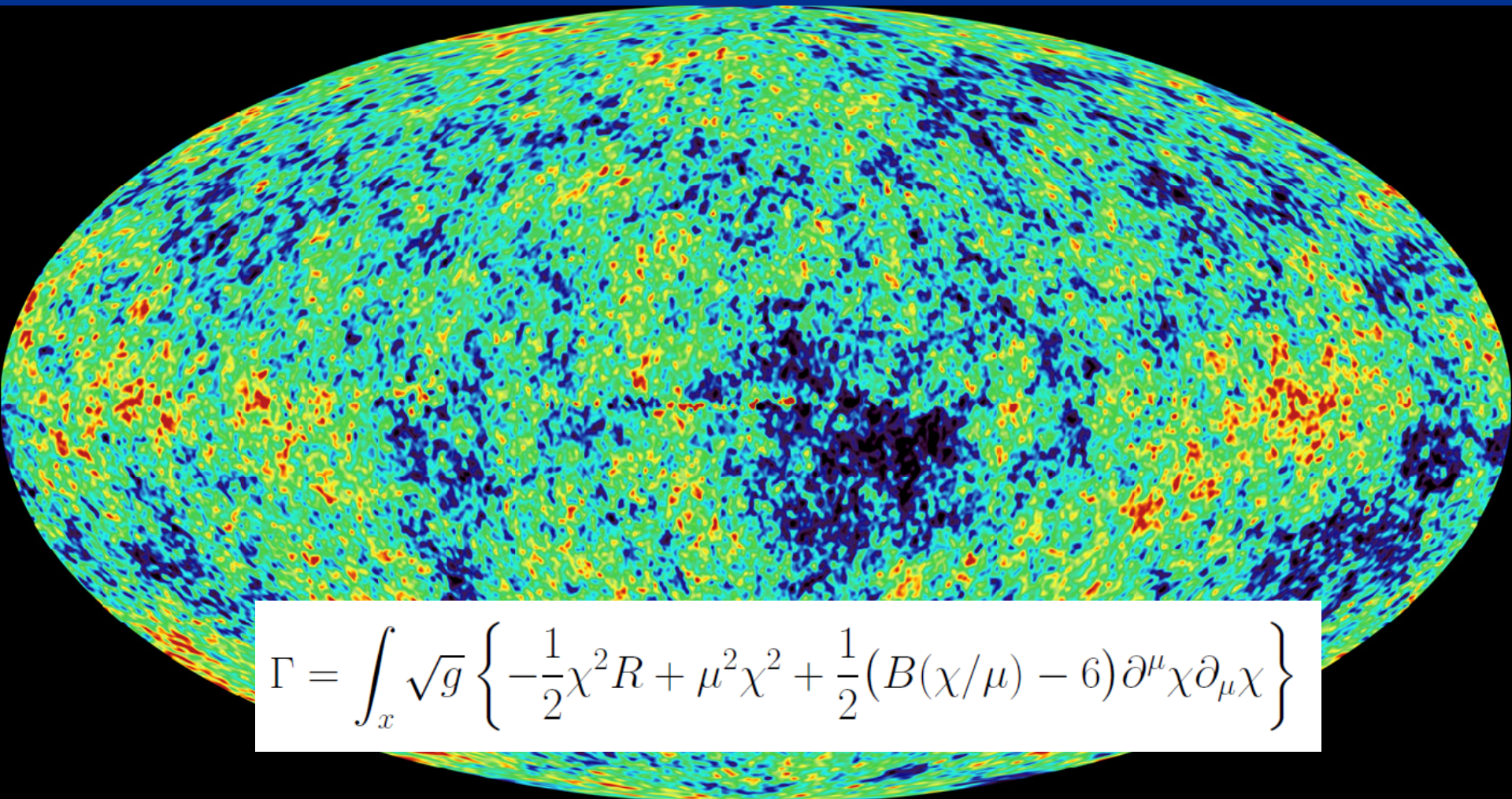


Graviton fluctuations erase the cosmological constant



$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

*In quantum gravity,
the graviton fluctuations can
play an important role on
distances as large as the
size of the Universe*

Graviton propagator

effective action

$$\Gamma = \int_x \sqrt{g} \left(-\frac{M^2}{2} R + V \right)$$

flat space:

$$G^{-1} = \frac{M^2 q^2}{4} - \frac{V}{2}$$

Instability for $V > 0$: "tachyonic mass term" $-\frac{2V}{M^2}$

curved space:

$$G^{-1} = \sqrt{g} \left\{ \frac{M^2}{4} \left(-D^2 + \frac{2R}{3} \right) - \frac{V}{2} \right\}$$

On shell graviton propagator

$$G^{-1} = \sqrt{g} \left\{ \frac{M^2}{4} \left(-D^2 + \frac{2R}{3} \right) - \frac{V}{2} \right\}$$

on shell :
(for solution of field equations)

$$R = \frac{4V}{M^2}$$

homogenous isotropic metric,
conformal time

$$g_{\mu\nu} = a^2(\eta) \delta_{\mu\nu}$$

$$\mathcal{H} = \frac{\partial \ln a}{\partial \eta}$$

inverse graviton
propagator in
de Sitter space

$$a^{-2} \left(-D^2 + \frac{R}{6} \right) a^2 = \frac{1}{a^2} (\partial_\eta^2 + 2\mathcal{H}\partial_\eta + \vec{q}^2)$$

milder instability, not tachyonic, absent for
cosmologies close to de Sitter space

IR – instability for graviton fluctuations

problem solved ?

