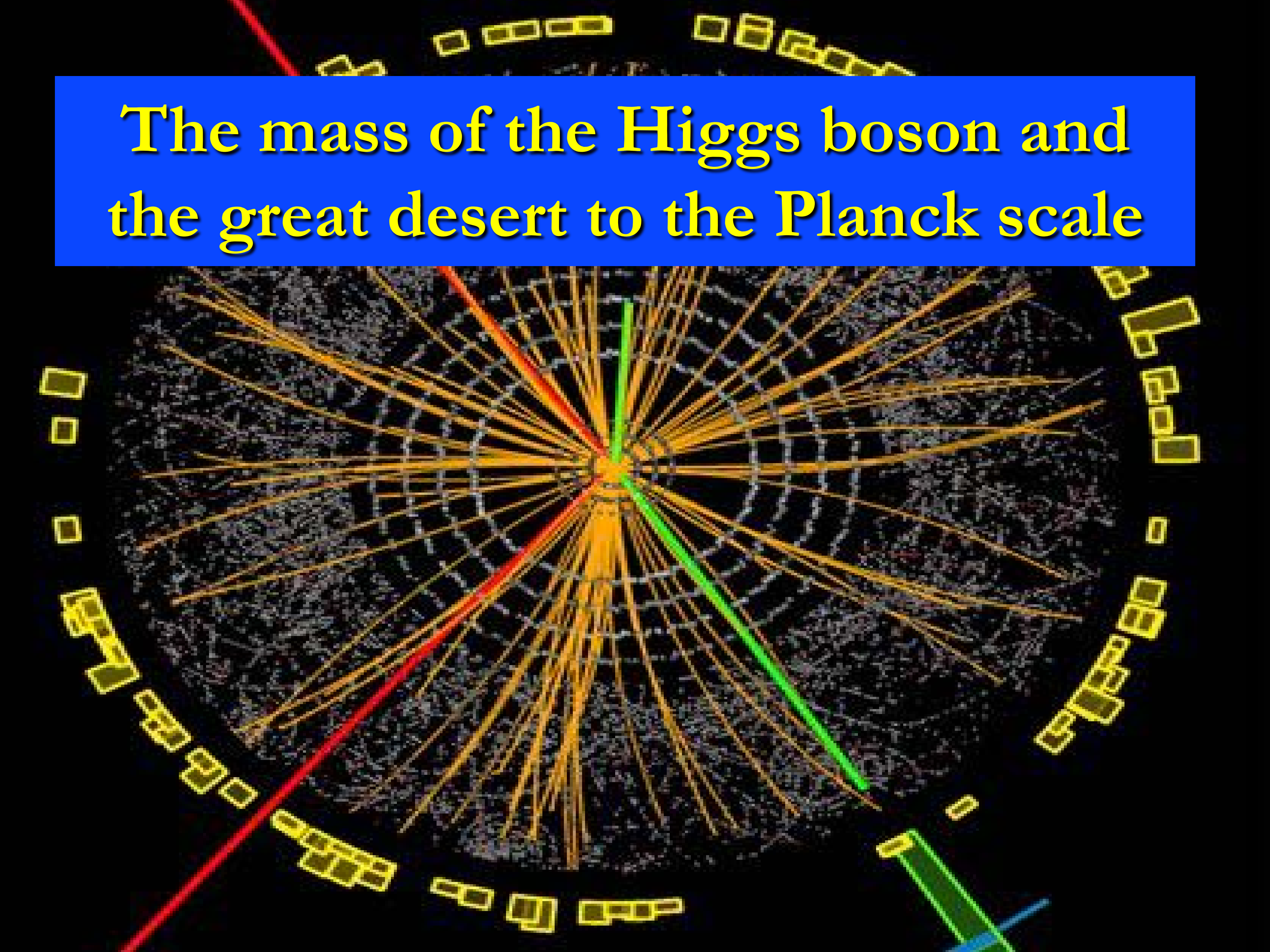
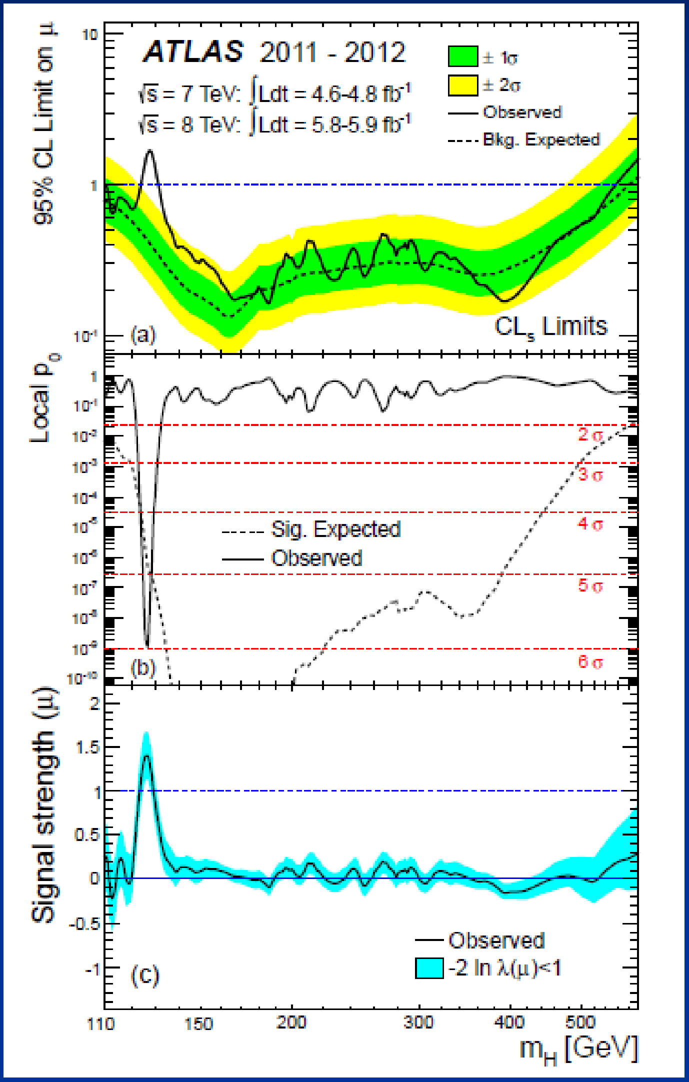
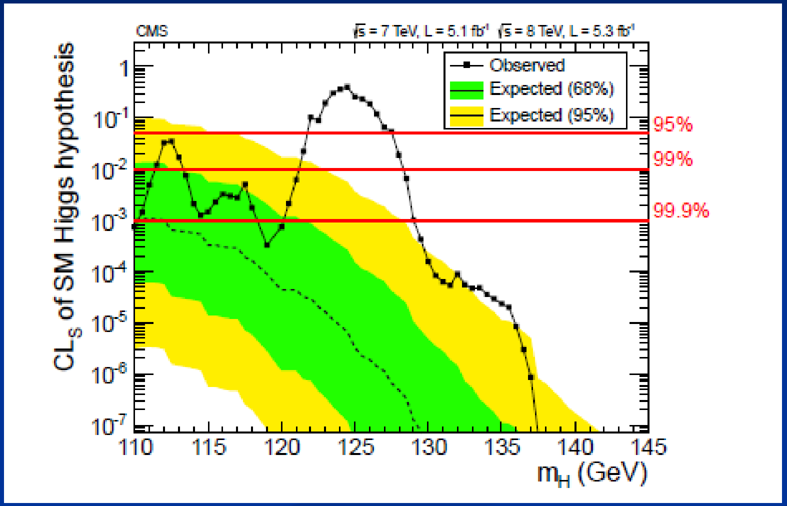
The mass of the Higgs boson and the great desert to the Planck scale



LHC : Higgs particle observation





CMS 2011/12

ATLAS 2011/12

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\rm min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\rm min} = 126$ GeV, with o

too good to be true ?

