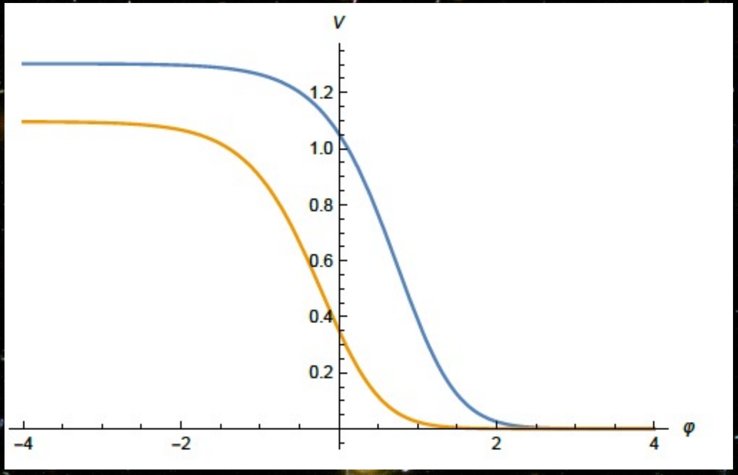
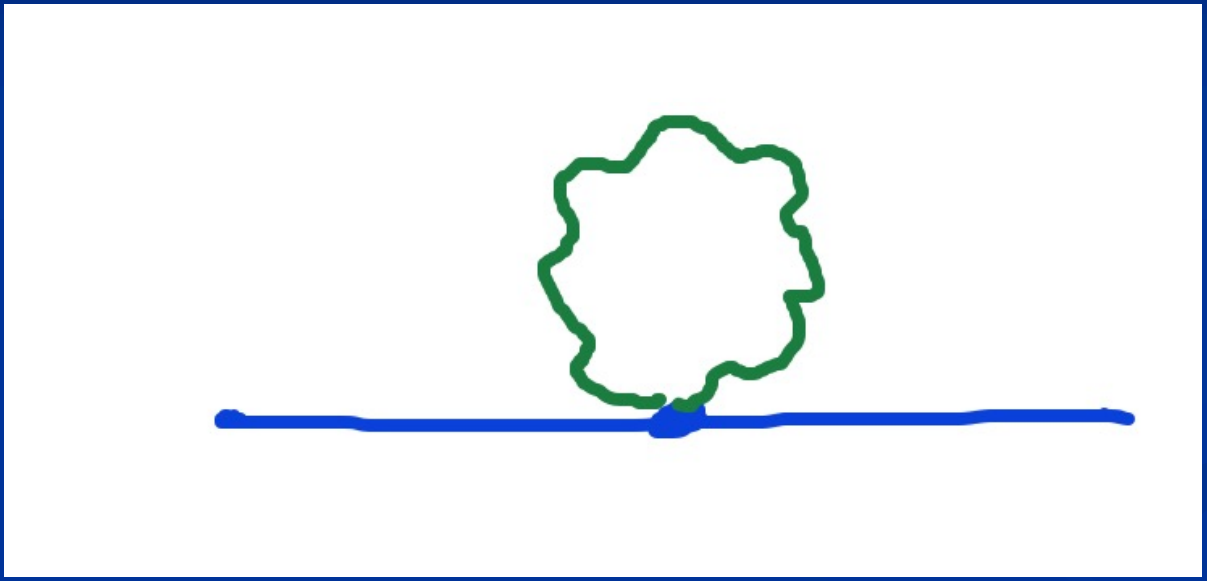
Quantum gravity from the beginning to the present Universe



Graviton fluctuations matter



Quantum gravity needs method to take them into account

Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral

Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors
- Difference: Quantum gravity is not perturbatively renormalizable
- no small coupling, effective coupling q^2/M^2

