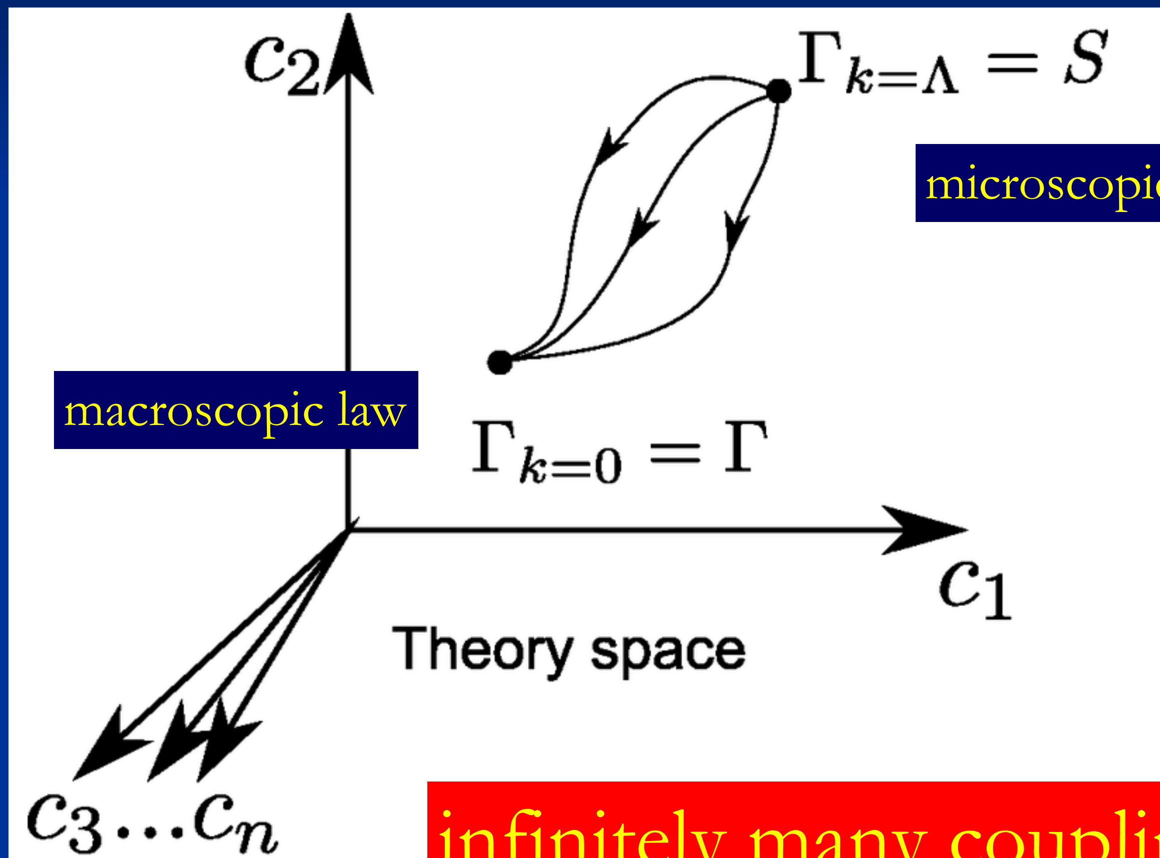
(fermions : STr)

R_k : cutoff function
does not affect
high momentum fluctuations
cuts off
"infrared fluctuations"

Flowing action



Flowing action



microscopic law

macroscopic law

infinitely many couplings

Effective potential

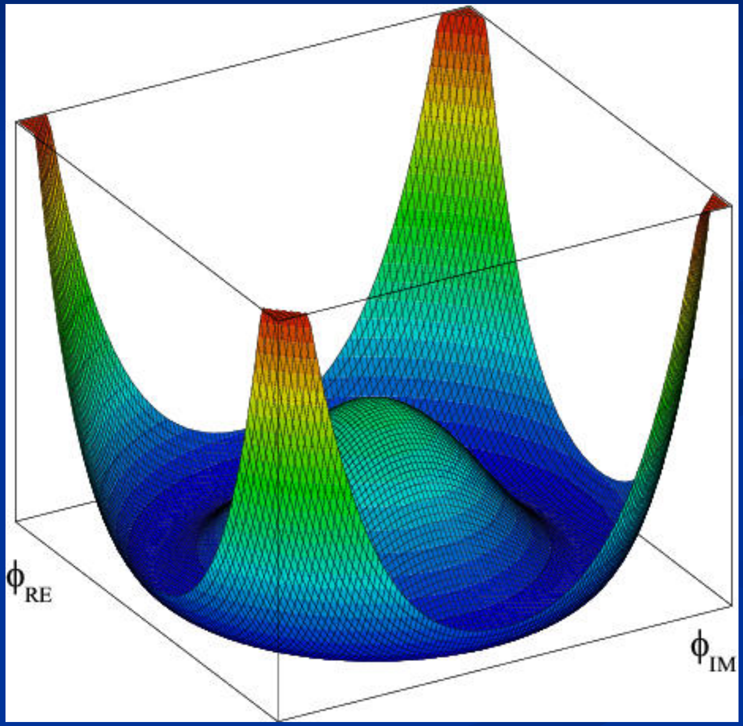
Effective potential

=

non – derivative

part of

effective action



Effective potential includes all fluctuations

Average potential U_k

$\equiv$ scale dependent effective potential
$\equiv$ coarse grained free energy

$$\boxed{\text{Only fluctuations with momenta } q^2 > k^2 \text{ included}}$$

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

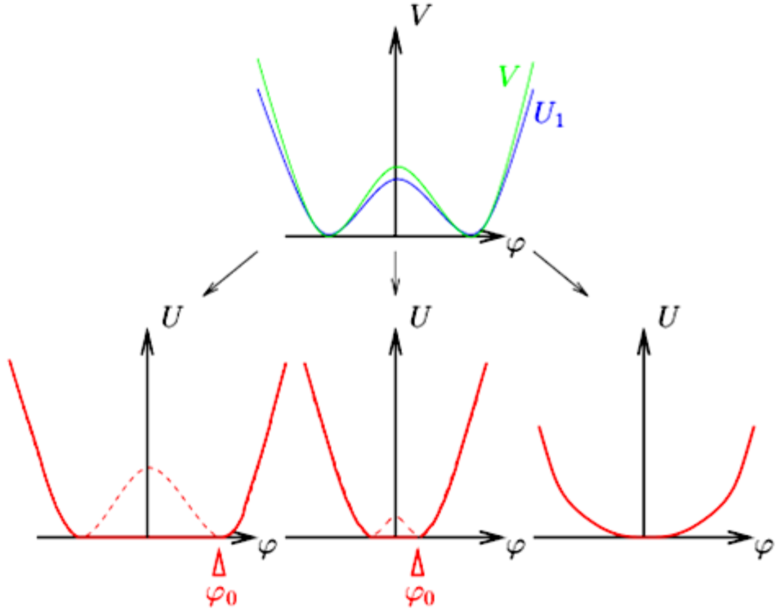
$$U_\Lambda \approx S \to U_0 \equiv U$$

