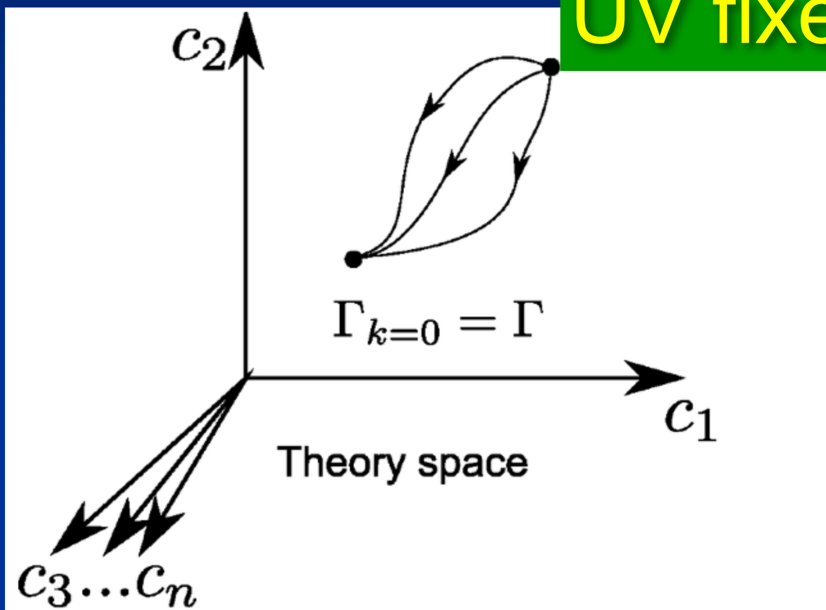
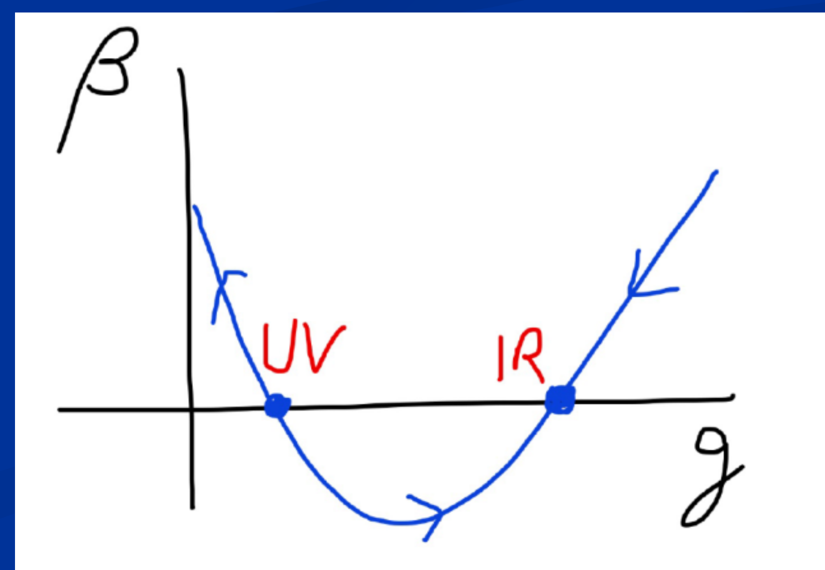
Fundamental scale invariance

UV fixed point





Renormalizable theories

1) Ultraviolet fixed point :

scaling solution

2) Flow away from fixed point :

relevant parameters

Functional renormalization

Leo Kadanoff

Kenneth Wilson

Franz Wegner

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\boxed{\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}}$$