- yes for primordial cosmic fluctuations (on shell)
- no for quantum gravity (off shell)
- Computation of effective action is an off-shell problem.
- example : one needs the effective potential for the Higgs field in the vicinity of its minimum (off shell), not only at the minimum (on shell)

Quantum gravity with scalar field

M^2 and V depend on scalar field χ

$$M^2 = c_1 + c_2 \chi^2$$

$$V = d_1 + d_2 \chi^2 + d_3 \chi^4$$

question : behavior of V for $\chi \rightarrow \infty$

- $d_3 \neq 0$ excluded!
- $d_3 < 0$ unstable potential
- $d_3 > 0$ instability of graviton propagator

Graviton barrier

Quantum gravity computation :

For $\chi \rightarrow \infty$

V cannot increase stronger than M^2 !

Instability of graviton propagator is avoided

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ , and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

Normalization of scalar field

If M increases monotonically with χ :

choose normalization of scalar

$$M = \chi$$

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \rightarrow \infty$!

Quintessence

Dynamical dark energy ,
generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... modifications
(different growth of neutrino mass)

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}(B(\chi/\mu) - 6)\partial^\mu\chi\partial_\mu\chi \right\}$$

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp \left(-\frac{\alpha \varphi}{M} \right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k , such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \text{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

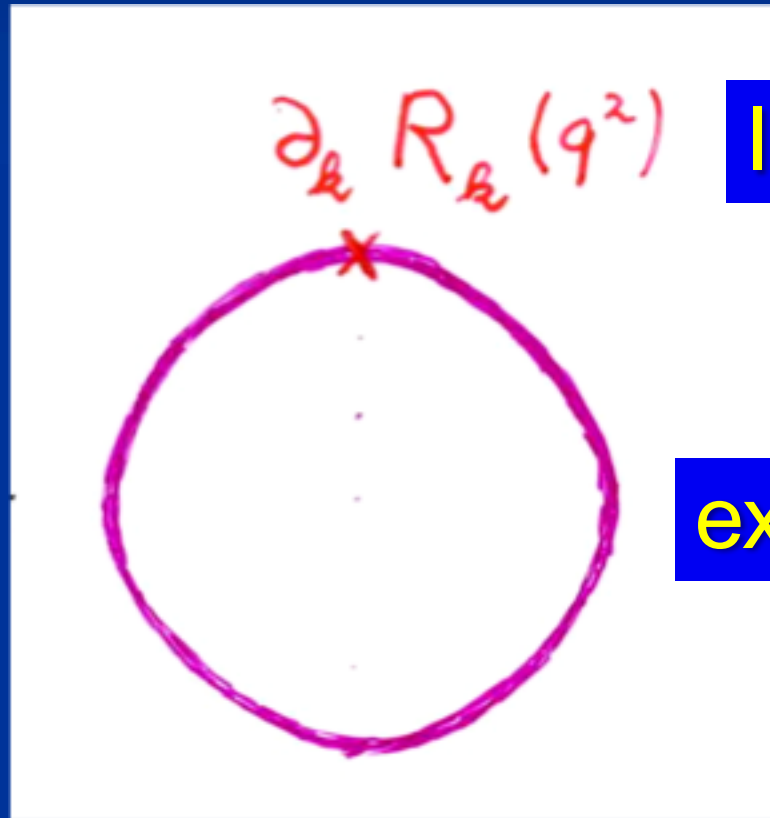
'92

$$\left(\Gamma_k^{(2)} \right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$$\text{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

Functional flow equation for scale dependent effective action



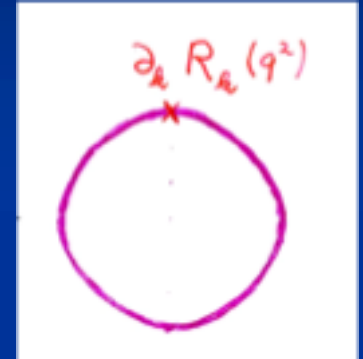
IR cutoff

exact propagator

Graviton contribution to flow of scalar potential

$$\partial_t V = k \partial_k V = 5I_k \left(-\frac{2V}{M^2} \right)$$

$$I_k(m^2) = \frac{1}{2} \int_q (q^2 + R_k(q) + m^2)^{-1} \partial_t R_k(q)$$



Litim cutoff:

$$R_k(q) = (k^2 - q^2) \theta(k^2 - q^2)$$

$$I_k(m^2) = \frac{1}{32\pi^2} \frac{k^6}{k^2 + m^2}$$

$$m^2 = -\frac{2V}{M^2}$$

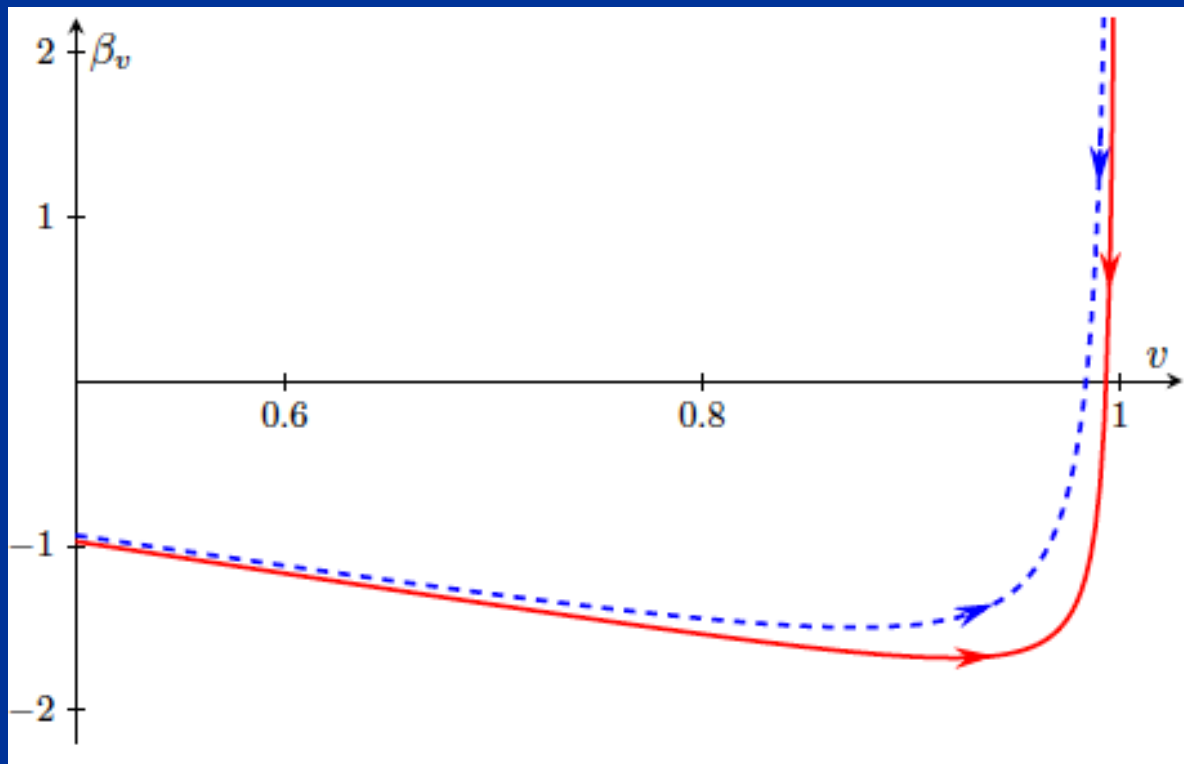
crucial dimensionless quantity

$$v = \frac{2V}{M^2 k^2}$$

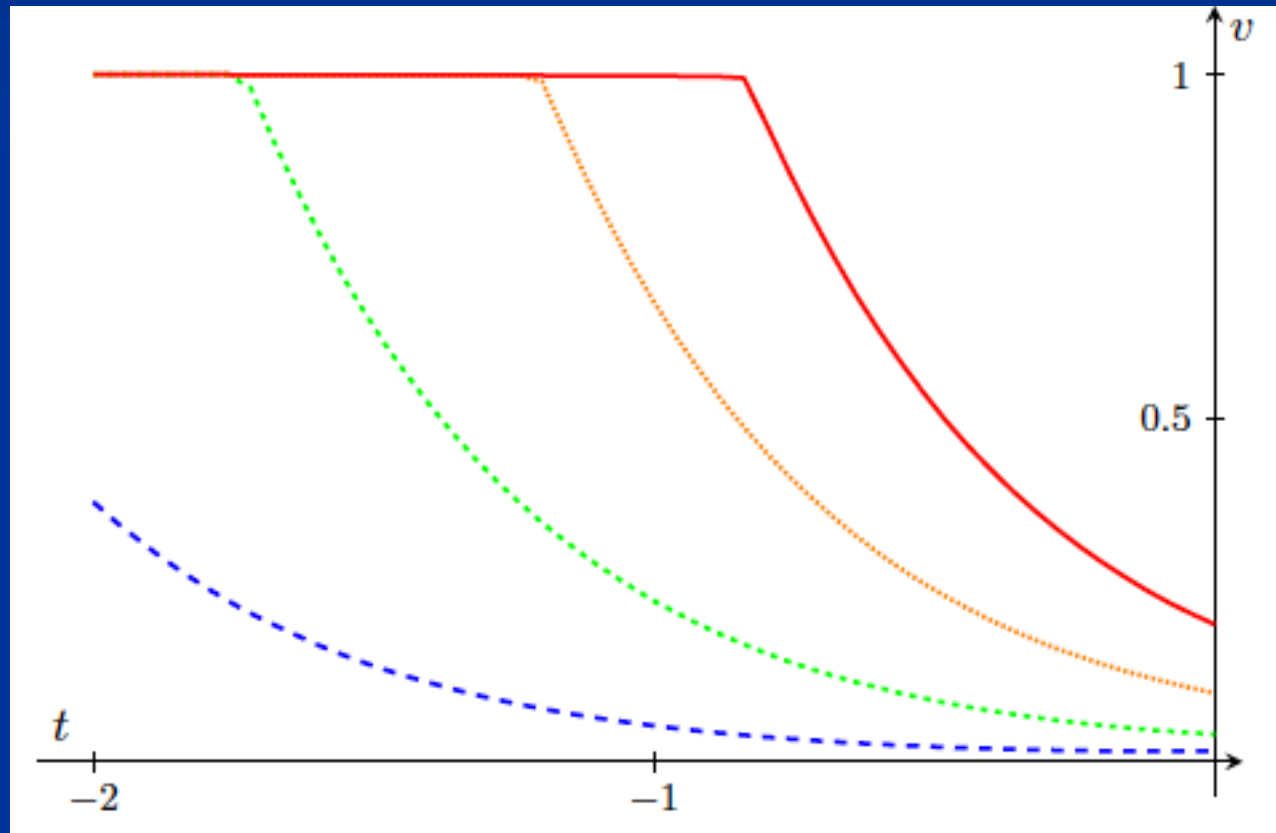
Flow equation for v

$$\partial_t v = \beta_v = -2v + \frac{5k^2}{16\pi^2 M^2} (1-v)^{-1}.$$

$$v = \frac{2V}{M^2 k^2}$$



Flow of v for different initial conditions



Infrared value of effective scalar
potential for $k/\chi \rightarrow 0$

$$v=1$$

$$U = \frac{\bar{k}^2}{2} M^2(\chi).$$

graviton barrier !