Quantum gravity

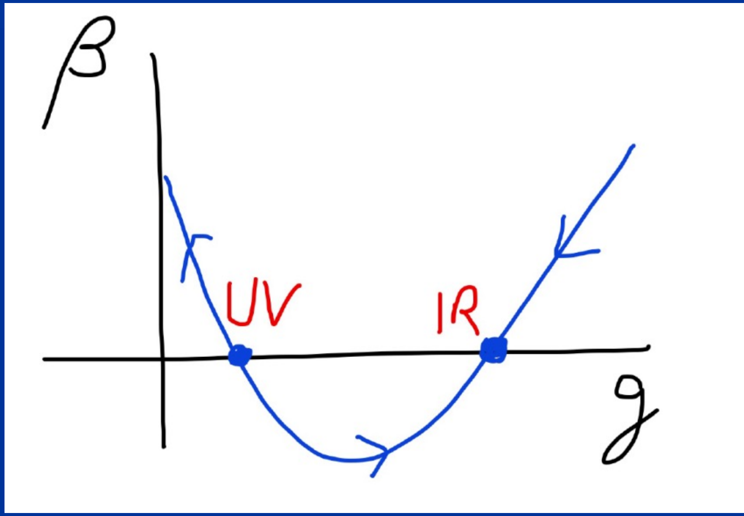
Quantum gravity is
non-perturbatively renormalizable

Asymptotic safety

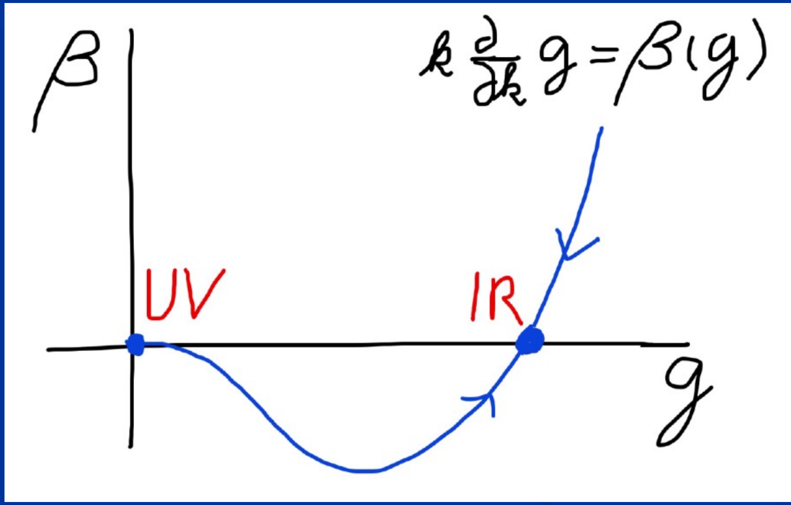
Weinberg, Reuter, …

Uses functional renormalization

Asymptotic safety



Asymptotic freedom



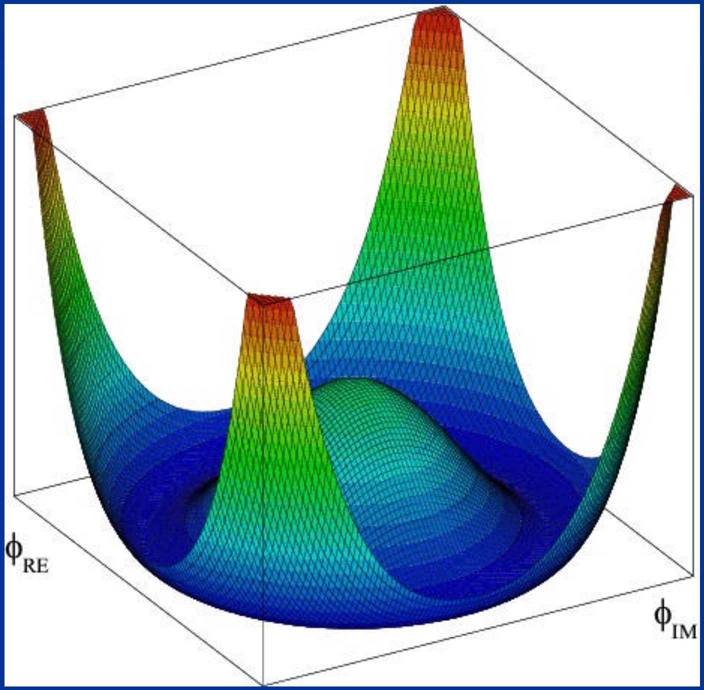
Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included. The scale k can be momenta, geometric quantities, or just be introduced "by hand".

Flow of k to zero : all fluctuations included, IR
Flow of k to infinity : UV

Flow of quartic Higgs coupling



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \tfrac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \tfrac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included.

