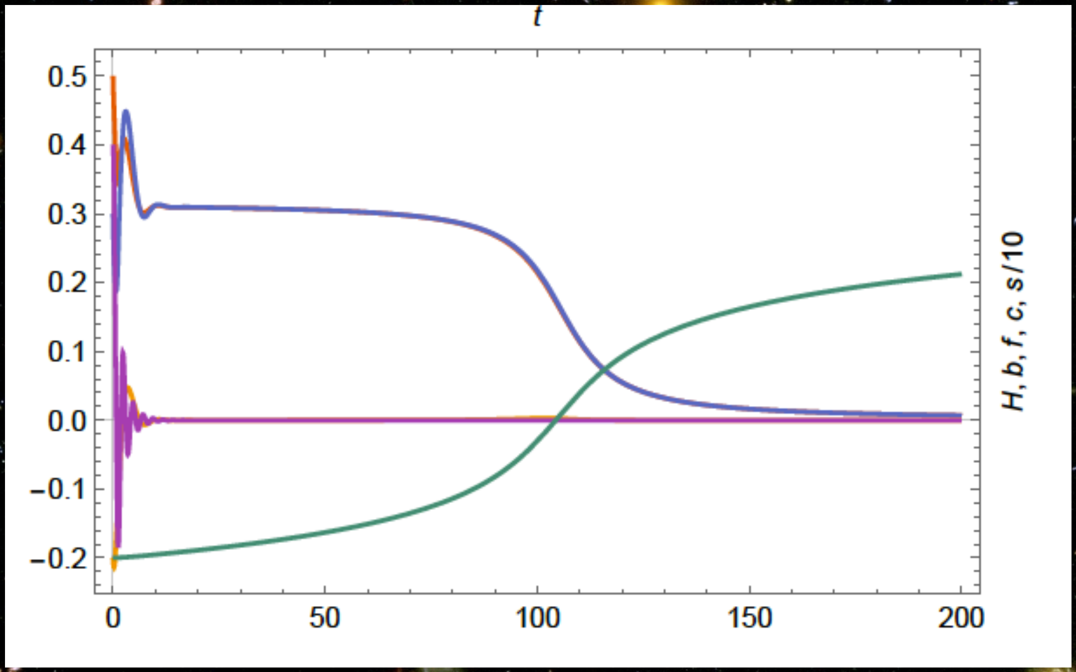
Pregeometry and emergent general relativity



What is pregeometry ?

- Theory with diffeomorphism invariance without fundamental metric
- Metric emerges as composite object

- Geometry is emerging, not fundamental
- Diffeomorphism symmetry is fundamental

Degrees of freedom

- Metric is collective or composite degree of freedom
- similar to pions in QCD
- Basic theory formulated in terms of different degrees of freedom
- similar to quarks and gluons in QCD

Why pregeometry ?

- Problems with metric quantum gravity:
- Euclidean functional integral for metric gravity problematic
- Lattice approaches ?
- Asymptotic safety presumably correct, but no simple picture of microscopic formulation has emerged so far

Fermionic pregeometry

- Pregeometry based only on fundamental fermions Akama, Amati,Veneziano, Denardo,Spallucci

- Spinor gravity implements local Lorentz symmetry
- Spinor gravity with global Lorentz symmetry Hebecker,…

Pregeometry as Yang- Mills theory

Euclidean formulation

- Gauge symmetry : SO(4)
- Additional vector field in vector representation of SO(4) allows for diffeomorphism invariant action
- ”Generalized vierbein”

Fields and covariant derivatives

- Gauge fields $A_{\mu mn} = -A_{\mu nm}$ $n,\ m = 0, \cdots, 3$
- Vierbein $e_\nu{}^m$
- Field strength

$$F_{\mu\nu mn} = \partial_\mu A_{\nu mn} - \partial_\nu A_{\mu mn} + A_{\mu m}{}^p A_{\nu pn} - A_{\nu m}{}^p A_{\mu pn}$$

Covariant derivative of vierbein

$$U_{\mu\nu}{}^m = D_\mu e_\nu{}^m = \partial_\mu e_\nu{}^m - \Gamma_{\mu\nu}{}^\sigma e_\sigma{}^m + A_\mu{}^m{}_n e_\nu{}^n$$

Composite metric

- Restrict generalized vierbein to

$$e = \det(e_\mu{}^m) \; > 0$$

- Inverse vierbein

$$e_\mu{}^m e_m{}^\nu = \delta_\mu^\nu \,, \qquad e_m{}^\mu e_\mu{}^n = \delta_m^n$$

- Metric as vierbein bilinear (shorthand)

$$g_{\mu\nu} = e_\mu{}^m e_\nu{}^n \delta_{mn} \qquad g^{\mu\nu} g_{\nu\rho} = \delta_\rho^\mu$$

Classical action

- Kinetic terms

$$S = \int d^4x \, e(L_F + L_U)$$

- Gauge fields :

$$L_F = \frac{Z}{8} F_{\mu\nu mn} F^{\mu\nu mn}$$

- Vierbein:

$$L_U = \frac{m^2}{4} U_{\mu\nu m} U^{\mu\nu m}$$

Here world (space-time) indices μ, ν are raised and lowered with $g^{\mu\nu}$ and $g_{\mu\nu}$ given by eq. (9), while "Lorentz indices" m, n are raised and lowered with δ^{mn} and δ_{mn}. We can convert Lorentz indices to world indices by multiplication with ${e_\mu}^m$ or ${e_m}^\mu$,

Classical action

- Gauge and diffeomorphism invariant kinetic term for gauge fields and vierbein

  4x4 + 4x6 degrees of freedom

  4 + 6 gauge degrees of freedom

  30 physical fields

- all dynamical : difference to Cartan's geometry

Levi - Civita connection

- Formed from vierbein via composite metric

$$U_{\mu\nu\rho} = U_{\mu\nu}{}^{m} e_{\rho m}$$

$$D_\sigma U_{\mu\nu\rho} = (D_\sigma U_{\mu\nu}{}^{m})\, e_{\rho m} + U_{\mu\nu}{}^{m} U_{\sigma\rho m}$$

$$U_{\mu\nu\rho} = \omega_{\mu\nu\rho} - A_{\mu\nu\rho}$$

$$\omega_{\mu\nu\rho} = \frac{1}{2}\Big\{ e_{\mu m}\,(\partial_\rho e_\nu{}^{m} - \partial_\nu e_\rho{}^{m}) + e_{\nu m}\,(\partial_\rho e_\mu{}^{m} - \partial_\mu e_\rho{}^{m}) + e_{\rho m}\,(\partial_\mu e_\nu{}^{m} - \partial_\nu e_\mu{}^{m}) \Big\} = e_\nu{}^{m} e_\rho{}^{n} \omega_{\mu m n}\,. \quad (16)$$

$$\omega_{\mu\nu\rho} = \Gamma_{\mu\rho}{}^{\sigma} g_{\sigma\nu} - e_{\nu m} \partial_\mu e_\rho{}^{m}$$

$$\omega_{\mu\rho\nu} = -\omega_{\mu\nu\rho}\,, \quad A_{\mu\rho\nu} = -A_{\mu\nu\rho}\,, \quad U_{\mu\rho\nu} = -U_{\mu\nu\rho}$$

Limit of general relativiy

$$U_{\mu\nu\rho} = 0$$ Cartan's geometry

$$U_{\mu\nu\rho} = \omega_{\mu\nu\rho} - A_{\mu\nu\rho}$$

Express action in terms of metric and U

$$F_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - V_{\mu\nu\rho\sigma}$$

$$V_{\mu\nu\rho}{}^{\sigma} = e_m{}^{\sigma} \left(D_\mu U_{\nu\rho}{}^m - D_\nu U_{\mu\rho}{}^m \right)$$

$$U_{\mu\nu\rho} = 0$$ results in higher derivative gravity

Stelle

Stability and generalized Higgs mechanism

Positivity of euclidean action

- Kinetic terms

$$S = \int \mathrm{d}^4x \, e(L_F + L_U)$$

- Gauge fields :

$$L_F = \frac{Z}{8} F_{\mu\nu mn} F^{\mu\nu mn}$$

- Vierbein:

$$L_U = \frac{m^2}{4} U_{\mu\nu m} U^{\mu\nu m}$$

Flat space

Flat space solves field equations

$$e_\mu{}^m = \delta_\mu^m \,,\; g_{\mu\nu} = \delta_{\mu\nu} \qquad A_{\mu mn} = 0$$

Stability of fluctuations: expand around flat space

$$e_\mu{}^m = \delta_\mu^m + \frac{1}{2} H_{\mu\nu} \delta^{\nu m}$$

Stable high momentum behavior

$$\int_x eL_F = \frac{Z}{4} \int_q A_{\mu mn}(-q) \left(q^2 \delta^{\mu\nu} - q^\mu q^\nu\right) A_\nu{}^{mn}(q) .$$

