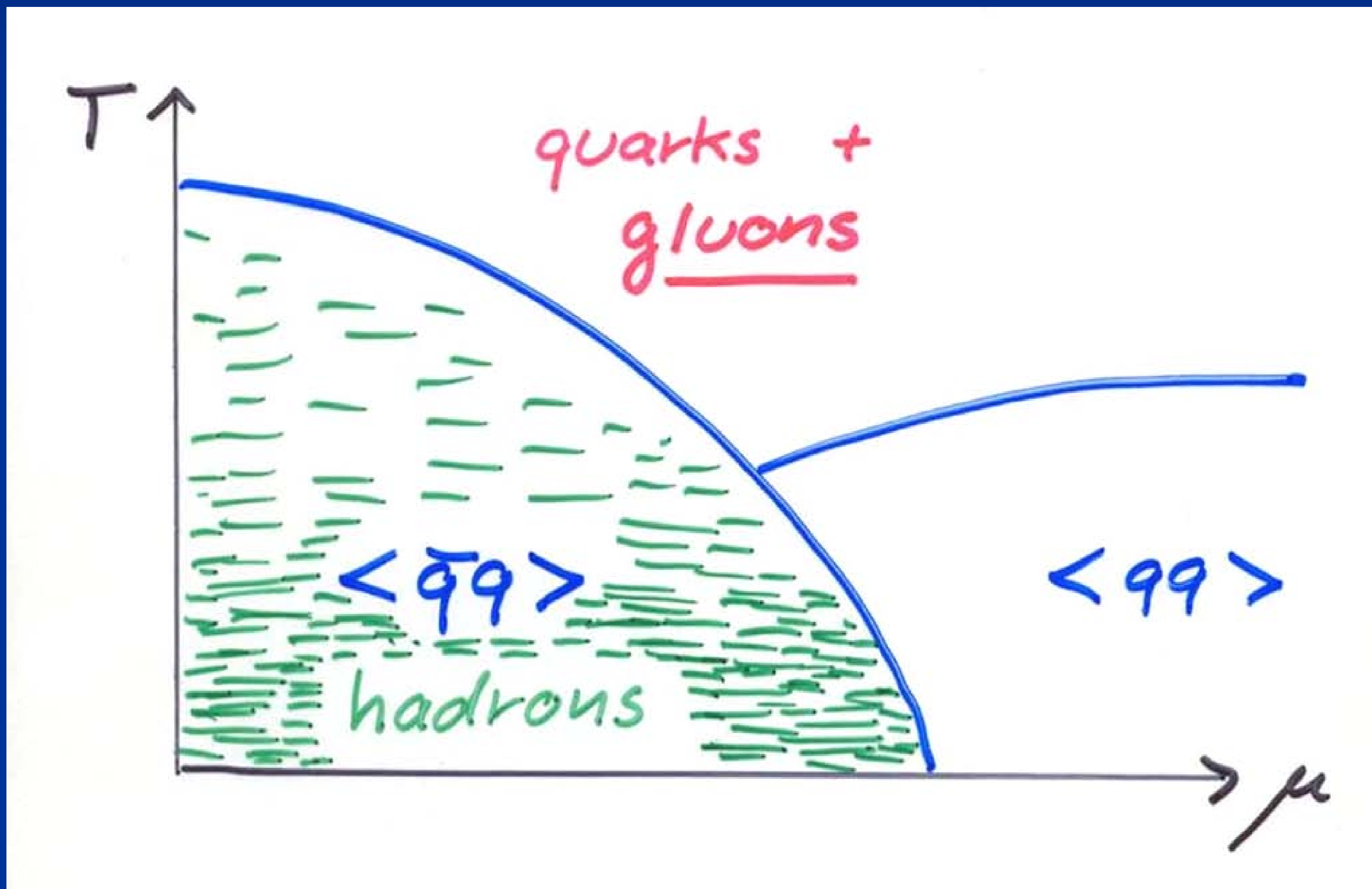
Aspects of the

QCD phase diagram

Understanding the phase diagram



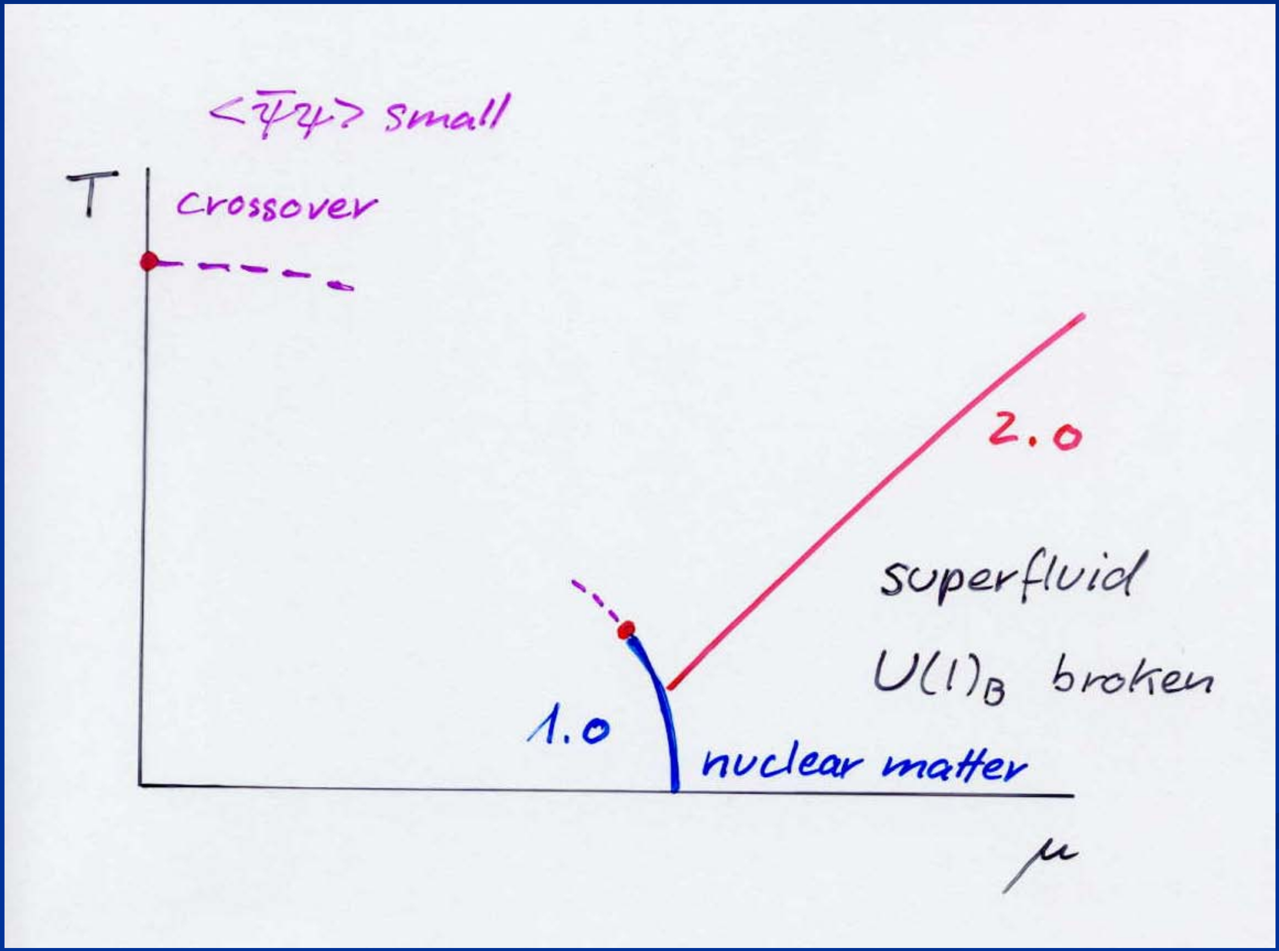
Order parameters

- Nuclear matter and quark matter are separated from other phases by true **critical lines**
- Different realizations of global symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin I_3

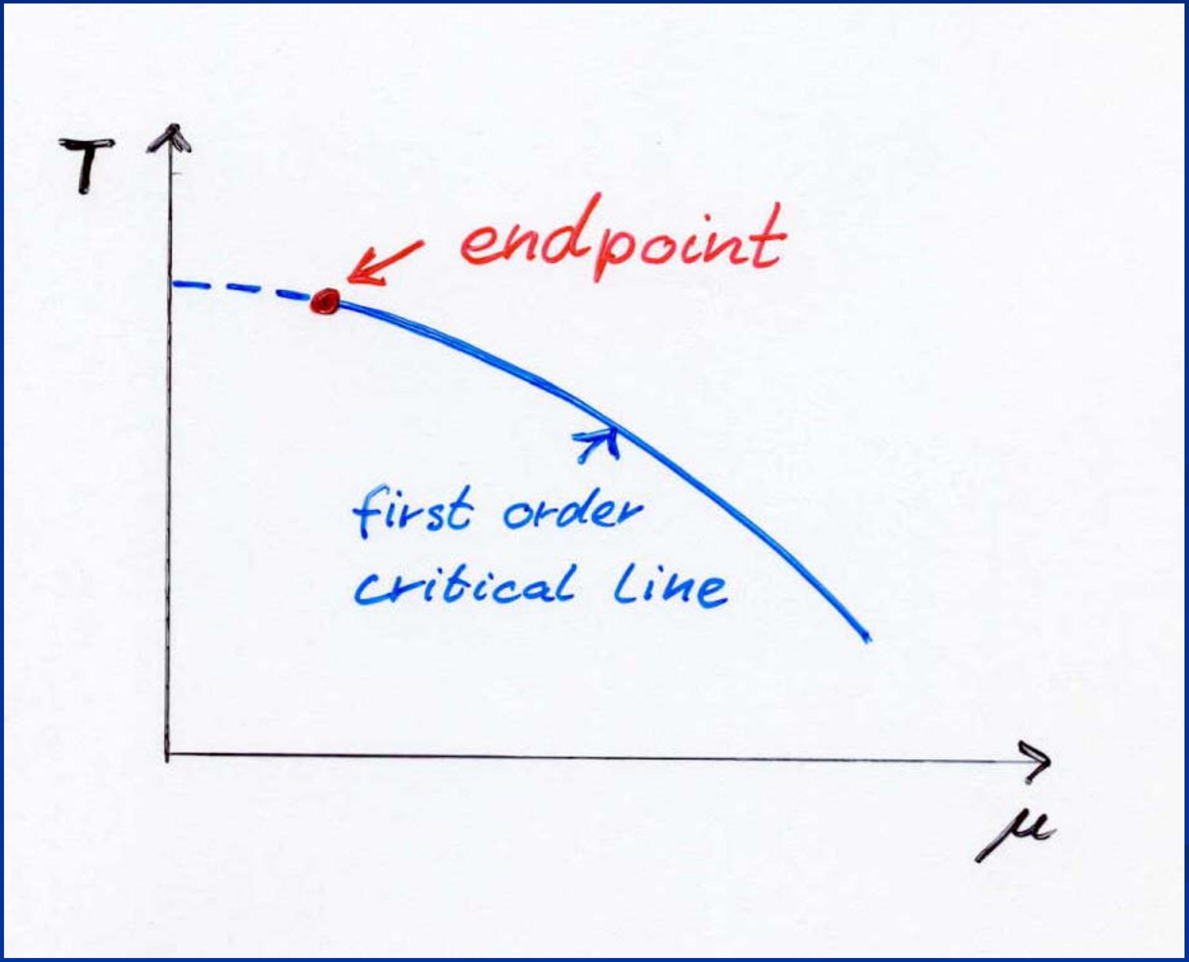
neutron-neutron condensate

“minimal” phase diagram

for nonzero quark masses



speculation : endpoint of critical line ?



How to find out ?