500 theoretical physicists = 500 models

equidistant predictions

range 100-600 GeV …

3 GeV bins : one expects several correct predictions ,

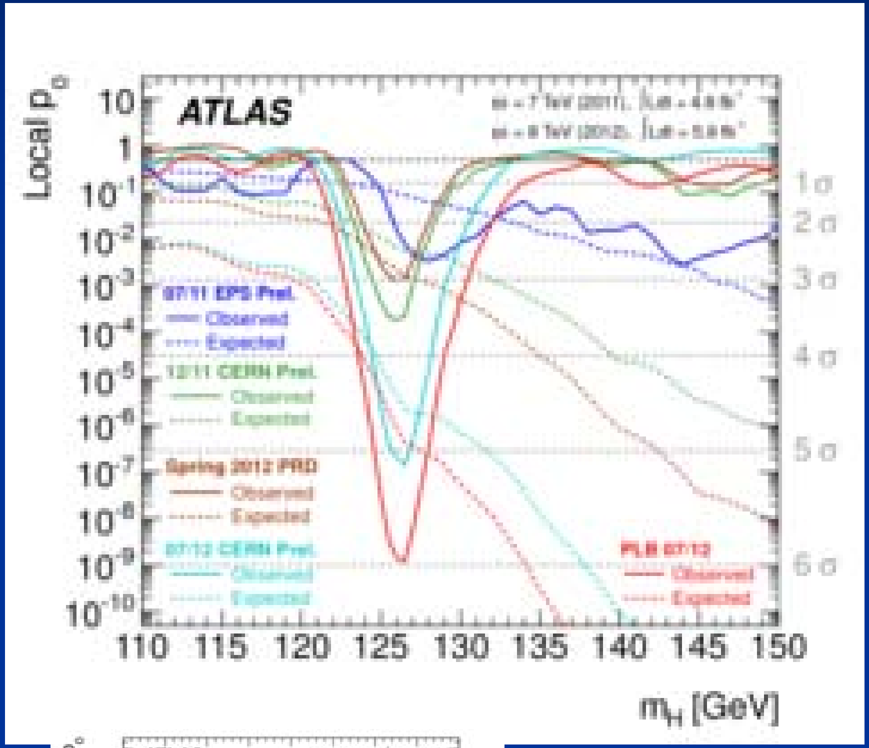
but for contradicting models

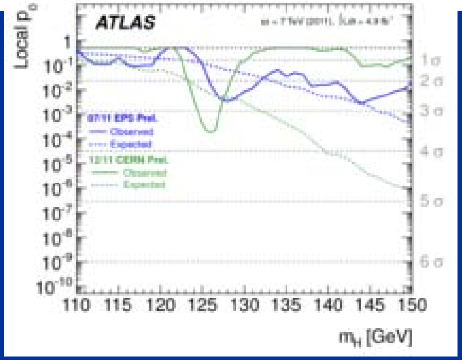
motivation behind prediction ?

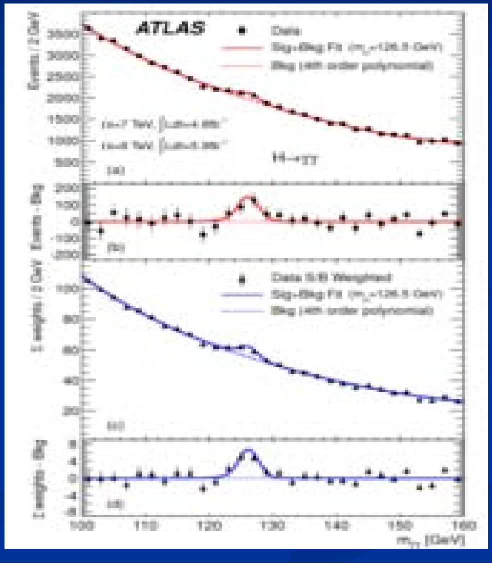
key points

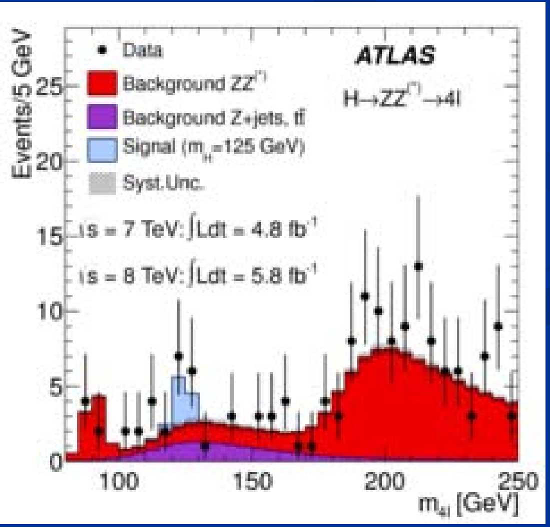
- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