'92

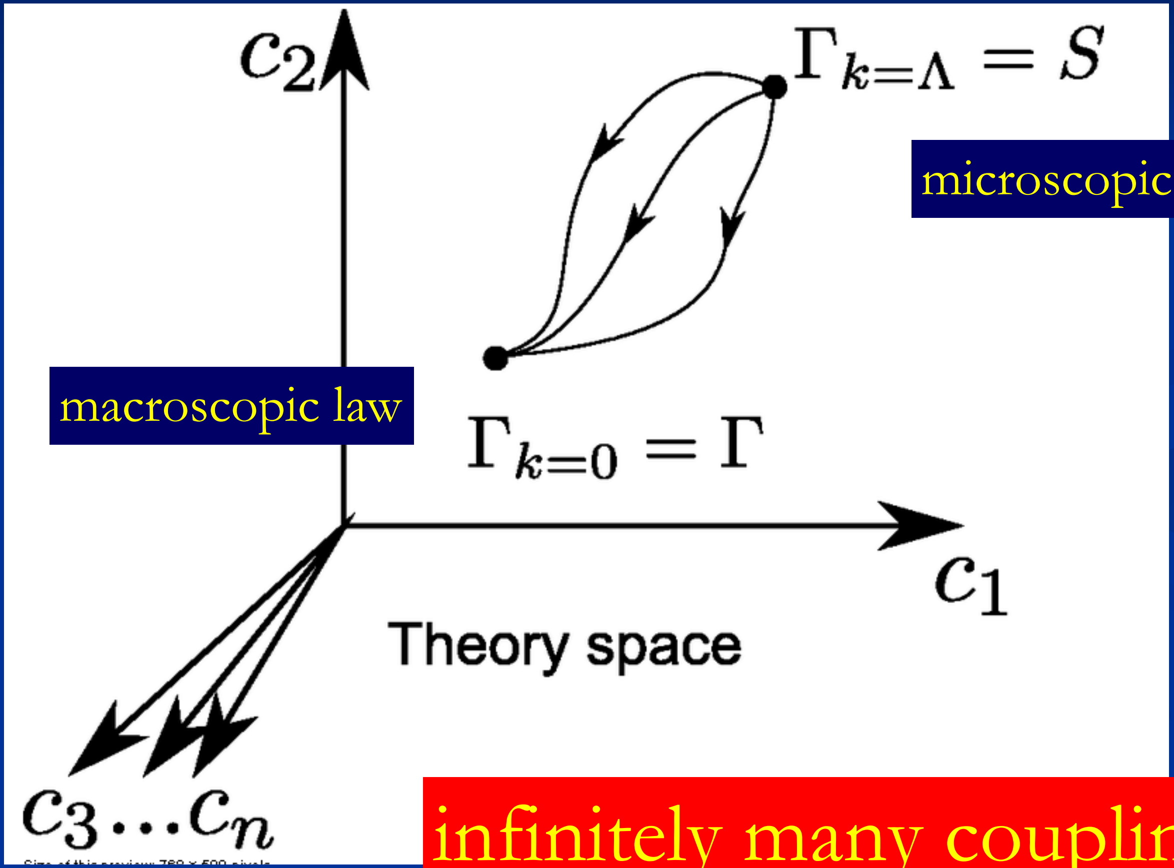
$$\left(\Gamma_k^{(2)}\right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta\varphi_a(-q)\delta\varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

R_k : cutoff function
does not affect
high momentum fluctuations
cuts off
"infrared fluctuations"