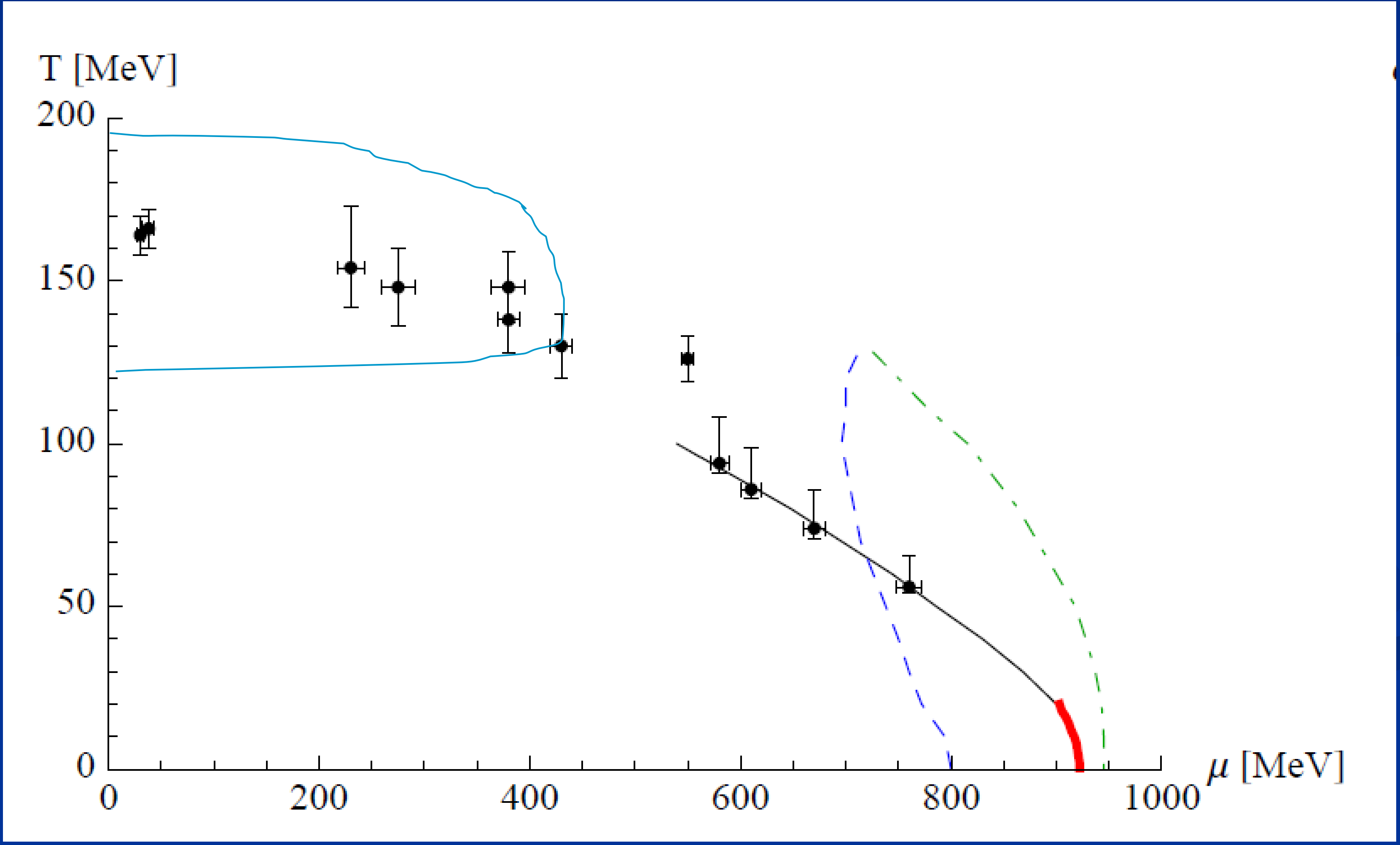
Methods

- Lattice : One has to wait until chiral limit is properly implemented ! Non-zero chemical potential poses problems.
- Functional renormalization : Not yet available for QCD with quarks and non-zero chemical potential. Nucleons ?
- Models : Simple quark meson models cannot work. Polyakov loops ? For low T : nucleons needed. Higgs picture of QCD ?

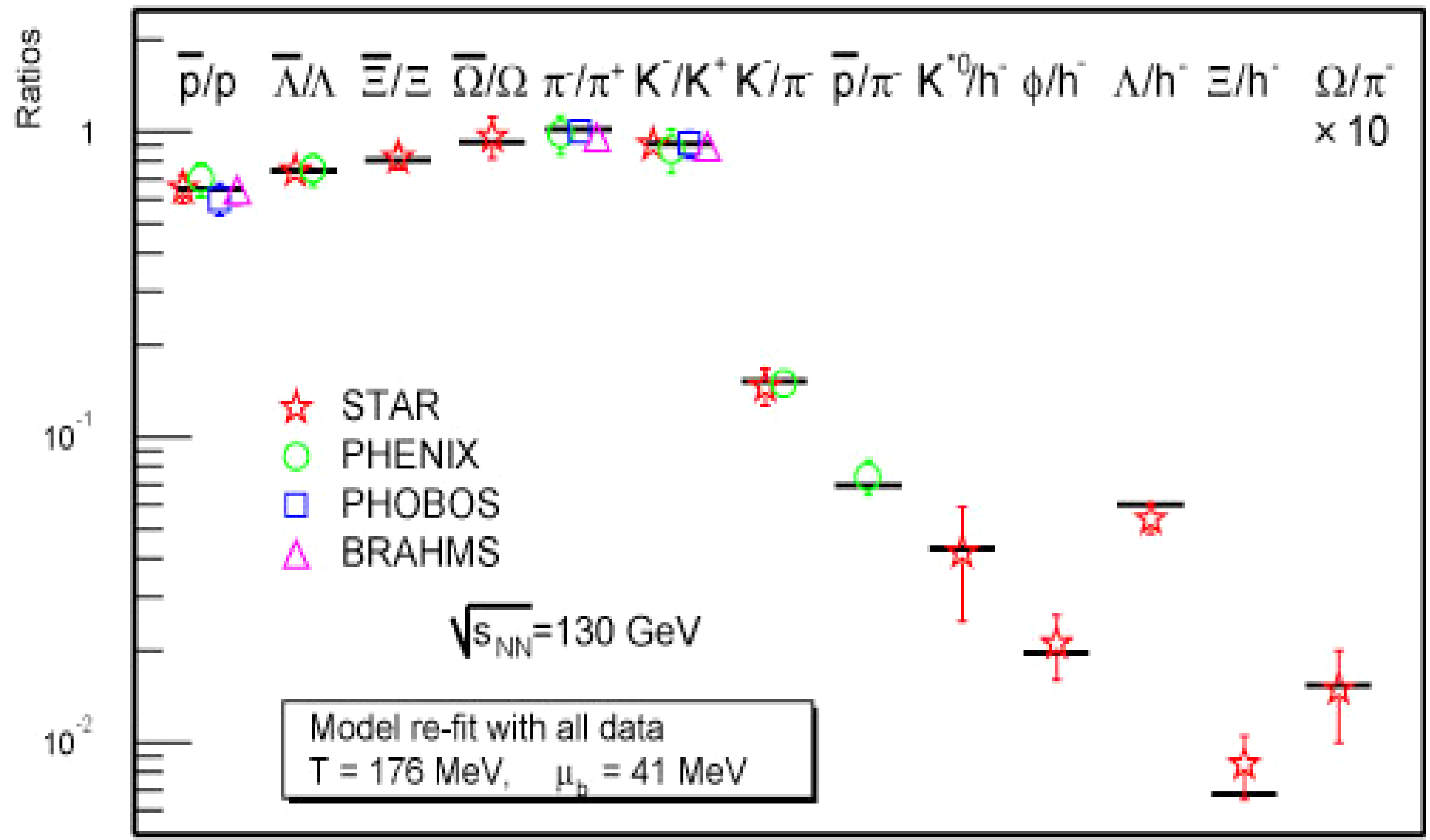
.

- Experiment : Has T_c been measured ?

Chemical freeze-out and phase diagram

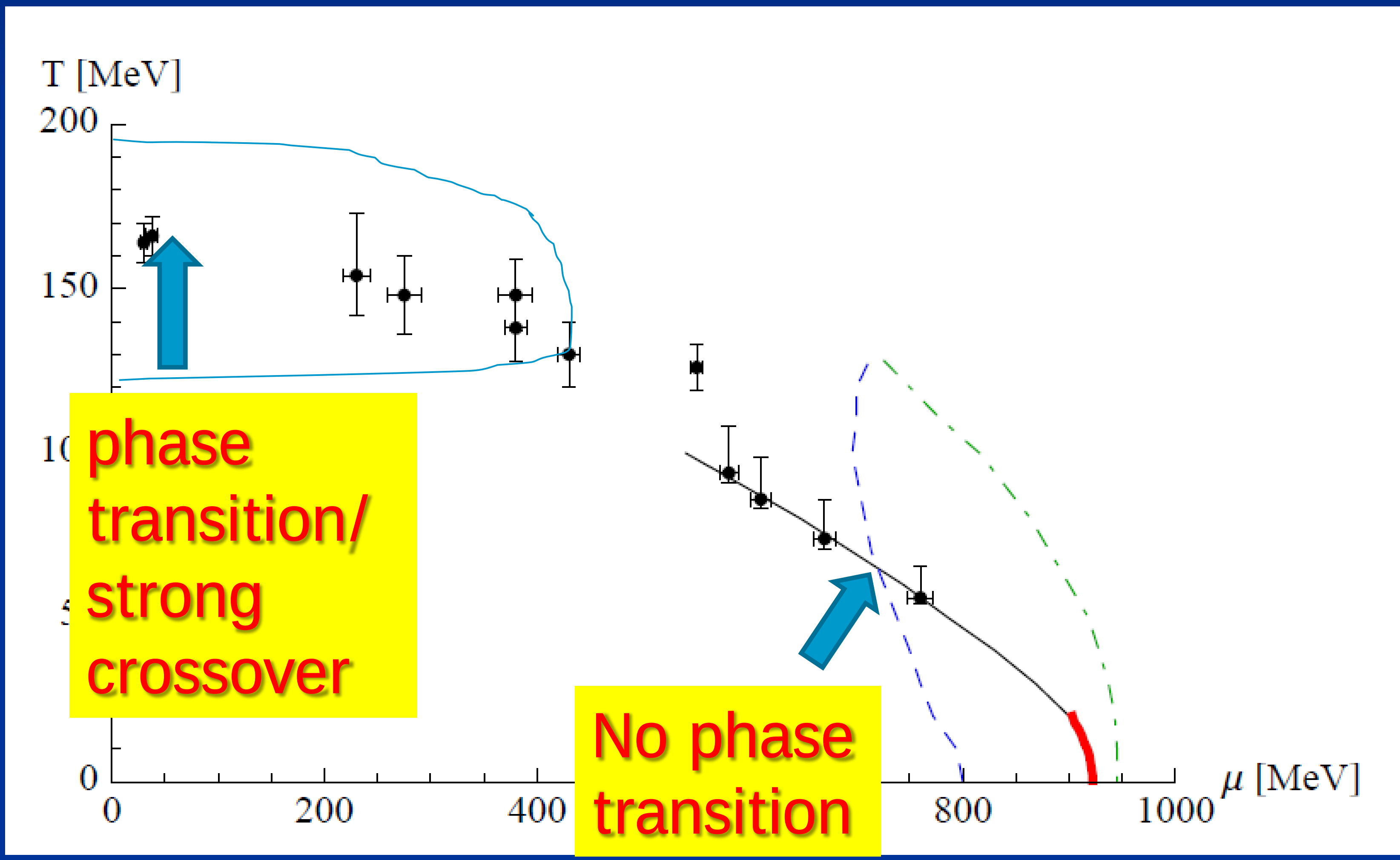


Hadron abundancies



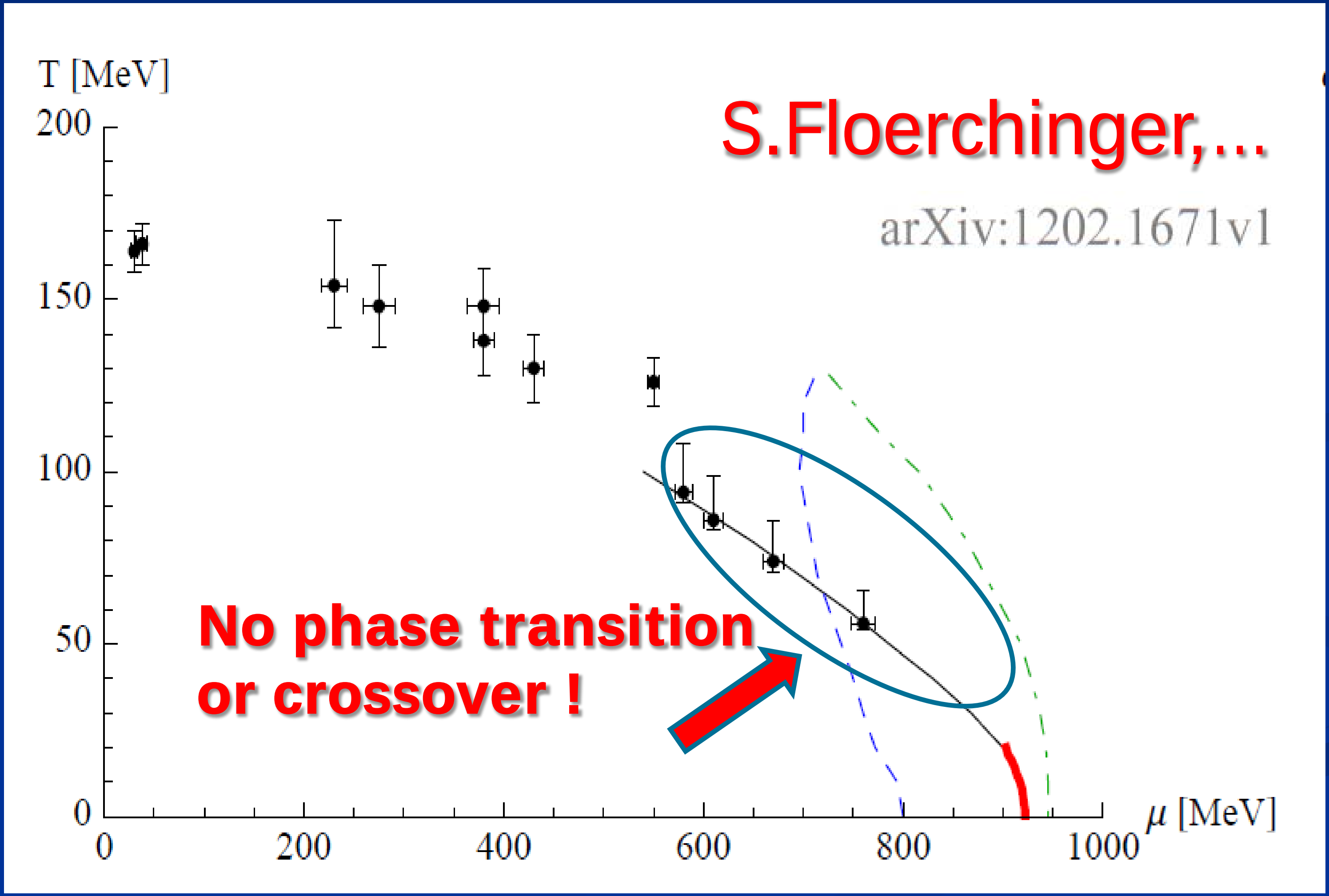
Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Chemical freeze-out