Higgs boson found

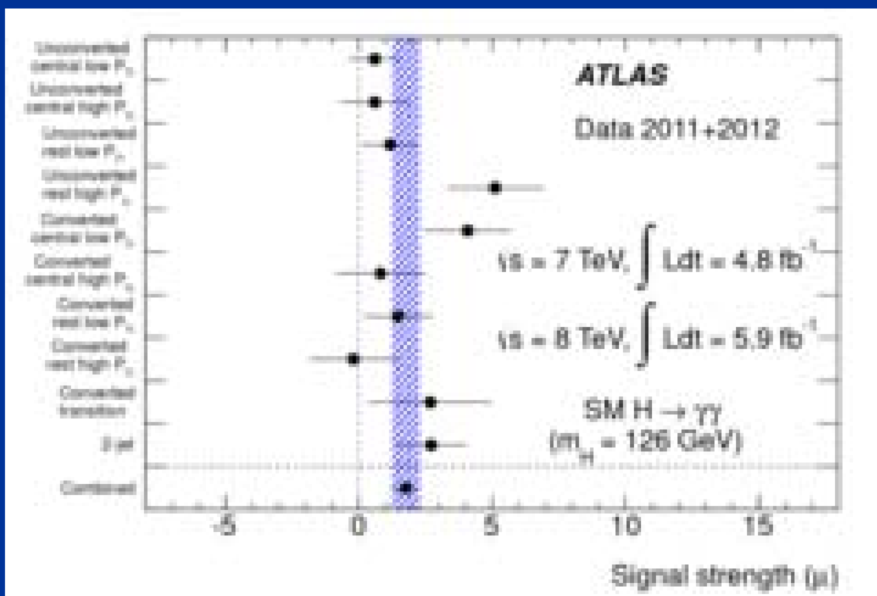


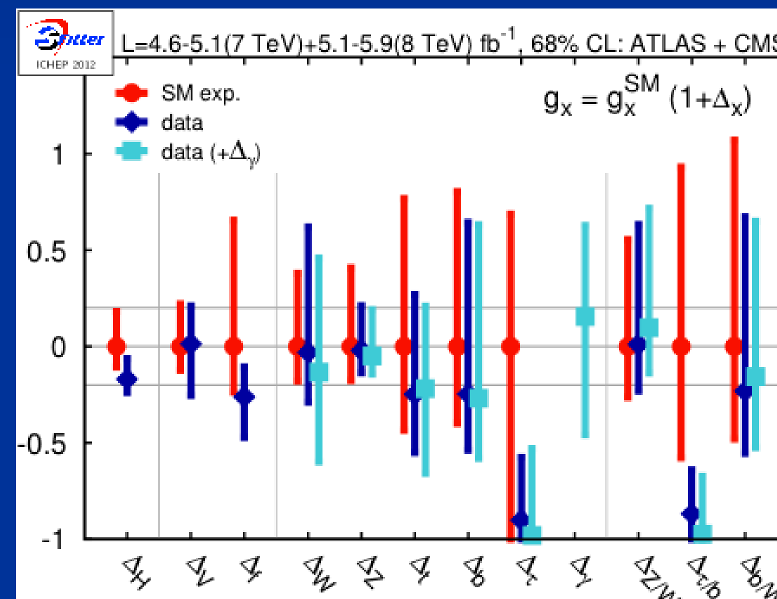






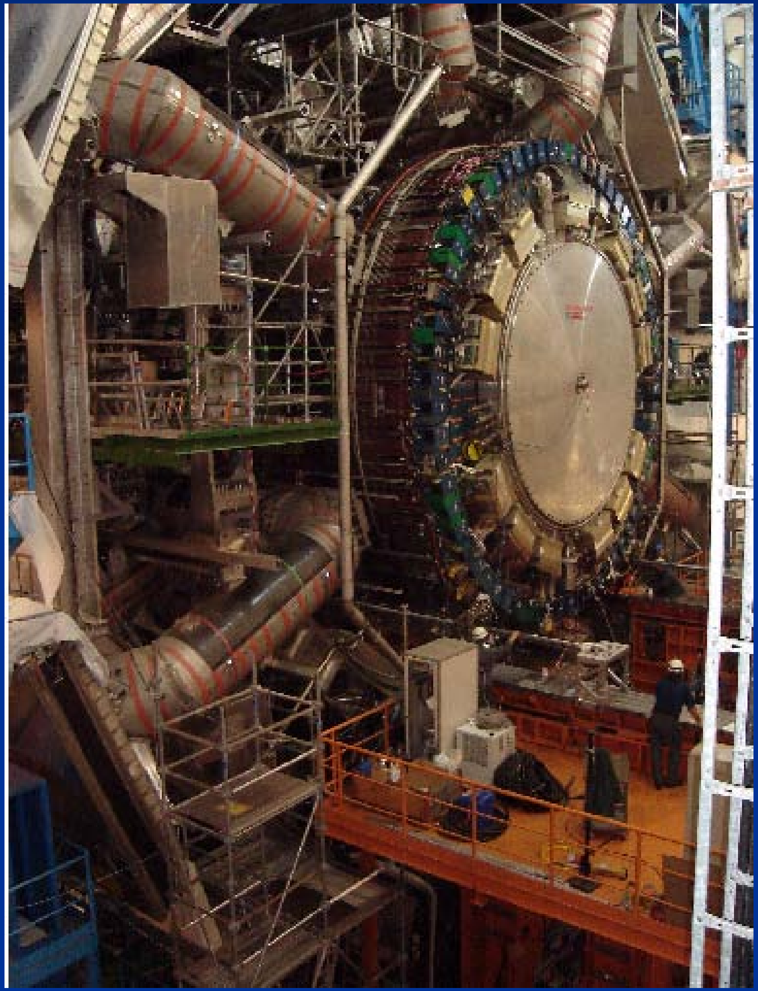
standard model Higgs boson



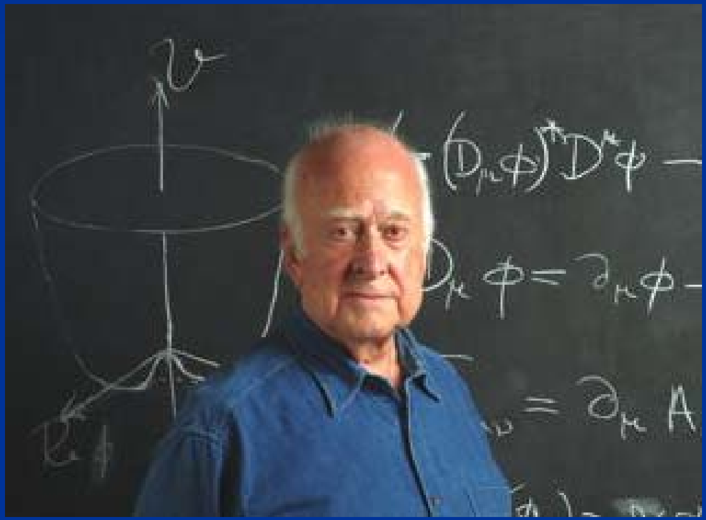


T.Plehn, M.Rauch

Spontaneous symmetry breaking confirmed at the LHC



Higgs mechanism verified



Higgs Brout
Englert

Spontaneous symmetry breaking

Spontaneous symmetry breaking

$$-\mathcal{L}_\varphi = \frac{1}{2}\partial^\mu \varphi^\dagger \partial_\mu \varphi + V(\varphi)$$

$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

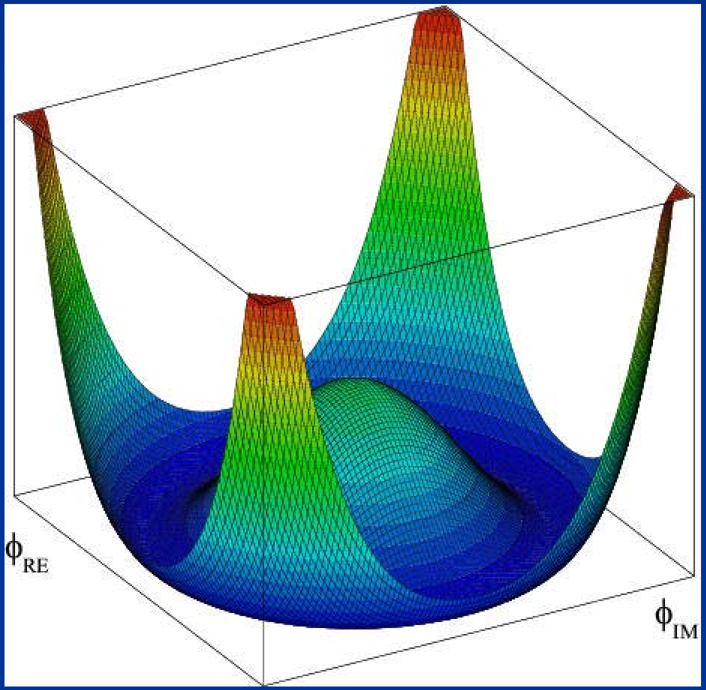
$\mu^2 > 0$

$\varphi_0^2 = \frac{\mu^2}{\lambda}$

Fermi scale

$\varphi_0 = 175 \text{ GeV}$

Scalar potential



$$V(\varphi) = -\mu^2\varphi^\dagger\varphi + \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger\varphi - \varphi_0^2)^2 + \text{const.})$$

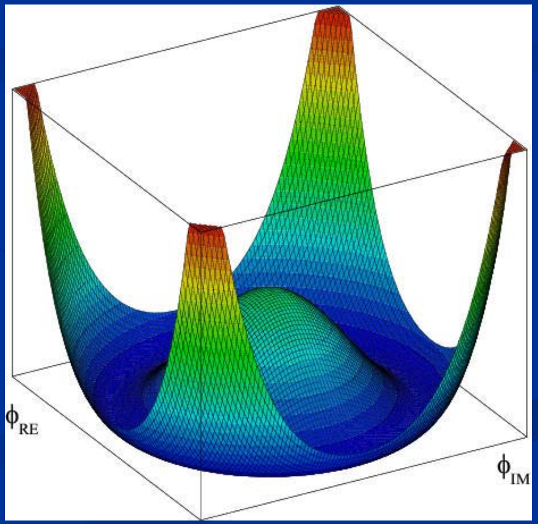
Radial mode and Goldstone mode

expand around minimum of potential

$$\varphi = \varphi_0 + \sigma + i\eta$$

$$\varphi_0, \sigma, \eta : \ \text{real}$$

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \tfrac{1}{2}\partial^\mu \eta \partial_\mu \eta + \tfrac{1}{2} m^2 \sigma^2 + \dots$$



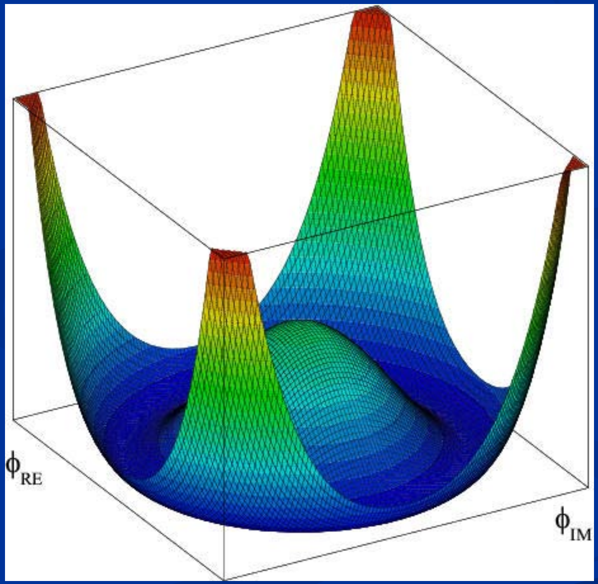
mass term for radial mode

$$m^2 = 2\lambda\varphi_0^2$$

massless Goldstone mode

$$-\mathcal{L}_\varphi = \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\partial^\mu\eta\partial_\mu\eta$$

$$+\frac{1}{2}m^2\sigma^2 + \ldots$$



Abelian Higgs mechanism supraconductivity

coupling of complex scalar field to photon

$$-\mathcal{L}_\varphi = \frac{1}{2}(D^\mu\varphi)^* D_\mu\varphi + V(\varphi)$$

$$+\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Abelian Higgs mechanism supraconductivity

$$\frac{1}{2}(D^\mu\varphi)^* D_\mu\varphi = \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma$$

$$\Rightarrow \quad +\frac{1}{2}e^2\varphi_0^2 A^\mu A_\mu$$

$$+\frac{1}{2}\partial^\mu\eta\partial_\mu\eta - e\varphi_0 A^\mu\partial_\mu\eta$$

$$+e^2\varphi_0\sigma A^\mu A_\mu + \ldots$$

$$\varphi = \varphi_0 + \sigma + i\eta$$

$$D_\mu = \partial_\mu - ieA_\mu$$

massive photon !