Expansion

Decompose fluctuations in representations of SO(4)

$$H^{(\mathrm{S})}_{\mu\nu} = (H_{\mu\nu} + H_{\nu\mu})/2 \qquad H^{(\mathrm{A})}_{\mu\nu} = (H_{\mu\nu} - H_{\nu\mu})/2$$

$$H^{(\mathrm{S})}_{\mu\nu} = t_{\mu\nu} + \partial_\mu \kappa_\nu + \partial_\nu \kappa_\mu + \frac{1}{3}\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\sigma + \frac{\partial_\mu\partial_\nu}{\partial^2} u$$

$$H^{(\mathrm{A})}_{\mu\nu} = b_{\mu\nu} + \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu$$

$$\partial^\mu t_{\mu\nu} = \partial^\nu t_{\mu\nu} = \partial^\mu b_{\mu\nu} = \partial^\nu b_{\mu\nu} = 0$$

$$\partial^\mu \kappa_\mu = \partial^\mu \gamma_\mu = 0\,, \qquad {t_\mu}^\mu = 0\,.$$

Expansion

Decompose fluctuations in representations of SO(4)

$$A_{\mu\nu\rho} = B_{\mu\nu\rho} + \partial_\mu L_{\nu\rho}\,, \qquad \partial^\mu B_{\mu\nu\rho} = 0$$

$$B_{\mu\nu\rho} = \frac{1}{4}\varepsilon_{\nu\rho}{}^{\sigma\tau}(P_{\mu\sigma}v_\tau - P_{\mu\tau}v_\sigma) + \frac{1}{3}(P_{\mu\nu}w_\rho - P_{\mu\rho}w_\nu) + D_{\mu\nu\rho}$$

$$D_{\mu\nu\rho} = \frac{1}{2}(\partial_\nu E_{\mu\rho} - \partial_\rho E_{\mu\nu}) + C_{\mu\nu\rho}$$

$$\partial^\mu E_{\mu\nu} = 0\,, \qquad \delta^{\mu\nu}E_{\mu\nu} = 0\,, \qquad E_{\mu\nu} = E_{\nu\mu}$$

Stable high momentum behavior

$$L_F = \frac{Z}{4}\left\{\frac{1}{2}E^{\nu\rho}\partial^4 E_{\nu\rho} - C^{\mu\nu\rho}\partial^2 C_{\mu\nu\rho} - v^{(t)\mu}\partial^2 v_\mu^{(t)} + \frac{3}{2}\tilde{v}\partial^4\tilde{v} - \frac{4}{9}w^{(t)\mu}\partial^2 w_\mu^{(t)} + \frac{2}{3}\tilde{w}\partial^4\tilde{w}\right\}.$$

$$L_U = L_U^{(1)} + L_U^{(2)} + L_U^{(3)}$$

$$\int_x eL_U^{(1)} = \frac{m^2}{16}\int_q \Big\{2t_{\mu\nu}(-q)q^2 t^{\mu\nu}(q) + b_{\mu\nu}(-q)q^2 b^{\mu\nu}(q) + 2\left(\kappa_\mu(-q) - \gamma_\mu(-q)\right)q^4\left(\kappa^\mu(q) - \gamma^\mu(q)\right) + \frac{2}{3}\sigma(-q)q^2\sigma(q)\Big\}. \quad (33)$$

Generalized Higgs mechanism

- Flat space breaks local gauge symmetry "spontaneously"
- Gauge bosons become massive

$$L_U^{(2)} = \frac{m^2}{4} A^{\mu\nu\rho} A_{\mu\nu\rho} = \frac{m^2}{4} (B^{\mu\nu\rho} B_{\mu\nu\rho} - L^{\nu\rho} \partial^2 L_{\nu\rho})$$

$$= \frac{m^2}{4} \Big\{ -\frac{1}{2} E^{\nu\rho} \partial^2 E_{\nu\rho} + C^{\mu\nu\rho} C_{\mu\nu\rho} + v^{(t)\mu} v_\mu^{(t)} - \frac{3}{2} \tilde{v} \partial^2 \tilde{v}$$

$$+ \frac{4}{9} w^{(t)\mu} w_\mu^{(t)} - \frac{2}{3} \tilde{w} \partial^2 \tilde{w} - M^{\nu\rho} \partial^2 M_{\nu\rho} + 2 l^\mu \partial^4 l_\mu \Big\} .$$

Mode mixing

Vierbein fluctuations and gauge fields mix

$$\int_x \bar{e} L_U^{(3)} = -\frac{m^2}{4} \int_x A^{\mu\nu\rho} \left(\partial_\mu H_{\nu\rho}^{(A)} + \partial_\rho H_{\mu\nu}^{(S)} - \partial_\nu H_{\mu\rho}^{(S)} \right)$$

$$L_U = \frac{m^2}{4} \{ L_{tE} + L_{\sigma\tilde{w}} + L_{bM} + L_{\kappa\gamma l} + L'_m \}$$

Example: graviton modes

$$L_{tE} = -\frac{1}{2} (t^{\mu\nu} + E^{\mu\nu}) \partial^2 (t_{\mu\nu} + E_{\mu\nu})$$

Stable graviton sector

- Inverse propagator matrix

$$P_R = \begin{pmatrix} Zq^2 + m^2 & , \; mq \\ mq & , \; q^2 \end{pmatrix}$$

- Only zero eigenvalue for $q^2=0$

$$\det P_R = Zq^4 = 0$$

- Inverse propagator

$$\lambda_\pm = \frac{1}{2}\Big\{(Z+1)q^2 + m^2 \pm \sqrt{(Z-1)^2 q^4 + 2(Z+1)q^2 m^2 + m^4}\Big\}$$

Effective action

- Quantum fluctuations induce additional terms in effective action

$$\Gamma = \int d^4 x e (L_F + L_U + U + L_R + L_G)$$

- Invariant linear in field strength

$$L_R = -\frac{M^2}{2} F_{\mu\nu}{}^{\mu\nu}$$

$$F = F_{\mu\nu}{}^{\mu\nu} = F_{\mu\nu mn} e^{m\mu} e^{n\nu}$$

General relativity as effective low energy theory

Express field strength in terms of metric and U

$$F_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - V_{\mu\nu\rho\sigma}$$

$$\begin{aligned} V_{\mu\nu\rho}{}^{\sigma} &= e_m{}^{\sigma}\left(D_\mu U_{\nu\rho}{}^m - D_\nu U_{\mu\rho}{}^m\right) \qquad (107) \\ &= D_\mu U_{\nu\rho}{}^{\sigma} - D_\nu U_{\mu\rho}{}^{\sigma} - U_\mu{}^{\sigma\tau} U_{\nu\rho\tau} + U_\nu{}^{\sigma\tau} U_{\mu\rho\tau} \end{aligned}$$

$$L_F = \frac{Z}{8}\left\{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu\rho\sigma}V^{\mu\nu\rho\sigma} + V_{\mu\nu\rho\sigma}V^{\mu\nu\rho\sigma}\right\}$$

Einstein Hilbert term from $L_R = -\frac{M^2}{2}\left(R - V_{\mu\nu}{}^{\mu\nu}\right)$

Source for U: $2R_{\mu\nu\rho\sigma}V^{\mu\nu\rho\sigma} \rightarrow -4U_{\nu\rho}{}^m D_\mu R^{\mu\nu\rho}{}_m$

Effective theory for low curvature and low momentum

$$L_R = -\frac{M^2}{2}\left(R - V_{\mu\nu}{}^{\mu\nu}\right)$$

- Approximate solution

$$U_{\mu\nu\rho} = -\frac{Z}{m^2} D^\sigma R_{\sigma\mu\nu\rho}$$

- Insertion:

$$S = \int_x e\left\{-\frac{M^2}{2}R + \frac{Z}{8} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\right\}$$ + higher derivative terms

Graviton propagator

inverse propagator matrix

$$P_R^{(tE)} = \begin{pmatrix} Zq^2 + m^2 - M^2 \;, & \frac{m^2 - M^2}{m} q \\ \frac{m^2 - M^2}{m} q \;, & q^2 \end{pmatrix}$$

massless + massive graviton:

propagator has poles at

$$q^2 = 0 \,, \quad q^2 = -\mu^2$$

$$\mu^2 = \frac{M^2}{Z}\left(1 - \frac{M^2}{m^2}\right)$$

stable particle for

$$0 \leqslant M^2 \leqslant m^2$$

Graviton propagator

- inverse propagator

$$\lambda_{\pm} = \frac{1}{2}\Big\{(Z+1)q^2 + m^2 - M^2 \pm \sqrt{\left[(Z-1)q^2 + m^2 - M^2\right]^2 + 4\frac{q^2}{m^2}(m^2 - M^2)^2}\Big\} \qquad (131$$

