The mass of the Higgs boson, the great desert, and asymptotic safety of gravity

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

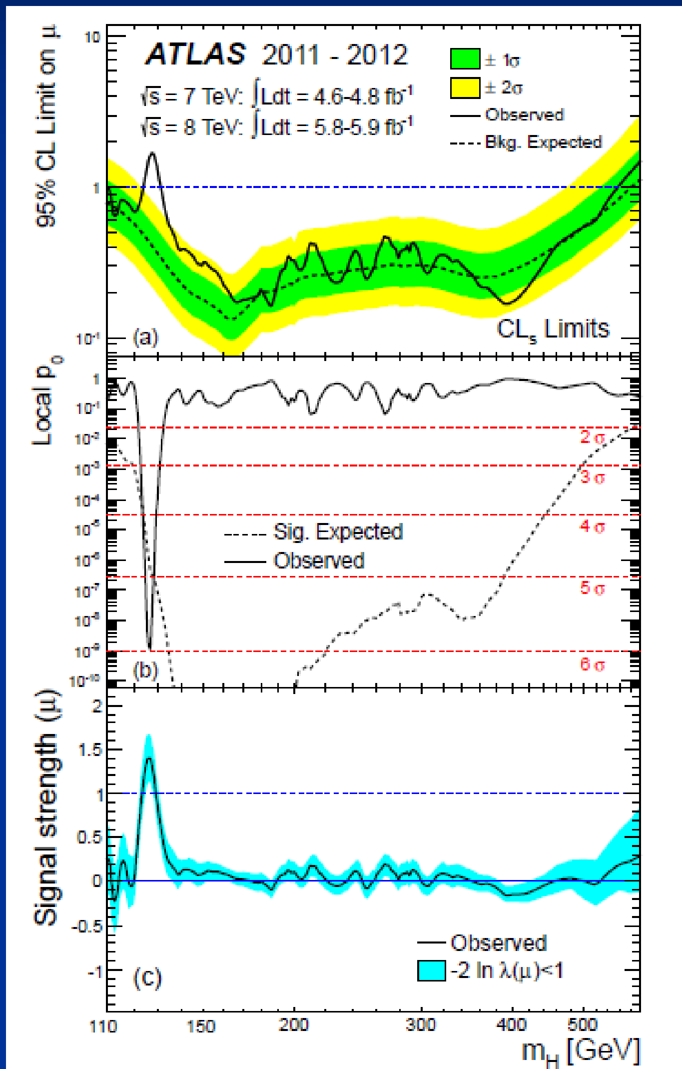
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

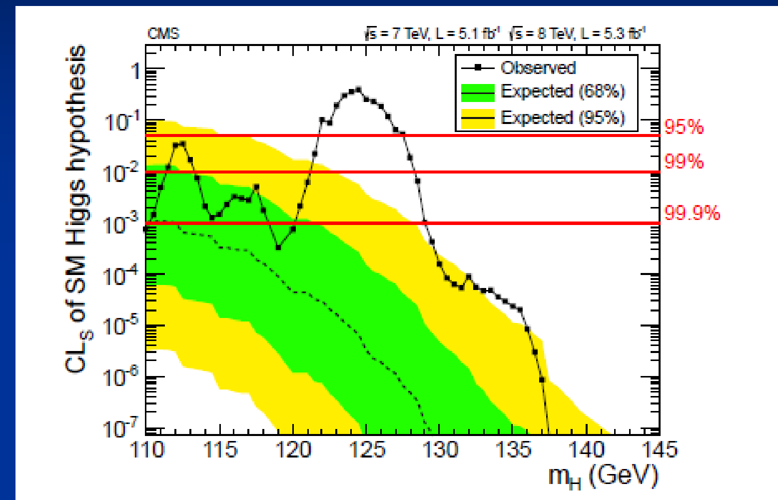
s in $m_H = m_{\min} = 126$ GeV, with o

a prediction …

In conclusion, we discussed the possibility that the SM, supplemented by the asymptotically safe gravity plays the role of a fundamental, rather than effective field theory. We found that this may be the case if the gravity contributions to the running of the Yukawa and Higgs coupling have appropriate signs. The mass of the Higgs scalar is predicted $m_H = m_{\text{min}} \simeq 126$ GeV with a few GeV uncertainty if all the couplings of the Standard Model, with the exception of the Higgs self-interaction λ, are asymptotically free, while λ is strongly attracted to an approximate fixed point $\lambda = 0$ (in the limit of vanishing Yukawa and gauge couplings) by the flow in the high energy regime. This can be achieved by a positive gravity induced anomalous dimension for the running of λ. A similar prediction remains valid for extensions of the SM as grand unified theories, provided the split between the unification and Planck-scales remains moderate and all relevant couplings are perturbatively small in the transition region. Detecting the Higgs scalar with mass around 126 GeV at the LHC could give a strong hint for the absence of new physics influencing the running of the SM couplings between the Fermi and Planck/unification scales.