flowing action

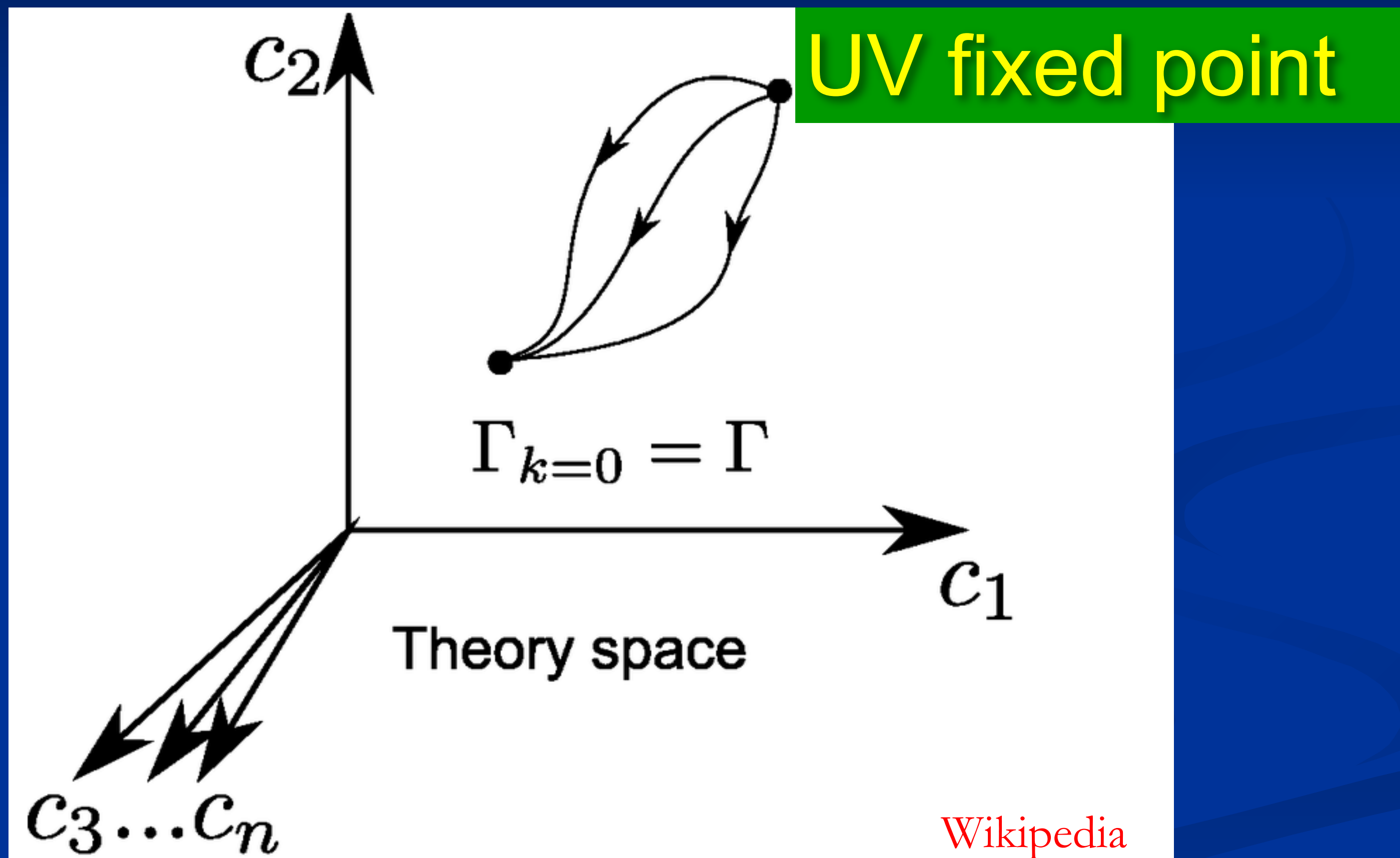


microscopic law

macroscopic law

infinitely many couplings

Ultraviolet fixed point

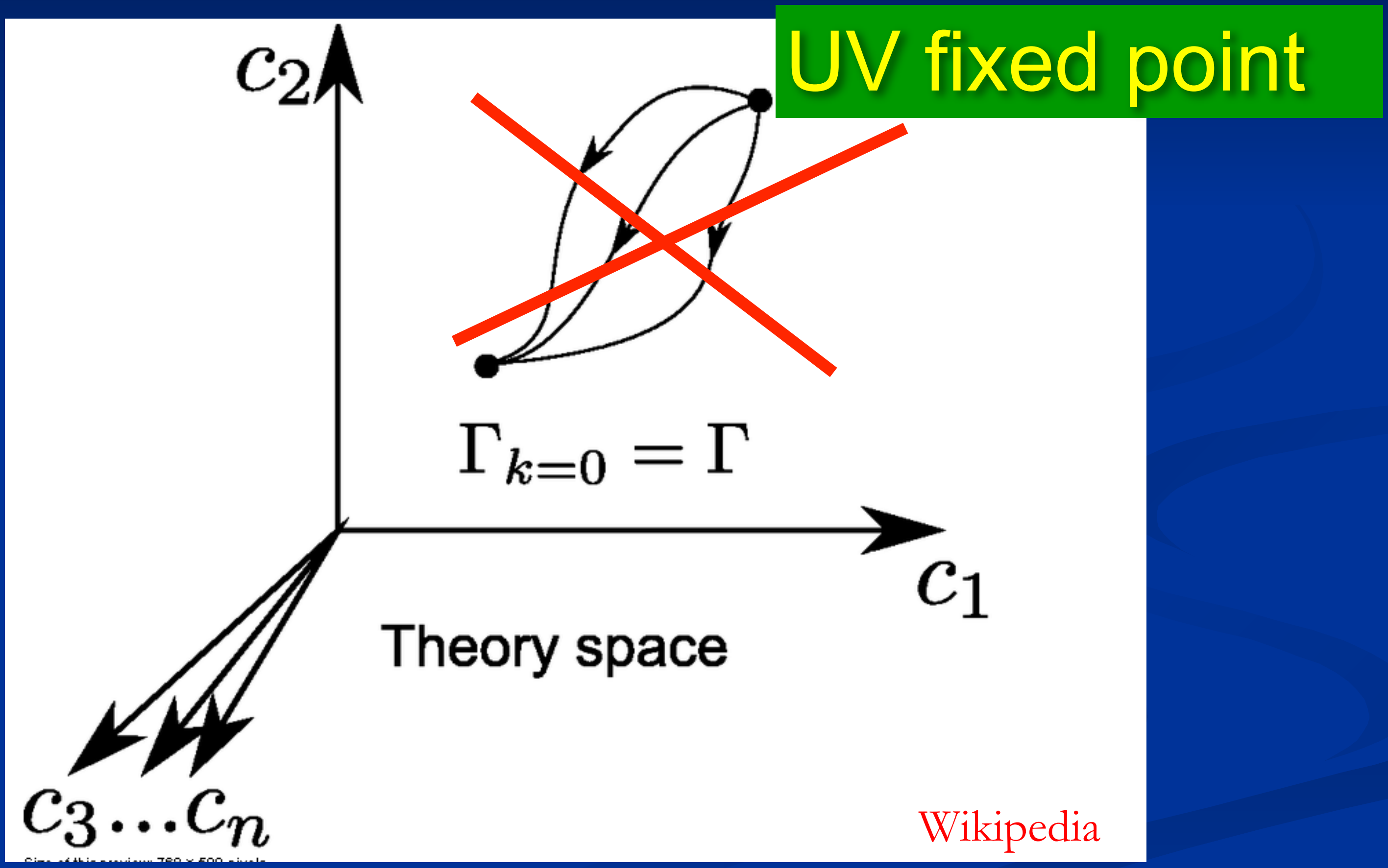


Fundamental scale invariance

Scaling solution

that's it

Ultraviolet fixed point



Condensed matter physics

Exactly on critical temperature of second order phase transition

Flow equation and scaling solution

- Flow equation for canonical fields

$$k\partial_k \Gamma_k[\varphi] = \zeta_k[\varphi],$$

- Scaling solution for scale invariant fields

$$k\partial_k \Gamma_k[\tilde{\varphi}] = 0.$$

Is the world described by a scaling solution ?