Scalar field theory

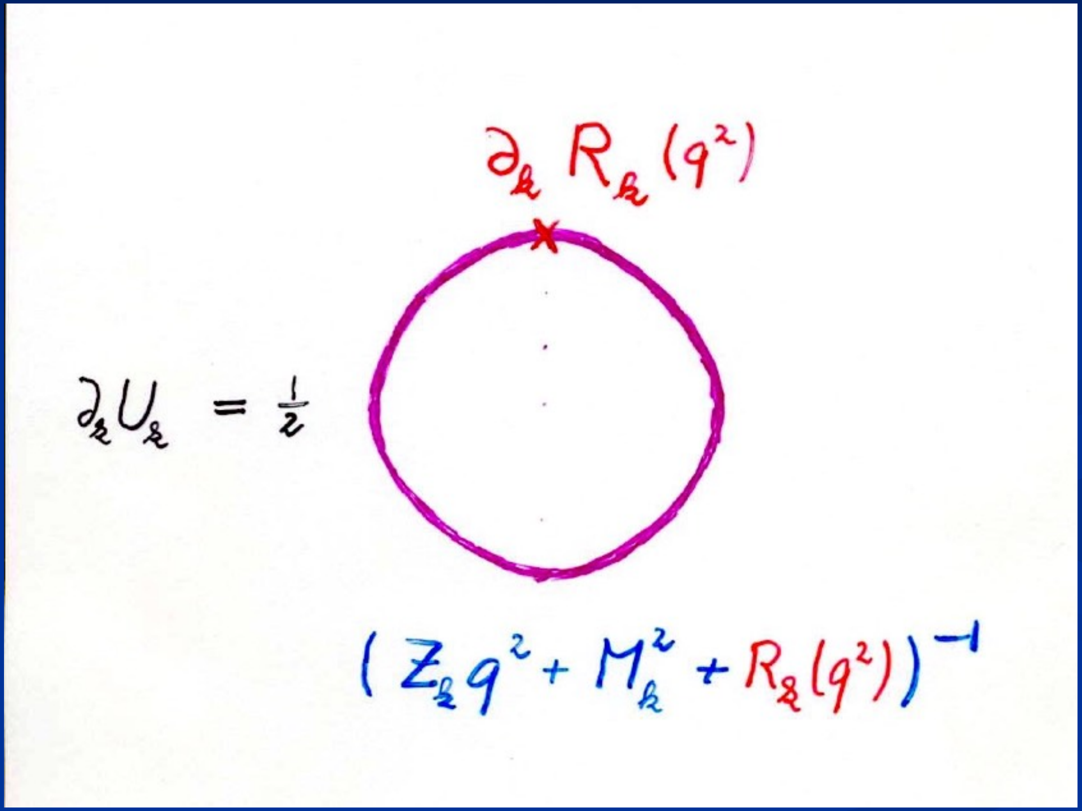
$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^dx \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Simple one loop structure – nevertheless (almost) exact



$$\boxed{\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}_{k,i}^2(\varphi)}}$$

Simple differential equation for O(N) – models , dimension d

$$
\begin{aligned}
\partial_t u|_{\tilde{\rho}} \;=\; & -du + (d-2+\eta)\tilde{\rho}u' \\
& +2v_d\{l_0^d(u'+2\tilde{\rho}u'';\eta) \\
& +(N-1)\,l_0^d(u';\eta)\}
\end{aligned}
$$

$$
\begin{aligned}
u &= \frac{U_k}{k^d} \\
\tilde{\rho} &= Z_k k^{2-d}\rho \\
u' &= \frac{\partial u}{\partial \tilde{\rho}} \quad \text{etc.}
\end{aligned}
$$

linear cutoff:

$$l_0^d(w;\eta) = \frac{2}{d}\left(1-\frac{\eta}{d+2}\right)\frac{1}{1+w}$$

$$v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma\left(\frac{d}{2}\right)$$

t = ln(k)

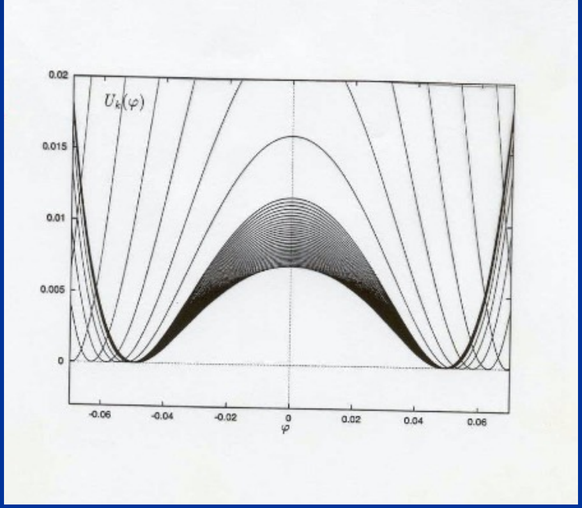
unified approach

- choose N
- choose d
- choose initial form of potential
- run !

unified description of scalar models for all d and N

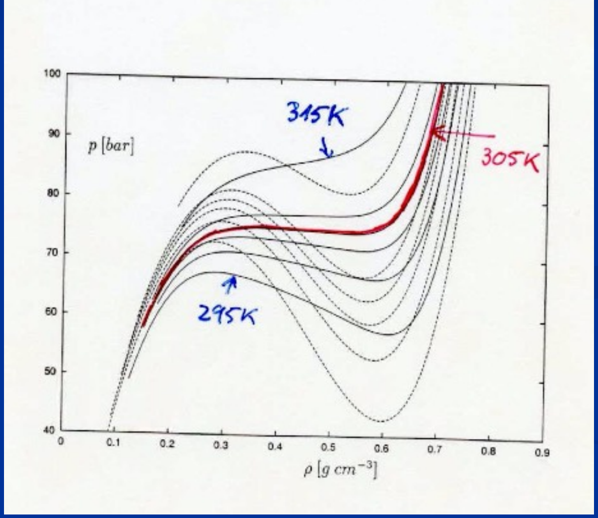
Flow of effective potential

Ising model



S.Seide …

CO_2



Experiment :

T_* =304.15 K
p_* =73.8.bar
ρ_* = 0.442 g cm-2

Critical exponents

$d = 3$

Critical exponents ν and η

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003

↑ ↑

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Critical exponents , d=3

N	ν ERGE	ν world	η ERGE	η world
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
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10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

More sophisticated approximations

Correlation-length exponent ν										
N	LPA	DE_2	DE_4	DE_6	LPA″	BMW	MC	PT	ϵ-exp	CB
0	0.5925	0.5879(13)	0.5876(2)	–	–	0.589	0.58759700(40)	0.5882(11)	0.5874(3)	0.5876(12)
1	0.650	0.6308(27)	0.62989(25)	0.63012(16)	0.631	0.632	0.63002(10)	0.6304(13)	0.6292(5)	0.629971(4)
2	0.7090	0.6725(52)	0.6716(6)	–	0.679	0.674	0.67169(7)	0.6703(15)	0.6690(10)	0.6718(1)
3	0.7620	0.7125(71)	0.7114(9)	–	0.725	0.715	0.7112(5)	0.7073(35)	0.7059(20)	0.7120(23)
4	0.805	0.749(8)	0.7478(9)	–	0.765	0.754	0.7477(8)	0.741(6)	0.7397(35)	0.7472(87)

Anomalous dimension η									
N	DE_2	DE_4	DE_6	LPA″	BMW	MC	PT	ϵ-exp	CB
0	0.0326(47)	0.0312(9)	–	–	0.034	0.0310434(30)	0.0284(25)	0.0310(7)	0.0282(4)
1	0.0387(55)	0.0362(12)	0.0361(11)	0.0506	0.039	0.03627(10)	0.0335(25)	0.0362(6)	0.0362978(20)
2	0.0410(59)	0.0380(13)	–	0.0491	0.041	0.03810(8)	0.0354(25)	0.0380(6)	0.03818(4)
3	0.0408(58)	0.0376(13)	–	0.0459	0.040	0.0375(5)	0.0355(25)	0.0378(5)	0.0385(13)
4	0.0389(56)	0.0360(12)	–	0.0420	0.038	0.0360(4)	0.0350(45)	0.0366(4)	0.0378(32)

Correction-to-scaling exponent ω								
N	LPA	DE_2	DE_4	BMW	MC	PT	ϵ-exp	CB
0	0.66	1.00(19)	0.901(24)	0.83	0.899(14)	0.812(16)	0.841(13)	–
1	0.654	0.870(55)	0.832(14)	0.78	0.832(6)	0.799(11)	0.820(7)	0.82968(23)
2	0.672	0.798(34)	0.791(8)	0.75	0.789(4)	0.789(11)	0.804(3)	0.794(8)
3	0.702	0.754(34)	0.769(11)	0.73	0.773	0.782(13)	0.795(7)	0.791(22)
4	0.737	0.731(34)	0.761(12)	0.72	0.765	0.774(20)	0.794(9)	0.817(30)

The nonperturbative functional renormalization group and its applications

N. Dupuis[a], L. Canet[b], A. Eichhorn[c,d], W. Metzner[e], J. M. Pawlowski[d,f], M. Tissier[a], N. Wschebor[g]

Solution of partial differential equation :

yields highly nontrivial non-perturbative results despite the one loop structure !