LHC : Higgs particle observation

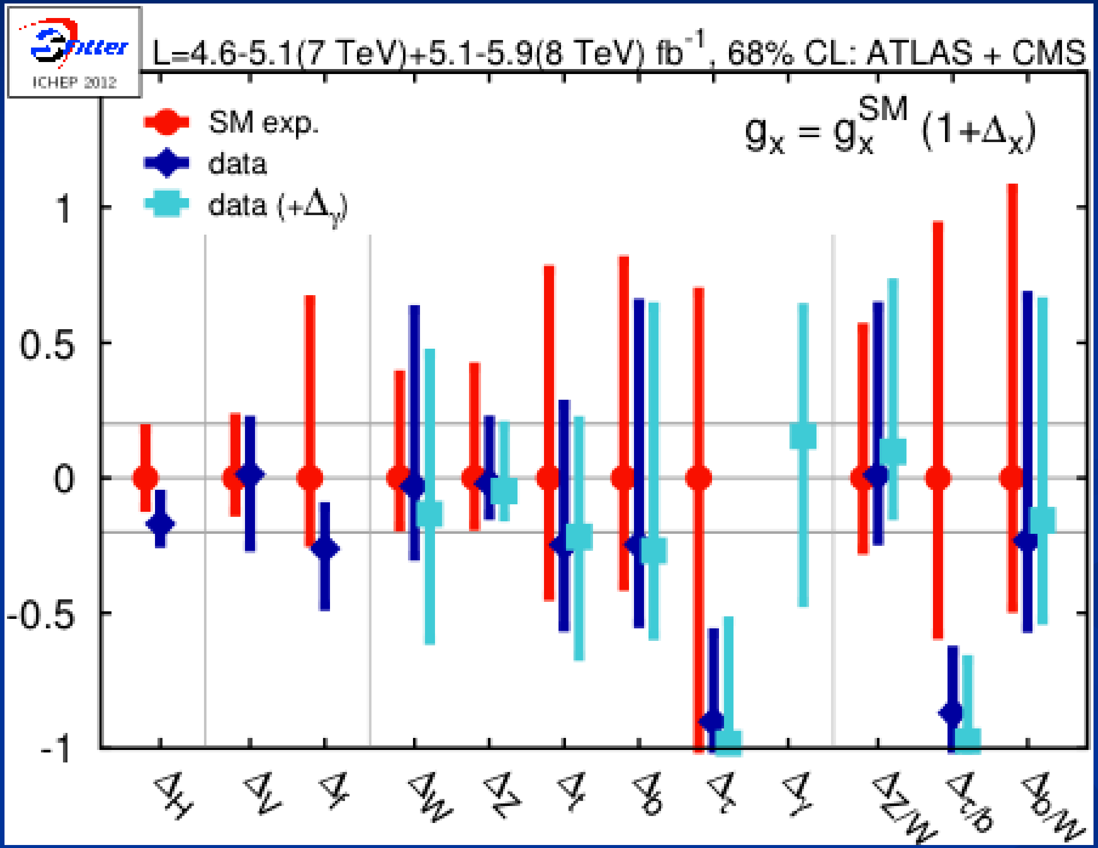




CMS 2011/12

ATLAS 2011/12

standard model Higgs boson



T.Plehn, M.Rauch

too good to be true ?

500 theoretical physicists = 500 models

equidistant predictions

range 100-600 GeV …

3 GeV bins : one expects several correct predictions ,

but for contradicting models

motivation behind prediction ?

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$M_s^2 = 2\lambda(\varphi_L^2)\varphi_L^2$$

Running couplings, Infrared interval, UV-IR mapping

Running quartic scalar coupling λ and Yukawa coupling of top quark h

$$\partial_t \lambda = \beta_\lambda(\lambda, h, g^2)$$

$$t = \ln(k/\chi)$$

neglect gauge couplings g

$$\beta_\lambda = \frac{3}{4\pi^2}(\lambda^2 + h^2\lambda - h^4)$$

$$\partial_t h = \beta_h = \frac{9}{32\pi^2} h^3$$

$$m_t = h(\varphi_0)\varphi_0$$

Partial infrared fixed point

$$\left(\frac{\lambda}{h^2}\right) = (\sqrt{65} - 1)/8$$

Gauge Hierarchy Due To Strong Interactions?.
C. Wetterich (Freiburg U.). Apr 1981. 20 pp.
Published in **Phys.Lett. B104 (1981) 269**

C. Wetterich, The mass of the Higgs particle, in "Superstrings, unified theory and cosmology, 1987", eds. G. Furlan, J. C. Pati, D. W. Sciama, E. Sezgin and Q. Shafi, World Scientific (1988) p. 403, DESY-87-154

B. Schrempp, M. Wimmer, Prog. Part. Nucl. Phys. **37** (1996) 1 become comparable. Indeed, for strong enough h_t the RGE for the ratio λ/h_t^2,

$$\frac{d}{dt}\left(\frac{\lambda}{h_t^2}\right) = \frac{h_t^2}{16\pi^2}\left\{12\left(\frac{\lambda}{h_t^2}\right)^2 + 3\,\frac{\lambda}{h_t^2} - 12\right\} \quad (16)$$

is governed by an infrared fixpoint.[13] The ratio λ/h_t^2 remains constant if the right hand side of (16) vanishes. This happens for

$$\frac{\lambda}{h_t^2} = \left(\frac{65}{64}\right)^{1/2} - \frac{1}{8} = x_0 \quad (17)$$

and corresponds to a mass ratio

$$\frac{M_H}{m_t} \approx 1.3 \quad (18)$$

infrared interval

allowed values of λ or λ/h^2 at UV-scale Λ :

between zero and infinity

are mapped to

finite infrared interval of values of

λ/h^2 at Fermi scale

infrared interval

deviation from partial fixed point

$$x = \frac{\lambda}{h_t^2} - x_0$$

flow parameter s

$$\frac{ds}{dt} = h_t^2(t)$$

$$s(\Lambda) - s(\mu) = \frac{32\pi^2}{9} \ln \frac{h_t(\Lambda)}{h_t(\mu)}$$

$$\frac{dx}{ds} = \frac{3}{4\pi^2} x \left(x + 2x_0 + \frac{1}{4} \right)$$

infrared interval

small x :