Gauge symmetry

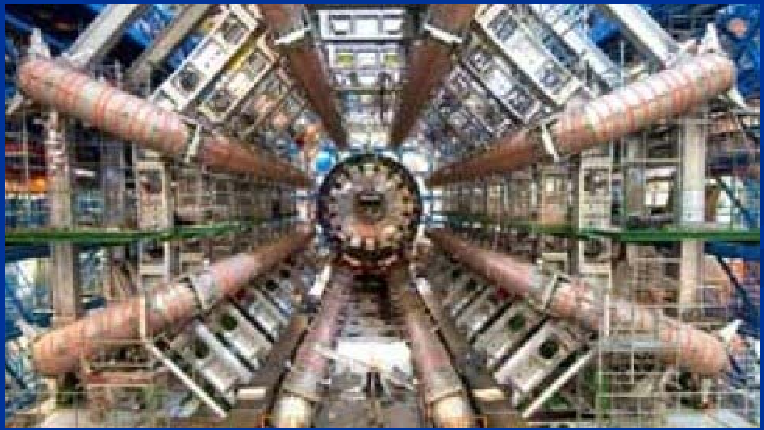
$$\delta\varphi(x) = -i\xi(x)\varphi(x)$$

$$\delta A_\mu(x) = -\frac{1}{e}\partial_\mu\xi(x)$$

$$\begin{aligned}\delta\Big(D_\mu\varphi(x)\Big) &= -i\xi(x)D_\mu\varphi_\mu\varphi(x)\\ &\quad -i\partial_\mu\xi(x)\varphi(x)\\ &\quad -ie\delta A_\mu(x)\varphi(x)\\ &= -i\xi(x)D_\mu\varphi(x)\end{aligned}$$

Goldstone boson is gauge degree of freedom
no physical particle
can be eliminated by gauge transformation
in favor of longitudinal component of massive photon

Photon mass m=e φ



Standard – Model of electroweak interactions : Higgs - mechanism

- The masses of all fermions and gauge bosons are proportional to the (vacuum expectation) value of a scalar field φ_H (Higgs scalar)
- For electron, quarks , W- and Z- bosons :

$$m_{electron} = h_{electron} * \varphi_H$$

etc.

lessons

1

Vacuum is complicated

mass generated by vacuum properties

particles: excitations of vacuum

Their properties depend on properties of vacuum

vacuum is not empty !

2

Fundamental “constants” are not constant

Have coupling constants in the early Universe other values than today ?

Yes !

Fundamental couplings in quantum field theory

Masses and coupling constants are determined by properties of ***vacuum*** *!*

Similar to Maxwell – equations in matter

Condensed matter physics : laws depend on state of the system

- Ground state , thermal equilibrium state …
- Example : Laws of electromagnetism in superconductor are different from Maxwells' laws

Standard model of particle physics :

Electroweak gauge symmetry is spontaneously broken by expectation value of Higgs scalar

Cosmology :

- Universe is not in one fixed state
- Dynamical evolution
- Laws are expected to depend on time

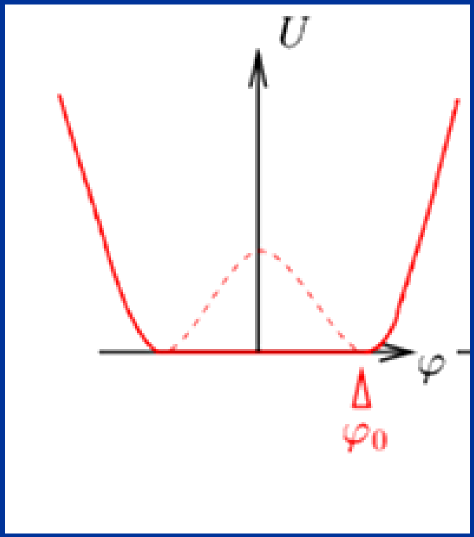
Restoration of symmetry at high temperature in the early Universe

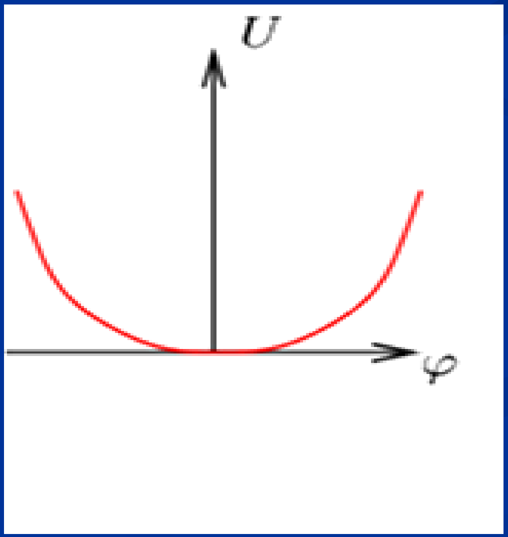
Low T
SSB
$<\varphi>=\varphi_0 \neq 0$

High T
SYM
$<\varphi>=0$

high T :
Less order
More symmetry

Example:
Magnets





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etc.

In hot plasma of early Universe :

masses of electron und muon not different!

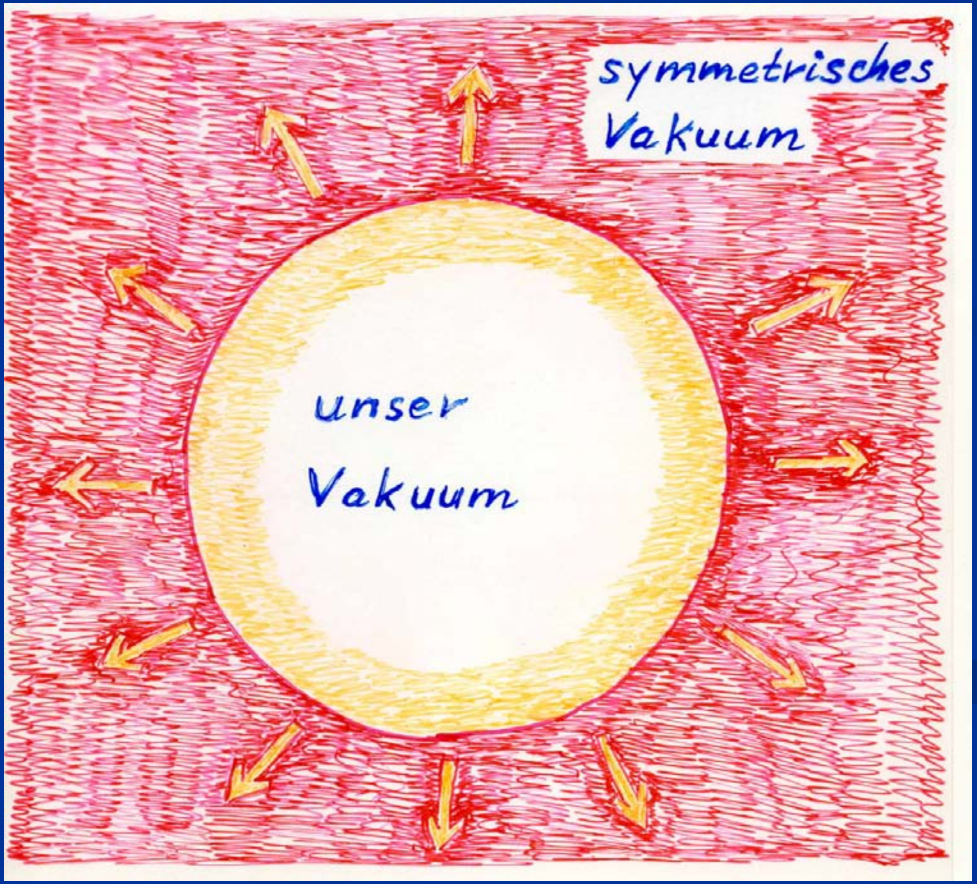
similar strength of electromagnetic and weak interaction

electromagnetic phase transition in early universe

10^{-12} s after big bang

most likely smooth crossover

could also be more violent first order transition



Varying couplings

How strong is **present** variation of couplings ?

Can variation of fundamental "constants" be observed ?