Example:
Kosterlitz-Thouless phase transition

Kosterlitz-Thouless phase transition (d=2,N=2)

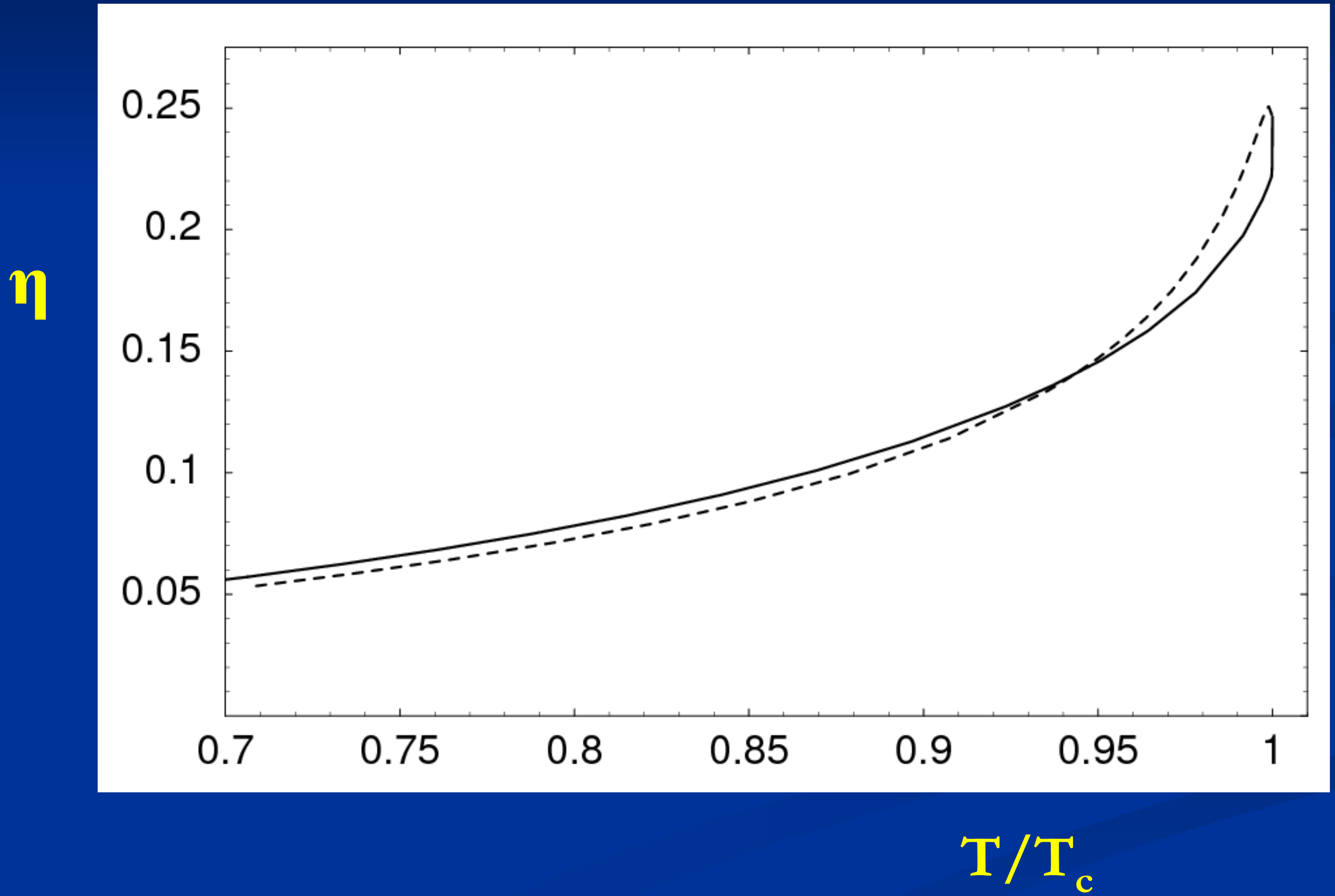
Correct description of phase with

Goldstone boson

(infinite correlation length)

for $T<T_c$

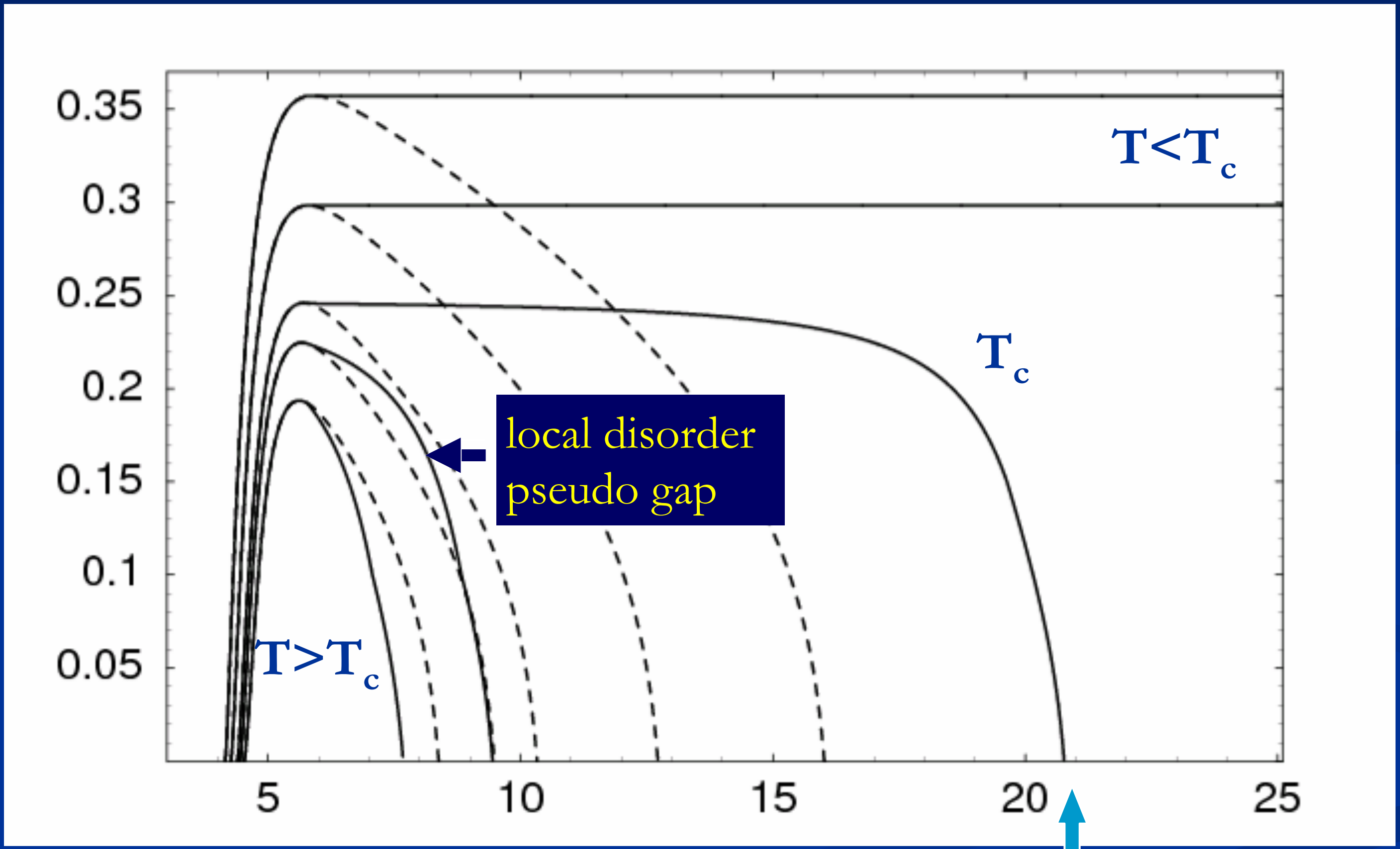
Temperature dependent anomalous dimension η