- no tachyon or ghost for

$$0 \leqslant y \leqslant 1$$

$$Z < Z_c\,, \quad Z_c = \frac{y}{1-y}$$

$$y = \frac{M^2}{m^2}$$

Analytic continuation

- Analytic continuation of euclidean theory well defined
- Continuation in field: phase factor for fields with Lorentz-index 0
- Analytic continuation of gauge symmetry **SO(4)** to **SO(1,3)**

Cosmology from pregeometry

Effective action

Add scalar singlet field χ

purpose: simple implementation of

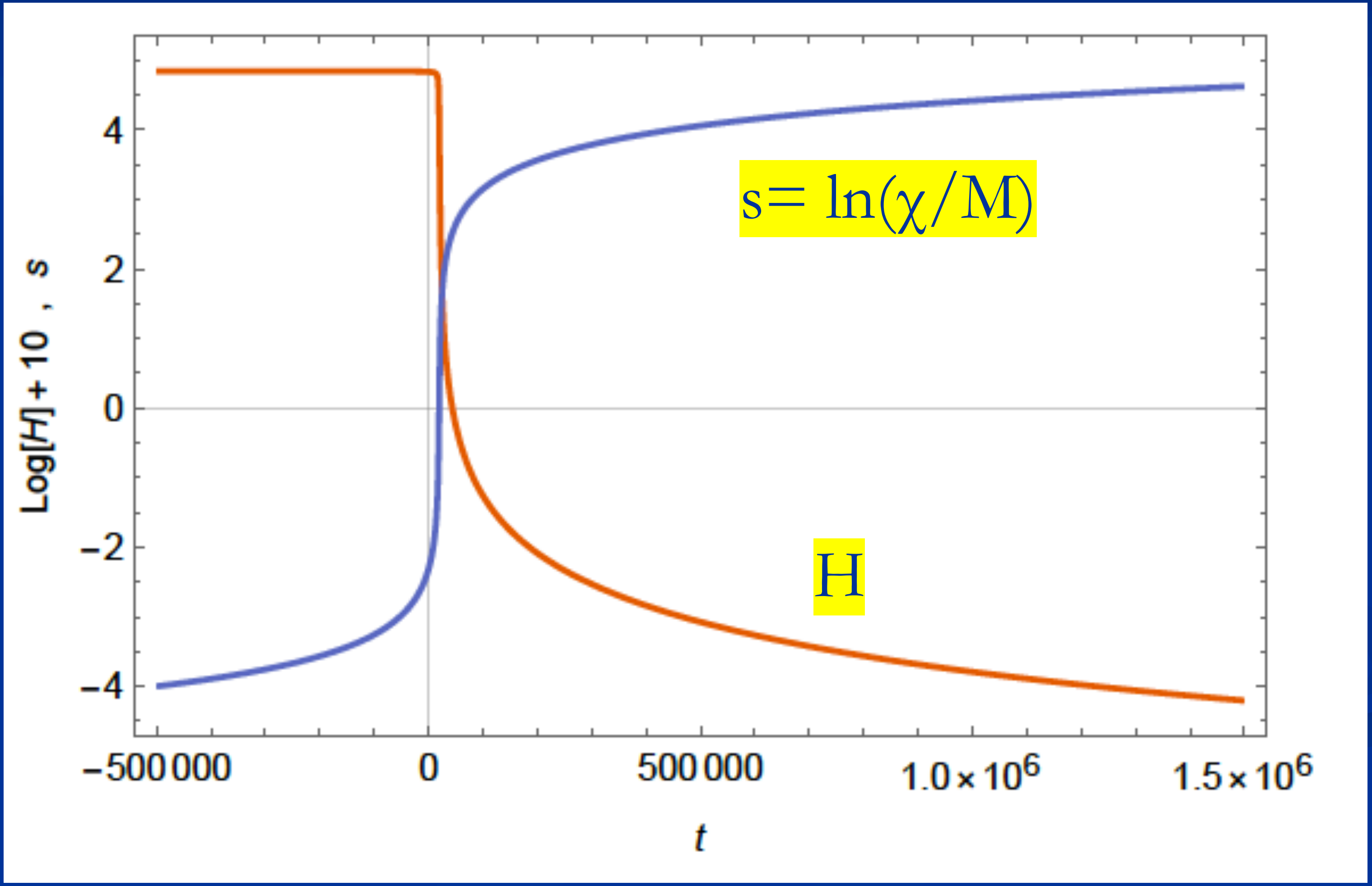
quantum scale symmetry

effective action

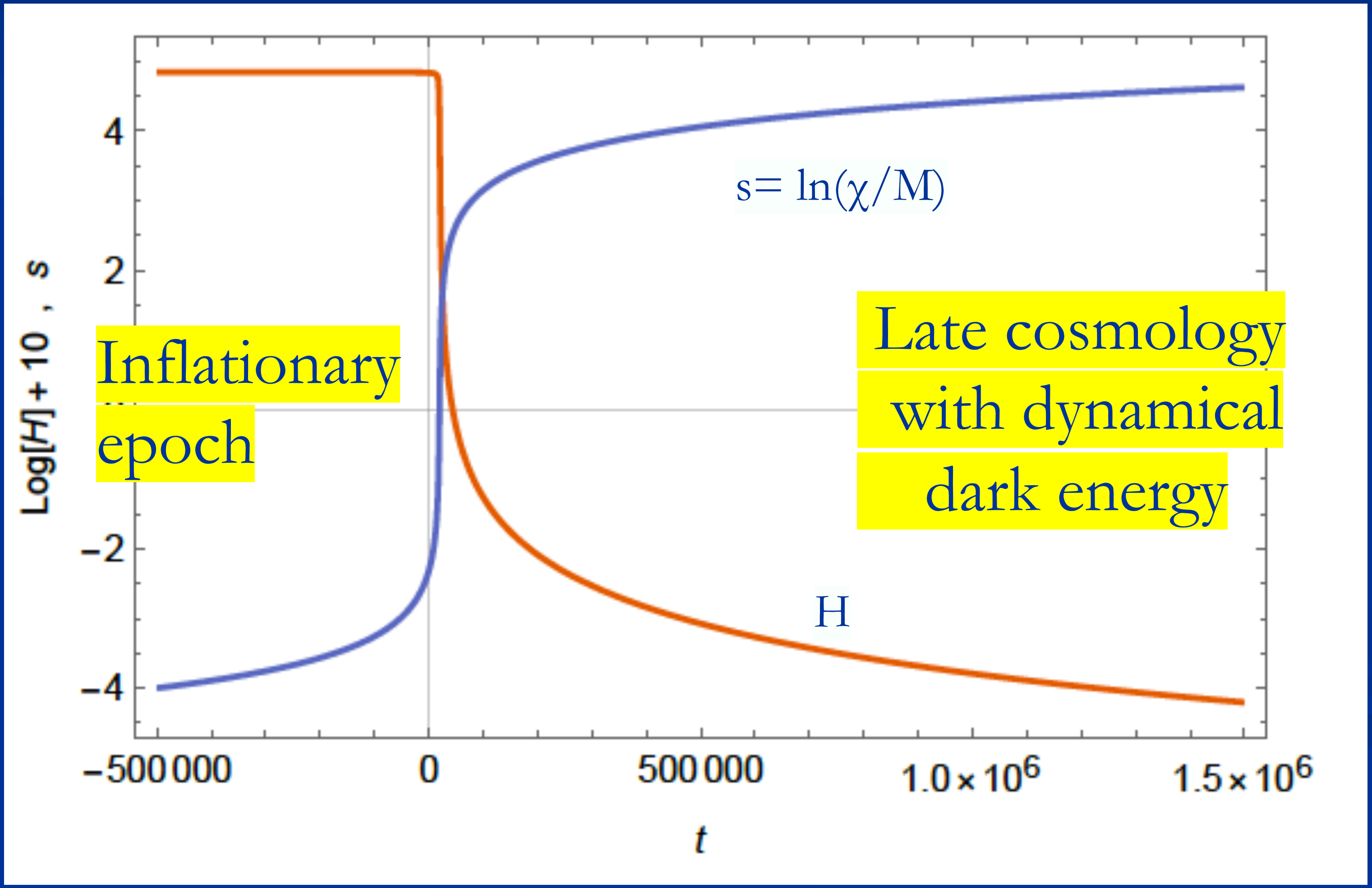
$$\Gamma = \int_x e\Big\{\frac{Z}{8}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + \frac{B}{2}F_{\mu\nu}F^{\mu\nu} + \frac{C}{2}F^2 - \frac{M^2}{2}F + \frac{m^2}{4}U_{\mu\nu\rho}U^{\mu\nu\rho} + \frac{n^2}{2}U_{\mu}{}^{\mu}{}_{\rho}U_{\nu}{}^{\nu\rho} + \frac{K}{2}\partial^\mu\chi\partial_\mu\chi + V + YU_{\mu}{}^{\mu\nu}\chi\partial_\nu\chi\Big\},$$