Consider first only fluctuations of metric or graviton :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced
anomalous dimension
$A > 0$

for
constant A :

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

$$k \to 0 \quad \Rightarrow \quad \lambda \to 0$$

Quantum gravity flattens scalar potentials

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

The quartic scalar coupling λ has a
fixed point at λ=0

For A>0 it flows towards the fixed point as k is lowered: irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_{\mathrm{p}}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t = k\partial_k$$

running Planck mass :

$$M_{\mathrm{p}}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$

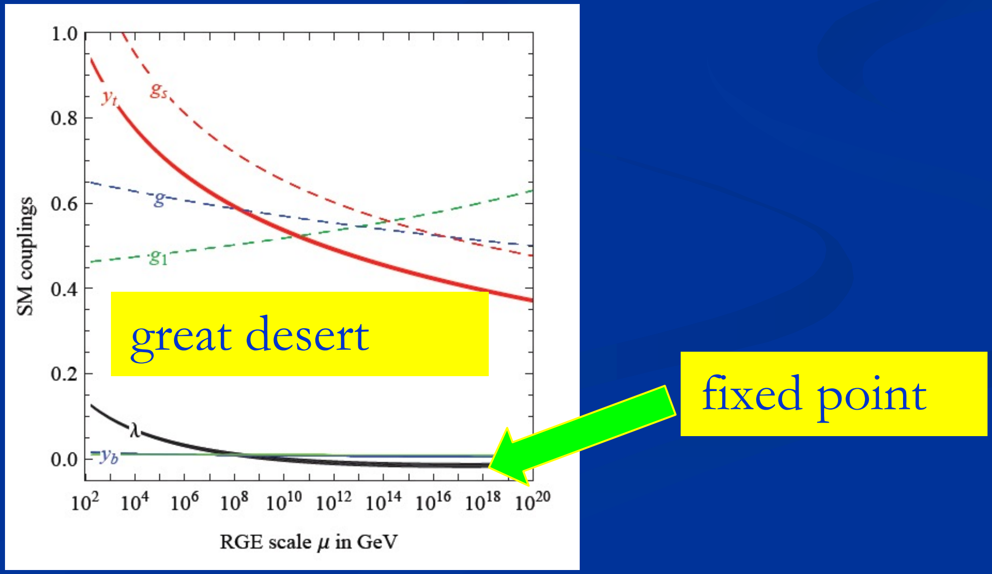
dimensionless
squared Planck mass

$$\tilde{M}_{\mathrm{p}}^2 = \frac{M_{\mathrm{p}}^2}{k^2}$$

for length scales smaller than the Planck length:
metric fluctuations dominate, constant A

Prediction for quartic Higgs coupling

- great desert
- high scale fixed point
- quartic scalar coupling predicted to be very small at transition scale where gravity decouples



Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

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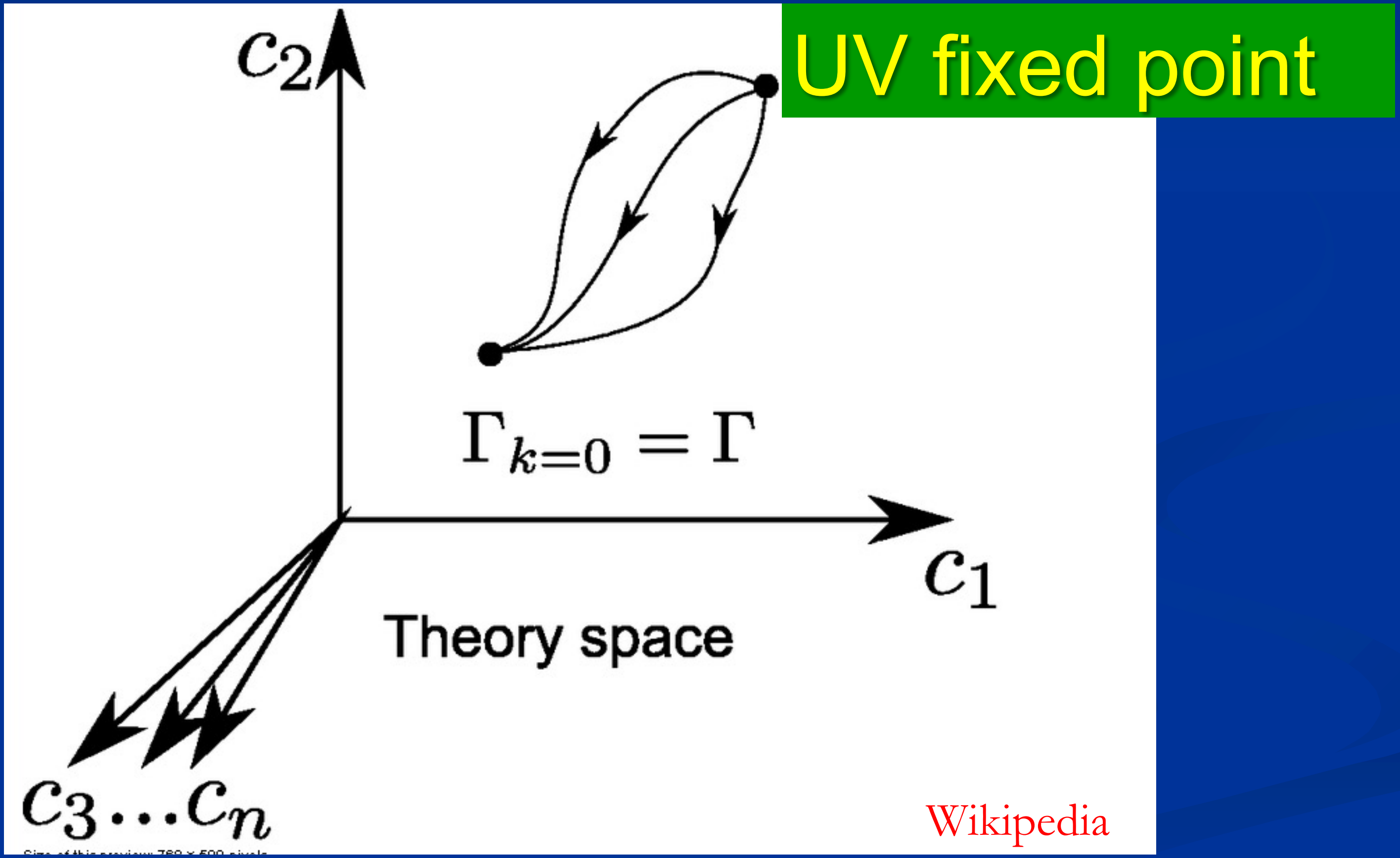
Scaling solutions