Running renormalized d-wave superconducting order parameter ϰ in doped Hubbard (-type) model

ϰ

location of minimum of u

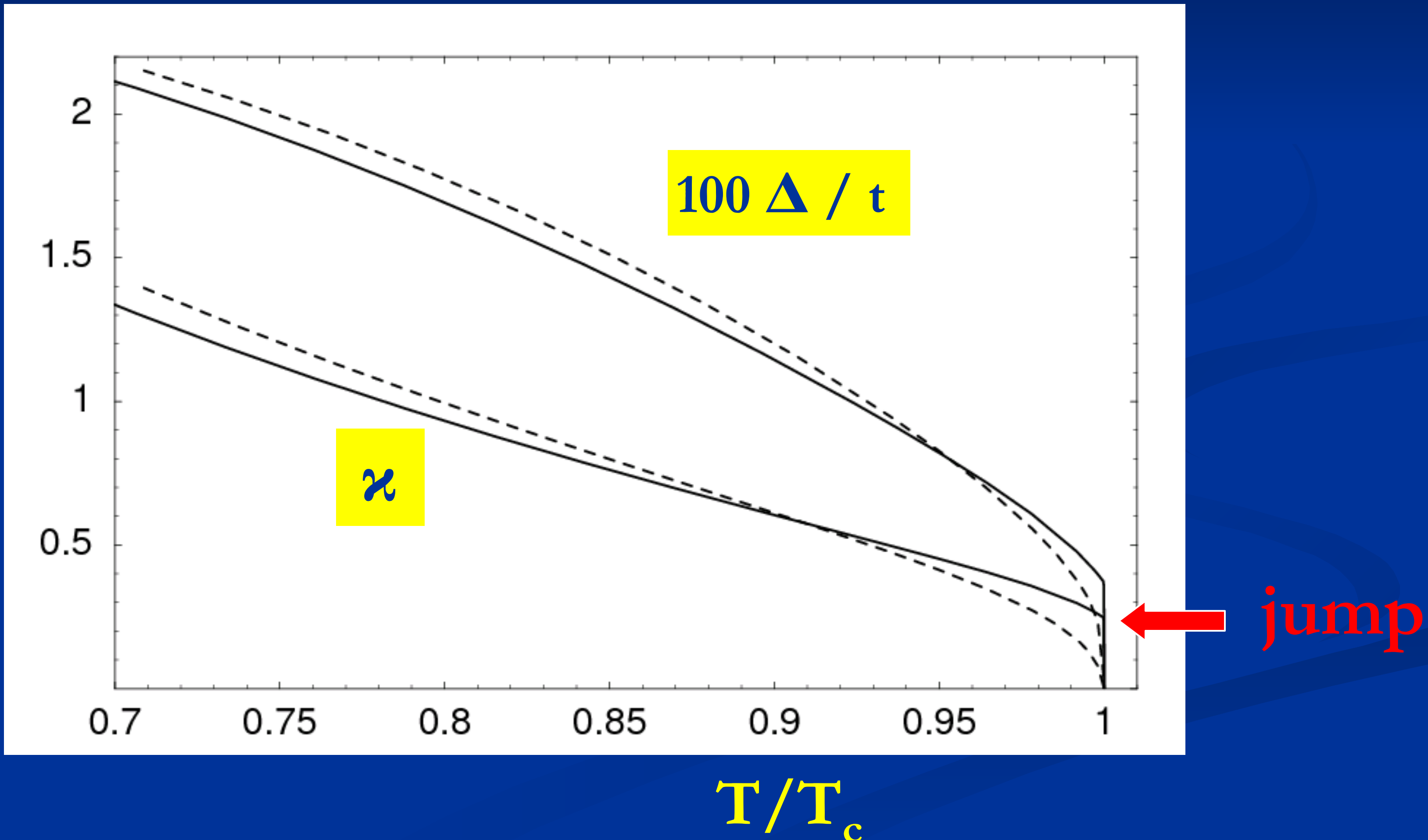


C.Krahl,…

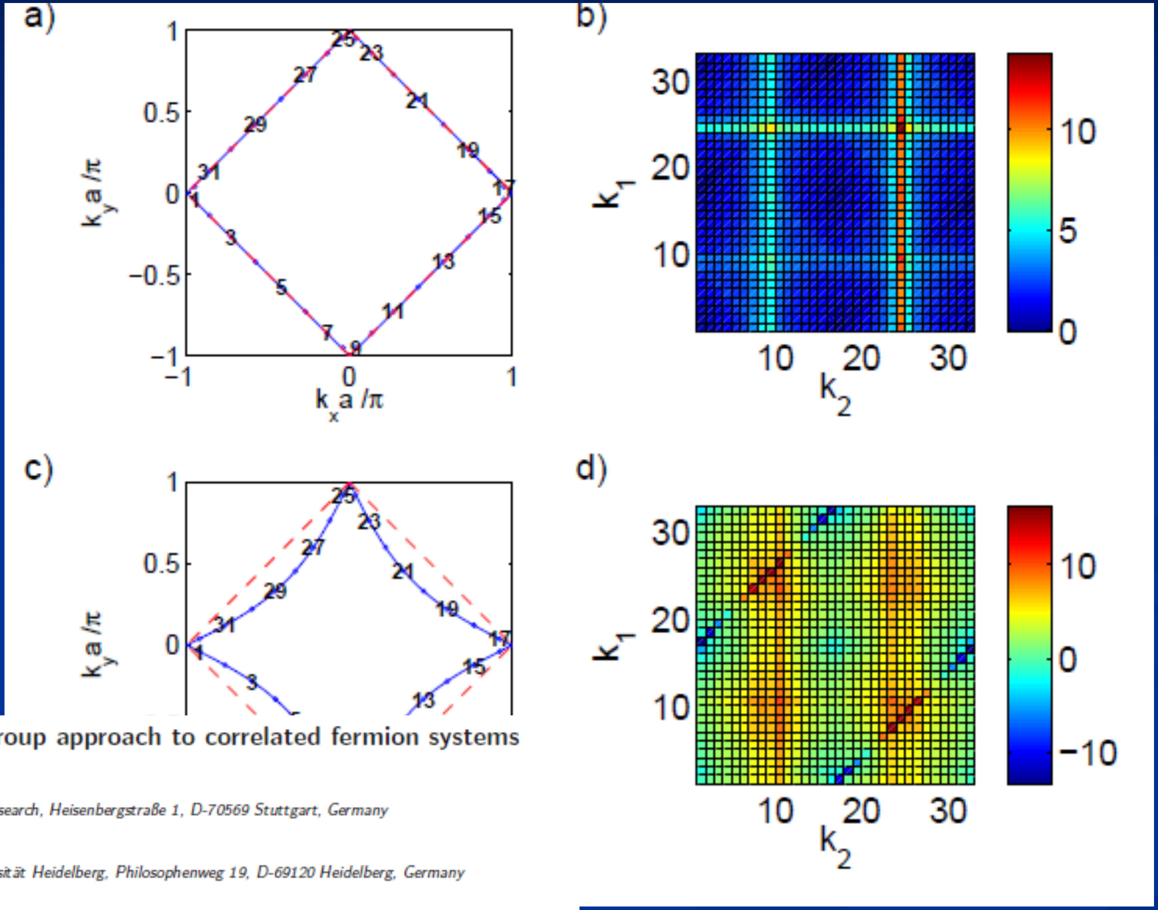
- ln (k/Λ)

macroscopic scale 1 cm

Renormalized order parameter ϰ and gap in electron propagator Δ in doped Hubbard model