Fine structure constant α (electric charge)

Ratio electron mass to proton mass

Ratio nucleon mass to Planck mass

Time evolution of couplings and scalar fields

- Fine structure constant depends on value of Higgs field : $\alpha(\varphi)$

- Time evolution of φ ⟹ Time evolution of α

Jordan,…

Static scalar fields

In Standard Model of particle physics :

- Higgs scalar has settled to its present value around 10^{-12} seconds after big bang.
- Chiral condensate of QCD has settled at present value after quark-hadron phase transition around 10^{-6} seconds after big bang .
- No scalar with mass below pion mass.
- No substantial change of couplings after QCD phase transition.
- Coupling constants are frozen.

Observation of time- or space-variation of couplings

→

Physics beyond Standard Model

Particle masses in quintessence cosmology

can depend on value of cosmon field

similar to dependence on value of Higgs field

3

Standard model of particle physics could be valid down to the Planck length

The mass of the Higgs boson, the great desert, and asymptotic safety of gravity

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

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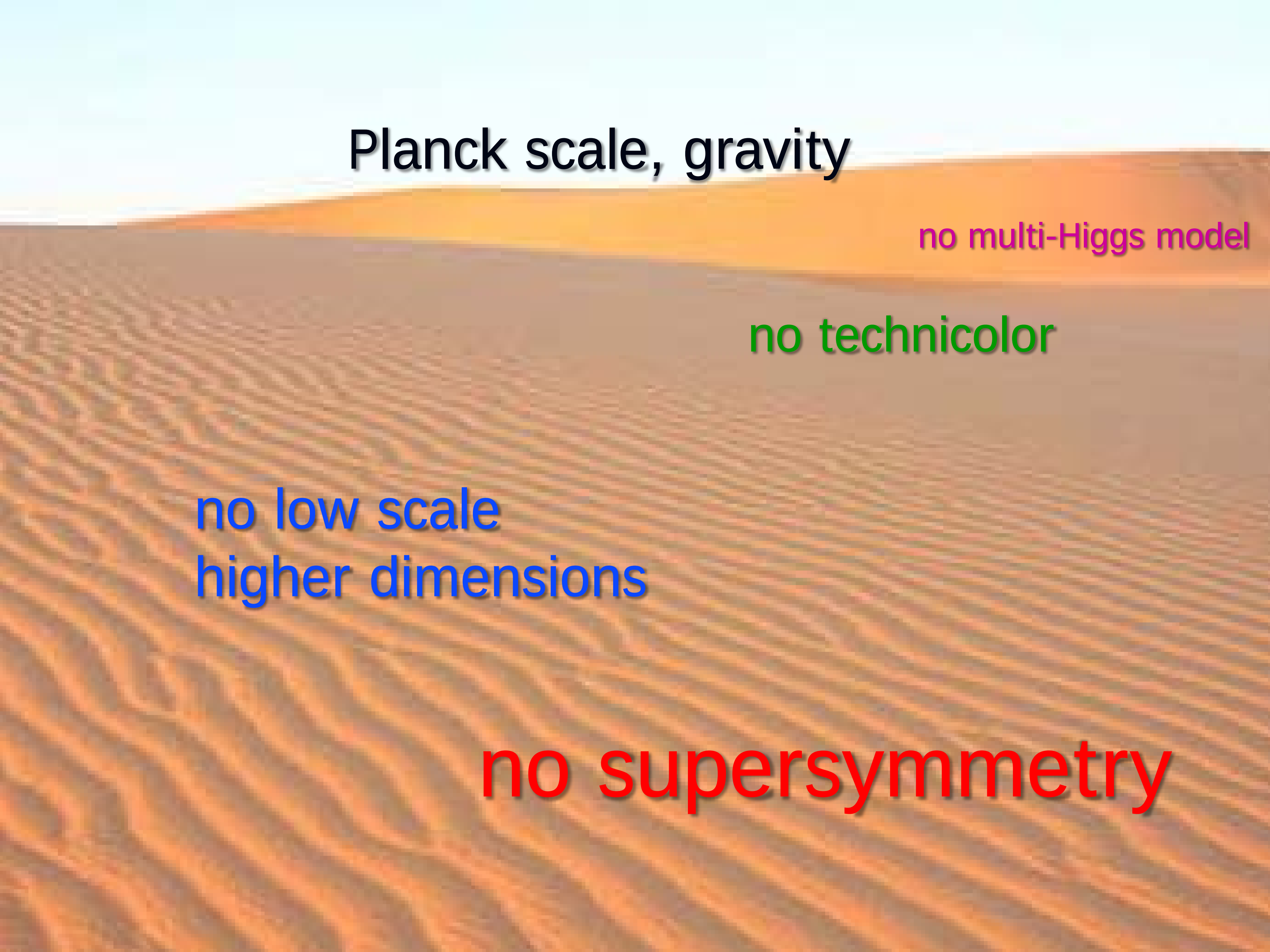
s in $m_H = m_{\text{min}} = 126$ GeV, with o

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point


Planck scale, gravity
no multi-Higgs model
no technicolor
no low scale
higher dimensions
no supersymmetry

Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$M_s^2 = 2\lambda(\varphi_L^2)\varphi_L^2$$

Radial mode = Higgs scalar

expansion around minimum of potential

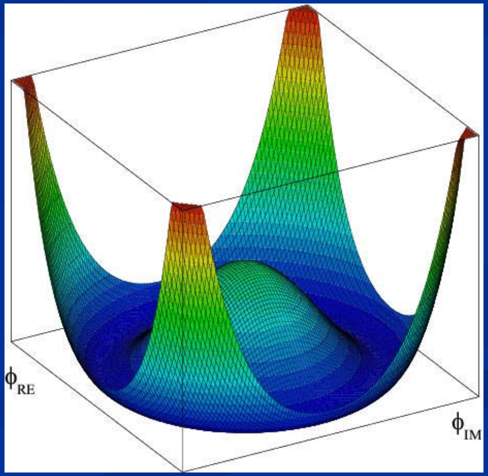
$$\varphi = \varphi_0 + \sigma + i\eta$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \tfrac{1}{2}\partial^\mu \eta \partial_\mu \eta$$