Effective action for scalar field and gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}F(\chi)R + U(\chi) + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi\right\}$$

- Scaling fields $\tilde{g}_{\mu\nu} = k^2 g_{\mu\nu}$ $\tilde{\chi} = \frac{\chi}{k},$ $\tilde{\rho} = \frac{1}{2}\tilde{\chi}^2$

- Scale invariant effective action

$$\Gamma = \int_x \sqrt{\tilde{g}}\left\{-w\tilde{R} + \frac{1}{2}K\partial_\mu\tilde{\chi}\partial_\nu\tilde{\chi}\tilde{g}^{\mu\nu} + u\right\}$$

$$w = \frac{F}{2k^2}, \quad u = \frac{U}{k^4}$$

Absence of relevant parameters for fundamental scale

$$U(\chi) = \frac{\mu^2}{2}\chi^2 + \frac{1}{8}\delta(\chi)\chi^4$$

$$u = \frac{\mu^2}{k^2}\tilde{\rho} + \frac{\delta}{2}\tilde{\rho}^2$$

$$F = M^2 + 2w_0k^2 + \xi\chi^2/2$$

$$w = \frac{M^2}{2k^2} + w_0 + \frac{\xi}{2}\tilde{\rho}$$

Predictivity

- Theories with fundamental scale symmetry are very predictive
- Absence of relevant parameters
- New criterion for fundamental theories
- Stronger than renormalizability

Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

Scaling potential in standard model

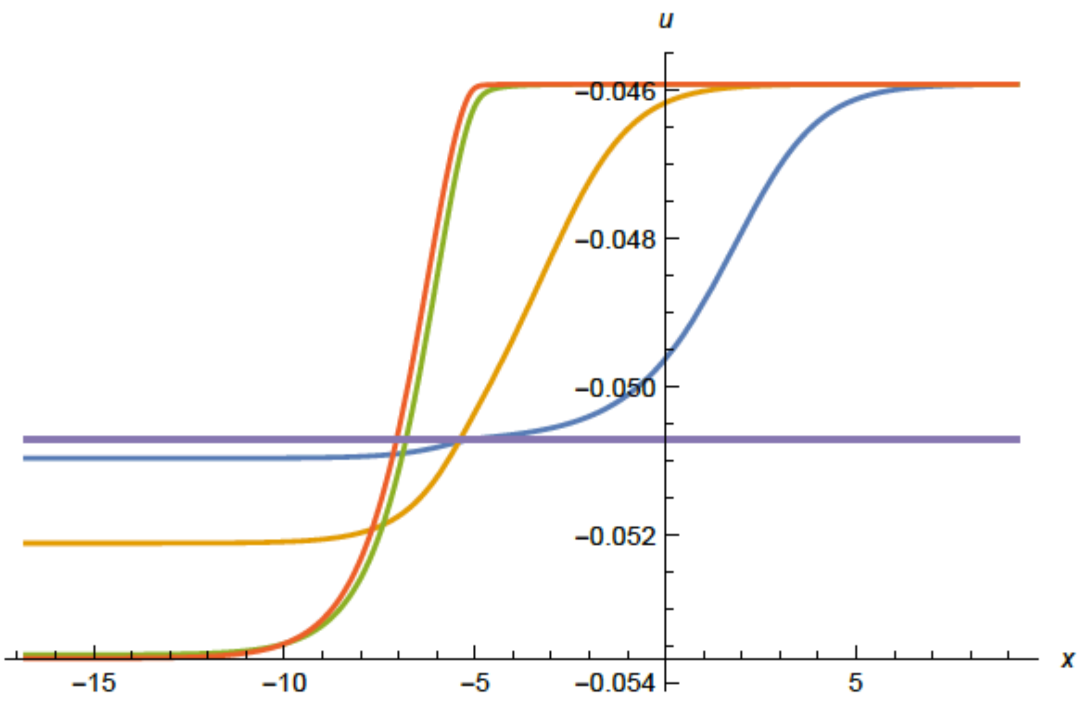


FIG. 19. Effective potential u as function of $x = \ln \tilde{\rho}$ for $\xi_\infty =$ 0.1 (blue), 1.0 (orange), 10^3 (green) and 10^4 (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model, $N_S = 4$, $N_V = 12$, $N_F = 45$.