Ultraviolet fixed point



Renormalizable theories

1) Ultraviolet fixed point :

scaling solution

2) Flow away from fixed point :

relevant parameters

Scaling solutions

- At fixed point: all (infinitely many) dimensionless couplings take fixed values
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

Scaling solutions and cosmology

- Cosmology involves scalar potentials over large range of field values
- Inflaton potential
- Higgs potential for Higgs inflation
- Cosmon potential for dynamical dark energy or quintessence

Quantum gravity :
these potentials are not arbitrary

Dilaton quantum gravity

quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert, Eichhorn, Pauly, Laporte, Pereira, Saueressig, Wang…

Scaling potential in standard model

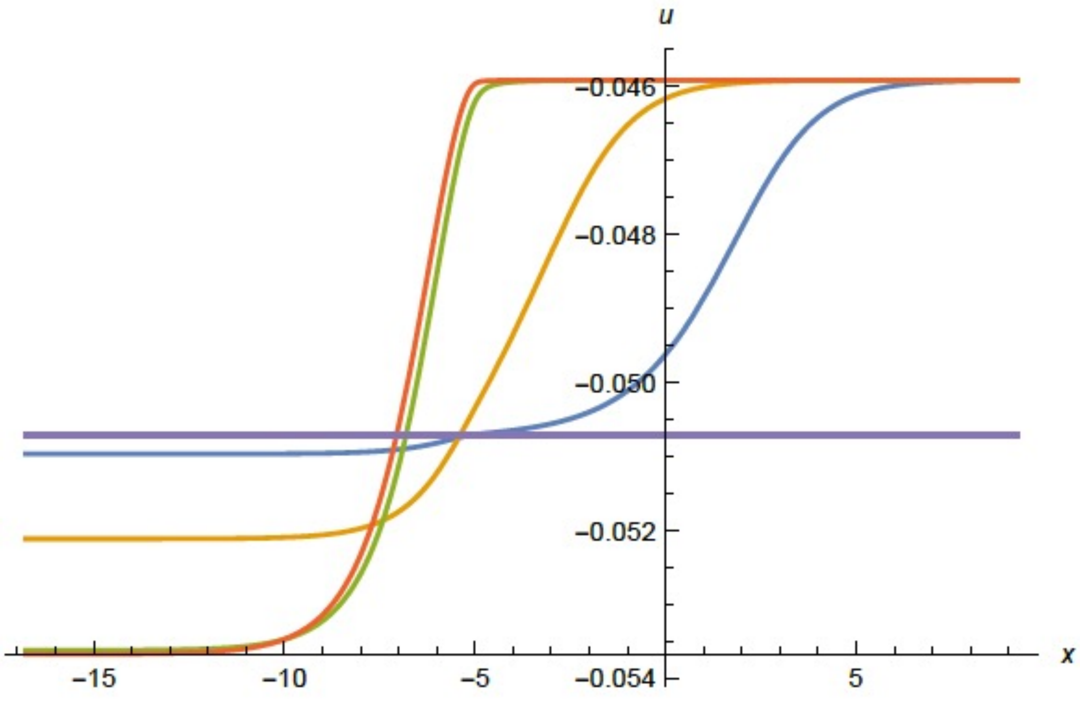
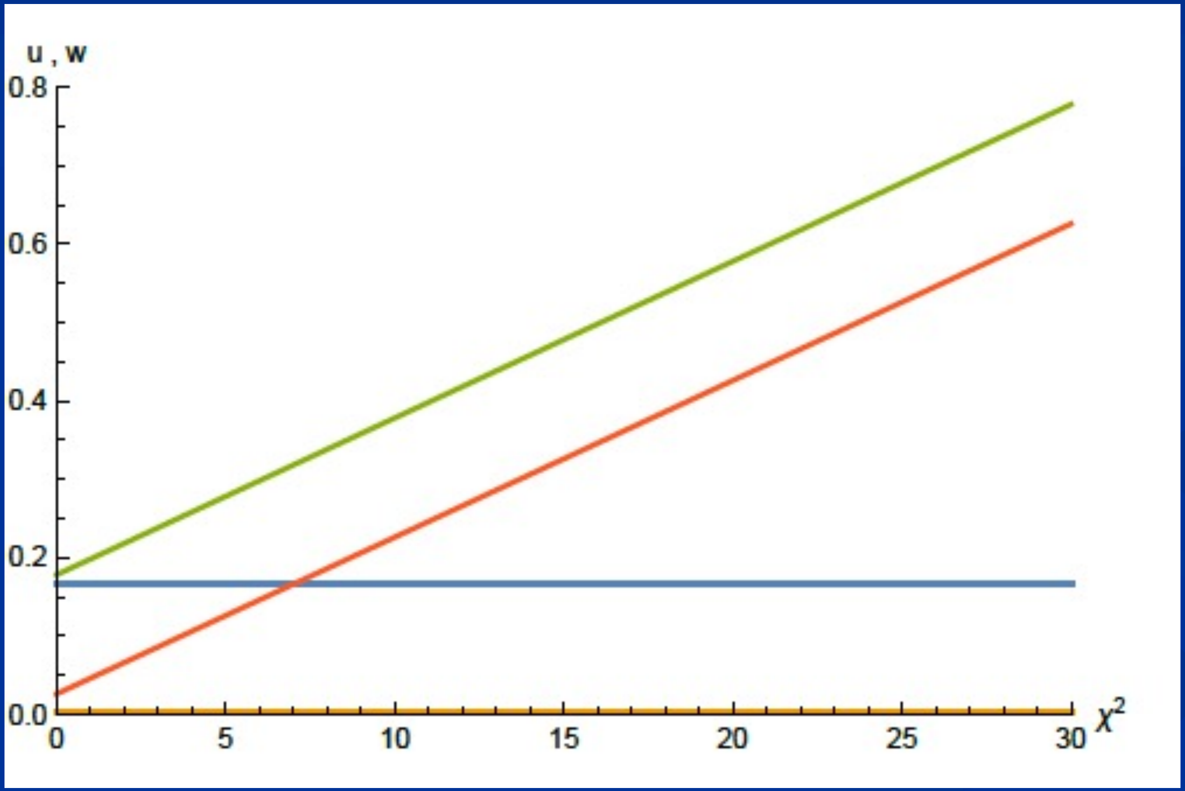


FIG. 19. Effective potential u as function of $x = \ln \tilde{\rho}$ for $\xi_\infty =$ 0.1 (blue), 1.0 (orange), 10^3 (green) and 10^4 (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model, $N_S = 4$, $N_V = 12$, $N_F = 45$.