$$+\tfrac{1}{2} m^2 \sigma^2 + \dots$$



mass term for radial mode

$$m^2 = 2\lambda\varphi_0^2$$

Running couplings, Infrared interval, UV-IR mapping

renormalization

couplings depend on length scale,
or mass scale k

Running quartic scalar coupling λ and Yukawa coupling of top quark h

$$\partial_t \lambda = \beta_\lambda(\lambda, h, g^2)$$

$$t = \ln(k/\chi)$$

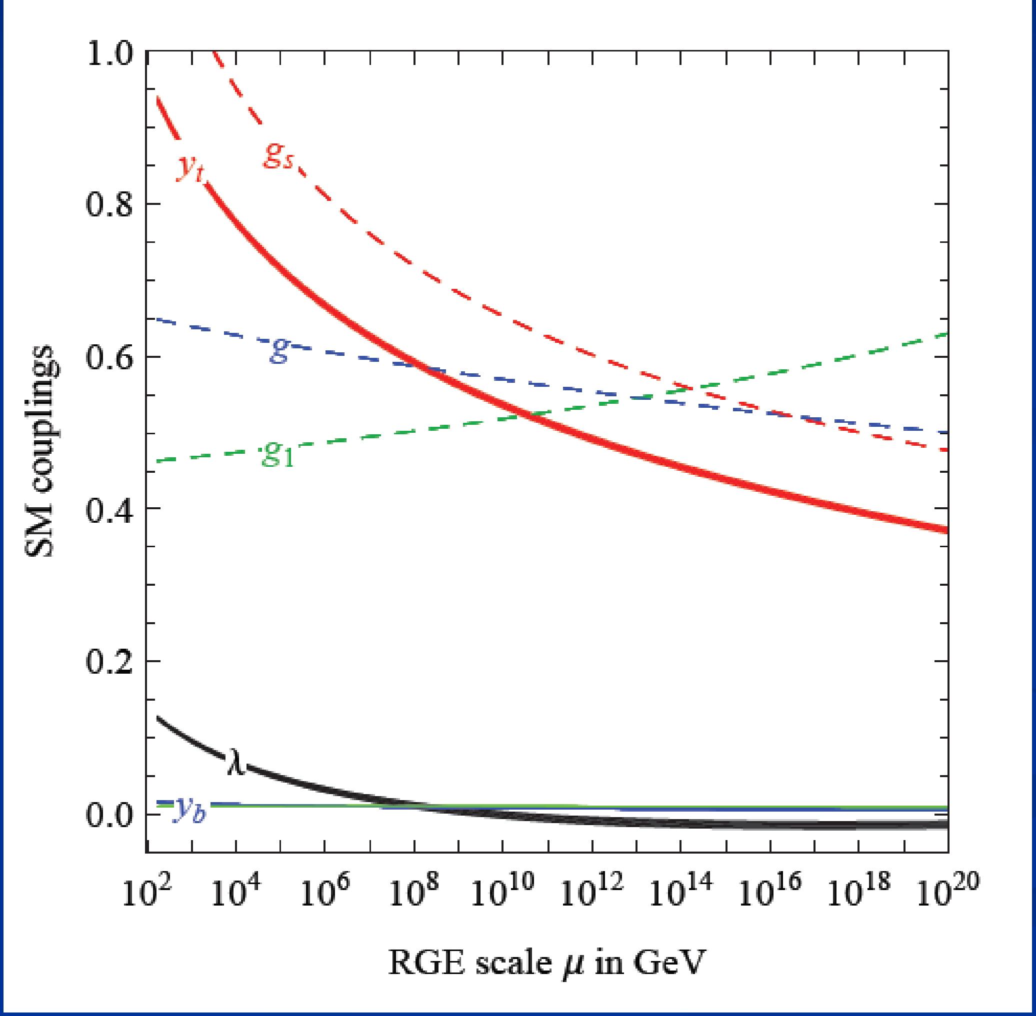
neglect gauge couplings g

$$\beta_\lambda = \frac{3}{4\pi^2}(\lambda^2 + h^2\lambda - h^4)$$

$$\partial_t h = \beta_h = \frac{9}{32\pi^2} h^3$$

$$m_t = h(\varphi_0)\varphi_0$$

running SM couplings



Degrassi et al

Partial infrared fixed point

become comparable. Indeed, for strong enough h_t the RGE for the ratio λ/h^2_t,

$$\frac{d}{dt}\left(\frac{\lambda}{h_t^2}\right) = \frac{h_t^2}{16\pi^2}\left\{12\left(\frac{\lambda}{h_t^2}\right)^2 + 3\,\frac{\lambda}{h_t^2} - 12\right\} \quad (16)$$

is governed by an infrared fixpoint.[13] The ratio λ/h^2_t remains constant if the right hand side of (16) vanishes. This happens for

$$\frac{\lambda}{h_t^2} = \left(\frac{65}{64}\right)^{1/2} - \frac{1}{8} = x_0 \quad (17)$$

and corresponds to a mass ratio

$$\frac{M_H}{m_t} \approx 1.3 \quad (18)$$

$$\left(\frac{\lambda}{h^2}\right) = (\sqrt{65} - 1)/8$$

infrared interval

allowed values of λ or λ/h^2 at UV-scale Λ :

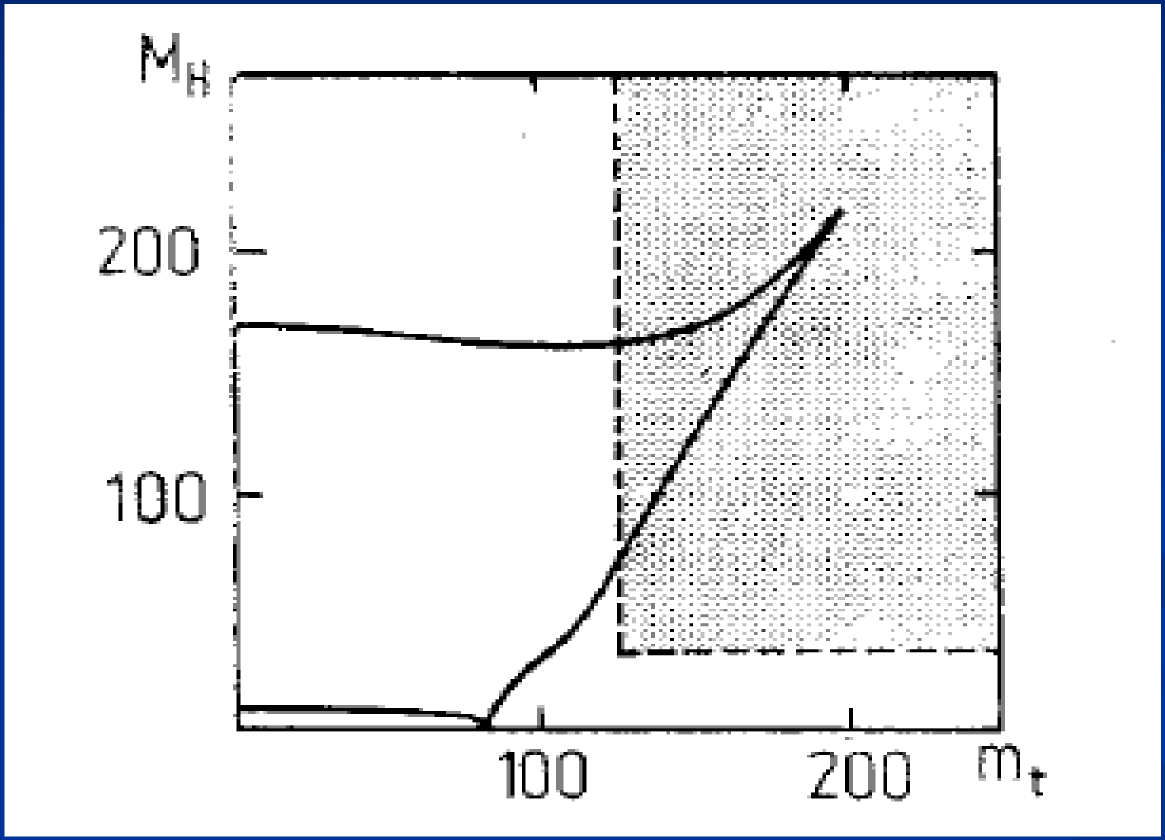
between zero and infinity

are mapped to

finite infrared interval of values of

λ/h^2 at Fermi scale

infrared interval



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N. Cabbibo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B158 (1979) 295

M. Lindner, Z. Phys. **C31** (1986) 295.

B. Grzadkowski and M. Lindner, Phys. Lett. **B178**, 81 (1986);

M. Lindner, M. Sher, and H. W. Zaglauer, Phys. Lett. **B228**, 139 (1989);

M. Sher, Phys. Rept. **179**, 273 (1989);

realistic mass of top quark (2010), ultraviolet cutoff: reduced Planck mass

$$M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\ \mathrm{GeV}$$

$$m_{min} = 126GeV\ ,\ m_{max} = 174GeV$$

ultraviolet- infrared map

Whole range of small λ

at ultraviolet scale is mapped by

renormalization flow

to lower bound of infrared interval !

Prediction of Higgs boson mass

close to 126 GeV

high scale fixed point

high scale fixed point

with small λ

predicts Higgs boson mass close to 126 GeV

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

fixed point in short-distance theory

- short-distance theory extends SM
- minimal: SM + gravity
- higher dimensional theory ?
- grand unification ?
- (almost) second order electroweak phase transition guarantees (approximate) fixed point of flow
- needed for gauge hierarchy: deviation from fixed point is an irrelevant parameter

asymptotic safety for gravity

Weinberg , Reuter

running Planck mass

$$M_P^2(k) = M_P^2 + 2\xi_0 k^2$$

infrared cutoff scale k ,

for k=0 : $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\ \mathrm{GeV}$

fixed point for dimensionless ratio M/k

$$M_P^2(k)/k^2 \approx 2\xi_0$$

scaling at short distances

$$q^2 \gg M_p^2$$

$G_N(q^2)$ scales as $\dfrac{1}{16\pi\xi_0 q^2}$

infrared unstable fixed point: transition from scaling to constant Planck mass

$$k_{tr} = \frac{M_P}{\sqrt{2\xi_0}} \approx 10^{19}\ \mathrm{GeV}$$

$$M_P^2(k) = M_P^2 + 2\xi_0 k^2$$

$$k \gtrsim k_{tr}$$

$$M_P^2(k)/k^2 \approx 2\xi_0$$

modified running of quartic scalar coupling in presence of metric fluctuations

$$\beta_\lambda = \frac{a_\lambda}{16\pi\xi_0}\lambda + \frac{1}{16\pi^2}(24\lambda^2 + 12\lambda h^2 - 6h^4) + \ldots$$

for a > 0 and small h :

λ is driven fast too very small values !