Coefficient of curvature scalar in standard model

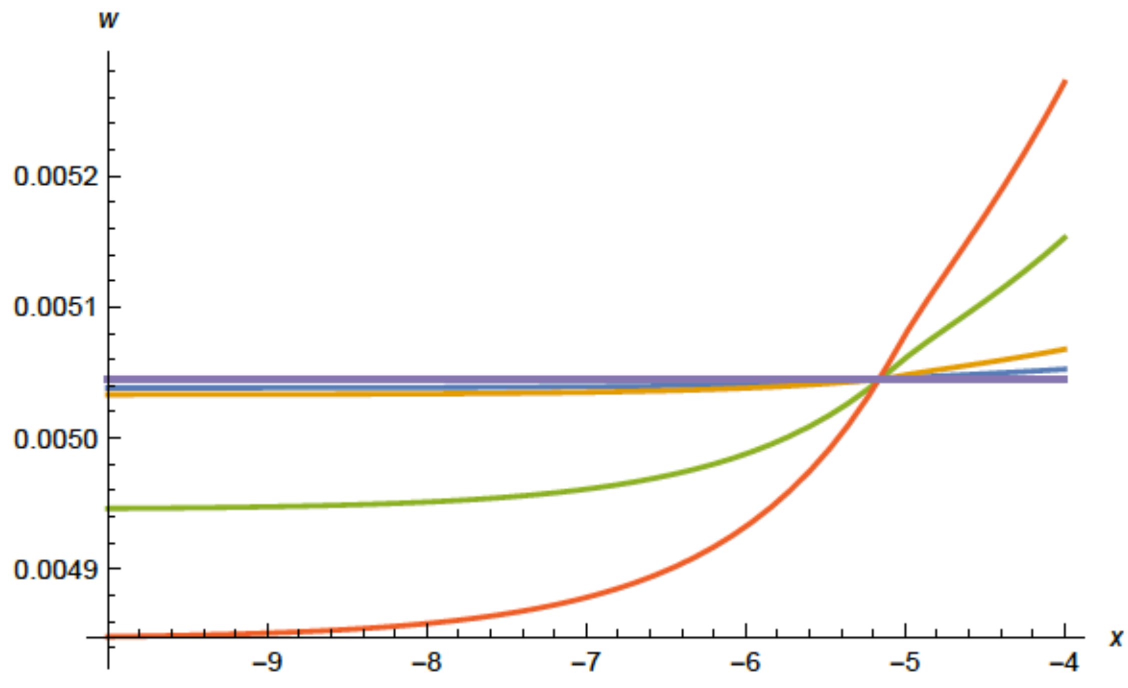


FIG. 21. Dimensionless squared Planck mass w as function of $x = \ln\tilde{\rho}$ for $\xi_\infty = 2\cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for $\xi_\infty \to 0$. All curves meet in a common point at $x \approx -5.05$.

Higgs inflation not compatible with asymptotic safety

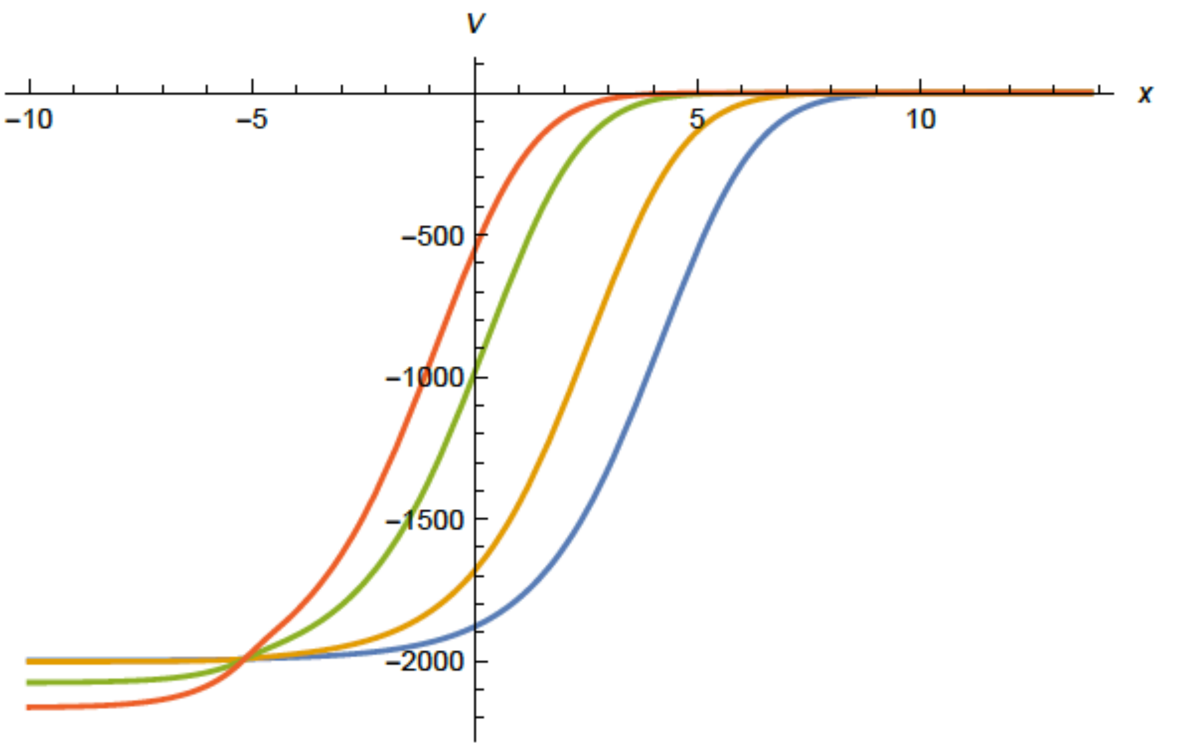


FIG. 23. Potential in the Einstein frame V_E as function of $x = \ln\tilde{\rho}$, for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from right to left.