*Graviton fluctuations erase
the cosmological constant*

Ultraviolet behavior for $k/\chi \rightarrow \infty$

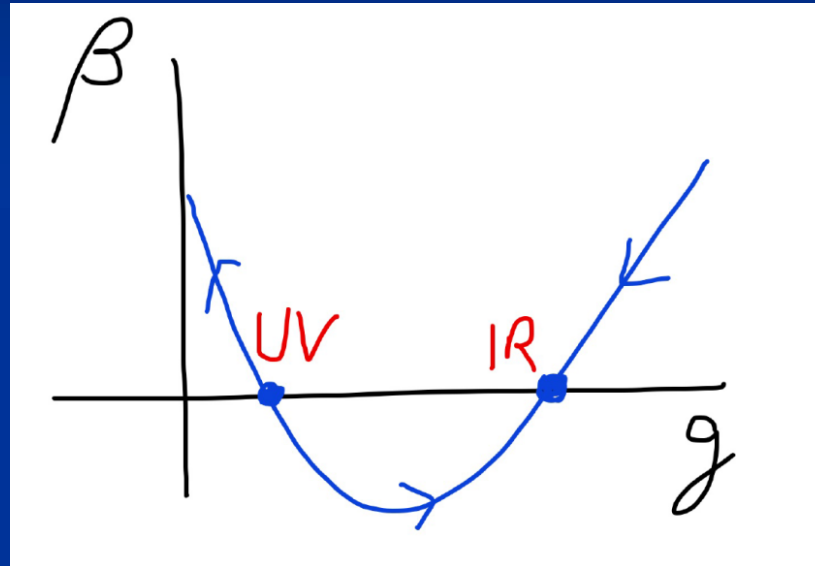
$$M^2 = f k^2$$

$$\partial_t v = -4v + \frac{5}{16\pi^2 f} (1 - v)^{-1}$$

UV – and IR –
fixed points

$$v_*(1 - v_*) = \frac{5}{64\pi^2 f}$$

Quantum gravity has UV- and IR- fixed point



Near UV fixed point:

Relevant parameters yield undetermined couplings.
Quartic scalar coupling is irrelevant and can
therefore be predicted.

Graviton contribution to flow of
quartic scalar coupling :
**positive and substantial
anomalous dimension**

$$\partial_t U = \frac{k^6}{32\pi^2} \left(\frac{5}{k^2 - 2U/M^2} + \frac{1}{k^2 + \partial^2 U / \partial^2 \chi} \right)$$

$$\partial_t \lambda = \frac{9\lambda^2}{16\pi^2} + \frac{5\lambda k^2}{16\pi^2 M^2}$$

$$\partial_t \lambda = A_\lambda \lambda + \frac{9\lambda^2}{16\pi^2}, \quad A_\lambda = \frac{5}{16\pi^2 f} = \frac{5g_*}{2\pi}$$

a prediction...

Asymptotic safety of gravity and the Higgs boson mass

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Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010

Abstract

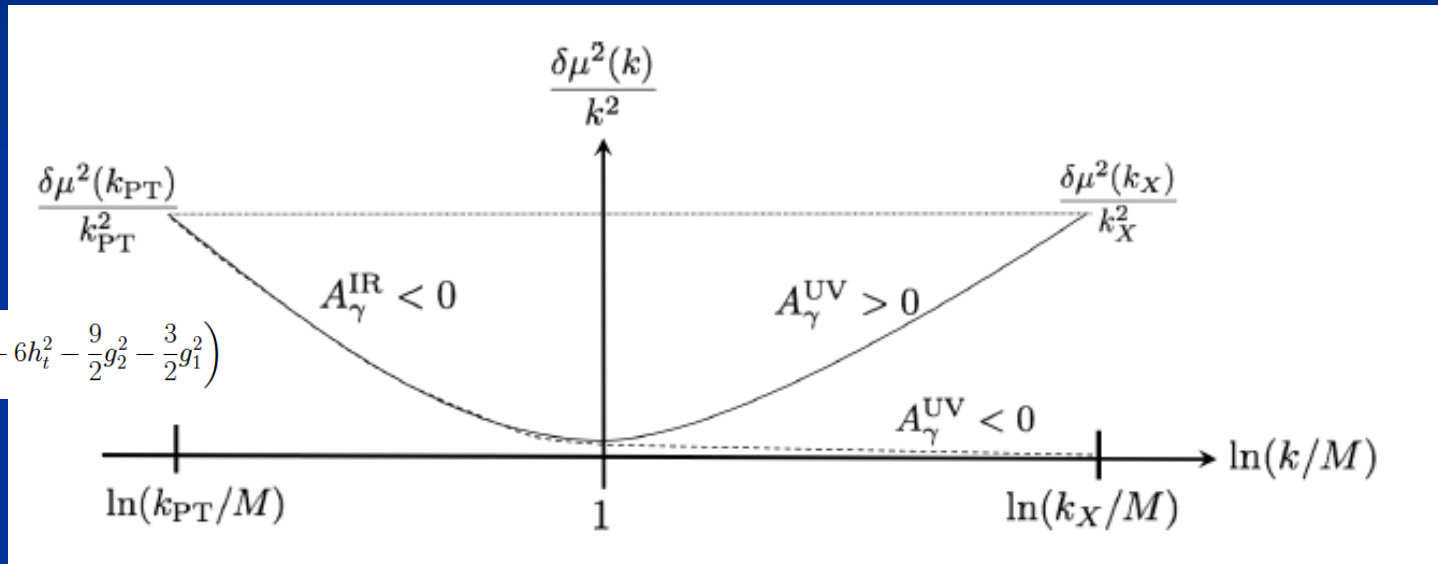
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Possible explanation of gauge hierarchy

$$A_{\gamma}^{\text{IR}} = -2 + A$$

$$A = \frac{1}{16\pi^2} \left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2 \right)$$