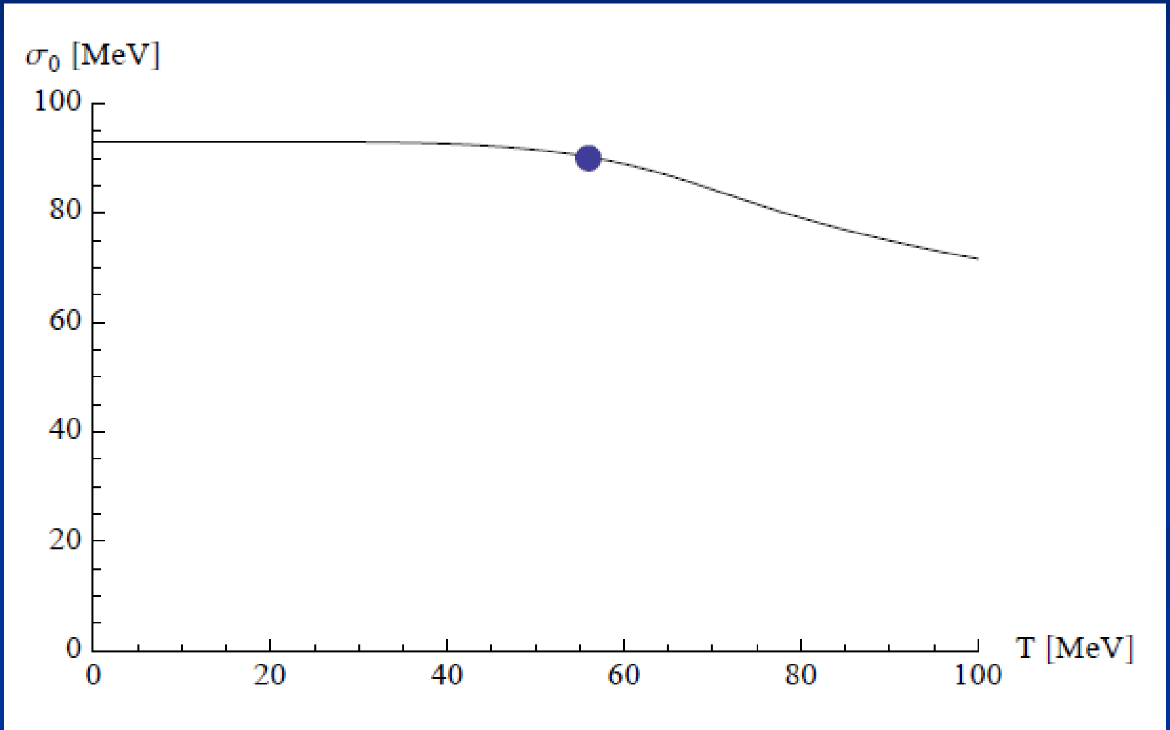


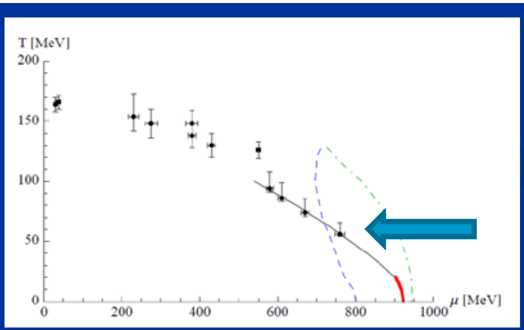
Lessons from the hadron world

Chemical freeze-out at high baryon density

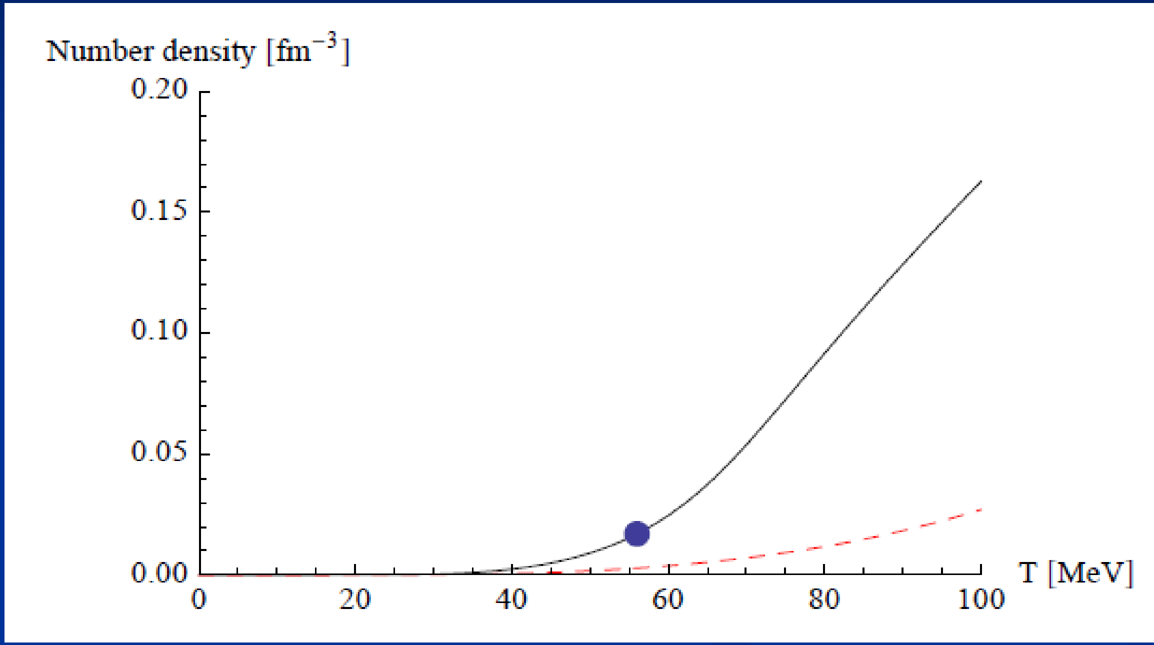


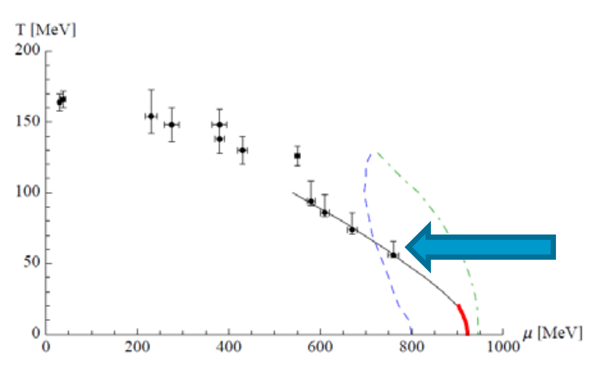
Chiral order parameter





Number density





Linear nucleon – meson model

- Protons, neutrons
- Pions , sigma-meson
- Omega-meson (effective chemical potential, repulsive interaction)
- Chiral symmetry fully realized
- Simple description of order parameter and chiral phase transition
- Chiral perturbation theory recovered by integrating out sigma-meson

Linear nucleon – meson model

$$\mathcal{L} = \bar{\psi}_a \, i\gamma^\nu (\partial_\nu - i\, g\, \omega_\nu - i\, \mu\, \delta_{0\nu})\, \psi_a$$
$$+ \sqrt{2}\, h \left[\bar{\psi}_a \left(\tfrac{1+\gamma_5}{2}\right) \phi_{ab} \psi_b + \bar{\psi}_a \left(\tfrac{1-\gamma_5}{2}\right) (\phi^\dagger)_{ab} \psi_b\right]$$
$$+ \tfrac{1}{2} \phi^*_{ab} (-\partial_\mu \partial^\mu) \phi_{ab} + U_{\rm mic}(\rho, \sigma)$$
$$+ \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)(\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \frac{1}{2} m_\omega^2\, \omega_\mu \omega^\mu .$$

$$\phi_{ab} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma + i\pi^0) & i\pi^- \\ i\pi^+ & \frac{1}{\sqrt{2}}(\sigma - i\pi^0) \end{pmatrix}$$

$$\rho = \frac{1}{2}\sigma^2,$$

$$U_{\rm mic}(\rho, \sigma) = \bar{U}(\rho) - m_\pi^2 f_\pi \sigma .$$

Effective potential and thermal fluctuations

$$U(\sigma, \omega_0) = U(\rho, \omega_0) - m_\pi^2 f_\pi \sigma \ , \ \rho = \frac{1}{2}\sigma^2,$$

$$\Delta \quad = U(\rho, \omega_0; T, \mu) - U(\rho, \omega_0; 0, \mu_c)$$

For high baryon density and low T :
dominated by nucleon fluctuations !

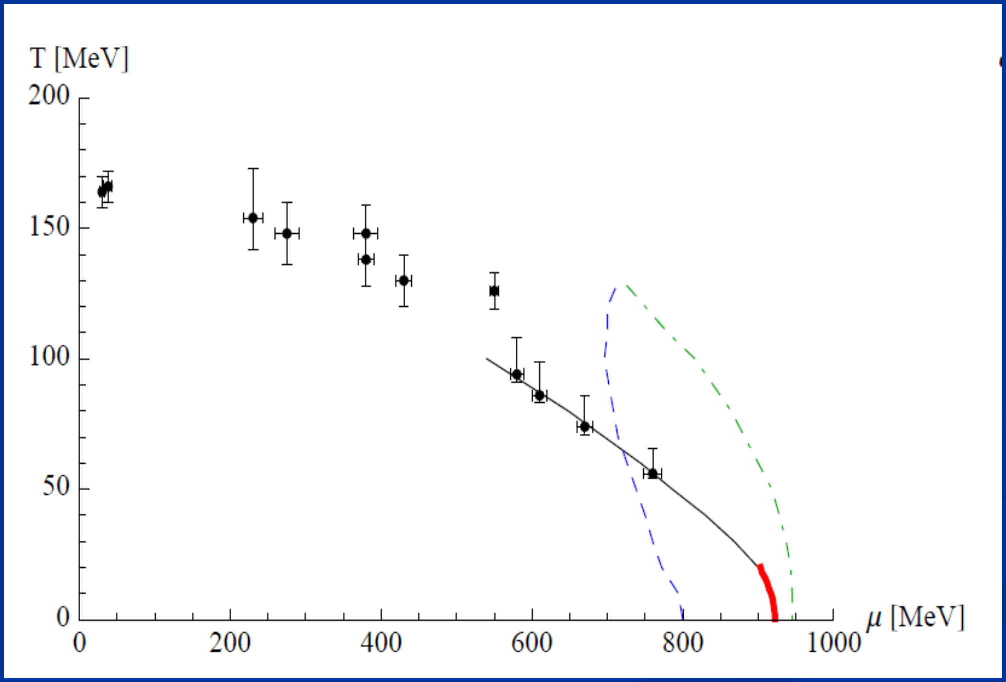
$$U(\sigma, \omega_0; T, \mu) = U_{\text{vac}}(\sigma, \omega_0) - 4\, P_{\text{FG}}(T, \mu + g\omega_0, h\sigma)$$