Flow of four point function Hubbard model



Functional renormalization group approach to correlated fermion systems

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Many applications

- Ultracold atoms (quantum statistics)
- Disorder
- Turbulence (non-equilibrium physics)
- Density functional
- Active matter (biophysics)
- Economics

Quantum gravity

Quantum Gravity can be a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

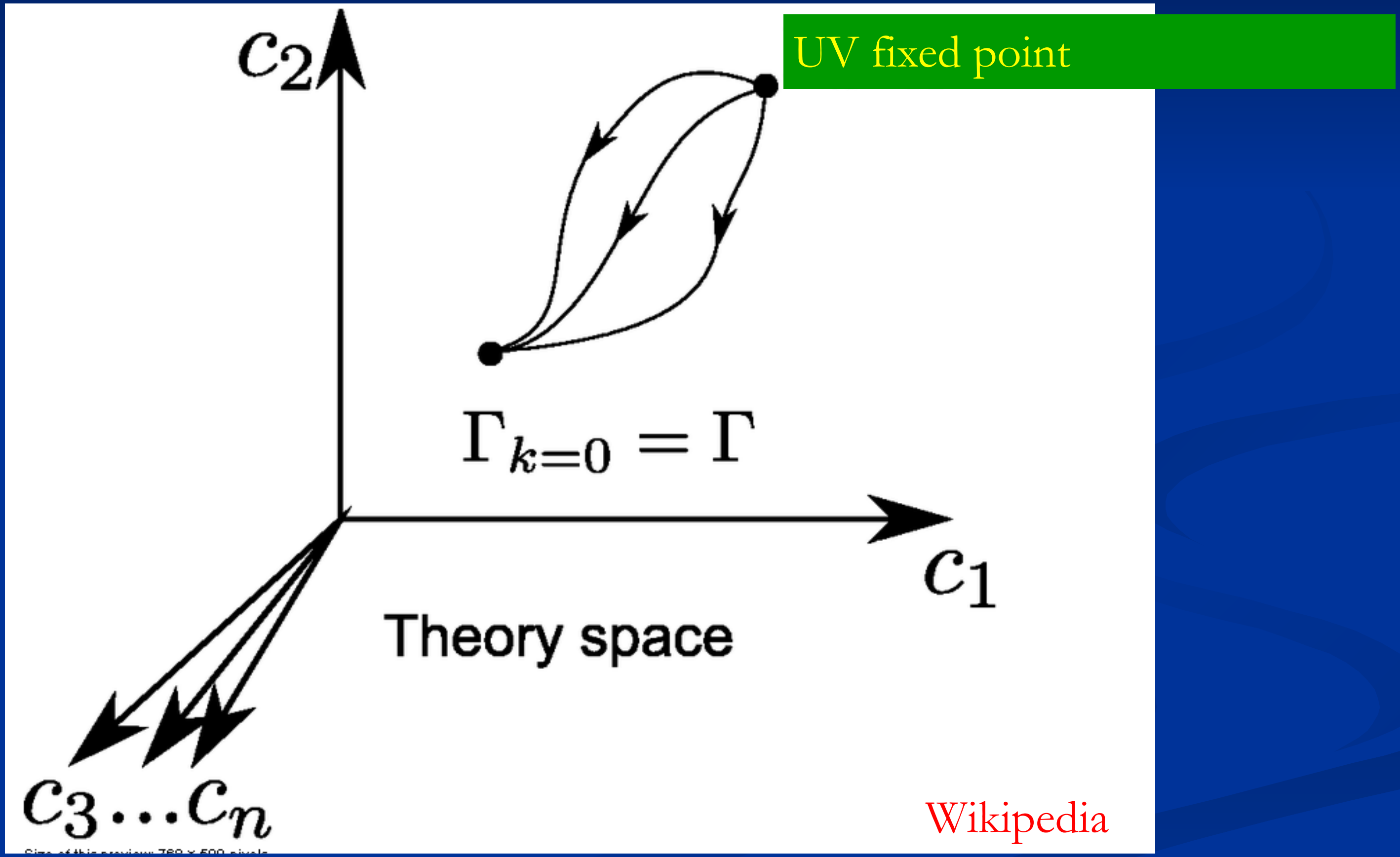
if UV fixed point exists :

quantum gravity is

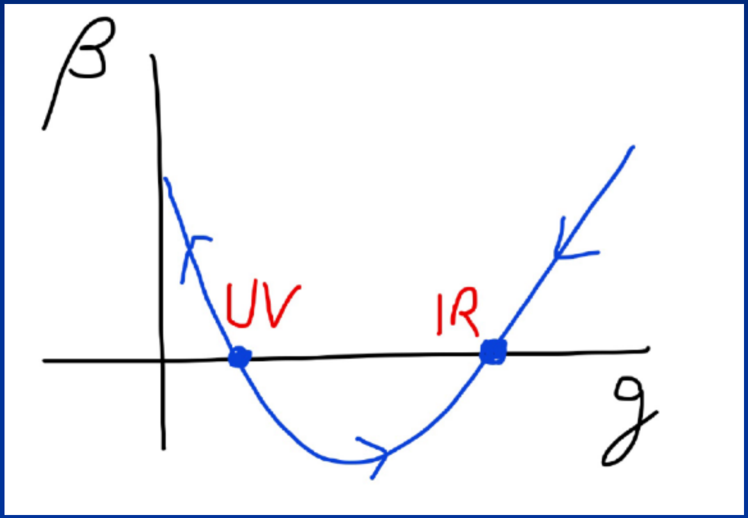
non-perturbatively renormalizable !

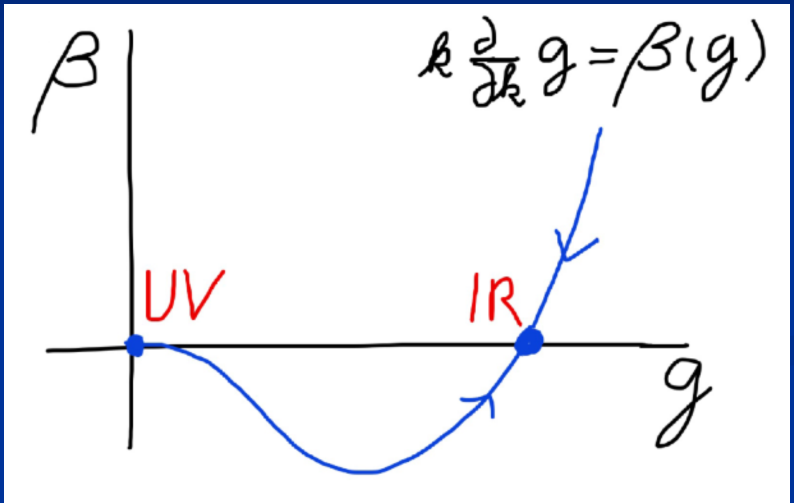
S. Weinberg , M. Reuter

Ultraviolet fixed point



Asymptotic safety   Asymptotic freedom





Relevant parameters yield undetermined couplings.
Quartic scalar coupling is not relevant and can therefore be predicted.

Irrelevant parameters

- Can be predicted !
- Standard model of particles has many renormalisable couplings as free parameters.
- Those corresponding to irrelevant parameters at UV-fixed point can be predicted !
- Quartic coupling for Higgs scalar is irrelevant parameter.