Gauge hierarchy problem in asymptotically safe gravity
–the resurgence mechanism

Christof Wetterich¹ and Masatoshi Yamada¹

Phys.Lett. B770 (2017) 268-271

Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

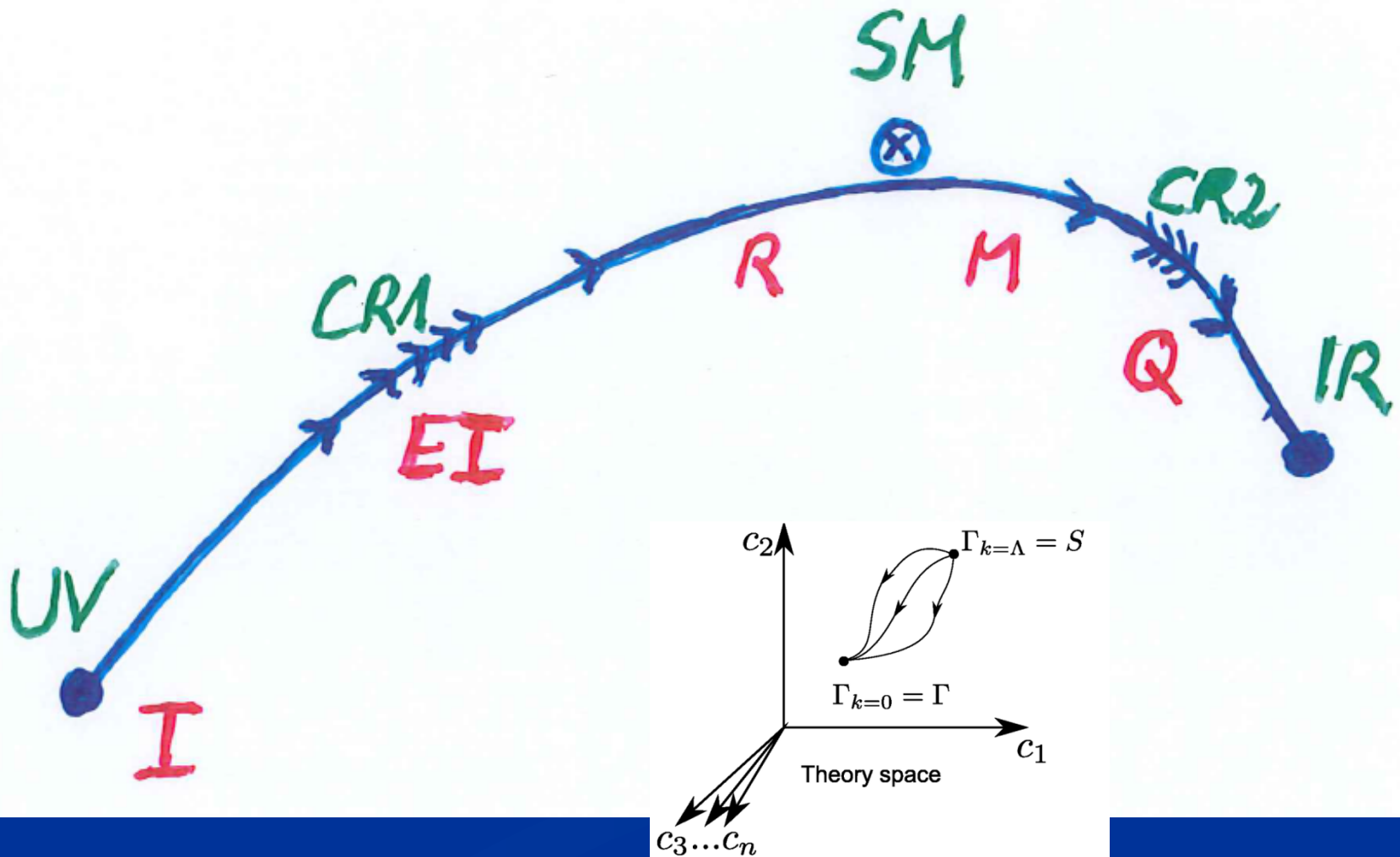
Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- all mass scales proportional to scalar field χ : electron mass, proton mass, Planck mass
- in presence of massive particles : sign of exact scale symmetry is exactly **massless Goldstone boson** – the dilaton

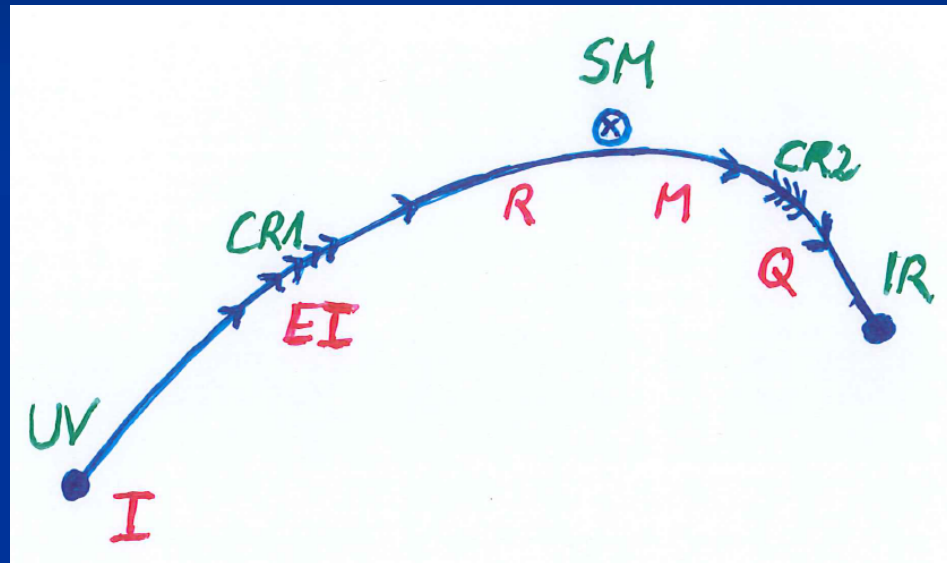
Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