Pressure of gas of nucleons with field-dependent mass

$$U(\sigma,\omega_0;T,\mu) = U_{\rm vac}(\sigma,\omega_0) - 4\,P_{\rm FG}(T,\mu+g\omega_0,h\sigma)$$

$$P_{\rm FG}(T,\mu,m) = \frac{1}{3}\int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{\sqrt{\vec{p}^2+m^2}} \times \left[\frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}-\mu)}+1} + \frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}+\mu)}+1}\right]$$

Valid estimate for Δ in indicated region



Input : T=0 potential includes complicated physics of quantum fluctuations in QCD

$$
\begin{aligned}
U_{\mathrm{vac}}(\sigma,\omega_0) =& \frac{1}{2}m_\pi^2(2\rho - f_\pi^2) + \frac{1}{8}\lambda(2\rho - f_\pi^2)^2 \\
&+ \frac{1}{3}\frac{\gamma_3}{f_\pi^2}(2\rho - f_\pi^2)^3 + \frac{1}{4}\frac{\gamma_4}{f_\pi^4}(2\rho - f_\pi^2)^4 \\
&- m_\pi^2 f_\pi(\sigma - f_\pi) - \frac{1}{2}m_\omega^2\omega_0^2.
\end{aligned}
$$

parameters

$$f_\pi,\ m_\pi,\ \lambda,\ \gamma_3,\ \gamma_4,\ m_\omega,\ h \text{ and } g.$$

determined by phenomenology of nuclear matter. Droplet model reproduced. Density of nuclear matter, binding energy, surface tension, compressibility, order parameter in nuclear matter.

other parameterizations : similar results

Effective potential (T=0)

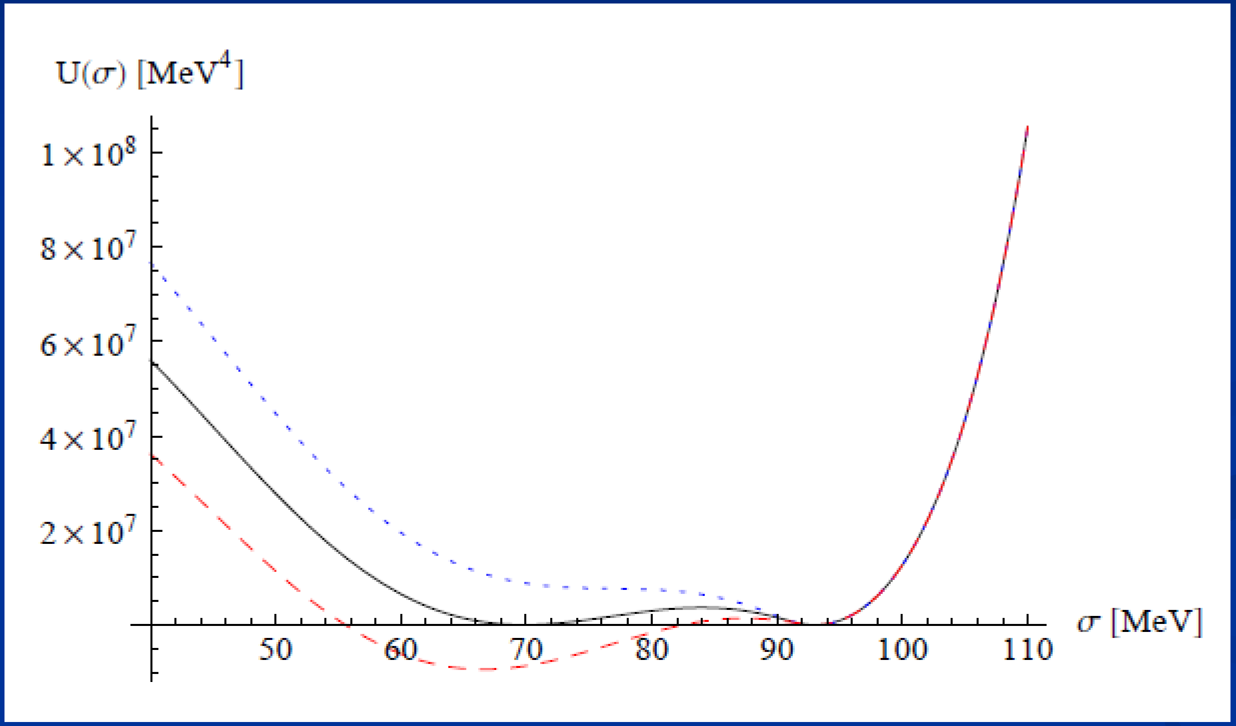


FIG. 4: Effective potential $U(\sigma)$ as a function of the chiral order parameter for $T = 0$ and chemical potential $\mu = 915$ MeV (dotted line), $\mu = 922.7$ MeV (solid line) and $\mu = 930$ MeV (dashed line).

Effective potential for different T

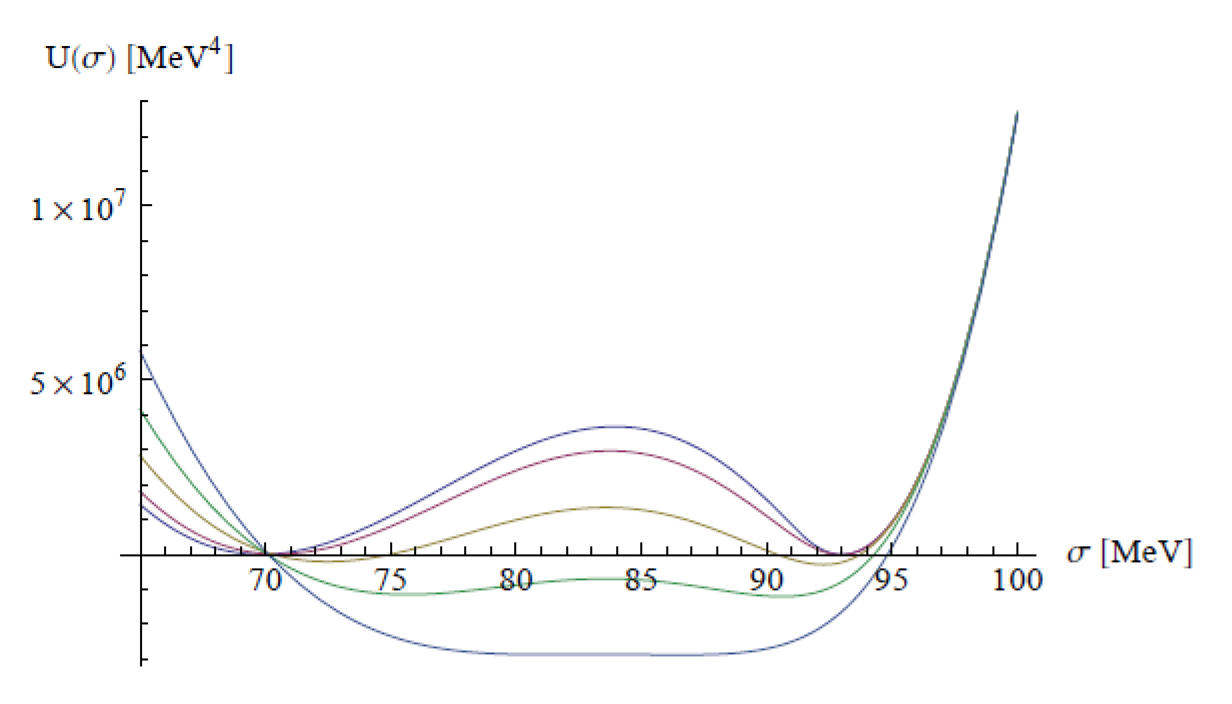


FIG. 5: Effective potential $U(\sigma)$ as a function of the chiral order parameter at the critical chemical potential of the first order phase transition $\mu = \mu_c(T)$ for temperatures $T = 0$, $T = 5\,\mathrm{MeV}$, $T = 10\,\mathrm{MeV}$, $T = 15\,\mathrm{MeV}$ and $T = 20$ MeV.

Chiral order parameter

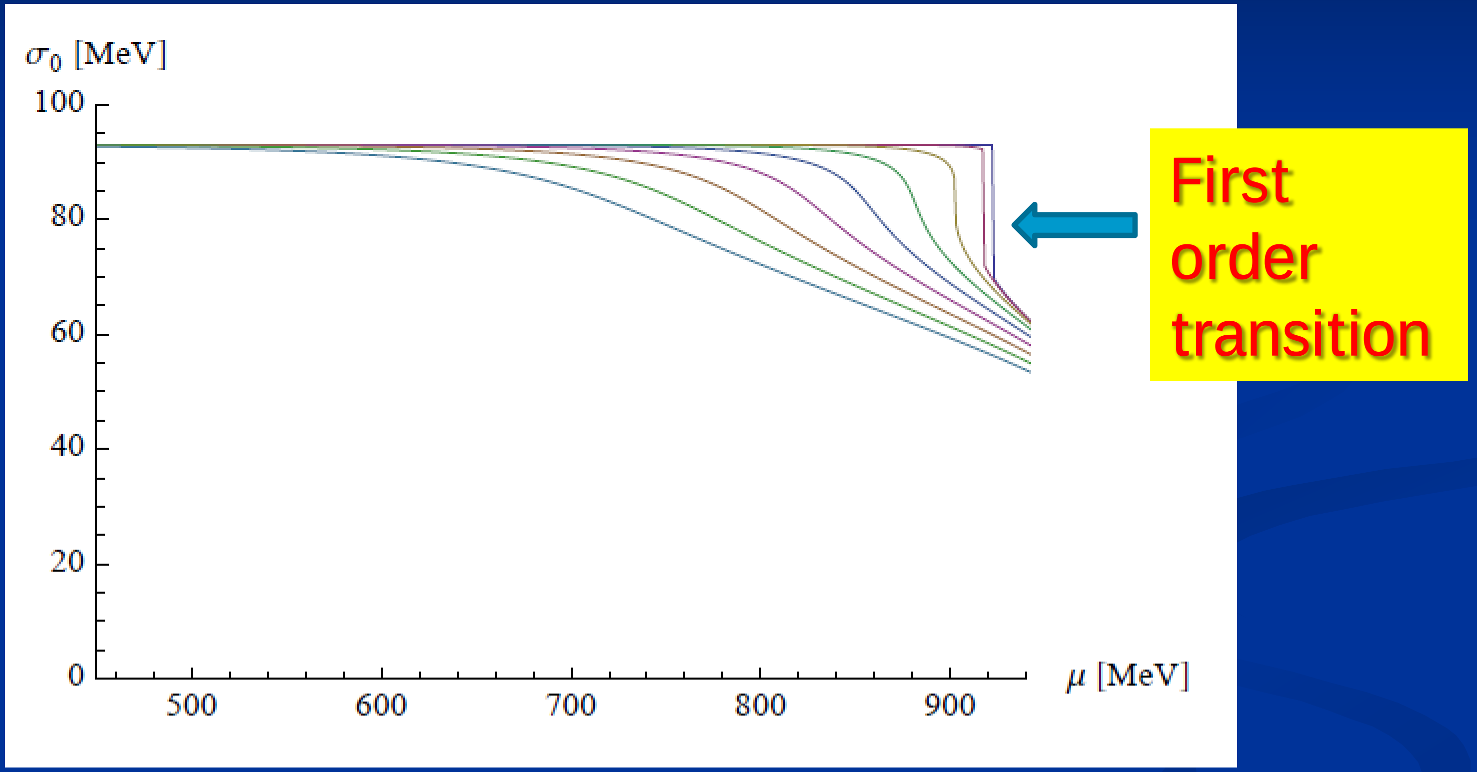


FIG. 6: Chiral order parameter σ_0 as a function of the chemical potential for $T = 0$ (uppermost curve), $T = 10$ MeV, $T = 20$ MeV, $T = 30$ MeV, $T = 40$ MeV, $T = 50$ MeV, $T = 60$ MeV, $T = 70$ MeV and $T = 80$ MeV (lowermost curve).