$$\frac{dx}{ds} = A_x\, x$$

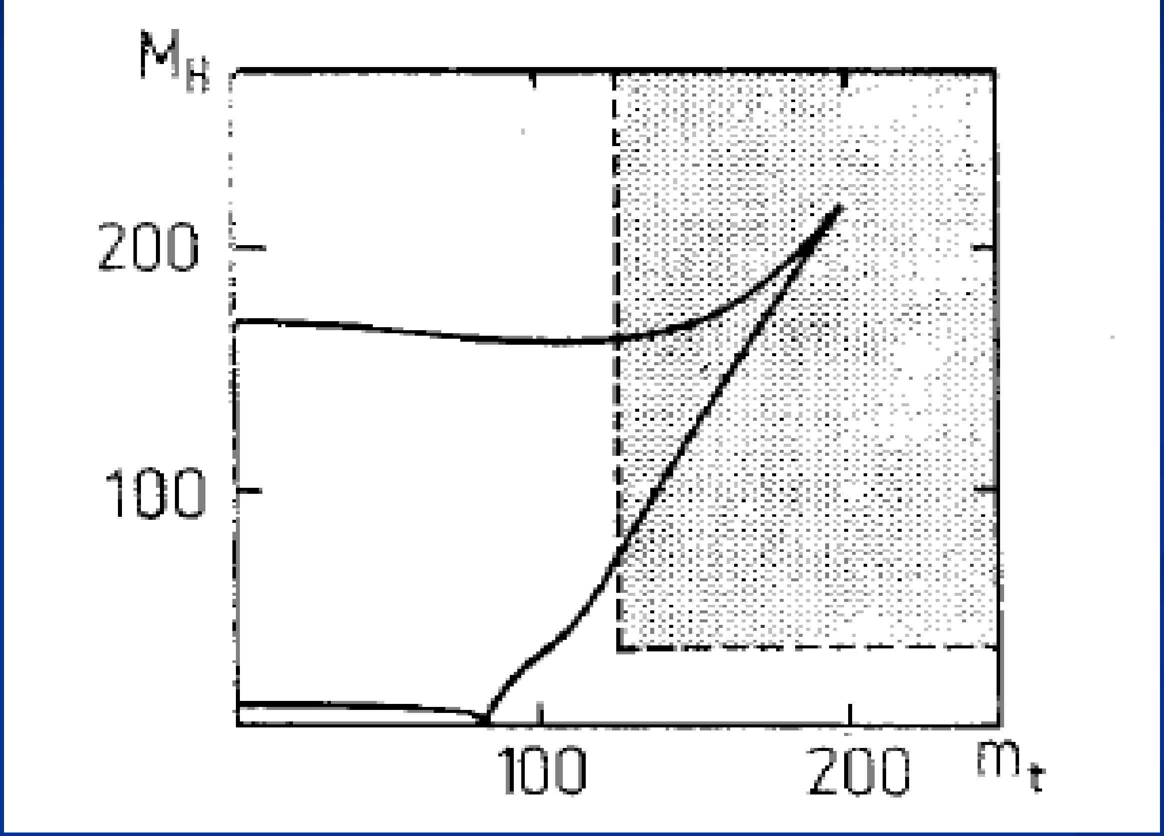
$$A_x = \frac{3}{4\pi^2}\left(2x_0 + \frac{1}{4}\right)$$

solution

$$\frac{x(\mu)}{x(\Lambda)} = \left(\frac{h_t(\mu)}{h_t(\Lambda)}\right)^{\frac{16x_0+2}{3}}$$

infrared interval shrinks as μ/Λ decreases

infrared interval



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C. Wetterich (DESY). Nov 1987. 52 pp.
DESY-87-154, C87/07/23
Talk presented at Conference: C87-07-23
(Trieste HEP Workshop 1987:0403)