Crossover in quantum gravity



Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy
with single scalar field

Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

intrinsic scale μ replaces k

quantum effective action,
variation yields field equations,
solve for cosmology

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

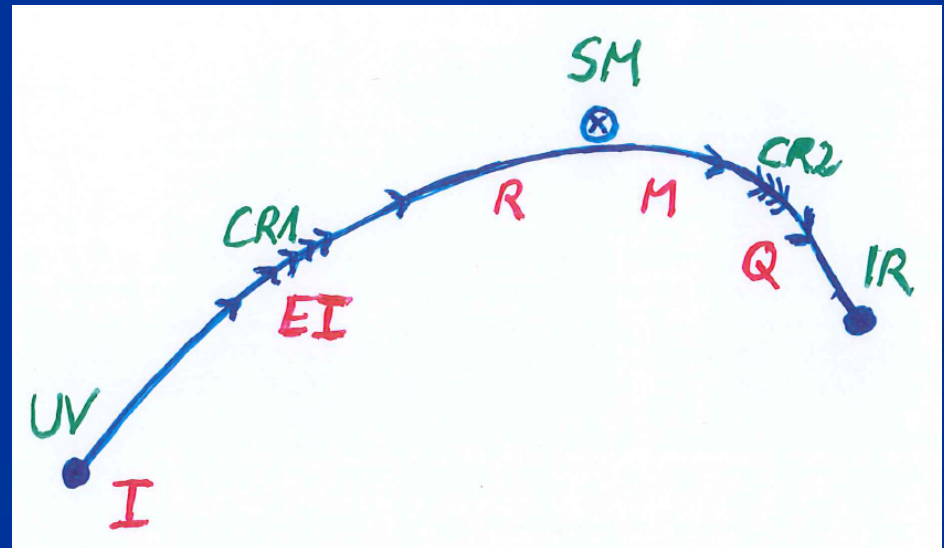
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}(B(\chi/\mu) - 6)\partial^\mu\chi\partial_\mu\chi \right\}$$

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \rightarrow 0$, $\chi \rightarrow \infty$
- UV: $\mu \rightarrow \infty$, $\chi \rightarrow 0$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

**Cosmology makes
crossover between
fixed points by
variation of χ .**



renormalization flow and cosmological evolution

- renormalization flow as function of μ
is mapped by dimensionless functions to
- field dependence of effective action on scalar field χ
translates by solution of field equation to
- dependence of cosmology on time t or η

Simplicity

simple description of **all** cosmological epochs

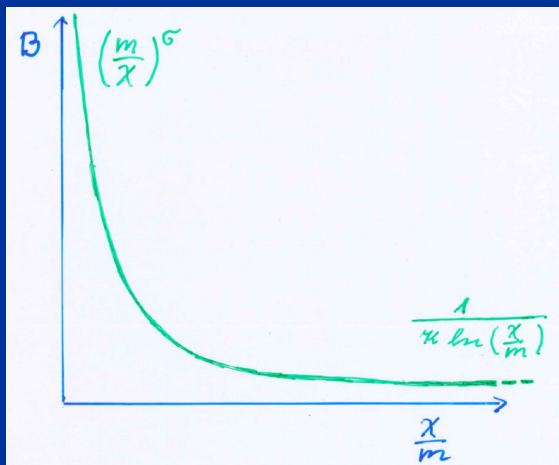
natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Kinetic B :

Crossover between two fixed points

Ansatz , not computed !



running
coupling obeys
flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover
can be exponentially larger than intrinsic scale μ

Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}(B(\chi/\mu) - 6)\partial^\mu \chi \partial_\mu \chi \right\}$$

No tiny dimensionless parameters (except matter sector, e.g. gauge hierarchy)

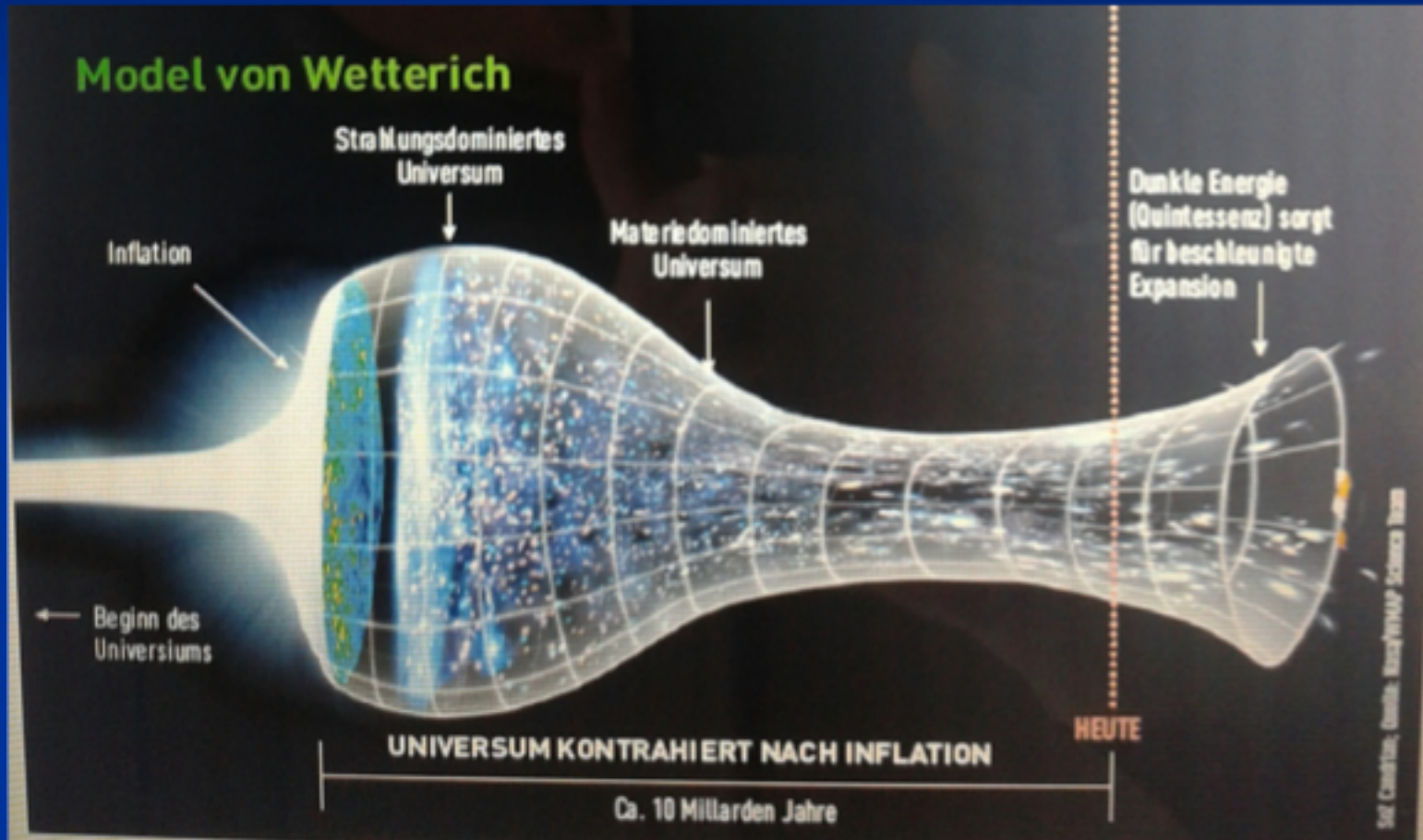
- one mass scale $\mu = 2 \cdot 10^{-33} \text{ eV}$
- one time scale $\mu^{-1} = 10^{10} \text{ yr}$
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Particle masses change with time

