# Quantum gravity from the beginning to the present Universe



#### Extension of general relativity

Einstein's general relativity is insufficient

Early Universe: cannot explain isotropy of primordial density fluctuations
Inflationary Universe
Late cosmology: Cannot explain accelerated expansion ?

(Dynamical ?) dark energy

#### Metric + scalar field

Inflation :
 add scalar field (inflaton)

 Dynamical dark energy or quintessence : add scalar field (cosmon)

inflaton = cosmon ?

### Can observations look back to inflation?

#### **Primordial fluctuations**

- inflaton field :  $\varphi$
- primordial fluctuations of inflaton field become observable in cosmic microwave background



## Amplitude of temperature fluctuations in dependence of size of spots ( angle )



## The structures in the Universe grow from quantum fluctuations during inflation







#### **Cosmological equations**

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$



same equations for inflation and dynamical dark energy

 $\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$ 

 $3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$ 

#### **Potential for inflation**

slow roll Lend of inflation 0

#### Potential for dynamical dark energy



#### Potential for quintessential inflation



#### **Quintessential inflation**



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini, Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio...

#### Prediction of scaling solution for quantum gravity



#### Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral

#### Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors
- Difference: Quantum gravity is not perturbatively renormalizable
- no small coupling, effective coupling  $q^2/M^2$

#### Quantum gravity

Quantum gravity is non-perturbatively renormalizable

Asymptotic safety

Weinberg, Reuter, ...

Uses functional renormalization

#### Asymptotic safety Asymptotic freedom





#### **Graviton fluctuations matter**



#### Quantum gravity needs method to take them into account

#### Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included. The scale k can be momenta, geometric quantities, or just be introduced "by hand".

Flow of k to zero : all fluctuations included, IR Flow of k to infinity : UV

#### Flow of quartic Higgs coupling



$$V(\varphi) = -\mu^2 \varphi^{\dagger} \varphi + \frac{1}{2} \lambda (\varphi^{\dagger} \varphi)^2$$
$$= \frac{1}{2} \lambda (\varphi^{\dagger} \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

#### Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included.

Consider first only fluctuations of metric or graviton :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension  $A \ge 0$ 

 $k \rightarrow 0 \Rightarrow \lambda \rightarrow 0$ 

for constant A :

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu}\right)^{A}$$



Fixed point  

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$
  
 $\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu}\right)^{A}$ 

The quartic scalar coupling  $\lambda$  has a fixed point at  $\lambda=0$ 

For A>0 it flows towards the fixed point as k is lowered: irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

#### Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$
  $A = \frac{1}{48\pi^2 \tilde{M}_p^2} \left[ \frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$   $\partial_t = R$ 

running Planck mass :

$$M_{\rm p}^2(k) = M^2 + \tilde{M}_{\rm p*}^2 k^2$$

dimensionless squared Planck mass

$$\tilde{M}_{\rm p}^2 = \frac{M_{\rm p}^2}{k^2}$$

for length scales smaller than the Planck length: metric fluctuations dominate, constant A

#### Prediction for quartic Higgs coupling

- great desert
- high scale fixed point
- quartic scalar coupling predicted to be very small at transition scale where gravity decouples



#### Prediction of mass of Higgs boson

#### Asymptotic safety of gravity and the Higgs boson mass

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#### Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson  $m_H$  can be predicted. For a positive gravity induced anomalous dimension  $A_{\lambda} > 0$  the running of the quartic scalar self interaction  $\lambda$  at scales beyond the Planck mass is determined by a fixed point at zero. This results in  $m_H = m_{\min} = 126$  GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.



### **Scaling solutions**

#### Ultraviolet fixed point



#### **Renormalizable theories**

1) Ultraviolet fixed point : scaling solution

2) Flow away from fixed point : relevant parameters

#### Scaling solutions

 At fixed point: all (infinitely many) dimensionless couplings take fixed values

Whole scalar potential is fixed, for arbitrary values of scalar field

Functional flow equations are needed

Scaling solutions are restrictive

 Scaling solutions are particular solutions of non-linear differential equations

 In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

#### Scaling solutions and cosmology

 Cosmology involves scalar potentials over large range of field values

Inflaton potential

Higgs potential for Higgs inflation

 Cosmon potential for dynamical dark energy or quintessence

## Quantum gravity : these potentials are not arbitrary

#### Dilaton quantum gravity

#### quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert, Eichhorn, Pauly, Laporte, Pereira, Saueressig, Wang...

# Scaling potential in standard model



FIG. 19. Effective potential u as function of  $x = \ln \tilde{\rho}$  for  $\xi_{\infty} = 0.1$  (blue), 1.0 (orange),  $10^3$  (green) and  $10^4$  (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model,  $N_{\rm S} = 4$ ,  $N_V = 12$ ,  $N_F = 45$ .