DEUTSCHES ELEKTRONEN – SYNCHROTRON DESY

DESY 87-154
November 1987

THE MASS OF THE HIGGS PARTICLE

by

C. Wetterich

Deutsches Elektronen-Synchrotron DESY, Hamburg

realistic mass of top quark (2010), ultraviolet cutoff: reduced Planck mass

$$M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\ \mathrm{GeV}$$

$$m_{min} = 126 GeV\ ,\ m_{max} = 174 GeV$$

ultraviolet- infrared map

Whole range of small λ

at ultraviolet scale is mapped by

renormalization flow

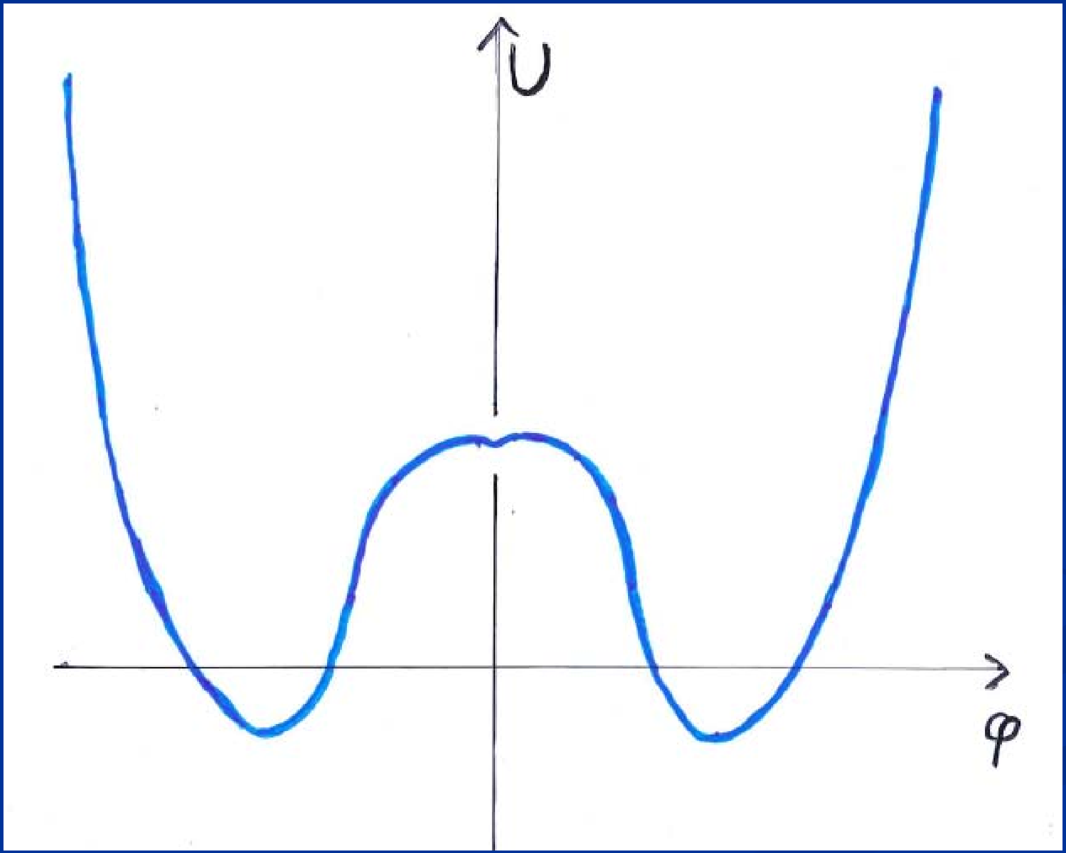
to lower bound of infrared interval !

Prediction of Higgs boson mass

close to 126 GeV

remark on metastable vacuum

no model known where this is realized in reliable way



key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

gauge hierarchy problem
and
fine tuning problem

quantum effective potential

$$U = \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2 + \gamma(\varphi^\dagger\varphi)\chi^2 + U_\chi,$$

$$\frac{\varphi_0}{M} = \sqrt{-\frac{\gamma}{\lambda}}.$$

scalar field χ with high expectation value M, say Planck mass

anomalous mass dimension

$$\partial_t \gamma \;=\; A_\mu(\lambda, h, g^2)\gamma$$

$$A_\mu \;=\; \frac{3}{8\pi^2}(\lambda^2 + h^2)$$

one loop,
neglect gauge couplings g

fixed point for $\gamma = 0$

- zero temperature electroweak phase transition (as function of γ) is essentially second order
- fixed point with effective dilatation symmetry
- no flow of γ at fixed point

$$\partial_t \gamma = A_\mu(\lambda, h, g^2)\gamma$$

- naturalness due to enhanced symmetry
- small deviations from fixed point due to running couplings: leading effect is lower bound on Fermi scale by quark-antiquark condensates

renormalization group improved perturbation theory

- no change of form of flow equation
- loop corrections to anomalous mass dimension

$$A_\mu = \frac{3}{8\pi^2}(\lambda^2 + h^2) + \ldots$$

- small γ at high scale remains small at low scale
- no need of tuning order by order in perturbation theory
- fine tuning problem artefact of bad expansion (perturbation theory does not see fixed point and associated effective dilatation symmetry)

critical physics

- second order phase transition corresponds to critical surface in general space of couplings
- flow of couplings remains within critical surface
- once couplings are near critical surface at one scale, they remain in the vicinity of critical surface
- gauge hierarchy problem : explain why world is near critical surface for electroweak phase transition
- explanation can be at arbitrary scale !

critical physics in statistical physics

use of naïve perturbation theory

(without RG – improvement)

would make the existence of critical temperature look “unnatural”

artefact of badly converging expansion

conclusions fine tuning problem

(a) Fine-tuning problems are always related to a particular expansion method and to a particular choice of the set of parameters used to describe the model.

(b) There is no fine-tuning problem for renormalization group improved perturbation theory if an appropriate set of short-distance parameters is used. All fine-tuning problems are artifacts either of badly chosen expansion methods or of a badly chosen parameter set.

Fine Tuning Problem And The Renormalization Group.
C. Wetterich (CERN). Feb 1983. 13 pp.
Published in **Phys.Lett. B140 (1984) 215**

(c) The relation between long-distance parameters and short-distance parameters is natural if we use an appropriate set of short-distance parameters. The problem of naturalness can therefore be postponed to the Planck scale and the gauge hierarchy problem is reduced to the search of a natural explanation of the scalar doublet mass normalized at the Planck scale.

(d) There is a possibility to solve the gauge hierarchy problem in the context of a unified theory for weak electromagnetic and strong interactions alone. This requires strong renormalization effects for the mass term of the scalar doublet. If this possibility is not realized, a satisfactory solution of the gauge hierarchy problem requires a theory of unification with gravity.

The gauge hierarchy problem is not of a technical nature. Technical problems can be solved satisfactorily once we start with appropriate short-distance parameters. What remains is the outstanding physical problem of naturalness: What is the explanation of the very small scale M_W?

SUSY vs Standard Model

natural predictions

baryon and lepton number conservation	SM
flavor and CP violation described by CKM matrix	SM
absence of strangeness violating neutral currents	SM
g-2 etc.	SM
dark matter particle (WIMP)	SUSY

key points

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high scale fixed point

large anomalous mass dimension ?