Endpoint of critical line of first order transition

$T = 20.7$ MeV
$\mu = 900$ MeV

Baryon density

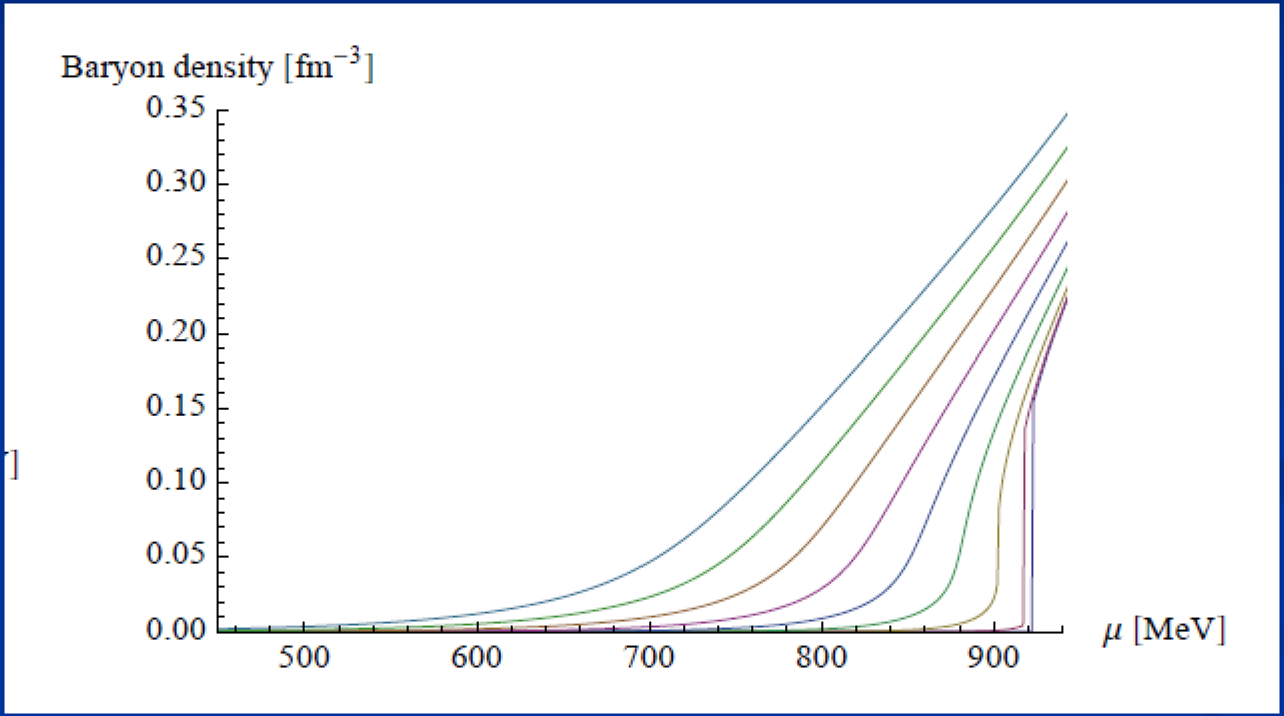


FIG. 7: Baryon number density as a function of the chemical potential for $T = 0$ (lowermost curve), $T = 10$ MeV, $T = 20$ MeV, $T = 30$ MeV, $T = 40$ MeV, $T = 50$ MeV, $T = 60$ MeV, $T = 70$ MeV and $T = 80$ MeV (uppermost curve).

Particle number density

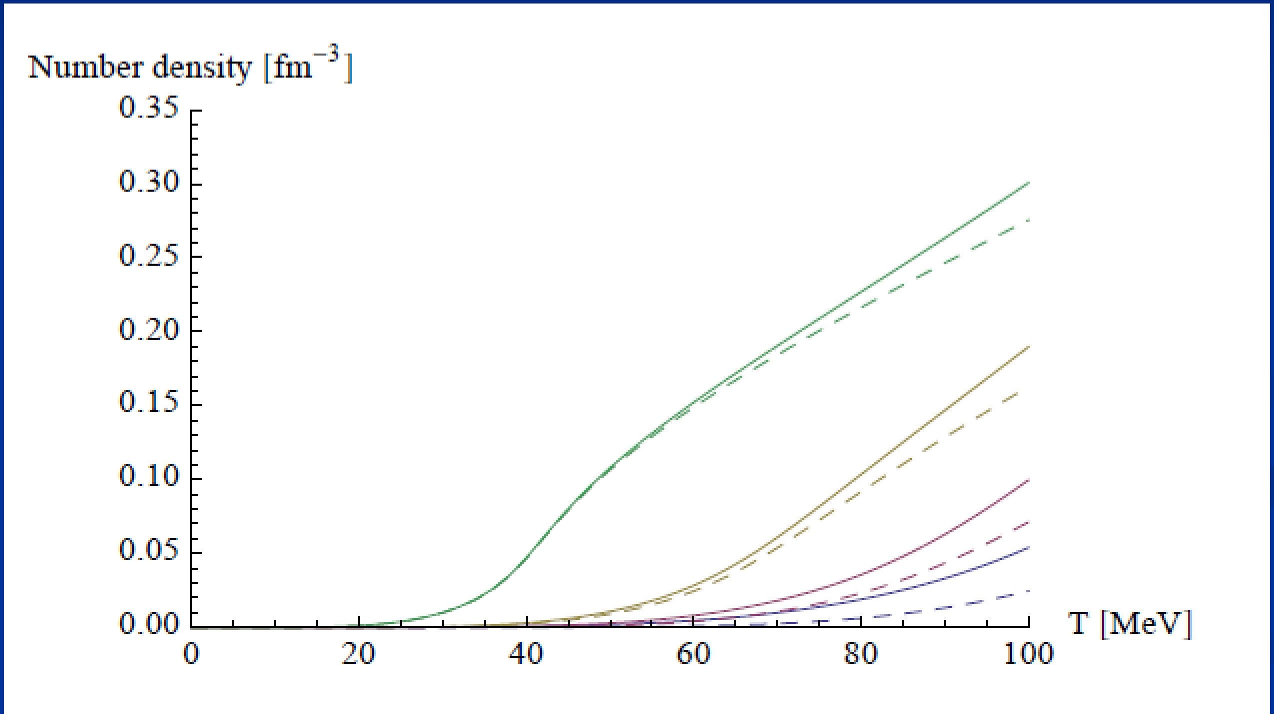


FIG. 8: Number density of baryons and pions (solid lines) as well as baryons only (dashed lines) as a function of temperature for the chemical potentials μ = 550 MeV (lowermost curves), μ = 650 MeV, μ = 750 MeV and μ = 850 MeV (uppermost curves).

Energy density

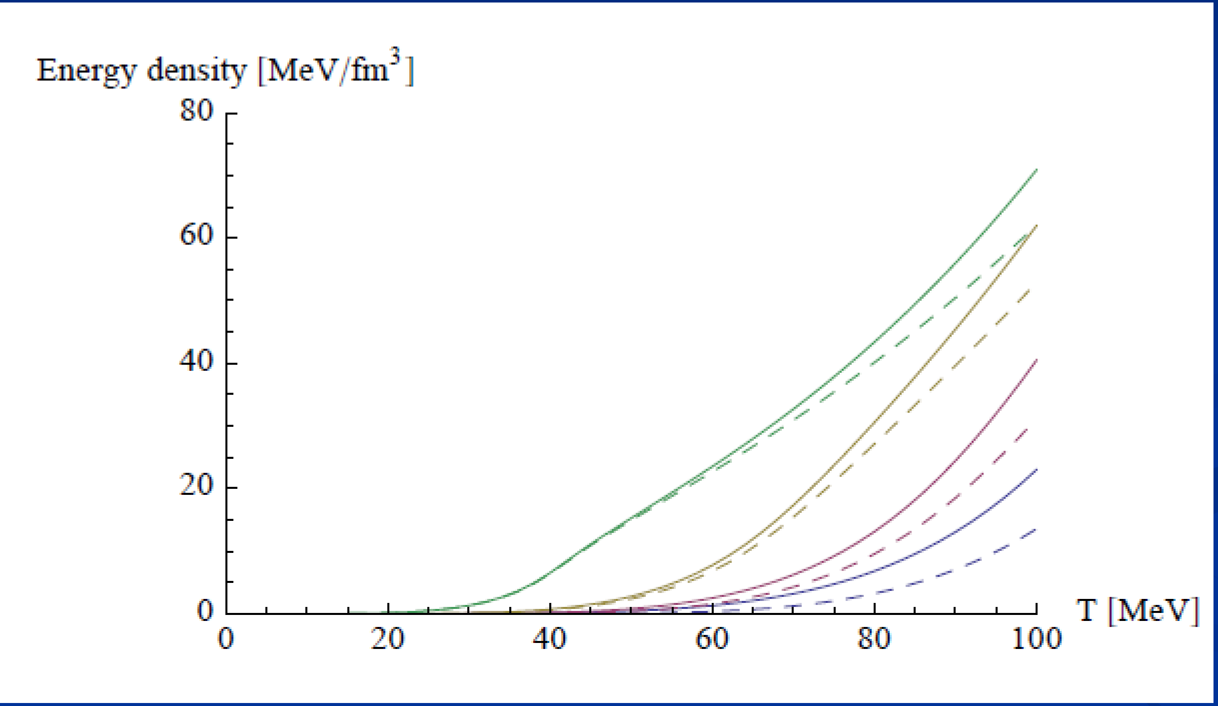
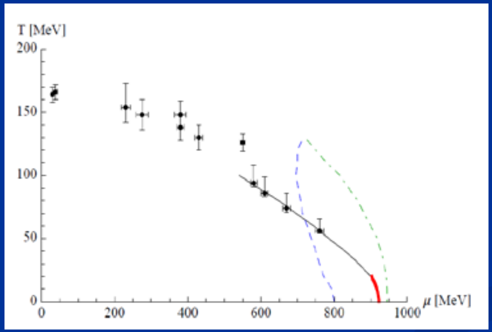


FIG. 9: Energy density of baryons and pions (solid lines) as well as baryons only (dashed lines) as a function of temperature for the chemical potentials μ = 550 MeV (lowermost curves), μ = 650 MeV, μ = 750 MeV and μ = 850 MeV (uppermost curves).

Conclusion (2)

- Thermodynamics reliably understood in indicated region of phase diagram

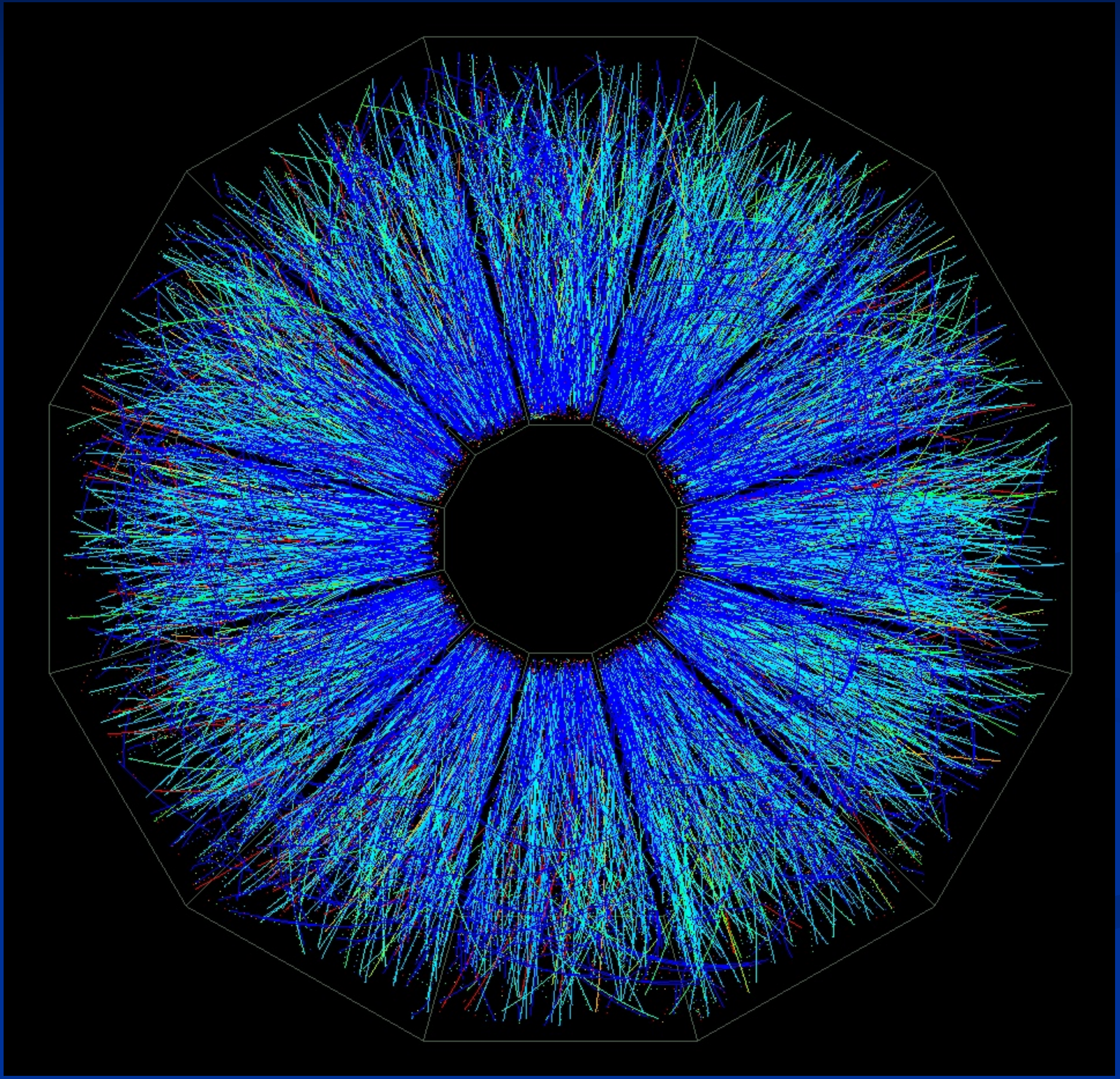


- No sign of phase transition or crossover at experimental chemical freeze-out points
- Freeze-out at line of constant number density

$$n = 0.15\ n_{\text{nuclear}}$$

Has the critical temperature of the QCD phase transition been measured ?