u : dimensionless scalar potential u= U/k<sup>4</sup>

x : logarithm of scalar field value
#### Scaling solution : flat potential



# Derivative expansion of effective action

$$\Gamma = \int_{\chi} \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^{\mu} \chi \partial_{\mu} \chi + U(\chi) \right\}$$

### Coefficient of curvature scalar in standard model





w : dimensionless field dependent squared Planck mass w =2 F/ k<sup>2</sup>

non-minimal coupling of scalar field to gravity

#### Scaling solution in Einstein frame



Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu} \quad \varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_{\chi} \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^{\mu} \chi \partial_{\mu} \chi + U(\chi) \right\}$$

$$\Gamma = \int_{\chi} \sqrt{g} \left\{ -\frac{M^2}{2} R' + \frac{1}{2} Z(\varphi) \partial^{\mu} \varphi \partial_{\mu} \varphi + V(\varphi) \right\}$$

$$V(\varphi) = \frac{UM^4}{F^2} \qquad Z(\varphi) = \frac{1}{16} \left\{ \frac{\chi^2 K}{F} + \frac{3}{2} \left( \frac{\partial \ln F}{\partial \ln \chi} \right)^2 \right\}$$

Dimensionless potential ( cosmological "constant" )

Dimensionless ratio

### $V / M^4 = U / F^2$

invariant under Weyl scaling Only this ratio is observable !

#### **Scaling solution**

$$U = u_0 k^4$$

$$F = 2w_0k^2 + \xi\chi^2$$



For low energy standard model :

$$u_{\infty} = \frac{7}{256\pi^2}$$

#### Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right)$$



#### Mass scales in Einstein frame

Renormalization scale k is no longer present Planck mass M not intrinsic: introduced only by change of variables

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right)$$



# Asymptotic solution of cosmological constant problem

$$\begin{split} V = & \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} \\ = & \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right) \end{split}$$



#### **Cosmological solution**

scalar field χ vanishes in the infinite past
 scalar field χ diverges in the infinite future



J.Rubio,...

### Dynamical dark energy

# Asymptotically vanishing cosmological "constant"

What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \to \frac{u_{\infty}}{\xi^2 \tilde{\rho}^2} \to \frac{4u_{\infty}k^4}{\xi^2 \chi^4}$$



vanishes for 
$$\chi \to \infty$$
 !

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + ck^4 \right\}$$

 $k = 2 \cdot 10^{-3} \, eV$ 



#### Dynamical dark energy, generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87 P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87



homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations .... modifications ( different growth of neutrino mass )

#### Metric frames

- "Weyl scaling" maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale k disappears !
  Standard gravity coupled to scalar field.

Exact equivalence of different frames !
 " different pictures"

#### Field relativity

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

#### changes geometry, not a coordinate transformation





# Why do we see redshift of photons emitted in the distant past ?

photons are more red because they have been emitted with longer wavelength



frequency ~ mass wavelength ~ atomsize

### Quantum gravity and

### the beginning of the Universe

#### **Beginning of Universe**

Zu Anfang war die Welt öd und leer und währte ewig.

In the beginning the Universe was empty and lasted since ever.

### Eternal light-vacuum

Everywhere almost nothing only fields and their fluctuations

All particles move with light velocity, similar to photons



#### Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.



#### The great emptiness story

In the beginning was light-like emptiness.

#### The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around 10<sup>-40</sup> seconds
- start with singularity : our whole observable Universe evolves from one point



#### Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

 different metrics related by Weyl transformation, which depends on scalar field (inflaton)

#### Field - singularity

 Big Bang is field - singularity
 similar ( but not identical with ) coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$





#### Conclusions

- Fluctuations of the metric matter
- They influence the behavior of scalar potentials for all field values
- Quantum gravity relevant for early cosmology and late cosmology
- Quantum scale symmetry is central ingredient for understanding cosmology
- Understanding of quantum gravity fluctuations still in beginning stage



#### Quantum scale symmetry

Exactly on fixed point: No parameter with dimension of length or mass is present in the quantum effective action. EVEN NOT k !



Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

#### Approximate scale symmetry near fixed points

UV : approximate scale invariance of primordial fluctuation spectrum from inflation

# Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe









#### Scale symmetry near UV- fixed point

kinetial: 
$$\frac{K}{16\xi} = \kappa \left(\frac{k}{\chi}\right)^{\sigma} + \frac{1}{\alpha^2} - \frac{3}{8}$$
  $Z(\varphi) = \kappa \exp\left(-\frac{\sigma\varphi}{4M}\right) + \frac{1}{\alpha^2(\varphi)}$ 

#### scalar anomalous dimension $\sigma$ , take $\sigma = 2$

$$\Gamma = \int_x \sqrt{g} \left\{ -w_0 k^2 R + \frac{8\xi \kappa k^2}{\chi^2} \partial^\mu \chi \partial_\mu \chi + u_0 k^4 \right\}$$

Weyl scaling  $g'_{\mu\nu} = \frac{k^2}{\chi^2} g_{\mu\nu}$ 

$$\Gamma = \int_{x} \sqrt{g'} \left\{ -w\chi^{2}R' + u\chi^{4} + \frac{1}{2} \left( \frac{\chi^{2}K}{k^{2}} - 12w - 12\frac{\partial w}{\partial \ln \chi} \right) \partial^{\mu}\chi \partial_{\mu}\chi \right\}$$

# Approximate scale symmetry near fixed points

UV : approximate scale invariance of primordial fluctuation spectrum from inflation

 IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

#### Crossover in quantum gravity



Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

# Approximate scale symmetry near fixed points

UV : approximate scale invariance of primordial fluctuation spectrum from inflation

 IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy


simple description of all cosmological epochs

natural incorporation of Dark Energy :
inflation
Early Dark Energy
present Dark Energy dominated epoch

#### Scale symmetric standard model

• Replace all mass scales by scalar field  $\chi$ 

(1) Higgs potential

(2) Strong gauge coupling, normalized at  $\mu =$ 

$$g(\chi) = \bar{g}$$
  $\longrightarrow$   $\Lambda_{\text{QCD}} = \chi \exp\left(-\frac{1}{b_0 \bar{g}^2}\right)$   $b_0 = \frac{1}{16\pi^2} \left(22 - \frac{4}{3}N_f\right)$ 

(3) Similar for all dimensionless couplings

Quantum effective action for standard model doesnot involve intrinsic mass or lengthQuantum scale symmetryCW'87For  $\chi_0 \neq 0$  : massless Goldstone boson

## Fundamental scale invariance

### Fundamental scale invariance

#### Scaling solution

that's it

#### Ultraviolet fixed point



### Fundamental scale invariance

Scaling solution is exact
All relevant parameters vanish

### Predictivity

Theories with fundamental scale symmetry are very predictive
Absence of relevant parameters
New criterion for fundamental theories
Stronger than renormalizability

### Scaling solutions and inflation

- Scaling solutions are interesting for inflation:
- flat potential
- non minimal scalar gravity coupling
   general properties do not depend on particular representation of scalar field
   scalar singlet = inflaton

# Higgs inflation not compatible with asymptotic safety



FIG. 23. Potential in the Einstein frame  $V_E$  as function of  $x = \ln \tilde{\rho}$ , for  $\xi_{\infty} = 2 \cdot 10^{-5}$  (blue),  $10^{-4}$  (orange),  $10^{-3}$  (green), 0.003 (red), from right to left.

# Scaling potential for GUT with non-zero gauge coupling



FIG. 9. Effective potential u(x) as function of  $x = \ln(\tilde{\rho})$ . Parameters are N = 20,  $\bar{N}_V = 3$ ,  $c_g = 1$ ,  $w_0 = 0.05$ ,  $\alpha = g^2/4\pi = 1/40$ . The initial conditions for the four curves from up to down are u(x = 0) = 0.037923, u(x = 0) = 0.0335, u(x = 0) = 0.25 and u(x = 0) = 0.24447. The initial values for the upper and lower curves limit the interval for which a scaling solution is found. For these solutions one has  $A_0 = 2.91$ .

Spontaneous symmetry breaking by scaling solution near Planck scale