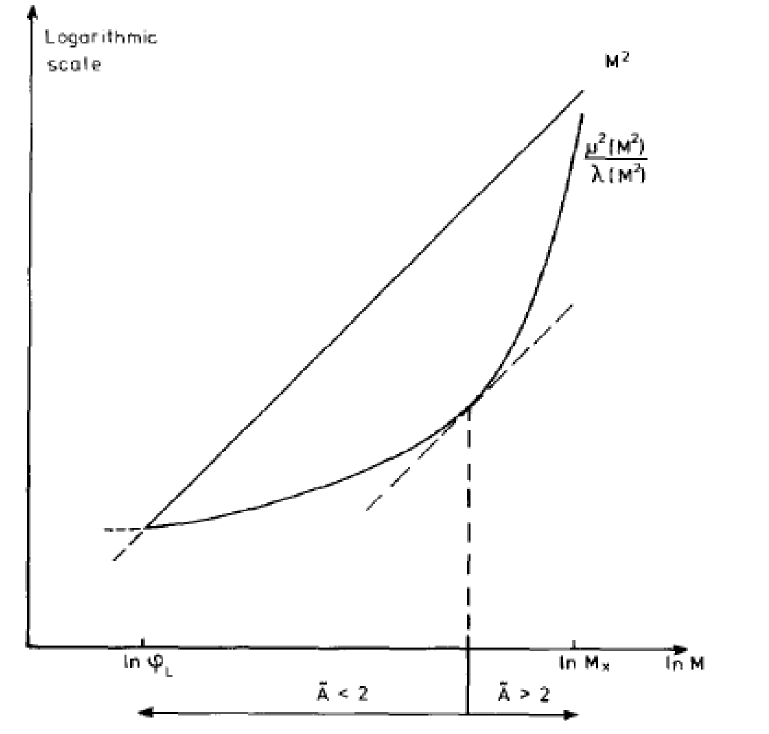

Fig. 1. General scenario for a natural gauge hierarchy.

Gauge Hierarchy Due To Strong Interactions?.
C. Wetterich (**Freiburg U.**).
Apr 1981. 20 pp.
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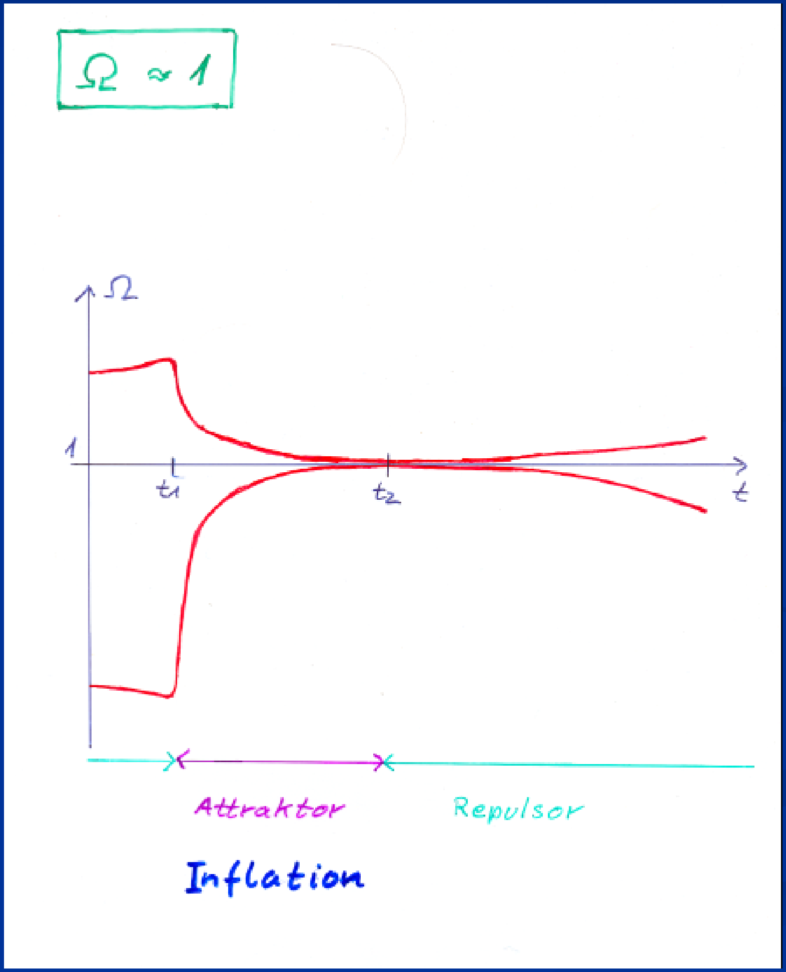
$$\ln \frac{M_X}{\varphi_L} = \frac{1}{2} \int_{\ln \varphi_L/M_X}^{0} \tilde{A}\, dt - \frac{1}{2} \ln \frac{\mu^2(M_X^2)}{\lambda(M_X^2) M_X^2}$$

$$\tilde{A} = A - d \ln \lambda / dt$$

relevant and irrelevant couplings

- $A < 2$ deviation from critical surface is relevant coupling
- $A > 2$ deviation from critical surface is irrelevant coupling
- parameters in effective action are attracted towards the critical surface as scale k flows towards the infrared
- self-tuned criticality

comparison with critical density in cosmology



fixed point in short-distance theory

- short-distance theory extends SM
- minimal: SM + gravity
- higher dimensional theory ?
- grand unification ?
- (almost) second order electroweak phase transition guarantees (approximate) fixed point of flow
- needed : deviation from fixed point is an irrelevant parameter (A>2)

self-tuned criticality

- deviation from fixed point is an irrelevant parameter (A>2)
- critical behavior realized for wide range of parameters
- in statistical physics : models of this type are known for d=2
- d=4: second order phase transitions found ,
  self-tuned criticality not yet found