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

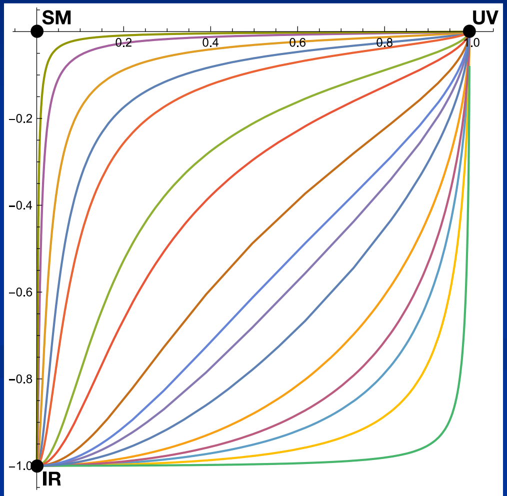
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\rm min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\rm min} = 126$ GeV, with o

Cosmology with quantum scale symmetry

Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.



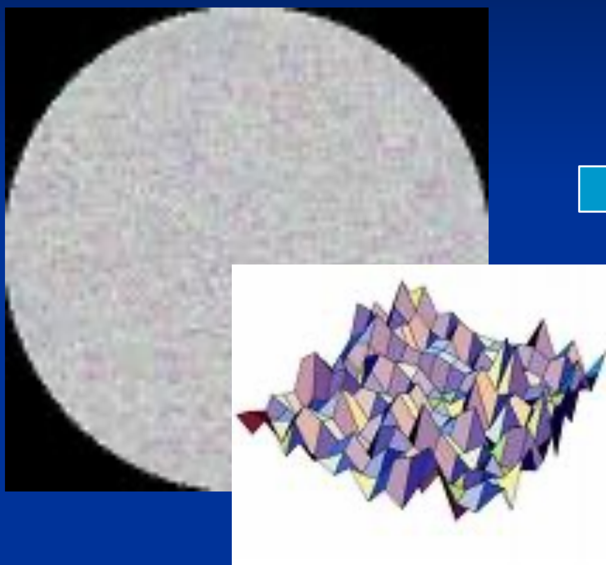
Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

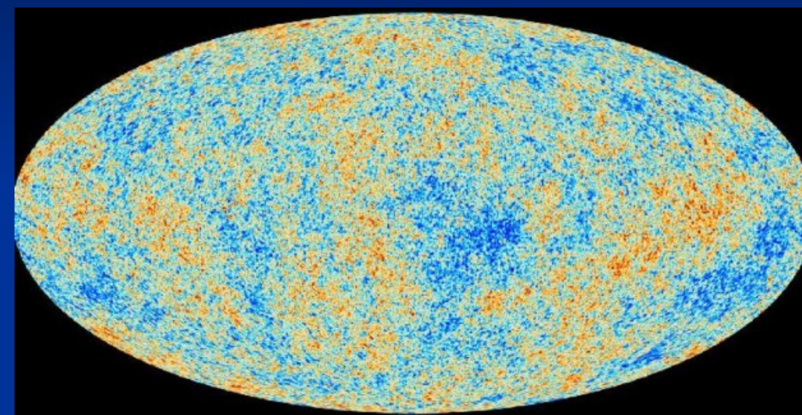
Continuous global symmetry

Approximate scale symmetry near fixed points

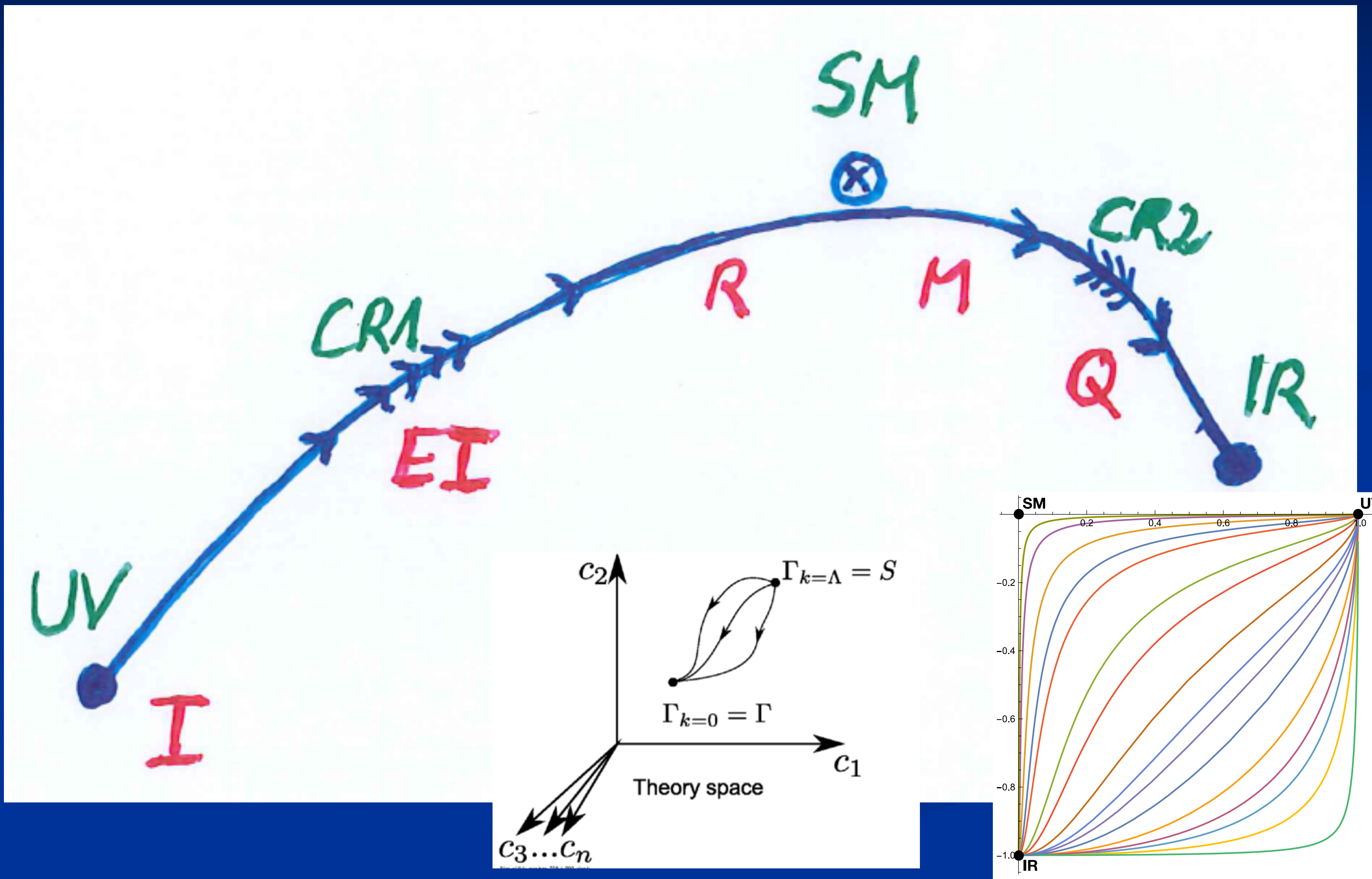
UV : approximate scale invariance of primordial fluctuation spectrum from inflation

Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe



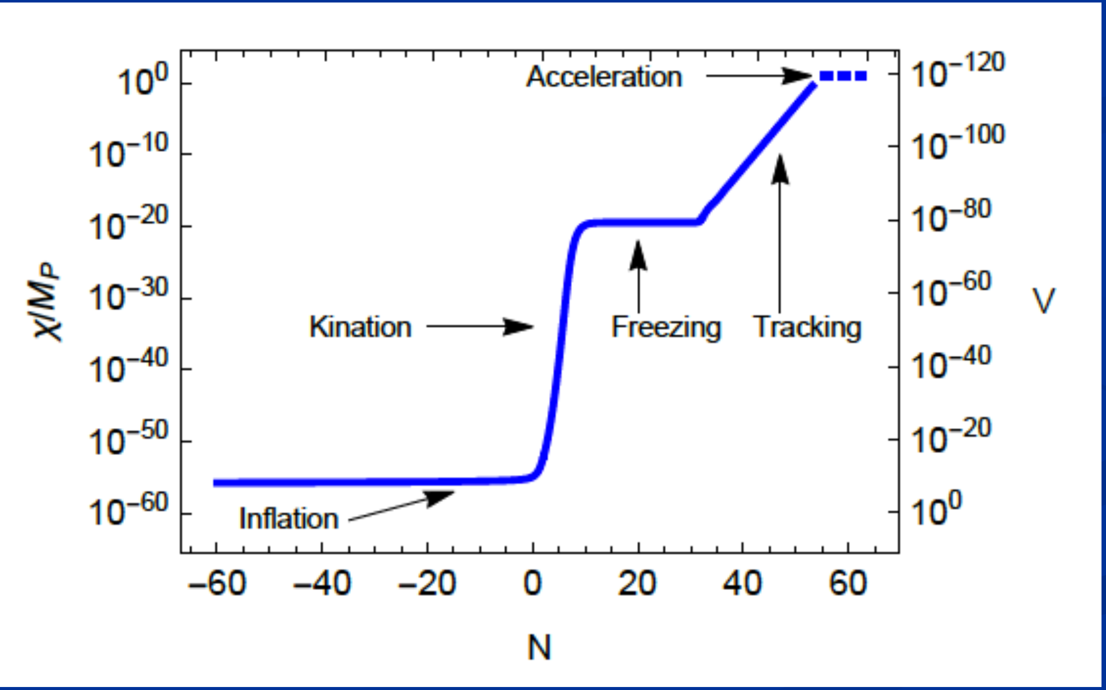


Crossover in quantum gravity



Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Fundamental scale invariance

- Scaling solution is exact
- All relevant parameters vanish

Predictivity

- Theories with fundamental scale invariance are very predictive
- Absence of relevant parameters
- New criterion for fundamental theories
- Stronger than renormalisability

Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

Scaling potential in standard model

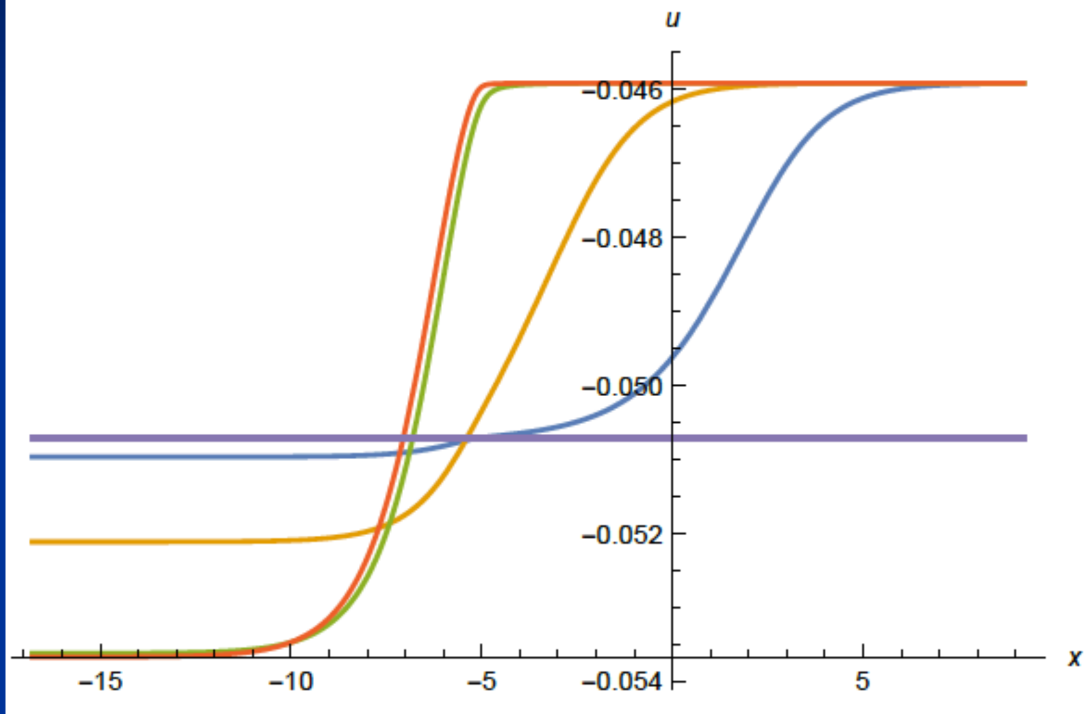


FIG. 19. Effective potential u as function of $x = \ln \tilde{\rho}$ for $\xi_\infty =$ 0.1 (blue), 1.0 (orange), 10^3 (green) and 10^4 (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model, $N_S = 4$, $N_V = 12$, $N_F = 45$.

Coefficient of curvature scalar in standard model

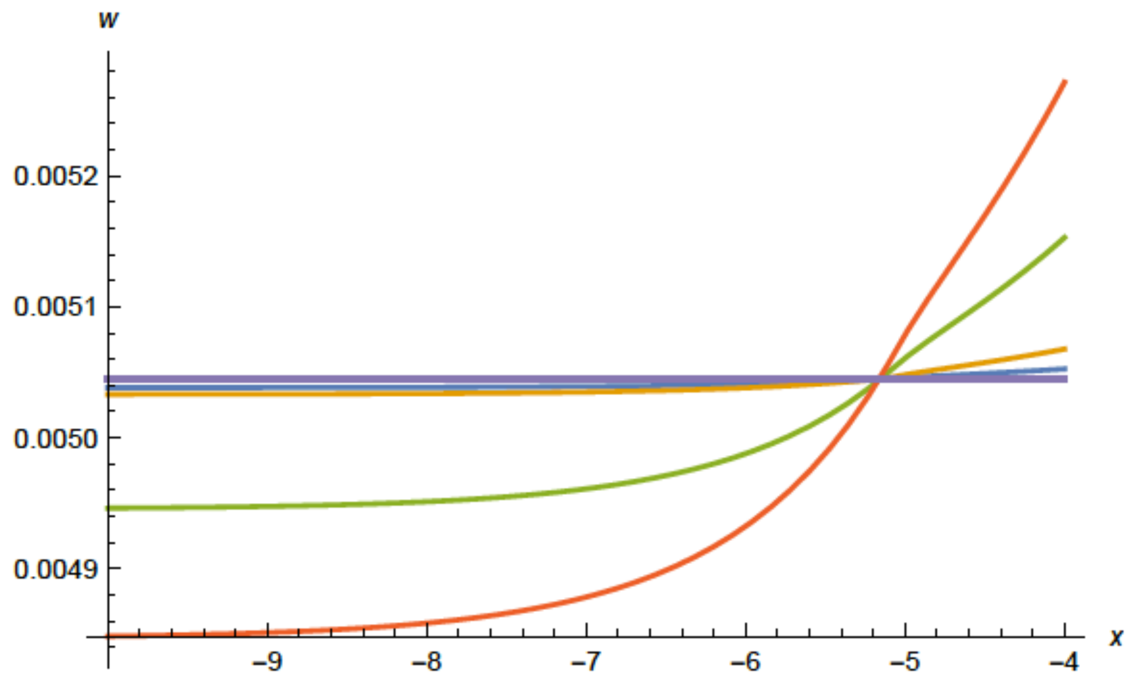


FIG. 21. Dimensionless squared Planck mass w as function of $x = \ln\tilde{\rho}$ for $\xi_\infty = 2\cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for $\xi_\infty \to 0$. All curves meet in a common point at $x \approx -5.05$.