Heavy ion collision



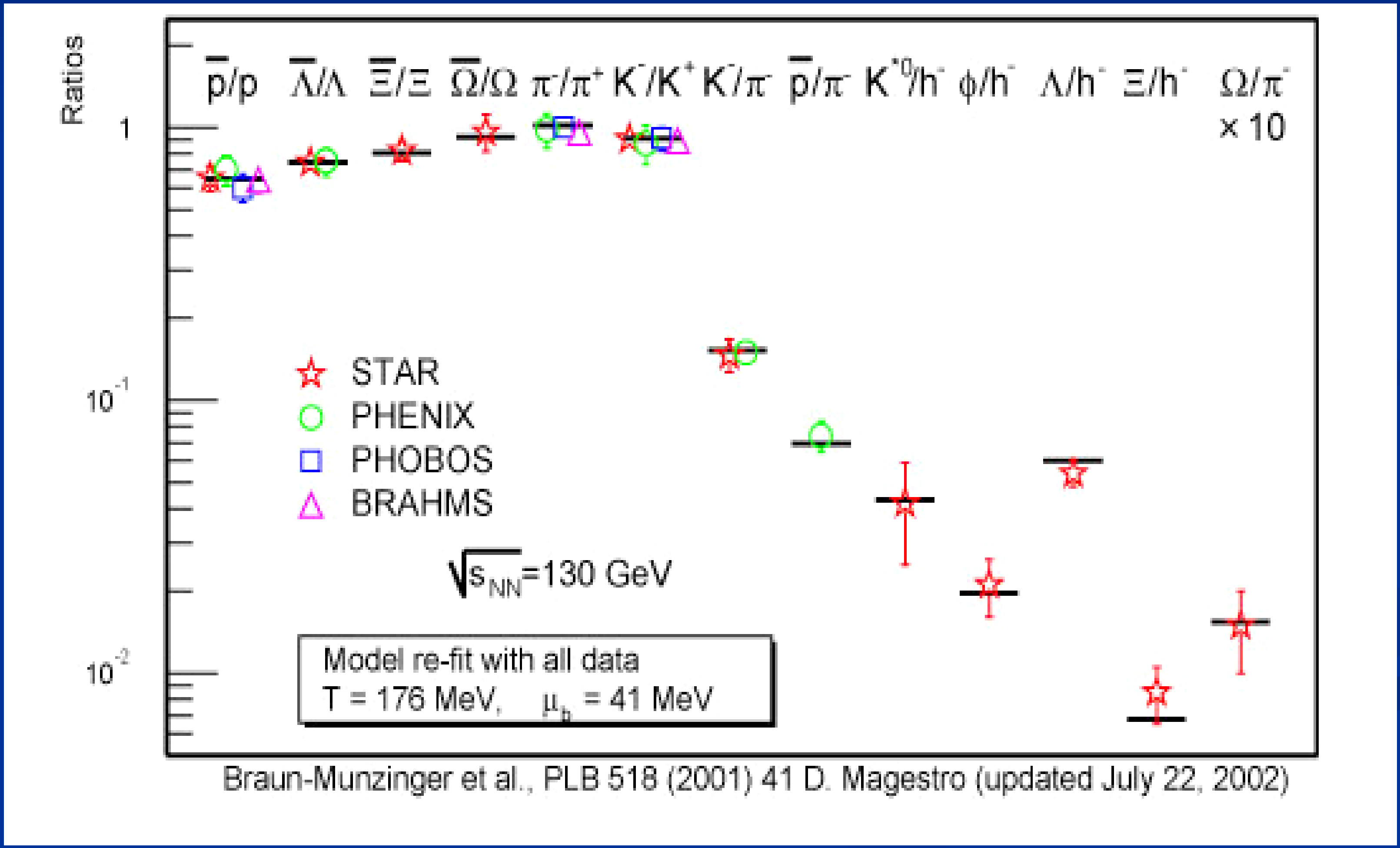
Yes !

$$0.95\ T_c < T_{ch} < T_c$$

- not : “ I have a model where $T_c \approx T_{ch}$ “
- not : “ I use T_c as a free parameter and find that in a model simulation it is close to the lattice value (or T_{ch}) “

$$T_{ch} \approx 176\ \text{MeV}$$

Hadron abundancies



Has T_c been measured ?

- Observation : statistical distribution of hadron species with "chemical freeze out temperature " T_{ch}=176 MeV
- T_{ch} cannot be much smaller than T_c : hadronic rates for $T< T_c$ are too small to produce multistrange hadrons (Ω,..)
- Only near T_c multiparticle scattering becomes important (collective excitations …) – proportional to high power of density

⇒ $T_{ch} \approx T_c$

P.Braun-Munzinger,J.Stachel,CW

Exclusion argument

Assume temperature is a meaningful concept - complex issue, to be discussed later

$T_{ch} < T_c$: $\Longleftrightarrow$ hadrochemical equilibrium

Exclude hadrochemical equilibrium at temperature much smaller than T_c : say for temperatures $< 0.95\ T_c$

$$0.95 < T_{ch}/T_c < 1$$

Estimate of critical temperature

For $T_{ch} \approx 176$ MeV :

$0.95 < T_{ch} / T_c$

- 176 MeV $< T_c <$ 185 MeV

$0.75 < T_{ch} / T_c$

- 176 MeV $< T_c <$ 235 MeV

Quantitative issue matters!

needed :

lower bound on T_{ch} / T_c

Key argument

- Two particle scattering rates not sufficient to produce Ω
- "multiparticle scattering for Ω-production " : dominant only in immediate vicinity of T_c

Mechanisms for production of multistrange hadrons

Many proposals

- Hadronization
- Quark-hadron equilibrium
- Decay of collective excitation (σ – field)
- Multi-hadron-scattering

Different pictures !

Hadronic picture of Ω - production

Should exist, at least semi-quantitatively, if $T_{ch} < T_c$

(for $T_{ch} = T_c$: $T_{ch} > 0.95\ T_c$ is fulfilled anyhow)

e.g. collective excitations $\approx$ multi-hadron-scattering

(not necessarily the best and simplest picture)

multihadron -> Ω + X should have sufficient rate

Check of consistency for many models

Necessary if $T_{ch} \neq T_c$ and temperature is defined

Way to give quantitative bound on T_{ch} / T_c

Rates for multiparticle scattering

2 pions + 3 kaons -> Ω + antiproton

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi.$$

Very rapid density increase

…in vicinity of critical temperature

Extremely rapid increase of rate of multiparticle scattering processes

(proportional to very high power of density)

Energy density

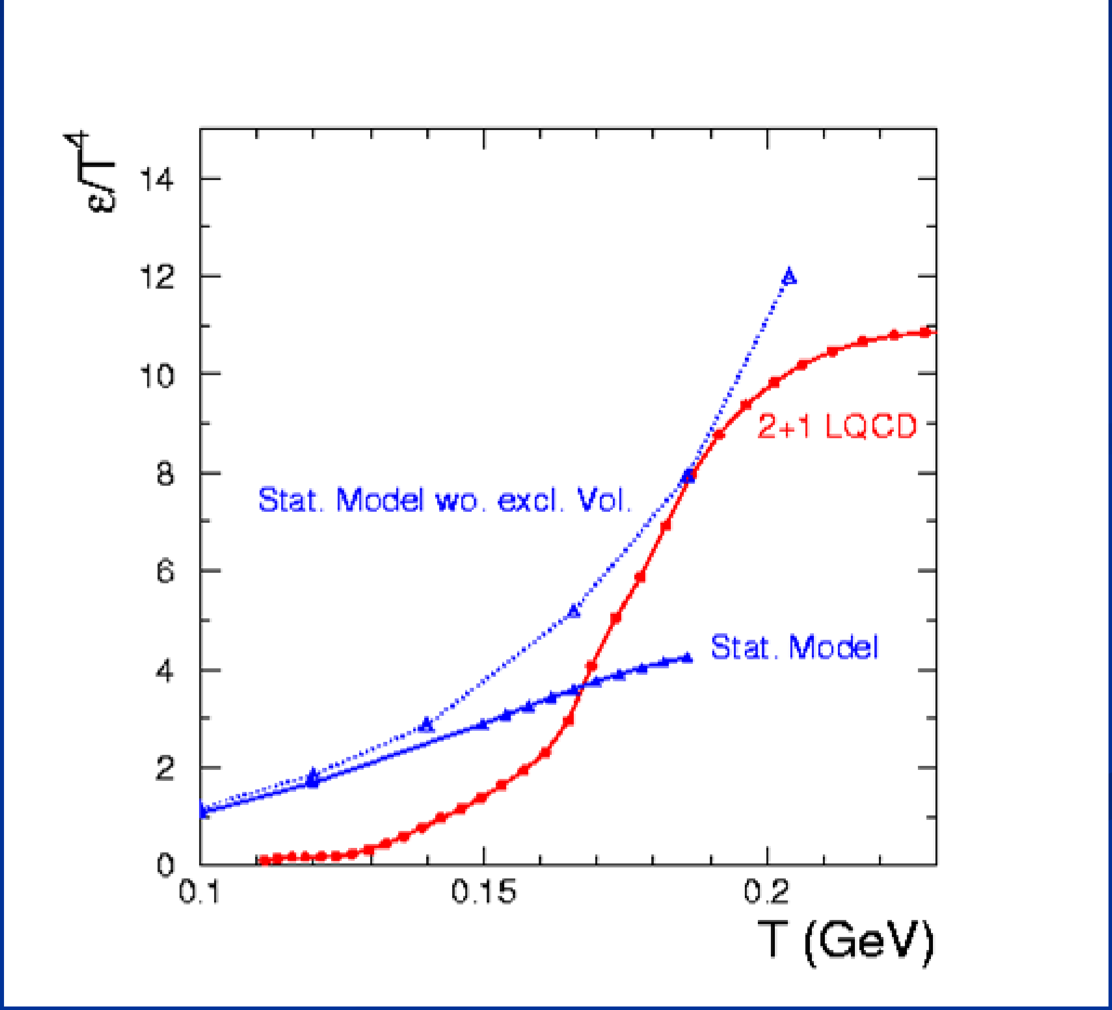
Lattice simulations

Karsch et al

(even more dramatic

for first order

transition)



Phase space

- increases very rapidly with energy and therefore with temperature
- effective dependence of time needed to produce Ω

$$\tau_{\Omega} \sim T^{-60} \ !$$

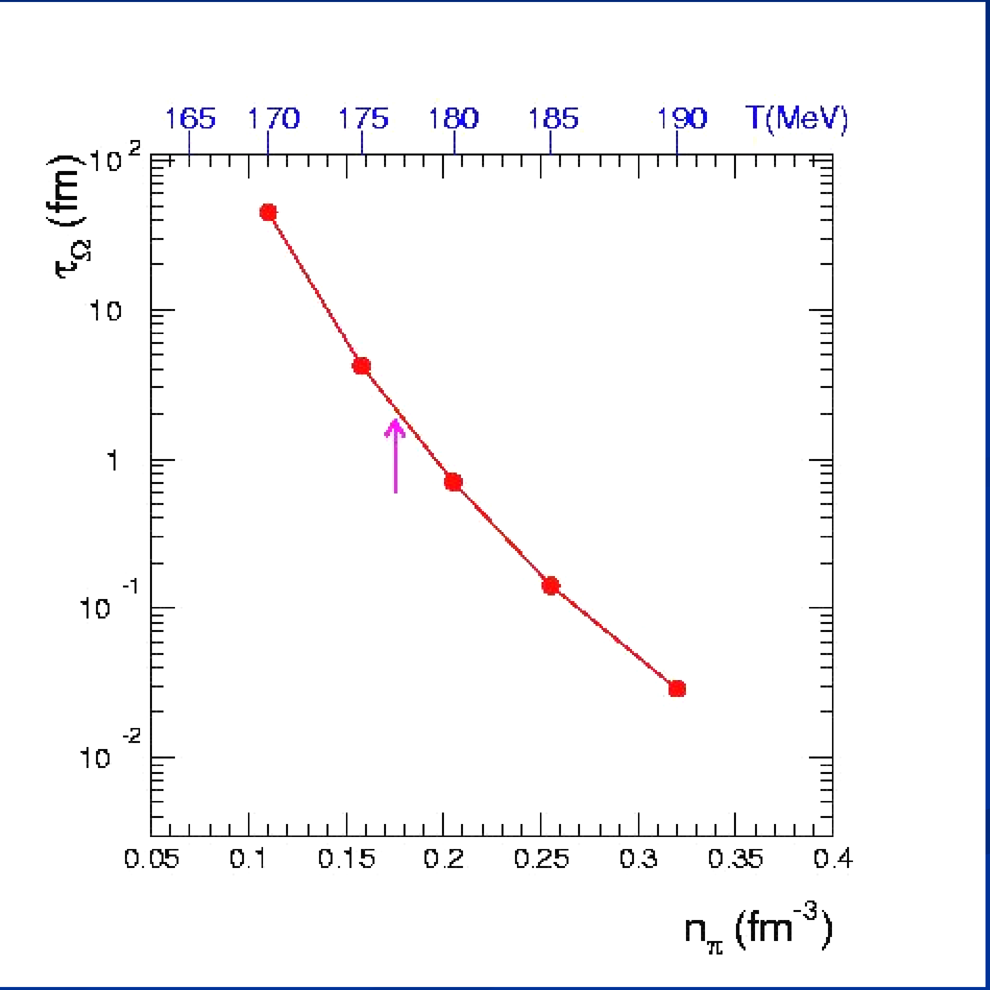
This will even be more dramatic if transition is closer to first order phase transition