Z, B, C, M^2, m^2, n^2, K, V, Y: coupling functions of field χ

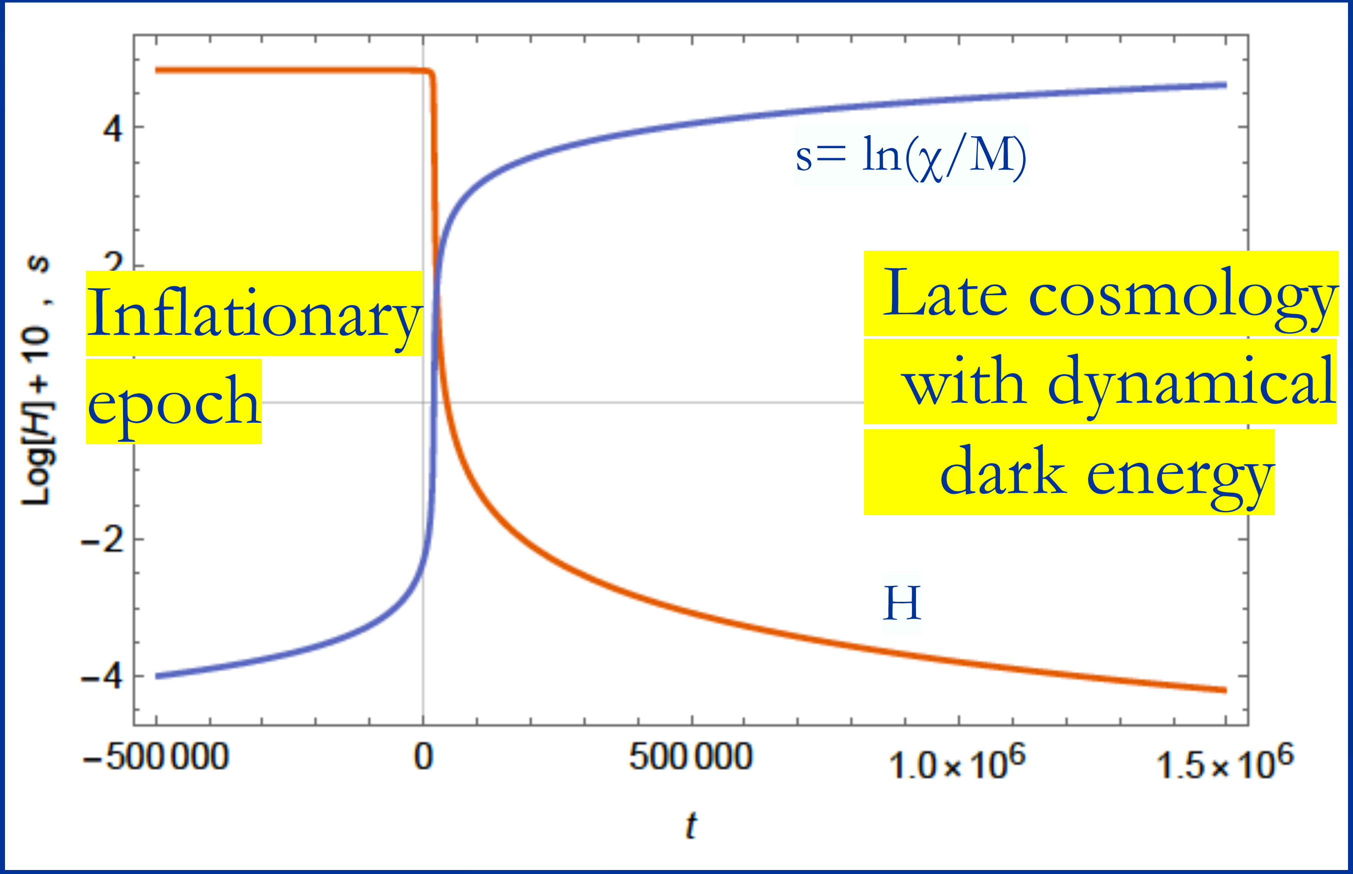
Crossover solution of homogenous field equations



Crossover solution of homogenous field equations



Crossover from UV – fixed point in infinite past to IR – fixed point in infinite future



Coupling functions

stability for: $Z > 0\,, \quad Z + 2B > 0\,, \quad \tilde{Z} < 0\,, \quad m^2 > 0\,, \quad M^2 > 0$

$$\tilde{Z} = Z + 4B + 12C\,, \quad \tilde{m}^2 = m^2 + 3n^2$$

ansatz motivated by scaling solutions of quantum gravity, similar to asymptotic safety,

quantum scale symmetry

$$V = u_0 k^4\,,\ M^2 = 2w_0 k^2 + \xi\chi^2$$

$$m^2 = m_0^2 k^2 + \zeta\chi^2\,, \quad \tilde{m}^2 = \tilde{m}_0^2 + \tilde{\zeta}\chi^2$$

Scaling solutions of functional flow equations

- At fixed point: all (infinitely many) couplings take fixed values
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

Scaling solutions and cosmology

- Cosmology involves scalar potentials over large range of field values
- Inflaton potential
- Higgs potential for Higgs inflation
- Cosmon potential for dynamical dark energy or quintessence

Quantum gravity :

these potentials are not arbitrary

Scaling potential in standard model

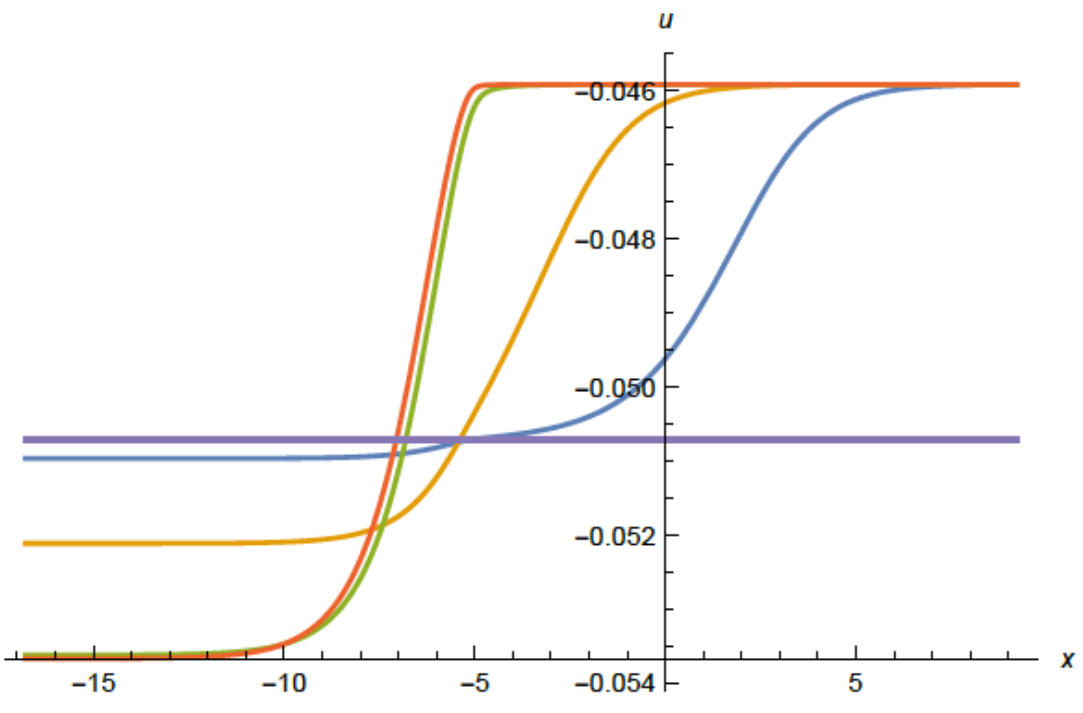


FIG. 19. Effective potential u as function of $x = \ln \tilde{\rho}$ for $\xi_\infty = 0.1$ (blue), 1.0 (orange), 10^3 (green) and 10^4 (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model, $N_S = 4$, $N_V = 12$, $N_F = 45$.