Scaling potential for GUT with non-zero gauge coupling

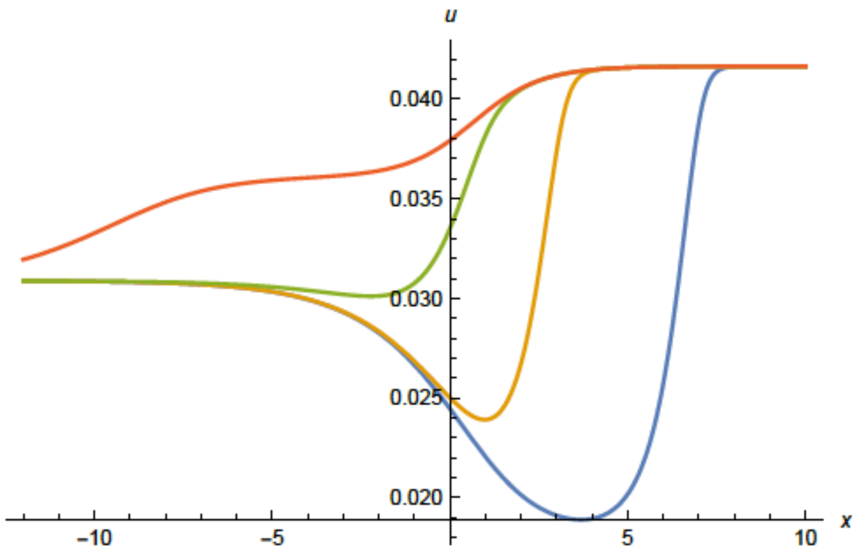


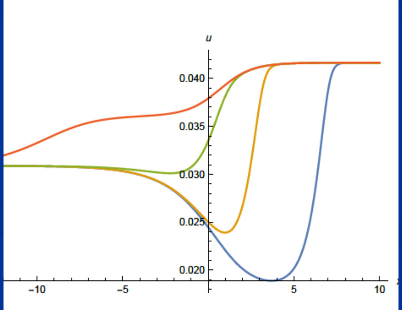
FIG. 9. Effective potential $u(x)$ as function of $x = \ln(\tilde{\rho})$. Parameters are $N = 20$, $\bar{N}_V = 3$, $c_g = 1$, $w_0 = 0.05$, $\alpha = g^2/4\pi = 1/40$. The initial conditions for the four curves from up to down are $u(x=0) = 0.037923$, $u(x=0) = 0.0335$, $u(x=0) = 0.25$ and $u(x=0) = 0.24447$. The inital values for the upper and lower curves limit the interval for which a scaling solution is found. For these solutions one has $A_0 = 2.91$.

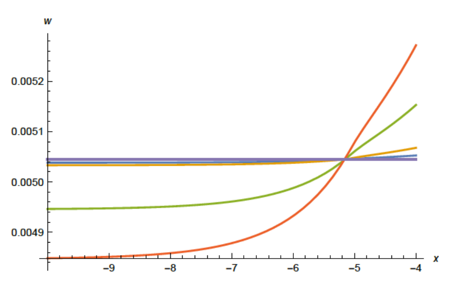
Spontaneous symmetry breaking by scaling solution near Planck scale

Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \rightarrow \frac{u_\infty}{\xi^2 \tilde{\rho}^2} \rightarrow \frac{4u_\infty k^4}{\xi^2 \chi^4}$$





vanishes for $\chi \rightarrow \infty$!

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + ck^4 \right\}$$

$k = 2 \bullet 10^{-3}$ eV

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale k disappears !
- Standard gravity coupled to scalar field.

- Exact equivalence of different frames !

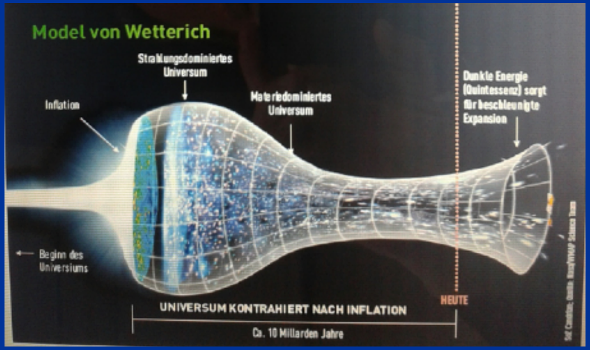
“ different pictures”