At SM fixed point :

- All particle masses (except for neutrinos) are proportional to scalar field χ .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Slow Universe

Asymptotic solution :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33} \text{ eV}$$

Expansion or shrinking always slow ,
characteristic time scale of the order of the age of the
Universe : $t_{\text{ch}} \sim \mu^{-1} \sim 10 \text{ billion years} !$

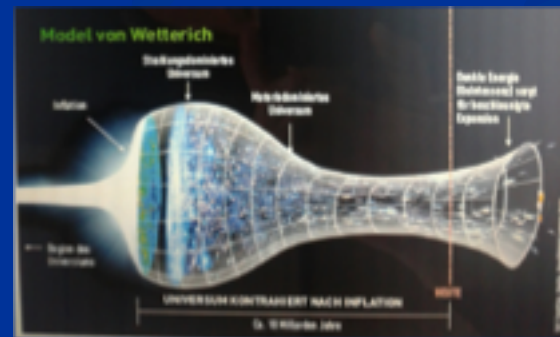
Hubble parameter of the order of **present** Hubble
parameter for all times , including inflation and big bang !
Slow increase of particle masses !

Eternal Universe

Asymptotic solution in
freeze frame :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite



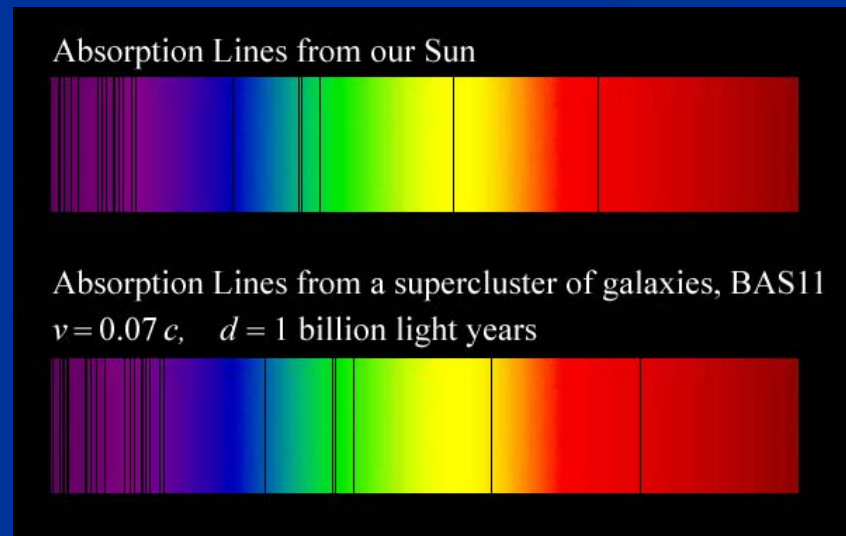
small dimensionless number ?

- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V : ratio vanishes asymptotically !

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past,
because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

Hot plasma ?

- Temperature in radiation dominated Universe :
 $T \sim \chi^{1/2}$ **smaller** than today
- Ratio temperature / particle mass :
 $T / m_p \sim \chi^{-1/2}$ **larger** than today
- T/m_p counts ! This ratio decreases with time.
- Nucleosynthesis , CMB emission as in standard cosmology !

Model is compatible with present observations

Together with variation of neutrino mass over
electron mass in present cosmological epoch :
model is compatible with all present observations