u : dimensionless scalar potential $u = U/k^4$

x : logarithm of scalar field value

Coefficient of curvature scalar in standard model

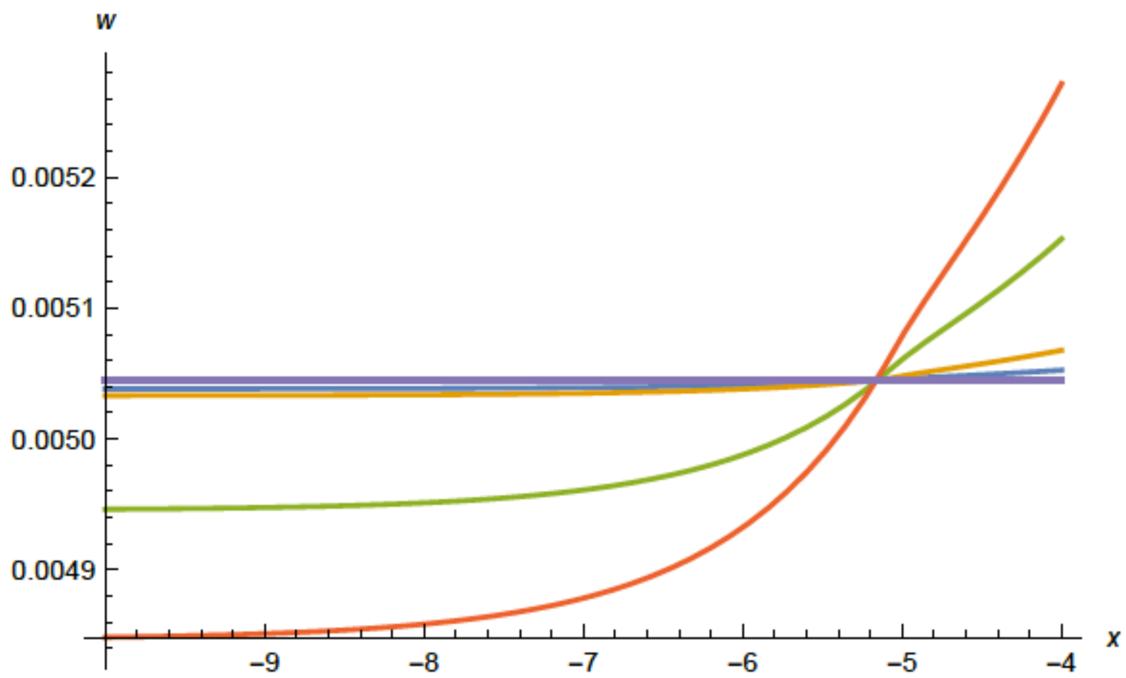


FIG. 21. Dimensionless squared Planck mass w as function of $x = \ln\tilde{\rho}$ for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for $\xi_\infty \to 0$. All curves meet in a common point at $x \approx -5.05$.

w : dimensionless field dependent squared Planck mass

$w = 2\ M^2/\ k^2$

non-minimal coupling of scalar field to gravity

Scaling solutions motivate choice of coupling functions

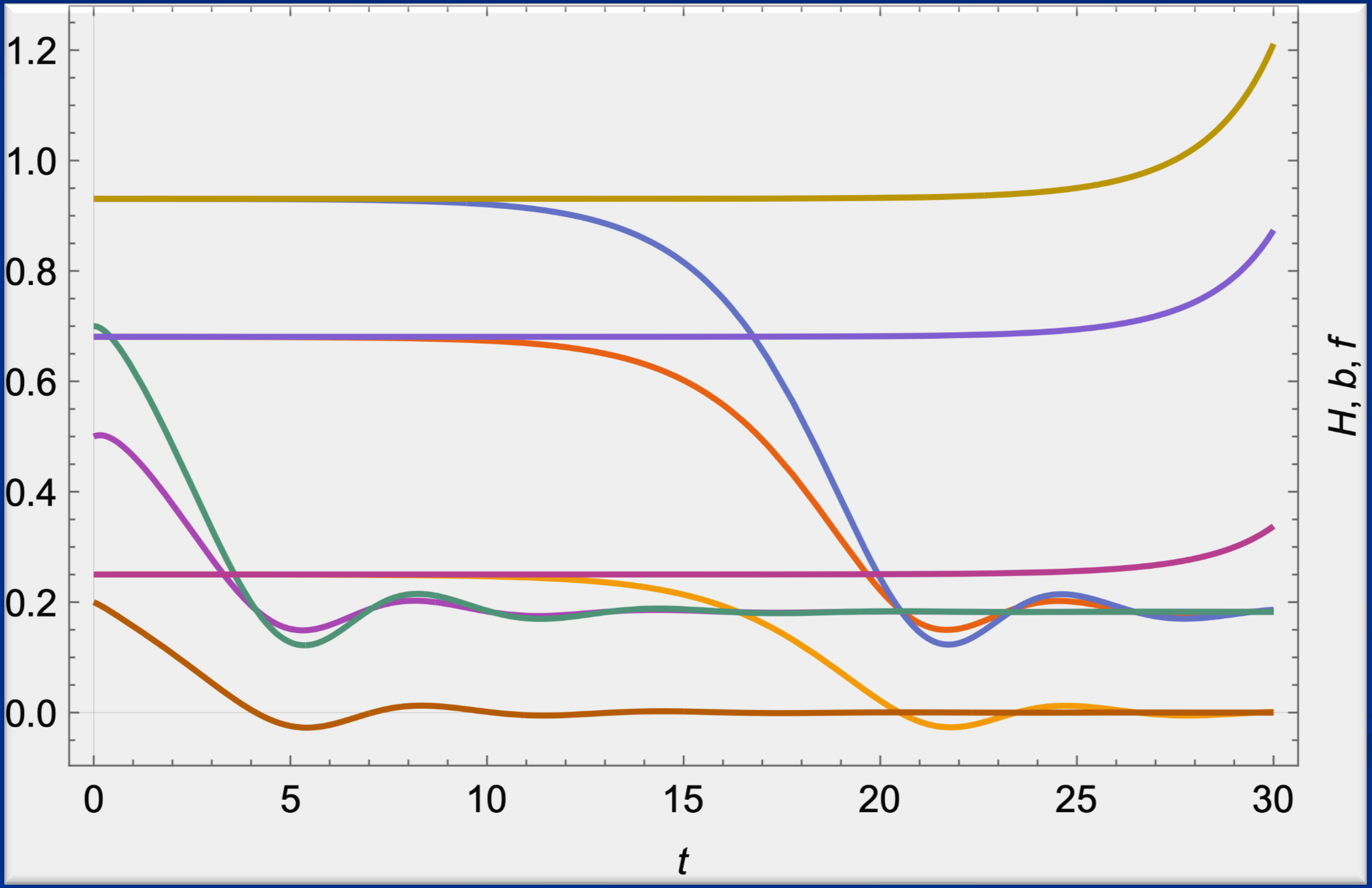
$$V = u_0 k^4 \ , \ M^2 = 2w_0 k^2 + \xi \chi^2$$

$$m^2 = m_0^2 k^2 + \zeta \chi^2 \ , \quad \tilde{m}^2 = \tilde{m}_0^2 + \tilde{\zeta} \chi^2$$

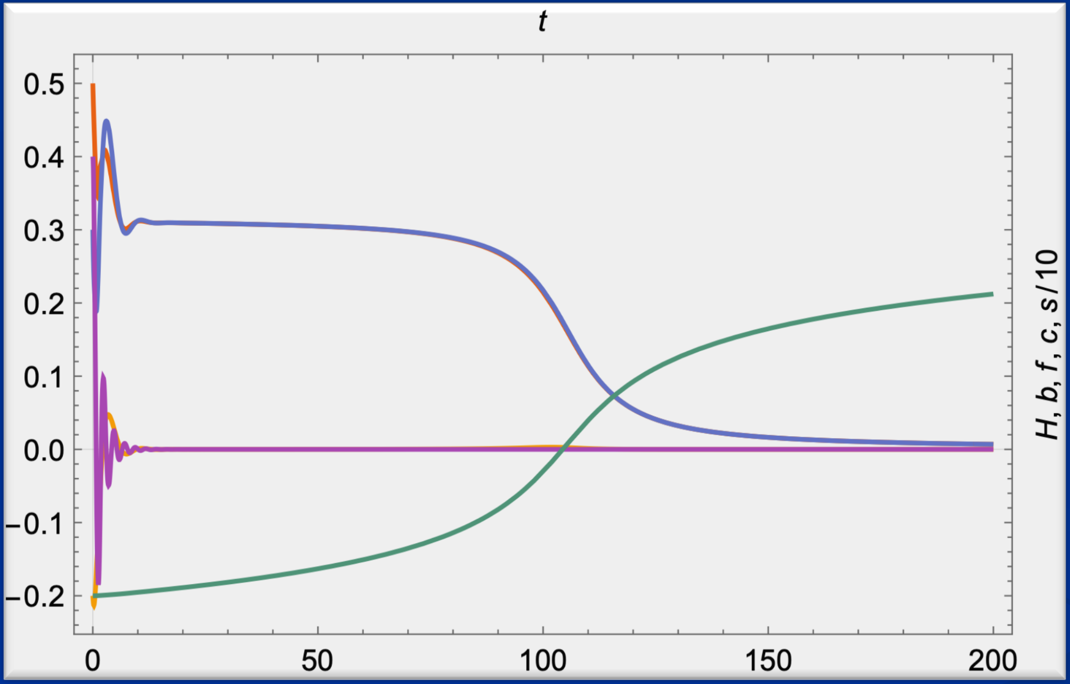
for given coupling functions:

derive field equations, solve them

Early approach to stable cosmic attractor
Basin of attraction



Crossover in scaling solution induces end of early attractor, end of inflation



$$V = u_0 k^4 \ , \ M^2 = 2 w_0 k^2 + \xi \chi^2$$

Variable gravity as effective theory

inflation, end of inflation, late cosmology :