asymptotic safety for gravity

Weinberg , Reuter

running Planck mass

$$M_P^2(k) = M_P^2 + 2\xi_0 k^2$$

infrared cutoff scale k ,

for k=0 : $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\ \mathrm{GeV}$

fixed point for dimensionless ratio M/k

$$M_P^2(k)/k^2 \approx 2\xi_0$$

scaling at short distances

$$q^2 \gg M_p^2$$

$G_N(q^2)$ scales as $\frac{1}{16\pi\xi_0 q^2}$

infrared unstable fixed point: transition from scaling to constant Planck mass

$$k_{tr} = \frac{M_P}{\sqrt{2\xi_0}} \approx 10^{19}\ \mathrm{GeV}$$

$$M_P^2(k) = M_P^2 + 2\xi_0 k^2$$

$$k \gtrsim k_{tr}$$

$$M_P^2(k)/k^2 \approx 2\xi_0$$

a prediction…

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s in $m_H = m_{\rm min} = 126$ GeV, with o

gravitational running

$$k\frac{dx_j}{dk} = \beta_j^{\mathrm{SM}} + \beta_j^{grav}$$

$$\beta_j^{grav} = \frac{a_j}{8\pi}\frac{k^2}{M_p^2(k)}x_j$$

$$x_j(k) \sim k^{A_j}$$

$$A_j = \frac{a_j}{16\pi\xi_0}$$

a < 0 for gauge and Yukawa couplings

➡ asymptotic freedom

modified running of quartic scalar coupling in presence of metric fluctuations

$$\beta_\lambda = \frac{a_\lambda}{16\pi\xi_0}\lambda + \frac{1}{16\pi^2}(24\lambda^2 + 12\lambda h^2 - 6h^4) + \ldots$$

for a > 0 and small h :

λ is driven fast too very small values !

e.g. a=3 found in gravity computations

short distance flow of λ

$$\lambda(k) = -\int_k^\infty \frac{dk'}{k'} \left(\frac{1 + 2\xi_0 k^2 / M_P^2}{1 + 2\xi_0 k'^2 / M_P^2} \right)^{\frac{a_\lambda}{32\pi\xi_0}} \times \beta_\lambda^{\mathrm{SM}}(x_j(k'))$$

integral dominated by small interval in k'

$$\lambda(k_{tr}) = -C\beta_\lambda^{\mathrm{SM}}\left(h(k_{tr}), g_i(k_{tr})\right)$$

prediction for mass of Higgs scalar

$$m_H = m_{\min}$$

$$m_{\min} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \text{GeV}\ ,$$

2010

M. Shaposhnikov, C. Wetterich, Phys. Lett. **B683** (2010) 196

M. Holthausen, K. S. Lim, M. Lindner, JHEP **1202** (2012) 037

uncertainties

- typical uncertainty is a few GeV
- central value has moved somewhat upwards , close to 129 GeV
- change in top-mass and strong gauge coupling
- inclusion of three loop running and two loop matching

K. G. Chetyrkin, M. F. Zoller JHEP **1206** (2012) 033;
F. Bezrukov, M. Kalmykov, B. Kniehl, M. Shaposhnikov, arXiv: 1205.2893;
G. Degrassi, S. Di Vita, J. Elias-Miro, J. Espinosa, G. Giudice, G. Isidori, A. Strumia, arXiv: 1205.6497;
S. Alekhin, A. Djouadi, S. Moch, arXiv: 1207.0980

bound on top quark mass

quartic scalar coupling has to remain positive during flow

(otherwise Coleman-Weinberg symmetry breaking at high scale)

$$\beta_\lambda^{SM} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda h^2 - 9\lambda\left(g_2^2 + \frac{1}{3}g_1^2\right) - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right] .$$