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

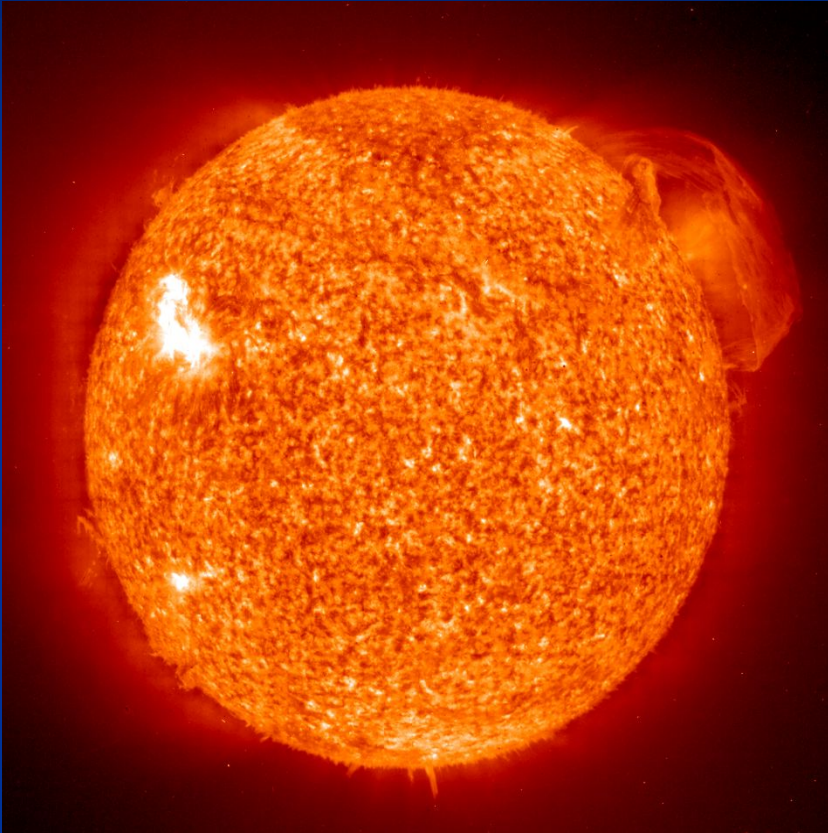
$$V'(\varphi) = M^4 \exp \left(-\frac{\alpha \varphi}{M} \right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

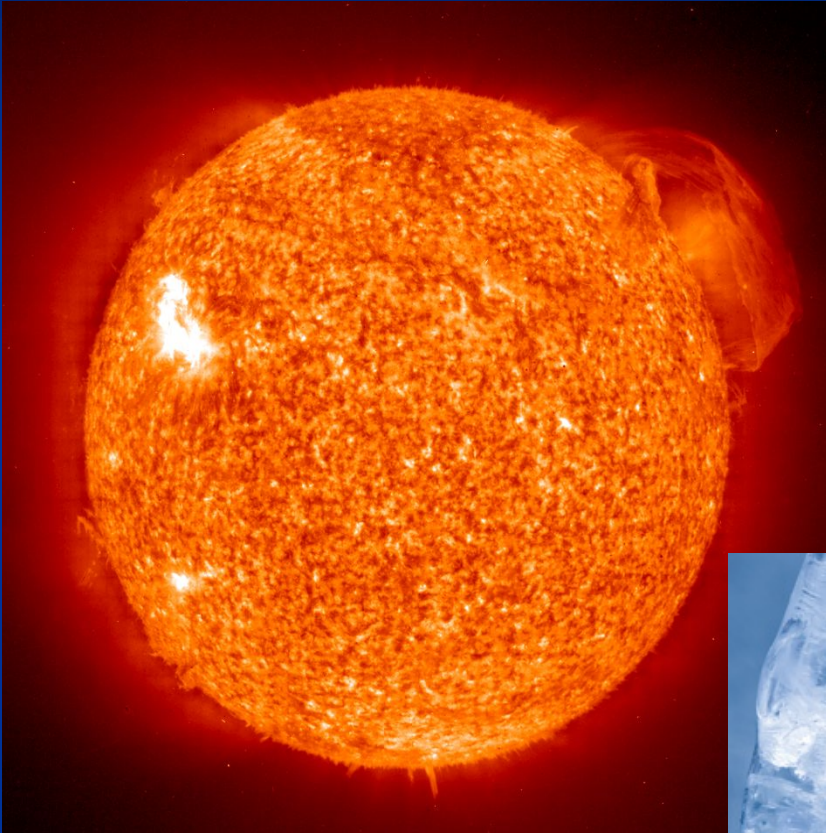
Field relativity : different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions ,
e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Big bang or freeze ?



Big bang or freeze ?



The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies and distant stars against a black background. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. Some are bright and clear, while others are faint and distant. The stars appear as small, bright points of light, some with visible diffraction patterns. The overall scene is a dense field of cosmic objects, representing the large-scale structure of the universe.

Big bang or freeze ?

just two ways of looking at same physics

*Big bang singularity
in Einstein frame is
field singularity !*

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

Field - singularity

- Big Bang is field - singularity
- similar (but not identical with)
coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$



Conclusions

- Graviton fluctuations play a role at large distances
- Graviton barrier for increase of scalar potential entails solution of cosmological constant problem
- Cosmology is described by crossover from UV-fixed point (inflation) to IR-fixed point (dynamical dark energy)

conclusions (2)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than Λ CDM : tests possible

end

Gauge invariant flow equation involves projection on physical fluctuations

$$k\partial_k\bar{\Gamma} = \zeta_k = \pi_k + \delta_k - \epsilon_k$$

$$\pi_k = \frac{1}{2} \text{Str}(k\partial_k \bar{R}_P G_P)$$

G_P : propagator for
physical fluctuations

$$P G_P = G_P P^T = G_P$$

$$\delta_\xi \bar{g} = (1 - P) \delta_\xi \bar{g}$$

measure contributions
on effective action

$$\delta_k - \epsilon_k$$

do not depend

Closed flow equation

projection on physical fluctuations
makes second functional derivative
invertible

$$\bar{\Gamma}_P^{(2)} = P^T \bar{\Gamma}^{(2)} P$$

$$\bar{\Gamma}^{(2)ij} = \frac{\partial^2 \bar{\Gamma}}{\partial \bar{g}_i \partial \bar{g}_j}$$

$$\left(\bar{\Gamma}_P^{(2)} + \bar{R}_P \right) G_P = P^T$$

Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \quad \left\{ \partial_\eta^2 + k^2 + a^2 \left(m^2 - \frac{R}{6} \right) \right\} \tilde{\varphi}_k = 0$$

determine **physical time** by counting number of oscillations

$$\tilde{t}_p = n_k$$

$$n_k = \frac{k\eta}{\pi}$$

(m=0)