e.g. a=3 found in gravity computations

short distance fixed point at λ=0

- interesting speculation

$$\lambda(k_{tr}) \approx 0 \ , \ \beta_{\lambda}(k_{tr}) \approx 0$$

- top quark mass **"predicted"** to be close to minimal value , as found in experiment

bound on top quark mass

quartic scalar coupling has to remain positive during flow

(otherwise Coleman-Weinberg symmetry breaking at high scale)

$$\beta_\lambda^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda h^2 - 9\lambda\left(g_2^2 + \frac{1}{3}g_1^2\right) - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right] .$$

$$m_t \geq m_t^{\min}$$

~170 GeV

prediction for mass of Higgs scalar

$$m_H = m_{\min}$$

$$m_{\min} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5] \text{ GeV} ,$$

2010

M. Shaposhnikov, C. Wetterich, Phys. Lett. **B683** (2010) 196

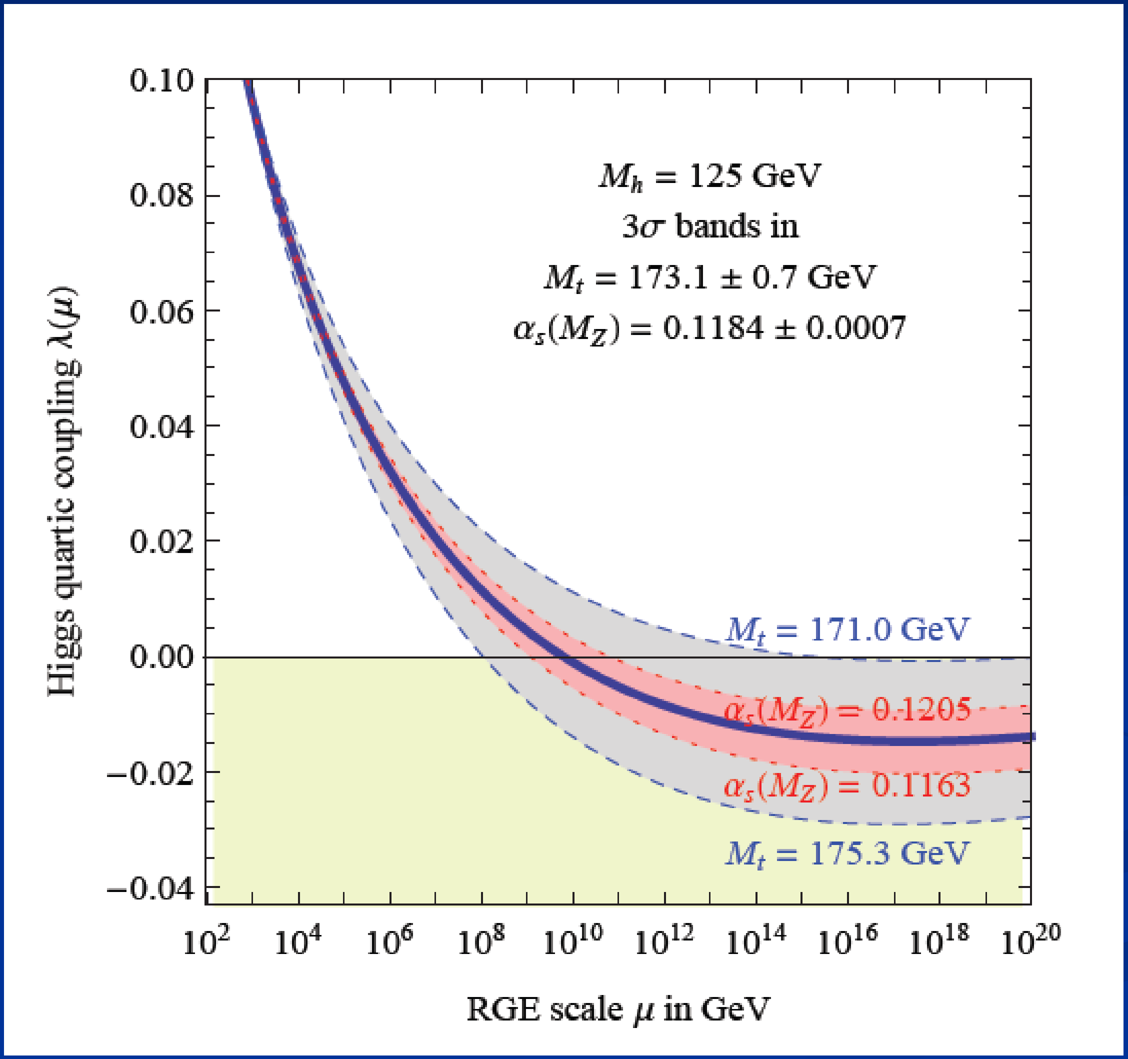
M. Holthausen, K. S. Lim, M. Lindner, JHEP **1202** (2012) 037

uncertainties

- typical uncertainty is a few GeV
- central value has moved somewhat upwards , close to 129 GeV
- change in top-mass and strong gauge coupling
- inclusion of three loop running and two loop matching

K. G. Chetyrkin, M. F. Zoller JHEP **1206** (2012) 033;
F. Bezrukov, M. Kalmykov, B. Kniehl, M. Shaposhnikov, arXiv: 1205.2893;
G. Degrassi, S. Di Vita, J. Elias-Miro, J. Espinosa, G. Giudice, G. Isidori, A. Strumia, arXiv: 1205.6497;
S. Alekhin, A. Djouadi, S. Moch, arXiv: 1207.0980

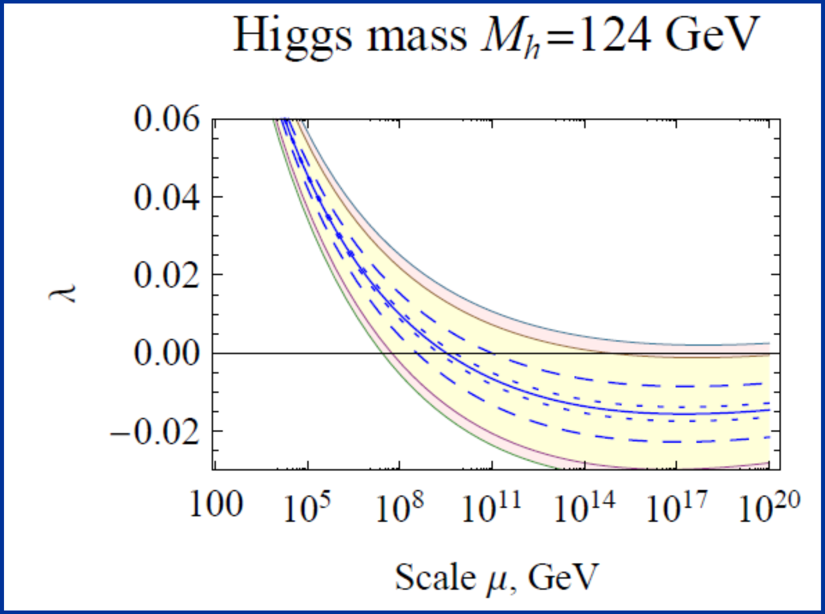
running quartic scalar coupling

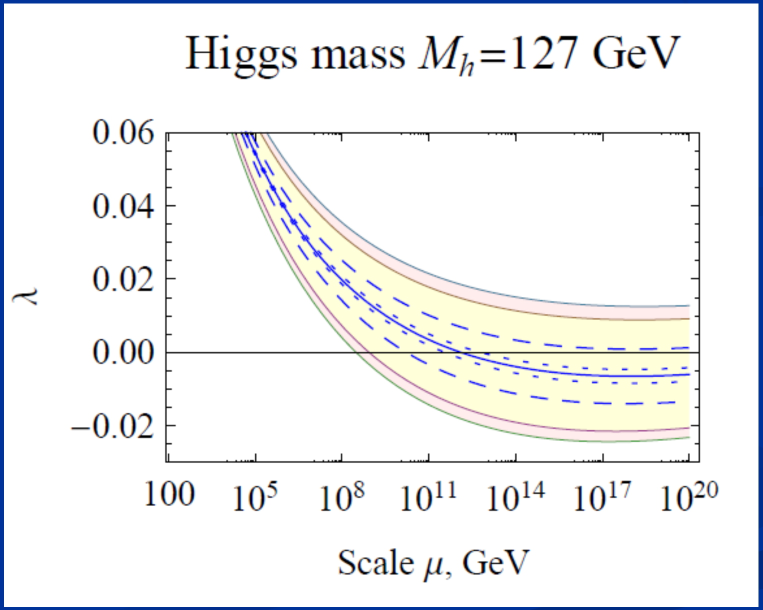


Degrassi
et al

Sensitivity to Higgs boson mass for given top quark mass

Fedor Bezrukov,[a,b] Mikhail Yu. Kalmykov,[c] Bernd A. Kniehl[c] and Mikhail Shaposhnikov[d]