$$m_t \geq m_t^{\min}$$

~170 GeV

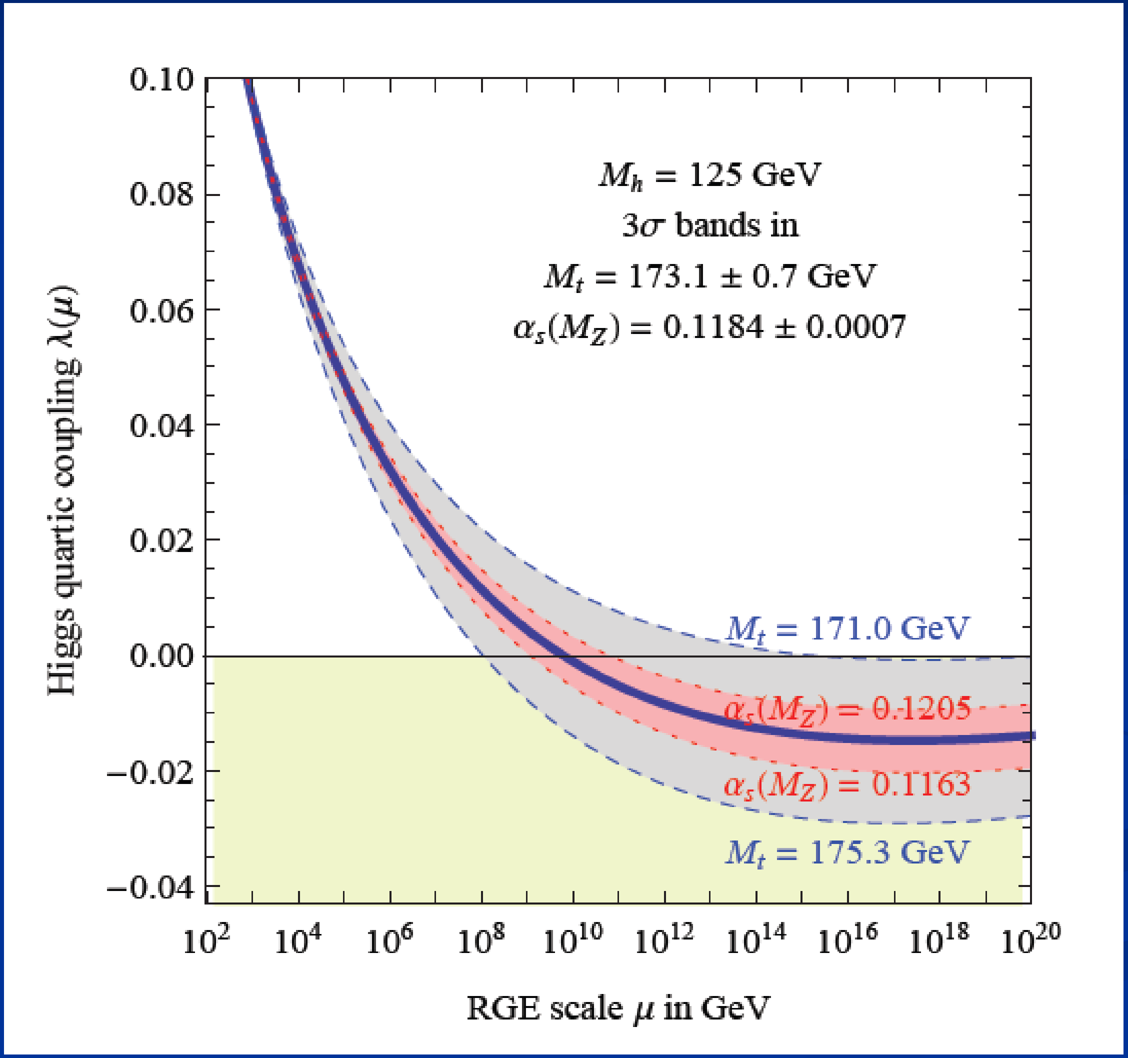
short distance fixed point at λ=0

- interesting speculation

$$\lambda(k_{tr}) \approx 0 \ , \ \beta_\lambda(k_{tr}) \approx 0$$

- top quark mass **"predicted"** to be close to minimal value , as found in experiment

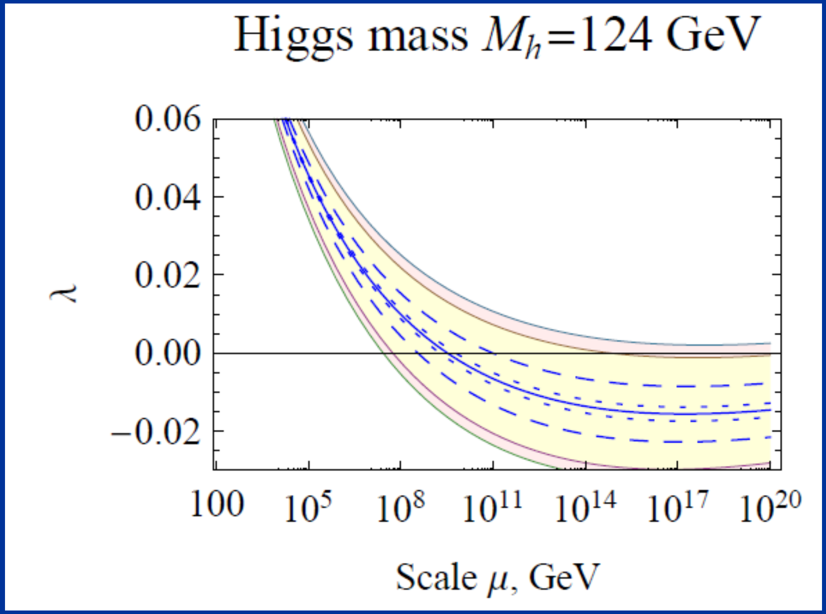
running quartic scalar coupling

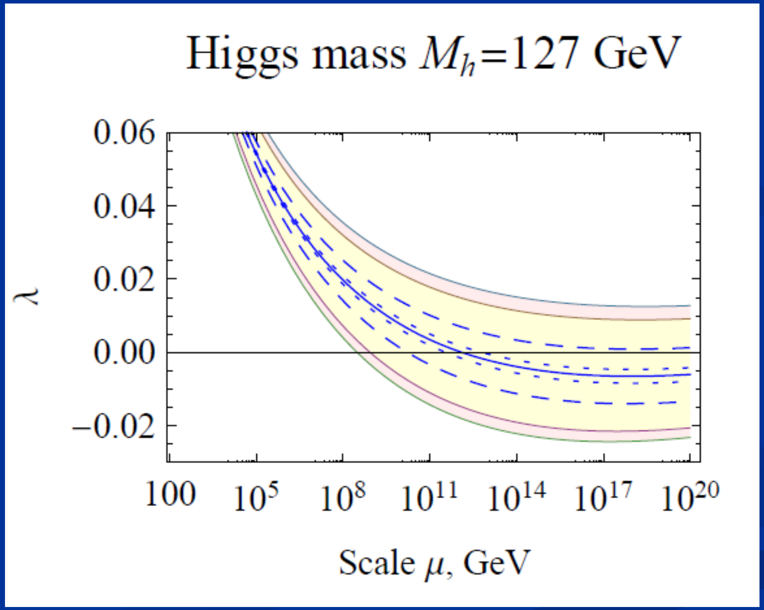


Degrassi et al

top "prediction" for known Higgs boson mass ?