Production time for Ω

multi-meson scattering

$\pi+\pi+\pi+K+K \rightarrow \Omega+p$

strong dependence on pion density



P.Braun-Munzinger,J.Stachel,CW

enough time for Ω - production

at T=176 MeV :

$$\tau_\Omega \sim 2.3 \text{ fm}$$

consistency !

extremely rapid change

lowering T by 5 MeV below critical temperature :

rate of Ω – production decreases by
factor 10

This restricts chemical freeze out to close vicinity of
critical temperature

$$0.95 < T_{ch} / T_c < 1$$

Relevant time scale in hadronic phase

rates needed for equilibration of Ω and kaons:

$$\bar{r}_j = \frac{\dot{N}_j}{V} = \dot{n}_j + n_j \dot{V}/V.$$

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = \frac{\ln F_{\Omega K}}{\tau_T}\frac{\mathrm{T_{ch}}}{\Delta \mathrm{T}} = (1.10\text{–}0.55)/\mathrm{fm}$$

$\Delta T = 5$ MeV,
$F_{\Omega K} = 1.13$,
$\tau_T = 8$ fm

two –particle – scattering :

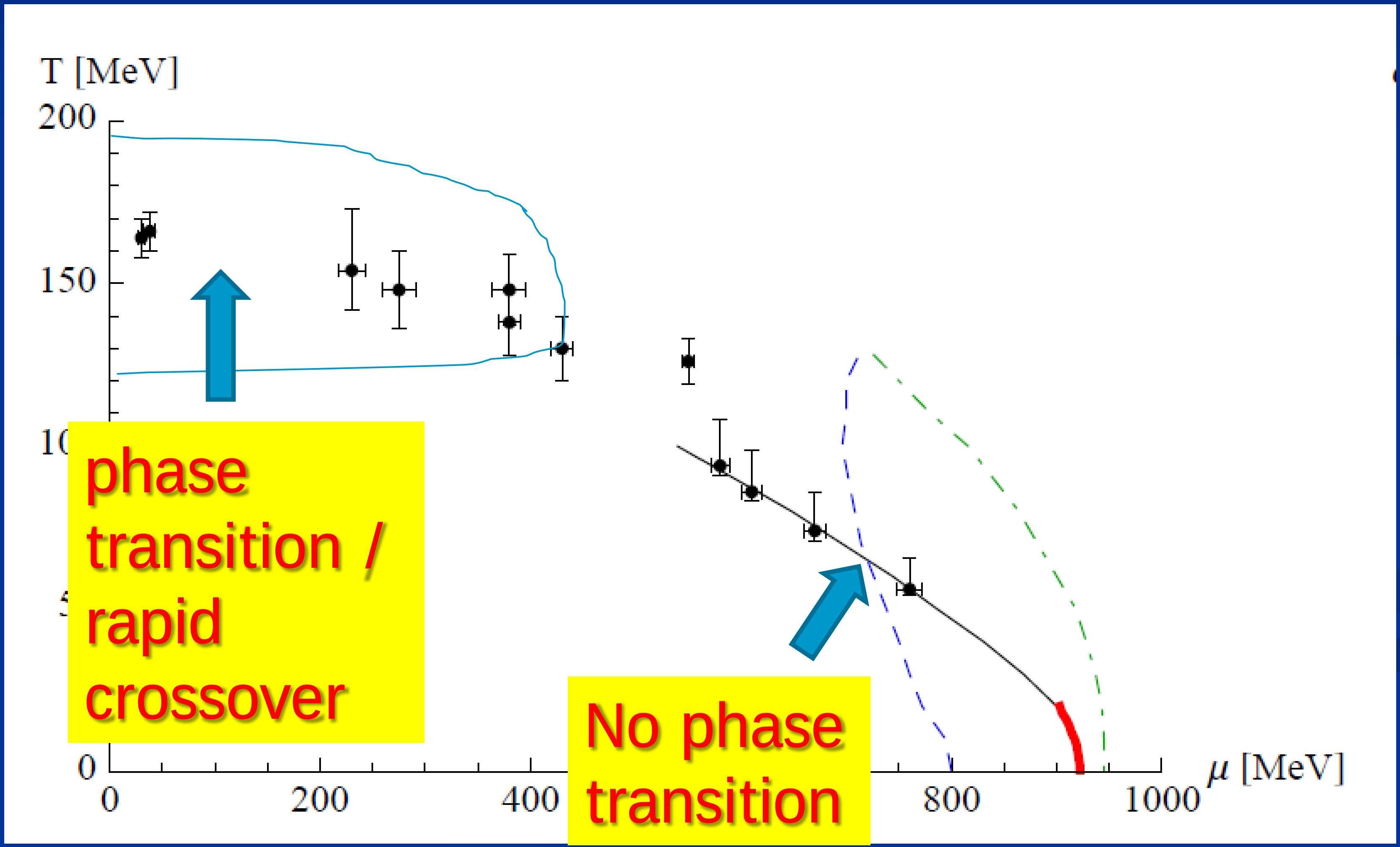
$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = (0.02\text{-}0.2)/\mathrm{fm}$$

$$T_{ch} \approx T_c$$

Conclusion (2)

- experimental determination of critical temperature may be more precise than lattice results
- error estimate becomes crucial

Chemical freeze-out



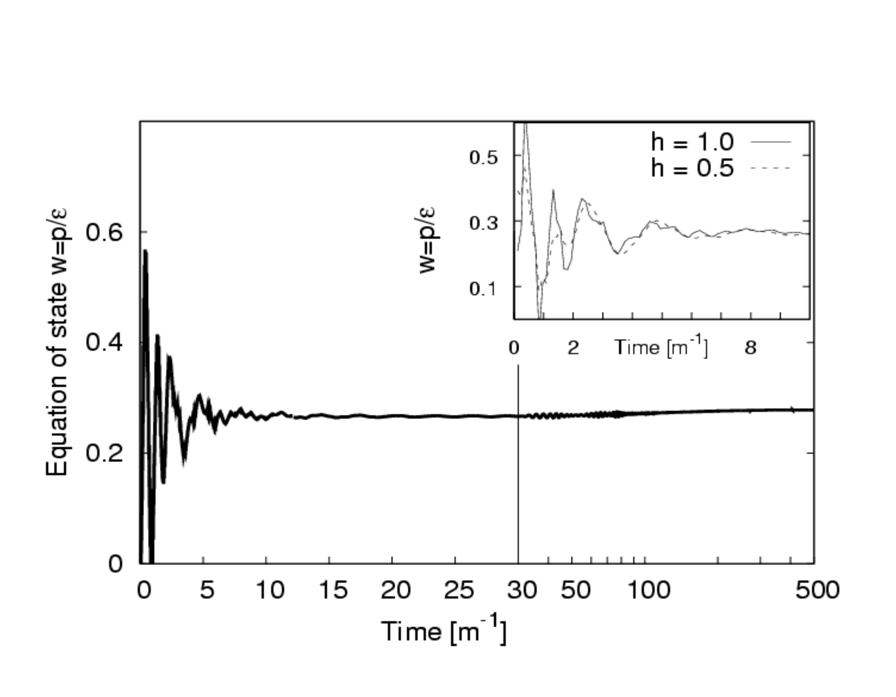
end

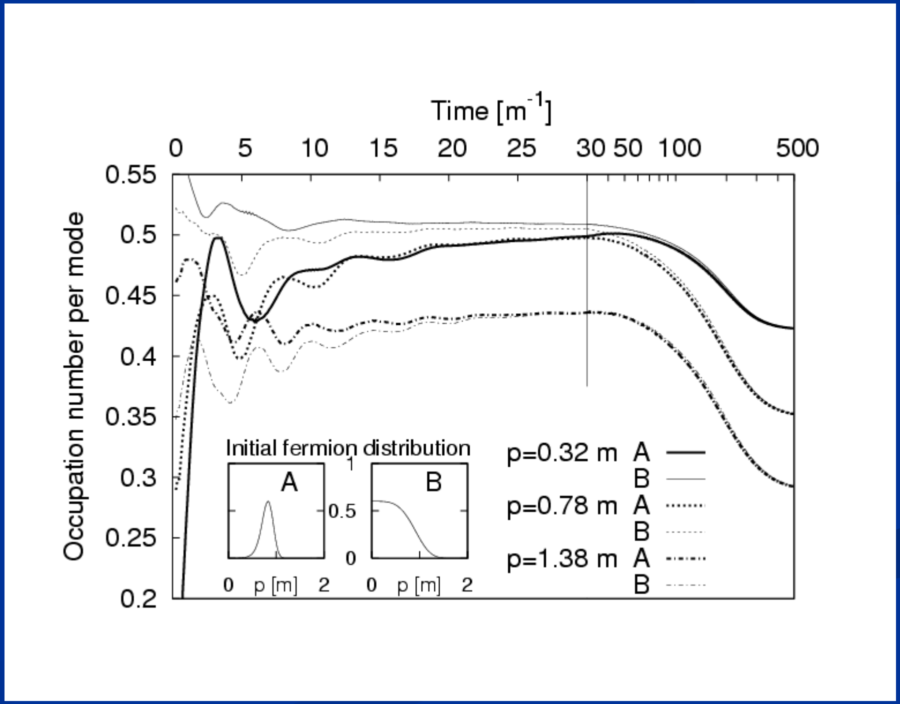
Is temperature defined ?

Does comparison with equilibrium critical temperature make sense ?