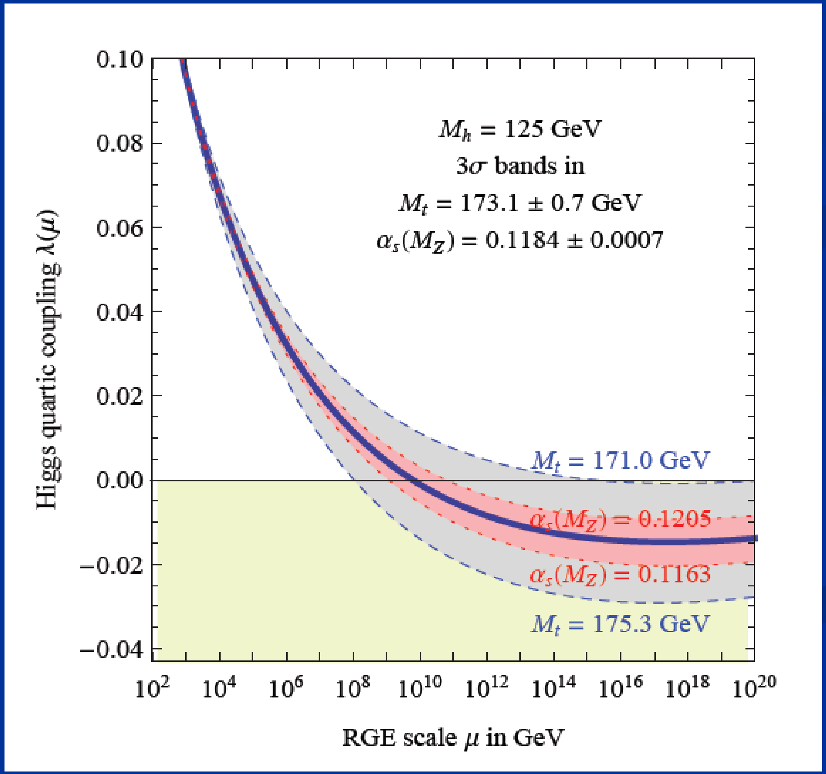
top “prediction” for known Higgs boson mass

for m_H =126 Gev :

m_t = 171.5 GeV

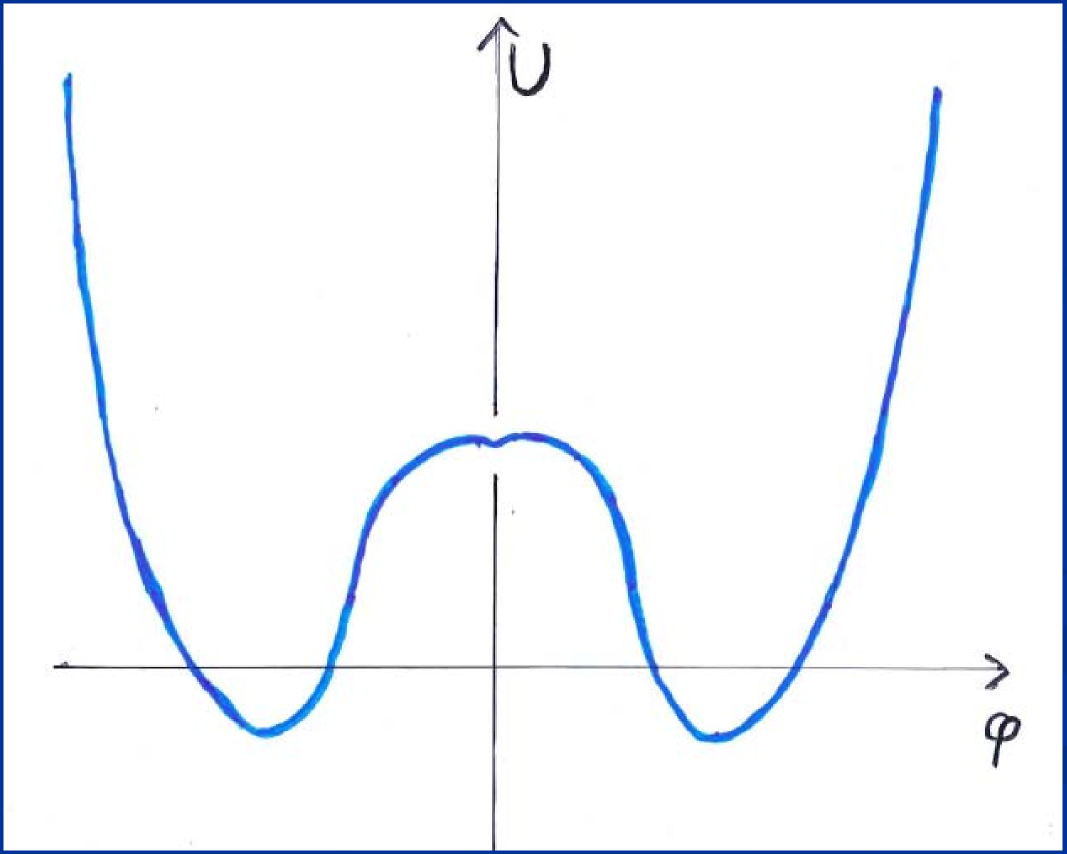
What if top pole mass is 173 GeV ?

- standard model needs extension around 10^{11} GeV
- scale of seesaw for neutrinos
- heavy triplet ?



remark on metastable vacuum

no model known where this is realized in reliable way



conclusions

- observed value of Higgs boson mass is compatible with great desert
- short distance fixed point with small λ predicts Higgs boson mass close to 126 GeV
- prediction in SM+gravity, but also wider class of models
- desert: no new physics at LHC and future colliders
- relevant scale for neutrino physics may be low or intermediate (say 10^{11} GeV) - oasis in desert ?

end

gauge hierarchy problem
and
fine tuning problem

quantum effective potential

$$U = \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2 + \gamma(\varphi^\dagger\varphi)\chi^2 + U_\chi,$$

$$\frac{\varphi_0}{M} = \sqrt{-\frac{\gamma}{\lambda}}.$$

scalar field χ with high expectation value M, say Planck mass

anomalous mass dimension

$$\partial_t \gamma = A_\mu(\lambda, h, g^2)\gamma$$

$$A_\mu = \frac{3}{8\pi^2}(\lambda^2 + h^2)$$

one loop,
neglect gauge couplings g

fixed point for $\gamma = 0$

- zero temperature electroweak phase transition (as function of γ) is essentially second order
- fixed point with effective dilatation symmetry
- no flow of γ at fixed point

$$\partial_t \gamma \;=\; A_\mu(\lambda, h, g^2)\gamma$$

- naturalness due to enhanced symmetry
- small deviations from fixed point due to running couplings: leading effect is lower bound on Fermi scale by quark-antiquark condensates

critical physics

- second order phase transition corresponds to critical surface in general space of couplings
- flow of couplings remains within critical surface
- once couplings are near critical surface at one scale, they remain in the vicinity of critical surface
- gauge hierarchy problem : explain why world is near critical surface for electroweak phase transition
- explanation can be at arbitrary scale !

critical physics in statistical physics

use of naïve perturbation theory

(without RG – improvement)

would make the existence of critical temperature look "unnatural"

artefact of badly converging expansion

self-tuned criticality

- deviation from fixed point is an irrelevant parameter (A>2)
- critical behavior realized for wide range of parameters
- in statistical physics : models of this type are known for d=2
- d=4: second order phase transitions found ,

  self-tuned criticality found in models of scalars coupled to gauge fields (QCD), Gies…

  realistic electroweak model not yet found

SUSY vs Standard Model

natural predictions

baryon and lepton number conservation	SM
flavor and CP violation described by CKM matrix	SM
absence of strangeness violating neutral currents	SM
g-2 etc.	SM
dark matter particle (WIMP)	SUSY

gravitational running

$$k \frac{dx_j}{dk} = \beta_j^{\mathrm{SM}} + \beta_j^{grav}$$

$$\beta_j^{grav} = \frac{a_j}{8\pi} \frac{k^2}{M_p^2(k)} x_j$$

$$x_j(k) \sim k^{A_j}$$

$$A_j = \frac{a_j}{16\pi\xi_0}$$

a < 0 for gauge and Yukawa couplings

➡ asymptotic freedom