Higgs inflation not compatible with asymptotic safety

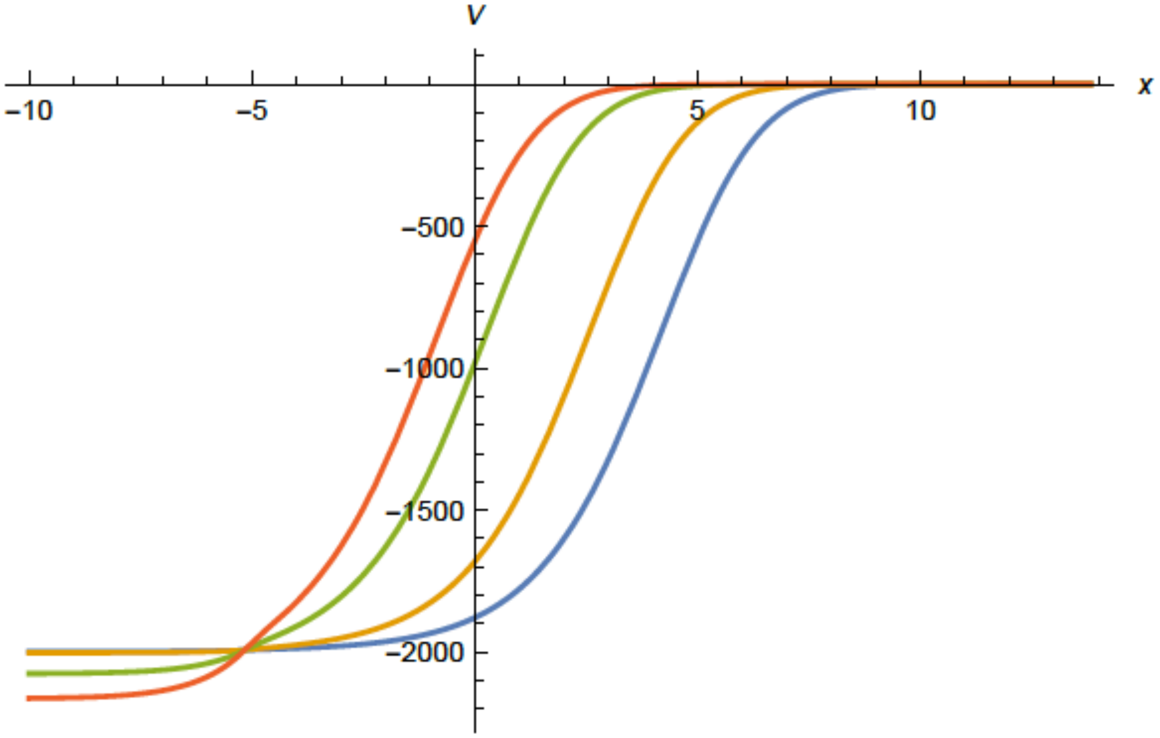


FIG. 23. Potential in the Einstein frame V_E as function of $x = \ln \tilde{\rho}$, for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from right to left.

Scaling potential for GUT with non-zero gauge coupling

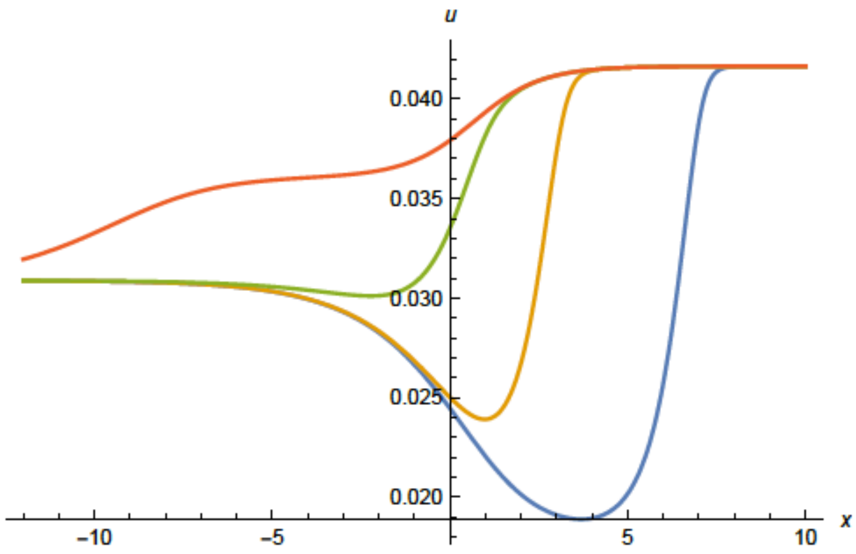


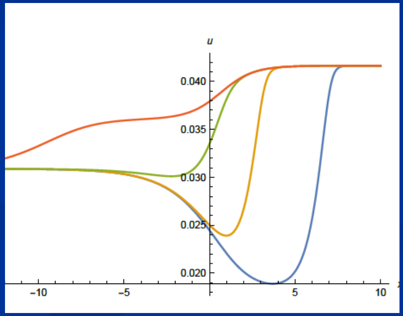
FIG. 9. Effective potential $u(x)$ as function of $x = \ln(\tilde{\rho})$. Parameters are $N = 20$, $\bar{N}_V = 3$, $c_g = 1$, $w_0 = 0.05$, $\alpha = g^2/4\pi = 1/40$. The initial conditions for the four curves from up to down are $u(x=0) = 0.037923$, $u(x=0) = 0.0335$, $u(x=0) = 0.25$ and $u(x=0) = 0.24447$. The inital values for the upper and lower curves limit the interval for which a scaling solution is found. For these solutions one has $A_0 = 2.91$.

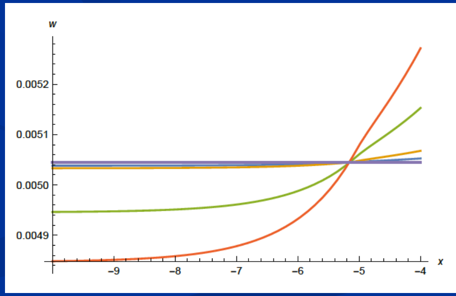
Spontaneous symmetry breaking by scaling solution near Planck scale

Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \to \frac{u_\infty}{\xi^2 \tilde{\rho}^2} \to \frac{4u_\infty k^4}{\xi^2 \chi^4}$$





vanishes for $\chi \to \infty$!

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + ck^4 \right\}$$

$k = 2 \bullet 10^{-3}\,\mathrm{eV}$

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale k disappears !
- Standard gravity coupled to scalar field.

- Exact equivalence of different frames !

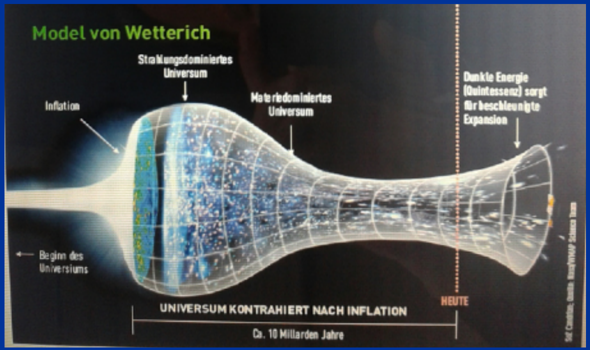
“ different pictures”

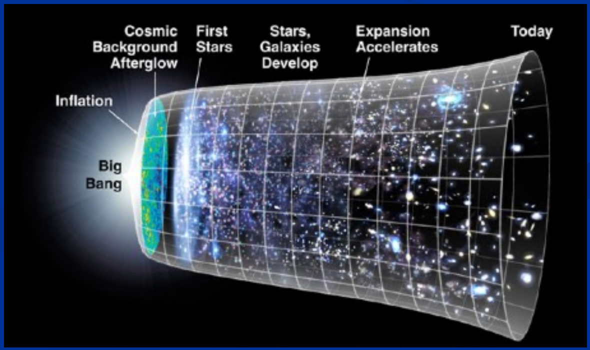
Field relativity

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,
not a coordinate transformation





Models of this type are compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations, including inflation and dark energy

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Conclusions

- Functional renormalisation has worked out in many areas of physics,

  even biology and economics…
- try it out !

end