Prethermalization

J.Berges,Sz.Borsanyi,CW





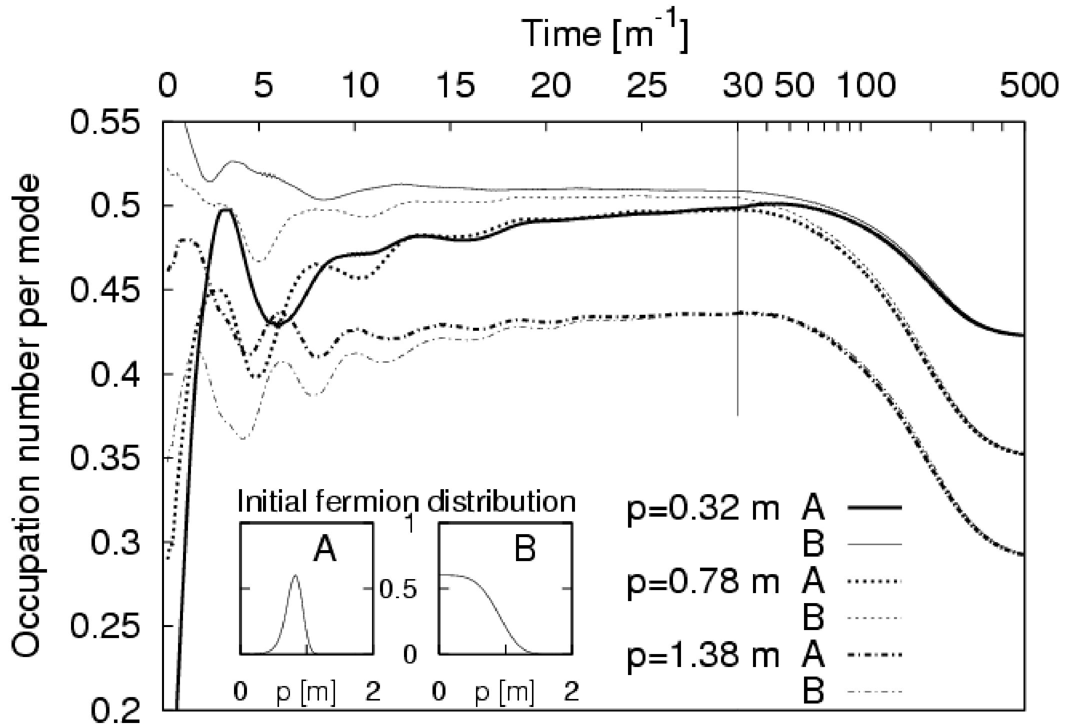
Vastly different time scales

for "thermalization" of different quantities

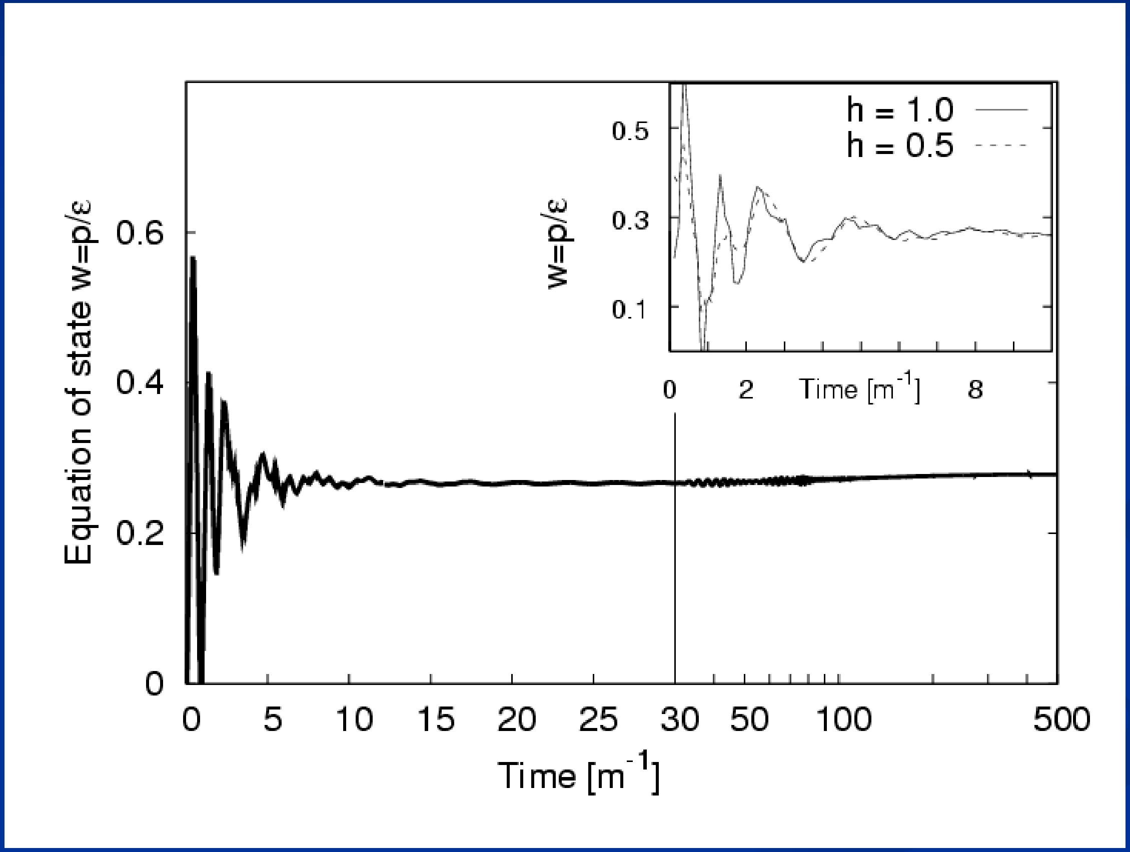
here : scalar with mass m coupled to fermions
(linear quark-meson-model)

method : two particle irreducible non- equilibrium effective action (J.Berges et al)

Thermal equilibration : occupation numbers



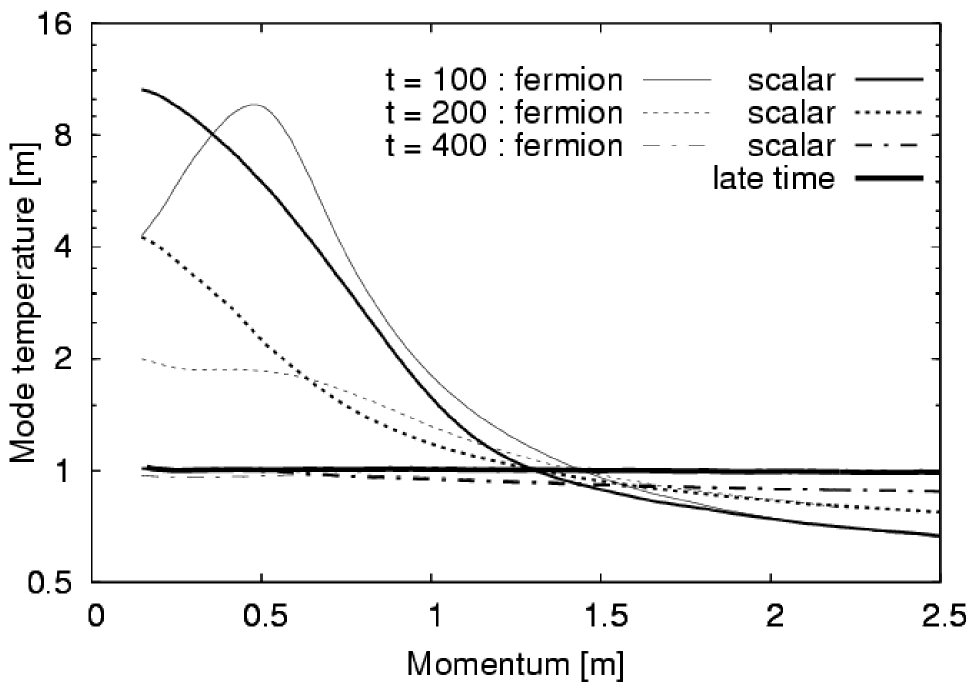
Prethermalization equation of state p/ε



similar for kinetic temperature

different “temperatures”

Mode temperature



$$n_p(t) \stackrel{!}{=} \frac{1}{\exp\left[\omega_p(t)/T_p(t)\right] \pm 1}$$

$\omega_p^{(f,s)}(t)$ determined by peak of spectral function

n_p :occupation number for momentum p

late time:
Bose-Einstein or Fermi-Dirac distribution

Global kinetic temperature $T_{\rm kin}$

Practical definition:

- association of temperature with average kinetic energy per d.o.f.

$$T_{\rm kin}(t) = E_{\rm kin}(t)/c_{\rm eq}$$

- $c_{\rm eq} = E_{\rm kin,eq}/T_{\rm eq}$ is given solely in terms of equilibrium quantities (E.g. relativistic plasma: $E_{\rm kin}/N = \epsilon/n = \alpha T$)

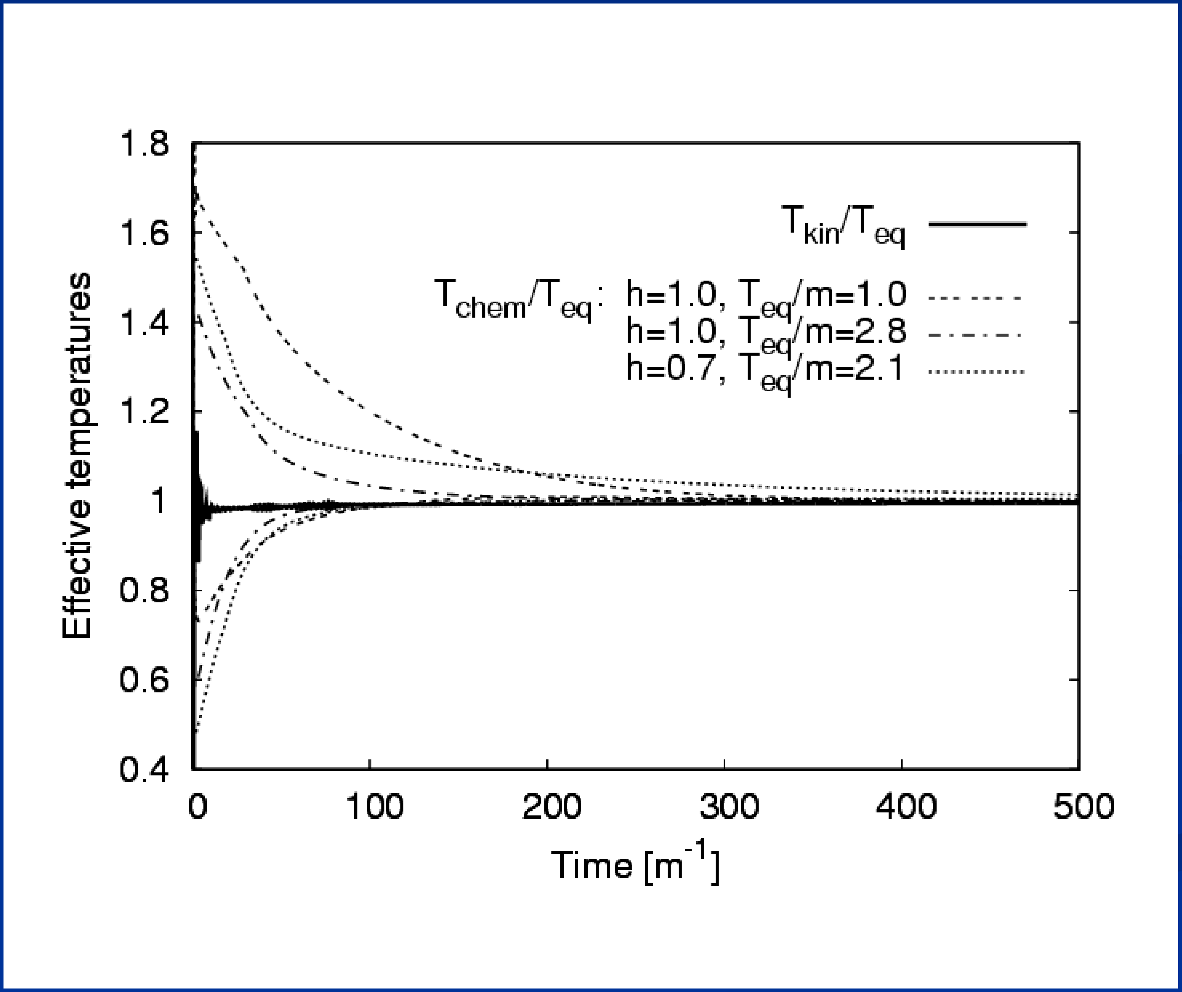
Kinetic equilibration: $T_{\rm kin}(t) \; = \; T_{\rm eq}$

Consider also *chemical temperatures* $T_{\rm ch}^{(f,s)}$ from integrated number density of each species, $n^{(f,s)}(t) = g^{(f,s)} \int {\rm d}^3p/(2\pi)^3 \, n_p^{(f,s)}(t)$:

$$n(t) \stackrel{!}{=} \frac{g}{2\pi^2} \int_0^\infty {\rm d}p p^2 \left[\exp\left(\omega_p(t)/T_{\rm ch}(t)\right) \pm 1\right]^{-1}$$

Chemical equilibration: $T_{\rm ch}^{(f)}(t) \; = \; T_{\rm ch}^{(s)}(t)$

Kinetic equilibration before chemical equilibration



Once a temperature becomes stationary it takes the value of the equilibrium temperature.

Once chemical equilibration has been reached the chemical temperature equals the kinetic temperature and can be associated with the overall equilibrium temperature.

Comparison of chemical freeze out temperature with critical temperature of phase transition makes sense

A possible source of error : temperature-dependent particle masses

Chiral order parameter σ depends on T

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

$$\frac{\sigma(\mathrm{T_{ch}},\mu)}{\mathrm{T_{ch}}} = \frac{\sigma(0,0)}{\mathrm{T_{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$

chemical freeze out measures T/m !

uncertainty in m(T)

→

uncertainty in critical temperature

Ratios of particle masses and chemical freeze out

at chemical freeze out :

- ratios of hadron masses seem to be close to vacuum values
- nucleon and meson masses have different characteristic dependence on σ
- $m_{nucleon} \sim \sigma$, $m_{\pi} \sim \sigma^{-1/2}$
- $\Delta\sigma/\sigma < 0.1$ (conservative)

systematic uncertainty :

$$\Delta\sigma/\sigma = \Delta T_c/T_c$$

Δσ is negative

$$M_j(\mathrm{T}) = h_j(\mathrm{T},\mu)\sigma(\mathrm{T},\mu)$$

$$\frac{\sigma(\mathrm{T_{ch}},\mu)}{\mathrm{T_{ch}}} = \frac{\sigma(0,0)}{\mathrm{T_{obs}}}.$$

$$\mathrm{T}_c = 176^{+5}_{-18}\ \mathrm{MeV}.$$