- well approximated by variable gravity
- (modified) general relativity

  + scalar field

Models of this type are compatible with present observations

Together with variation of neutrino mass over electron mass in present cosmological epoch :

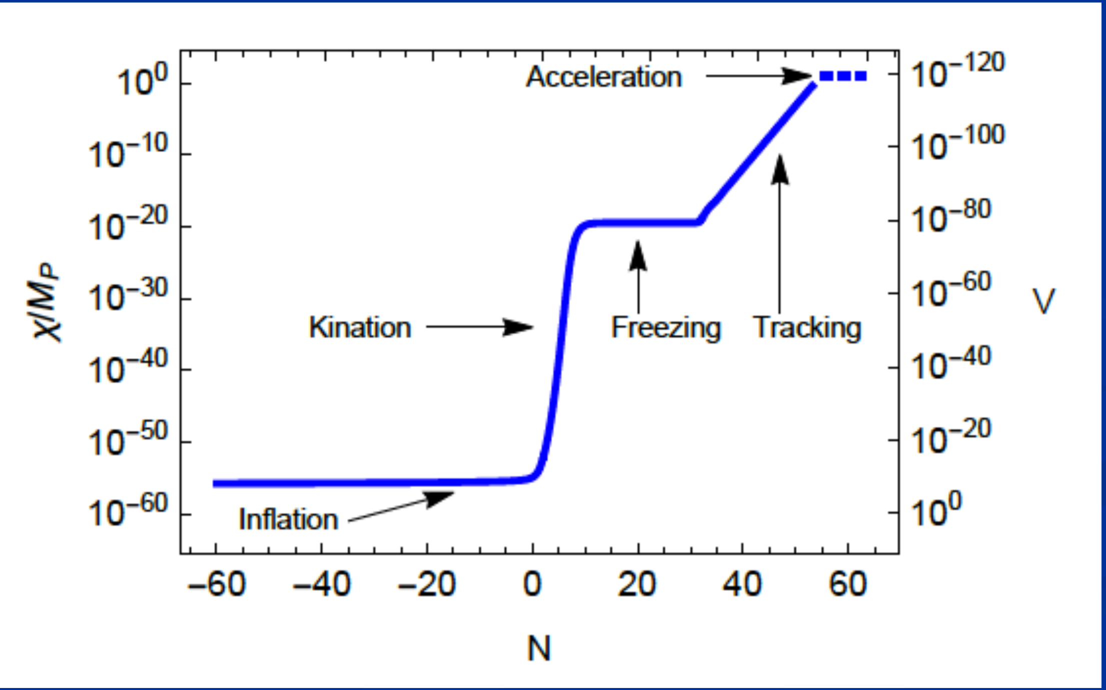
A model compatible with all present observations, including inflation and dark energy

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln\left(\frac{\chi}{\mu}\right) - c_t \right] = \kappa \ln\left(\frac{\chi}{m}\right)$$

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

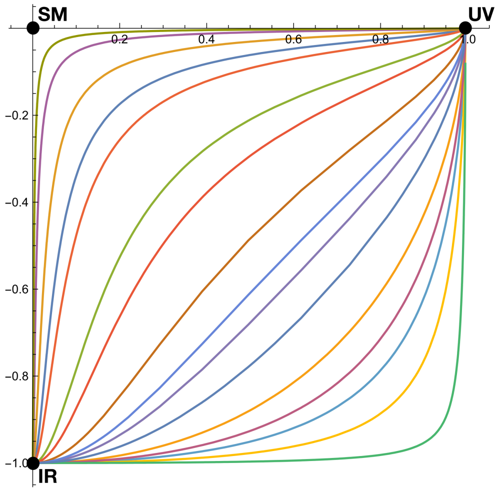
$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.



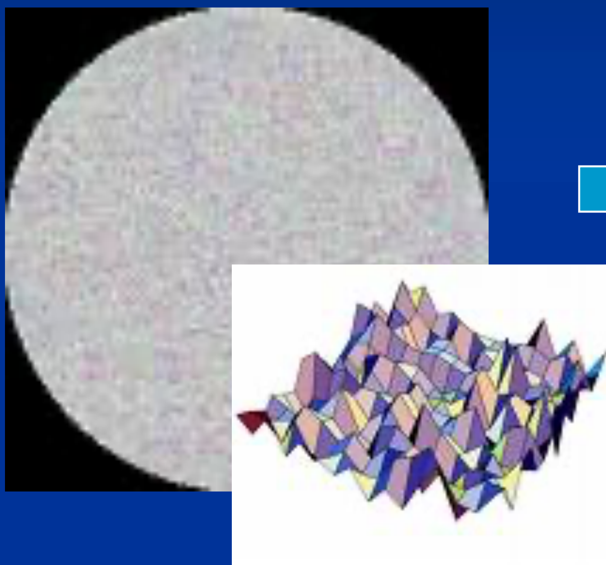
Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

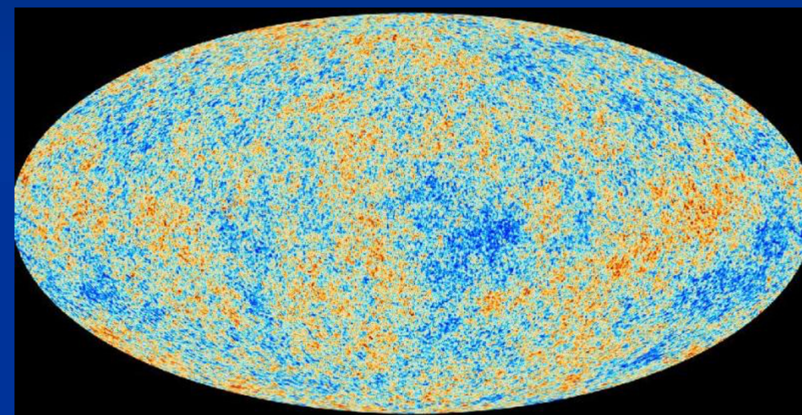
Continuous global symmetry

Approximate scale symmetry near fixed points

UV : approximate scale invariance of primordial fluctuation spectrum from inflation

Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe



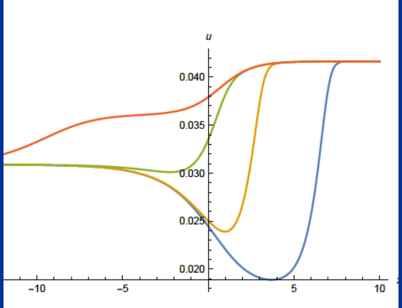


Dynamical dark energy

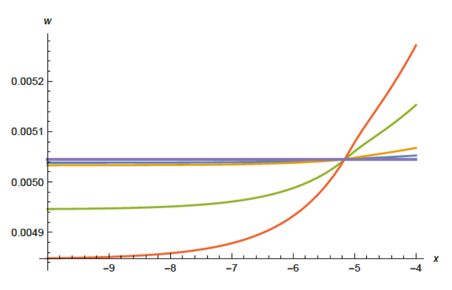
Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \to \frac{u_\infty}{\xi^2 \tilde{\rho}^2} \to \frac{4u_\infty k^4}{\xi^2 \chi^4}$$



vanishes for $\chi \to \infty$!