u : dimensionless scalar potential $u = U/k^4$

x : logarithm of scalar field value

Scaling solution : flat potential



Derivative expansion of effective action

$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

variable gravity

Coefficient of curvature scalar in standard model

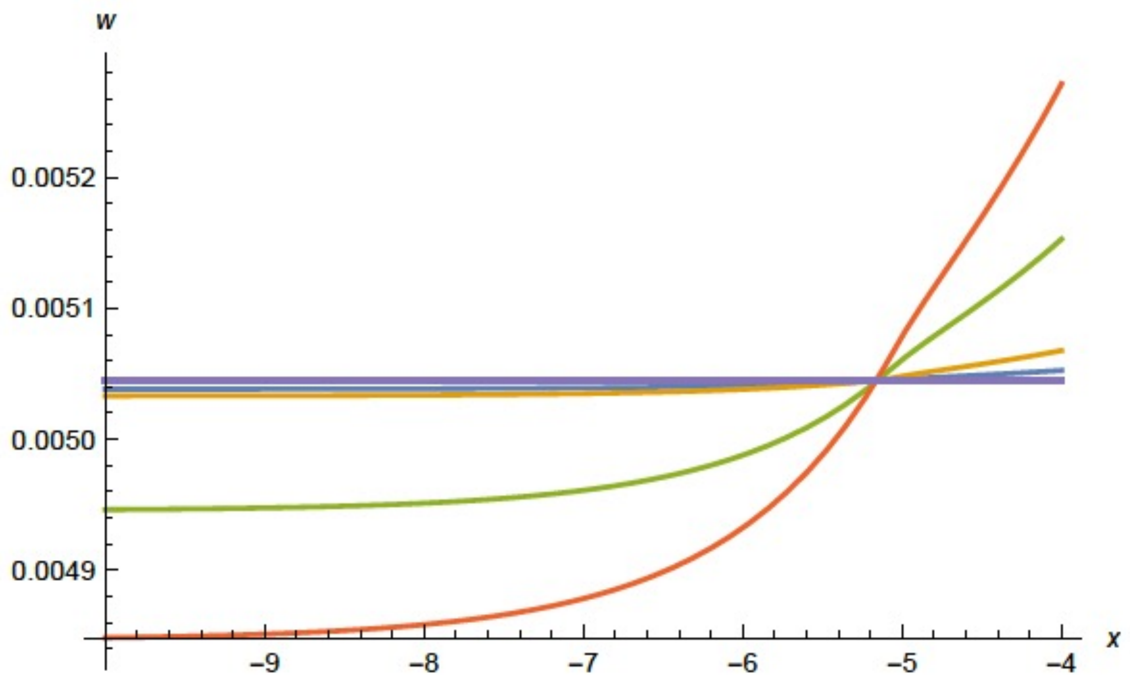


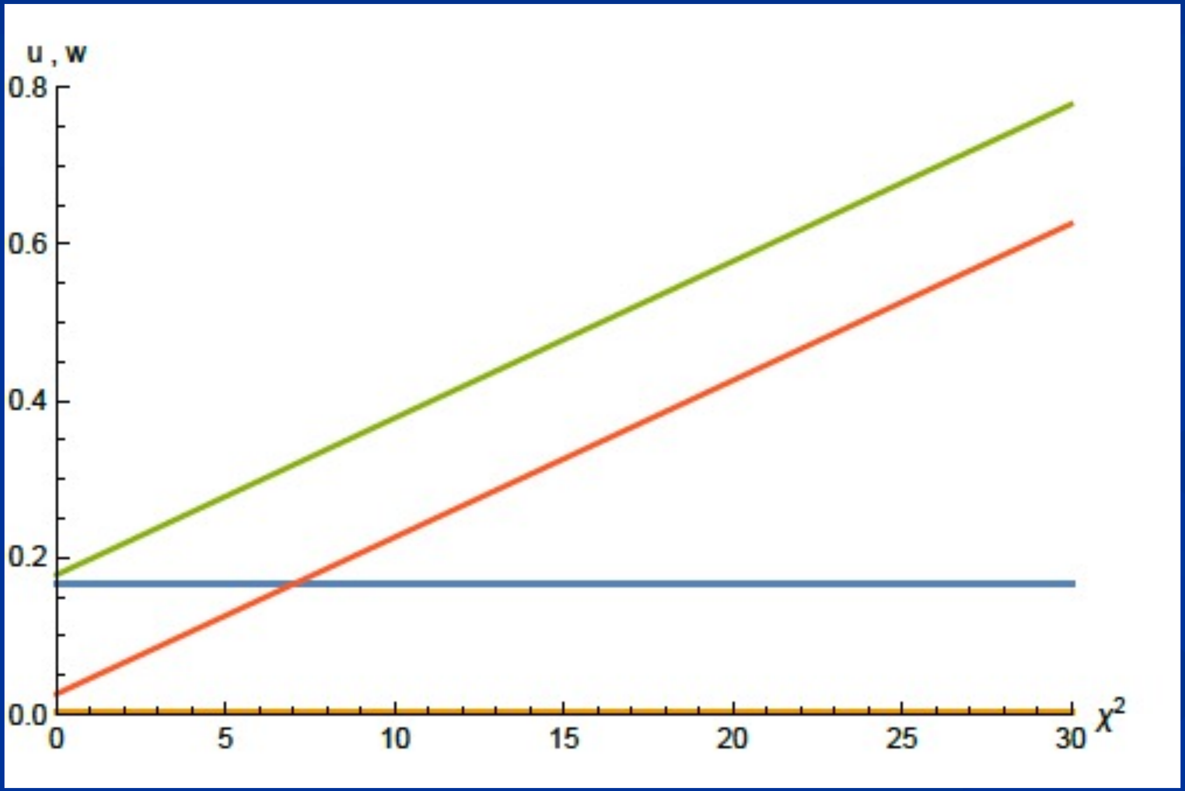
FIG. 21. Dimensionless squared Planck mass w as function of $x = \ln \tilde{\rho}$ for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for $\xi_\infty \to 0$. All curves meet in a common point at $x \approx -5.05$.