Fedor Bezrukov,[a,b] Mikhail Yu. Kalmykov,[c] Bernd A. Kniehl[c] and Mikhail Shaposhnikov[d]





conclusions

- observed value of Higgs boson mass is compatible with great desert
- short distance fixed point with small λ predicts Higgs boson mass close to 126 GeV
- prediction in SM+gravity, but also wider class of models
- desert: no new physics at LHC and future colliders
- relevant scale for neutrino physics may be low or intermediate (say 10^{11} GeV) - oasis in desert ?

end

one loop flow equations

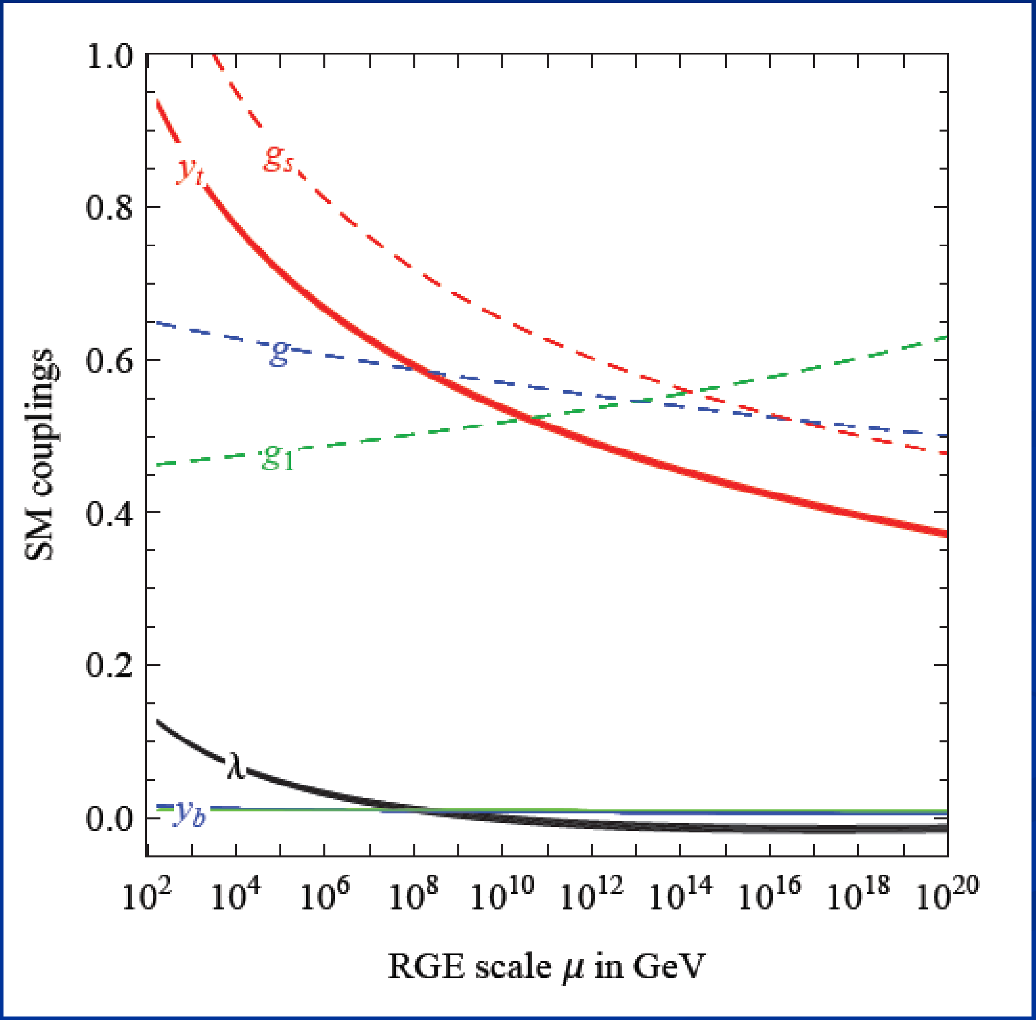
$$\beta_3^{SM} = -\frac{7}{16\pi^2} g_3^3 \,,$$

$$\beta_h^{SM} = \frac{1}{16\pi^2}\left[\frac{9}{2}h^3 - 8g_3^2 h - \frac{9}{4}g_2^2 h - \frac{17}{12}g_1^2 h\right] \,,$$

$$\beta_\lambda^{SM} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda h^2 - 9\lambda\,(g_2^2 + \frac{1}{3}g_1^2) - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right] \,.$$

$$\beta_1^{SM} = \frac{41}{96\pi^2} g_1^3 \,, \quad \beta_2^{SM} = -\frac{19}{96\pi^2} g_2^3$$

running SM couplings



Degrassi
et al

partial infrared fixed point for ratio quartic scalar coupling / squared Yukawa coupling (four generations)

$$\mathrm{d}\ln(\lambda/U_4^2)/\mathrm{d}t = (16\pi^2)^{-1}\ \{12\lambda + 3U_4^2 + 9D_4^2$$

$$-(12/\lambda)U_4^4 - (12/\lambda)D_4^4 + 16g_s^2 - \tfrac{9}{2}g_W^2 - \tfrac{1}{10}g_1^2$$

$$+ [3/(4-\lambda)]\,(3g_W^4 + \tfrac{6}{5}g_W^2 g_1^2 + \tfrac{9}{25}g_1^4)\}\ . \qquad (16)$$

Neglecting the gauge coupling this equation has a fixed point for $U_4^2 = D_4^2 = \lambda$. However, near the scale φ_L the

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infrared interval

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$$m_{\max} = [173.5 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.118}{0.002} \times 0.1]\ \mathrm{GeV}\ ,$$