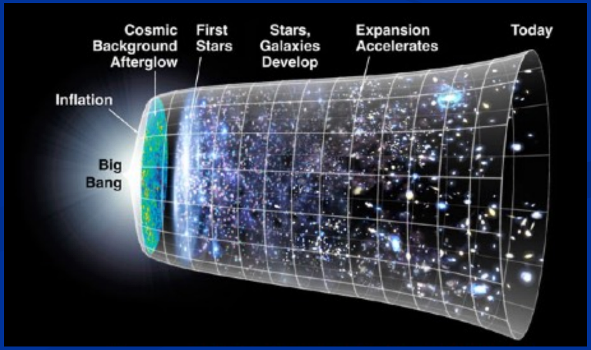
Field relativity

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,
not a coordinate transformation





Models of this type are compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

model is compatible with all present observations, including inflation and dark energy

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma}\ln B = \kappa\left[\ln\left(\frac{\chi}{\mu}\right) - c_t\right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

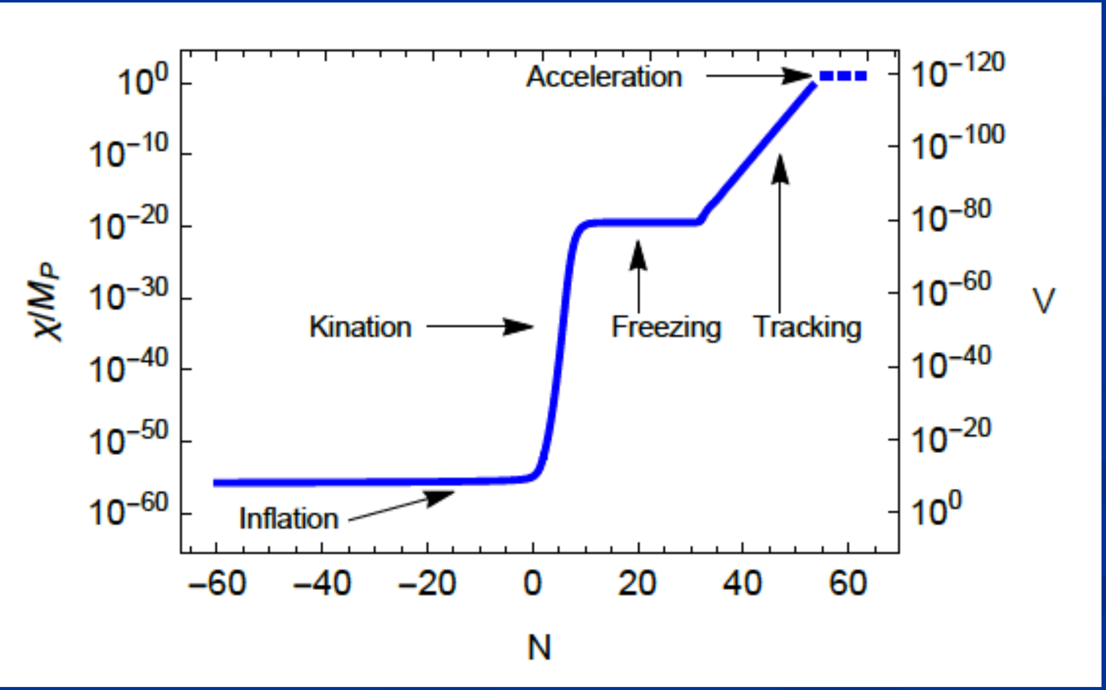
$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Why is the world described by a scaling solution ?

Fundamental theory without scale

Fundamental fields are dimensionless $\tilde{\psi}$

Length scale can be introduced for distances, mass=inverse length for derivatives $\hbar = c = 1$

Metric appears as composite object

$\tilde{g}_{\mu\nu} \sim f(\tilde{\psi})\partial_\mu\tilde{\psi}\partial_\nu\tilde{\psi}$ dimension: mass squared

Effective action for scale invariant fields

$$\exp(-\Gamma[\tilde{\varphi}]) = \int D\tilde{\chi} \exp\left\{-S[\tilde{\varphi}+\tilde{\chi}] + \int_x \frac{\partial\Gamma}{\partial\tilde{\varphi}}\tilde{\chi}\right\}$$

$$\tilde{J}_i(x) = \frac{\partial\Gamma}{\partial\tilde{\varphi}_i(x)}, \quad \tilde{J} = \frac{\partial\Gamma}{\partial\tilde{\varphi}}$$

- Assume that continuum limit exists for this effective action
- No intrinsic scale

Canonical fields

- Canonical metric is dimensionless
- Introduce renormalisation scale k

$$g_{\mu\nu} = k^{-2}\tilde{g}_{\mu\nu}$$

- Canonical scalar fields have dimension mass

$$\chi = k\tilde{\chi}$$

- General renormalized fields $\varphi_{\mathrm{R},i}(x) = k^{d_i} f_i(k)\tilde{\varphi}_i(x)$

Fundamental scale invariance

- Scale k introduced only for convenience - no need to do so

$$\exp(-\Gamma[\tilde{\varphi}]) = \int D\tilde{\chi} \exp\left\{-S[\tilde{\varphi}+\tilde{\chi}] + \int_x \frac{\partial \Gamma}{\partial \tilde{\varphi}} \tilde{\chi}\right\}$$

- Scale invariant flow equation

$$k\partial_k \Gamma_k[\tilde{\varphi}] = 0.$$

Continuum limit

Non- trivial part:

- Existence of continuum limit in terms of scaling fields : fixed couplings at microscopic scale (e.g. lattice cutoff)
- Not realised in standard QCD

Where do the observed scales in Nature come from ?