w : dimensionless field dependent squared Planck mass

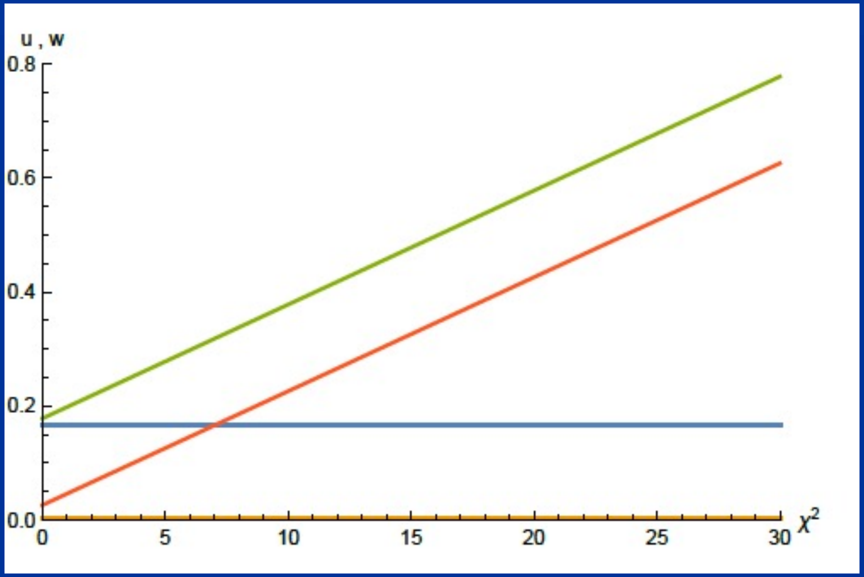
$w = 2\, F / k^2$

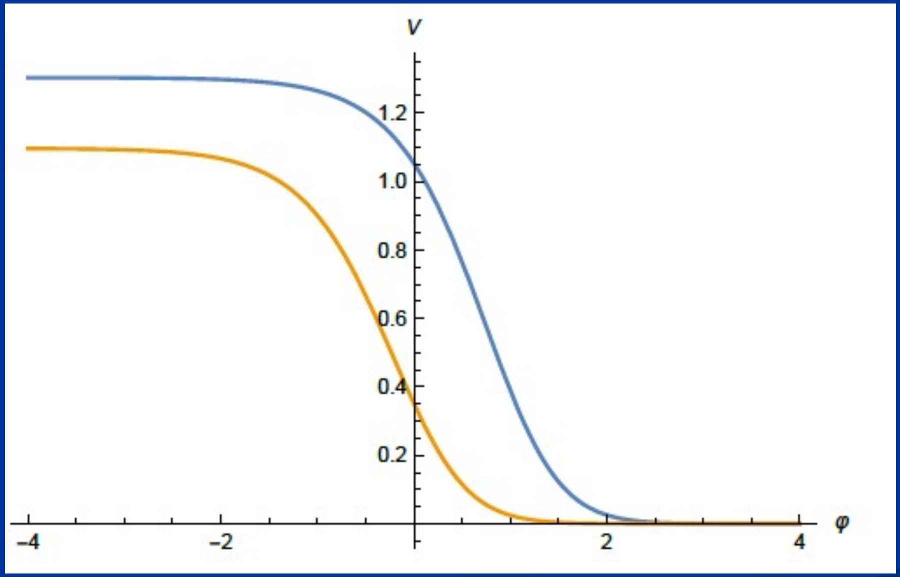
non-minimal coupling of scalar field to gravity