$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + ck^4 \right\}$$

$k = 2 \cdot 10^{-3}$ eV

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

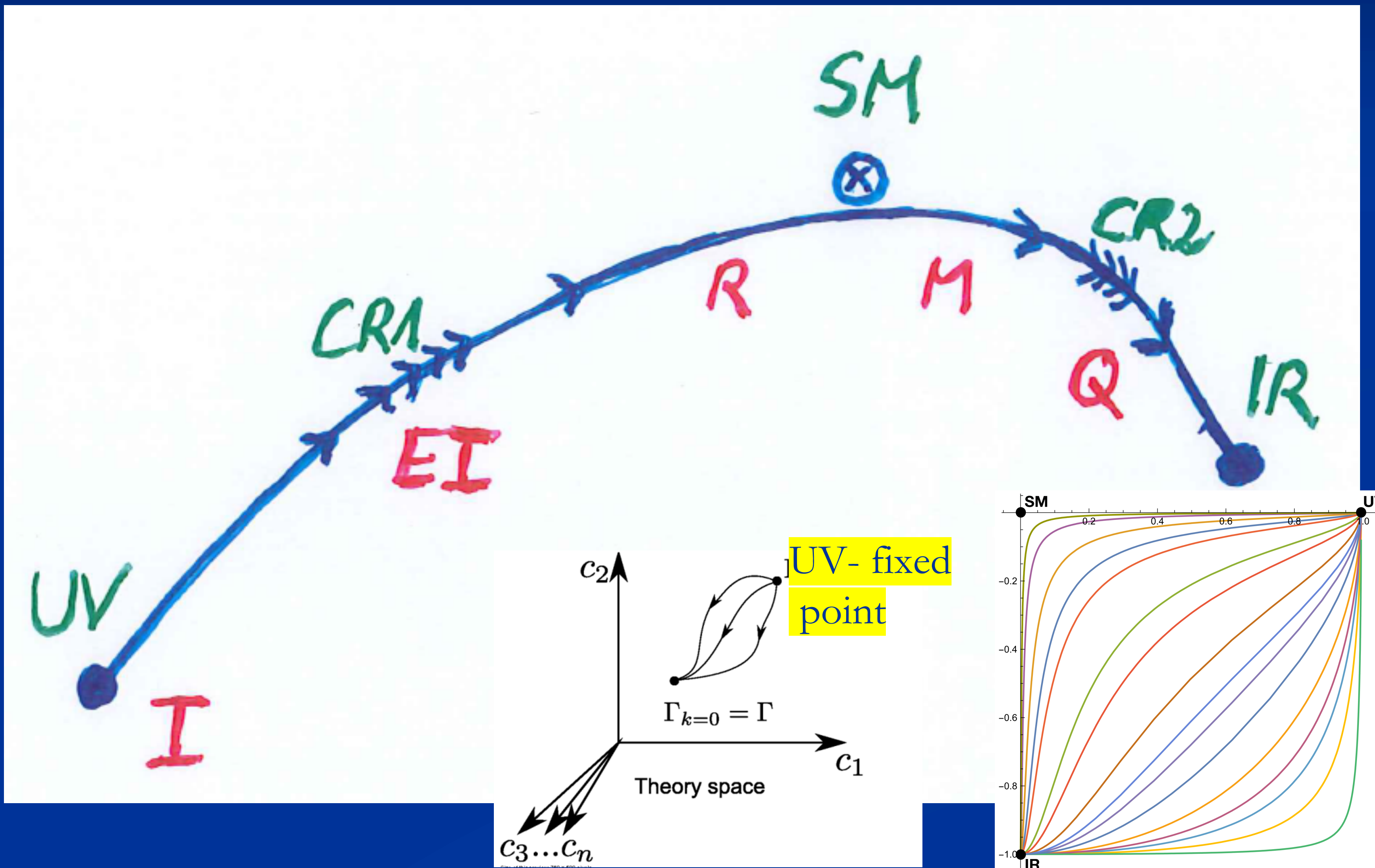
Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications
(different growth of neutrino mass)

Crossover in quantum gravity



Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

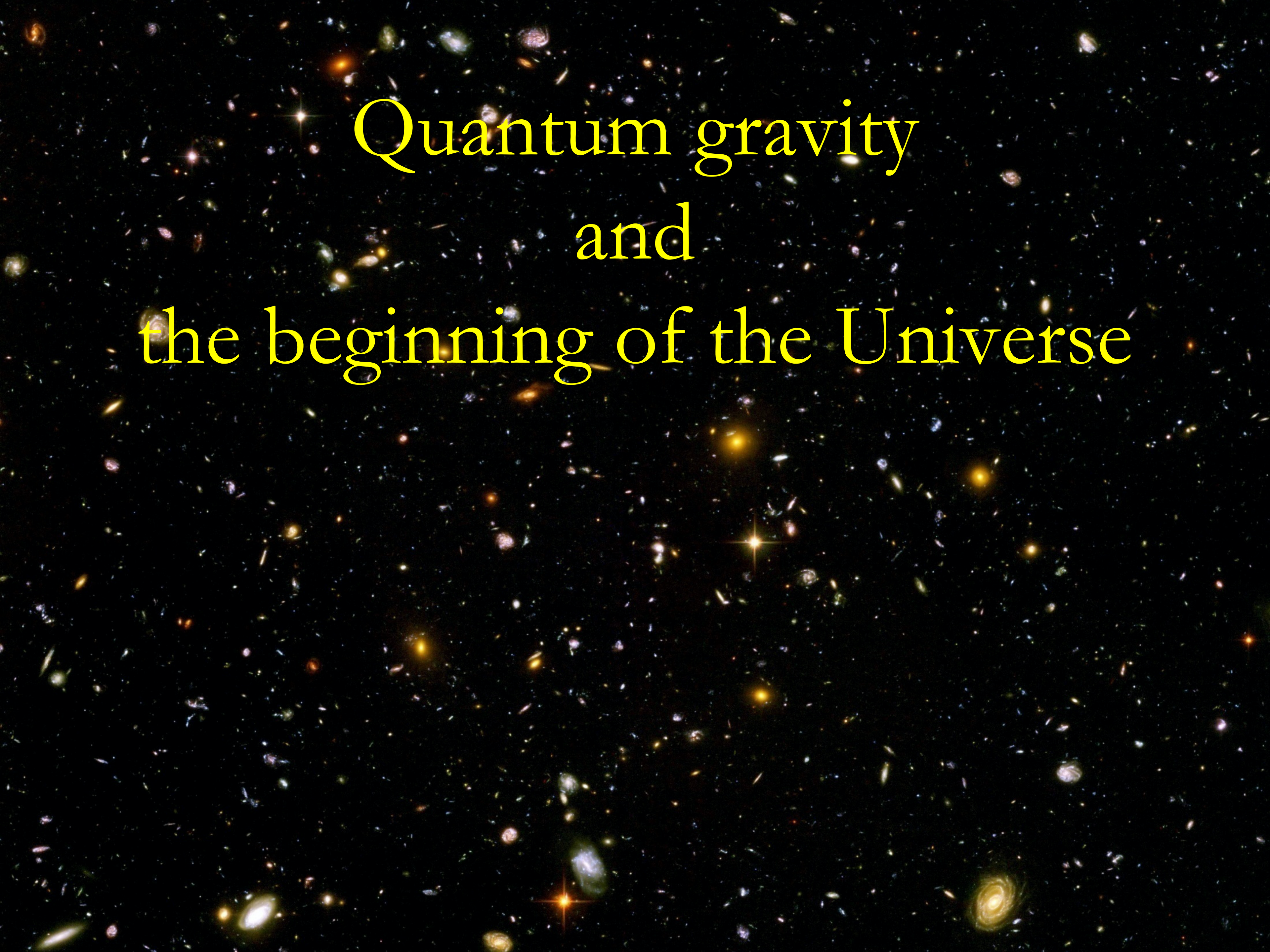
Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Quantum gravity
and
the beginning of the Universe



Beginning of Universe

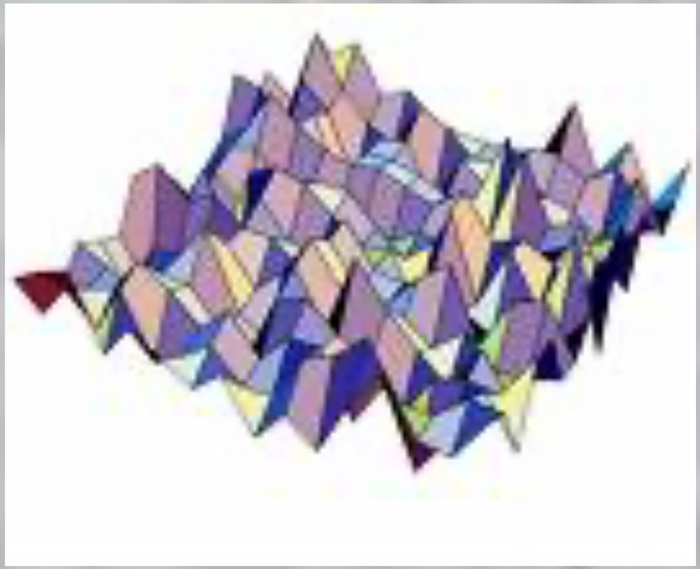
Zu Anfang war die Welt öd und leer und währte ewig.

In the beginning the Universe was empty and lasted since ever.