Quantum scale symmetry

No parameter with dimension of length or mass is present in the quantum effective action. Even not k!

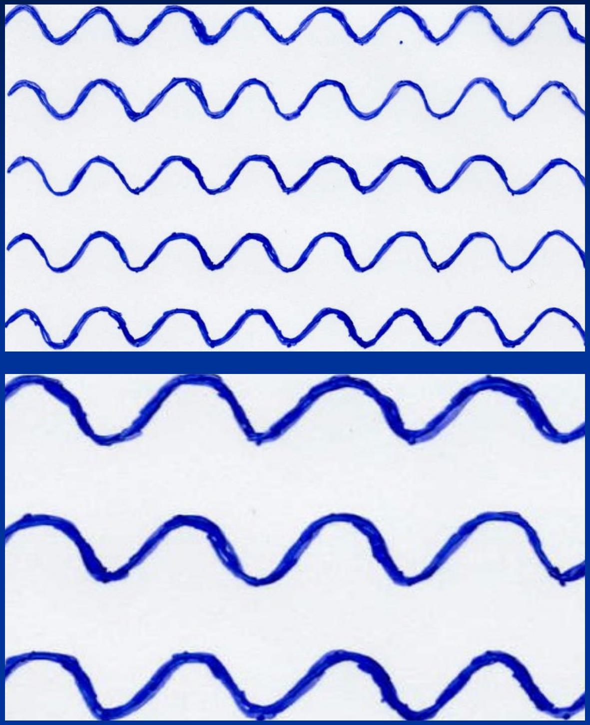
Then invariance under dilatations or global scale transformations is realized.

Continuous global symmetry

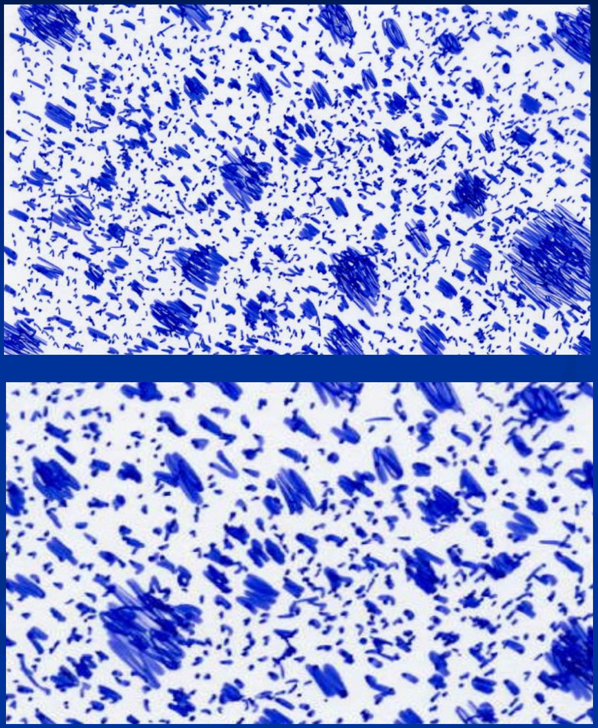
Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Scale symmetry



no scale symmetry



scale symmetry

only if no spontaneous symmetry breaking!

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Realistic particle physics

Compatible with

quantum scale symmetry

Compatible with

fundamental scale invariance

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \rightarrow \varphi_0^2 = \epsilon\chi^2$ Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \rightarrow \Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetry CW'87

For $\chi_0 \neq 0$: massless Goldstone boson

Running couplings

- Couplings run as function of $\tilde{\chi} = \frac{\chi}{k}$,

- Scale invariant flow equation

$$R_k = k^{-2}\tilde{e}r_k(-\tilde{D}^2;\tilde{\varphi})$$

$$\Delta_k = \frac{1}{2}\int_x \tilde{e}\tilde{\chi}^{\mathrm{T}} r(-\tilde{D}^2;\tilde{\varphi})\tilde{\chi}$$

$$Z_k[\tilde{\varphi}] = \int \mathcal{D}\tilde{\chi}\exp\left\{-S[\tilde{\varphi}+\tilde{\chi}] - \Delta_k[\tilde{\chi};\tilde{\varphi}] + \int_x \left(\frac{\partial \Gamma_k}{\partial \tilde{\varphi}} + \tilde{L}_k\right)\tilde{\chi}\right\}.$$

Conclusions

- Fundamental scale invariance is a new selection criterion for fundamental theories
- Particular renormalizable theories for which no scale is present at all
- Exact scaling solution of flow equation

end