Eternal light-vacuum

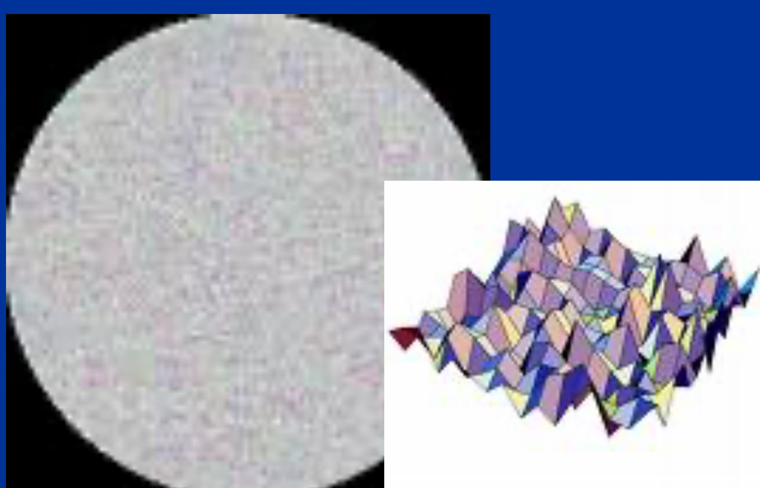
Everywhere almost nothing
only fields and their fluctuations

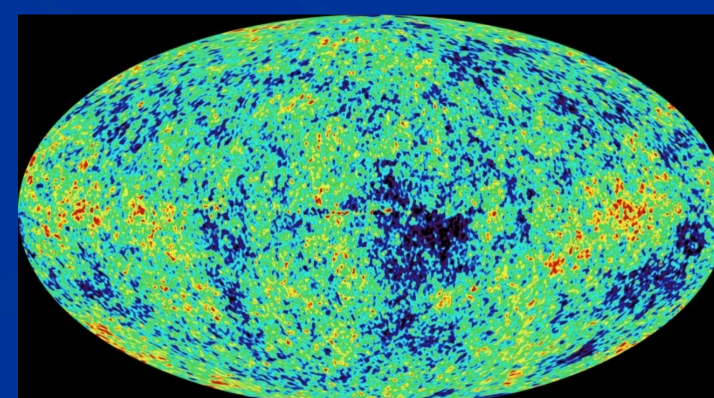
All particles move
with light velocity,
similar to photons



Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.





Einstein frame

- "Weyl scaling" maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale k disappears !
- Standard gravity coupled to scalar field.

- Exact equivalence of different frames !

" different pictures"

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

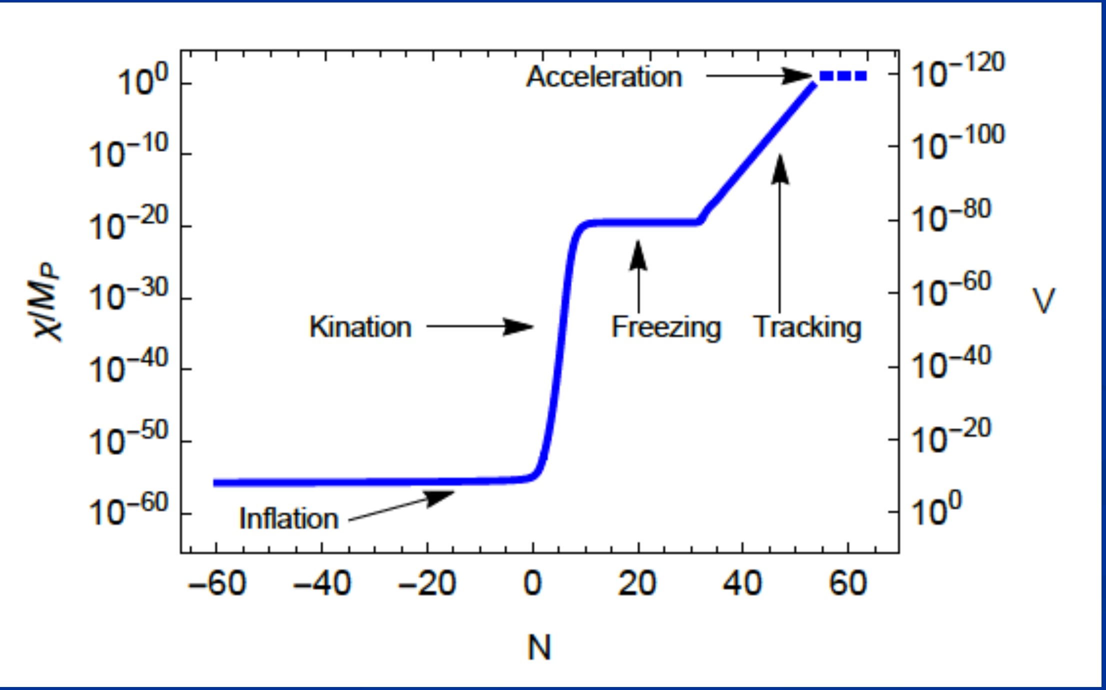
$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



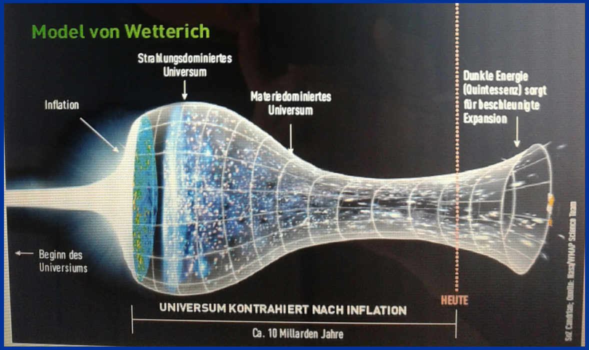
J.Rubio,…

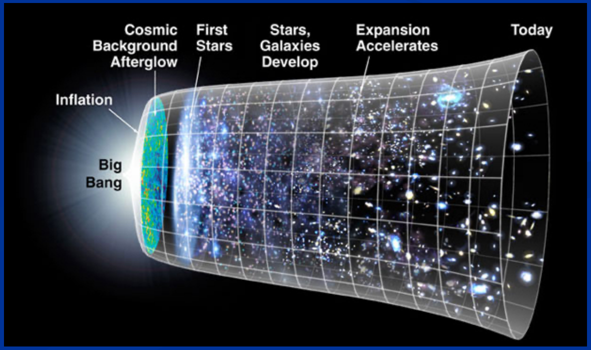
Field relativity

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,
not a coordinate transformation



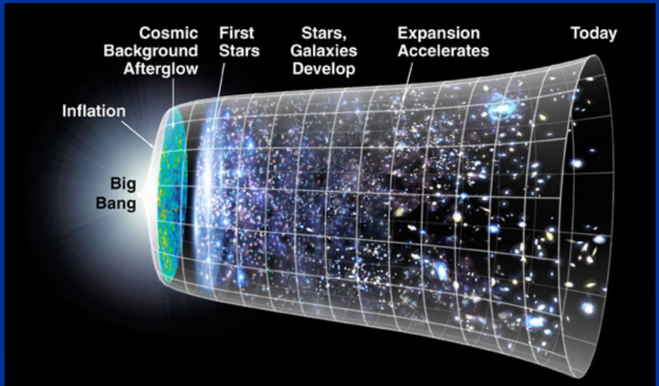


The great emptiness story

In the beginning was light-like emptiness.

The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around 10^{-40} seconds
- start with singularity : our whole observable Universe evolves from one point



Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

- different metrics related by Weyl transformation, which depends on scalar field (inflaton)

Conclusions

- geometry is not fundamental
- emergent general relativity
- interesting and realistic cosmology
- pregeometry: well defined Euclidean action
- flowing coupling functions in QFT :
  in principle calculable !
- candidate for quantum gravity

end

Fundamental scale invariance

- Scaling solution is exact
- All relevant parameters vanish

Predictivity

- Theories with fundamental scale symmetry are very predictive
- Absence of relevant parameters
- New criterion for fundamental theories
- Stronger than renormalizability

Fundamental theory without scale

Fundamental fields are dimensionless $\tilde{\psi}$

Length scale can be introduced for distances, mass=inverse length for derivatives $\hbar = c = 1$

Metric appears as composite object

$\tilde{g}_{\mu\nu} \sim f(\tilde{\psi})\partial_\mu\tilde{\psi}\partial_\nu\tilde{\psi}$ dimension: mass squared

Canonical fields

- Canonical metric is dimensionless
- Introduce renormalisation scale k

$$g_{\mu\nu} = k^{-2}\tilde{g}_{\mu\nu}$$

- Canonical scalar fields have dimension mass

$$\chi = k\tilde{\chi}$$

- General renormalized fields $\varphi_{\mathrm{R},i}(x) = k^{d_i} f_i(k)\tilde{\varphi}_i(x)$