Scaling solution : flat potential, non-minimal scalar- gravity coupling

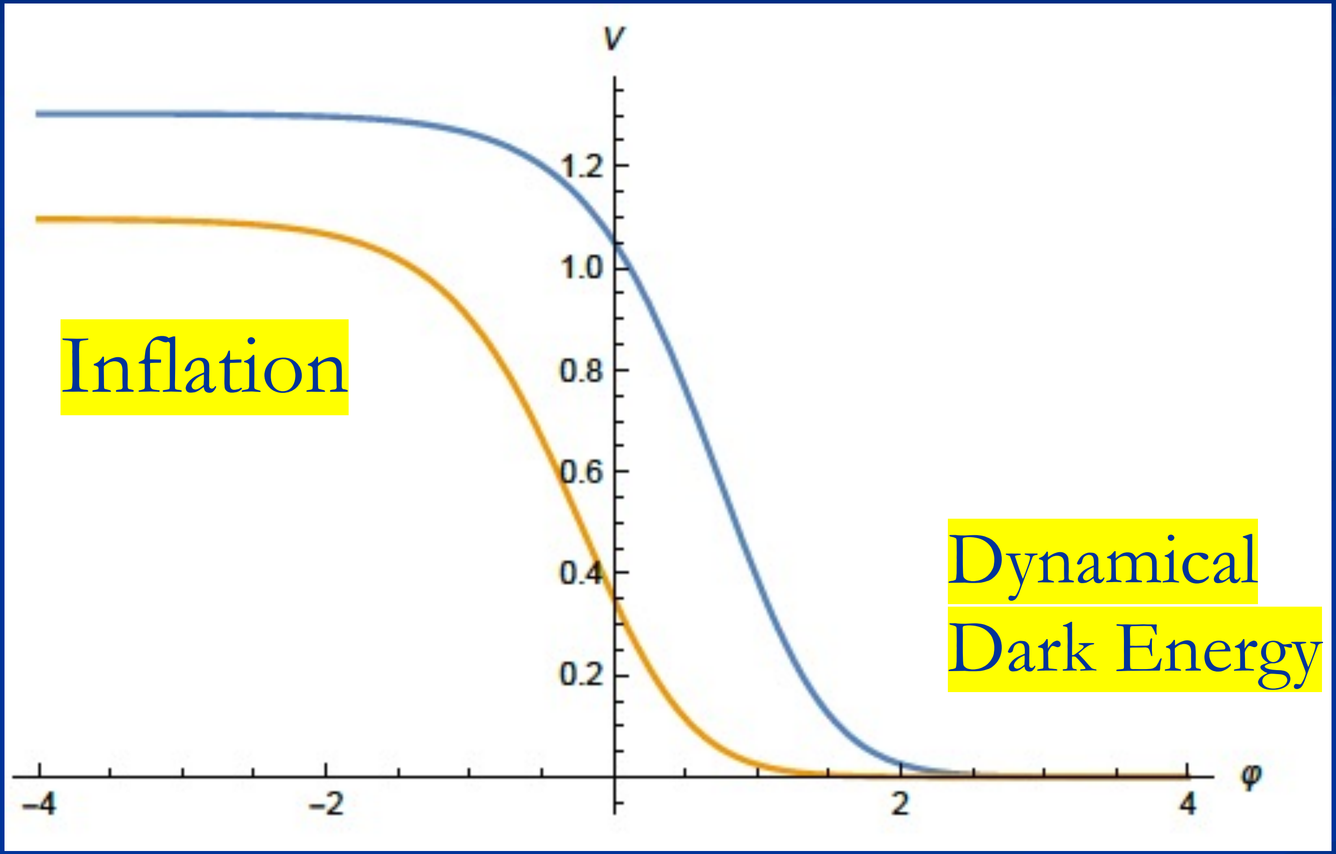


Scaling solution in Einstein frame





Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini, Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu} \qquad \varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_\chi \sqrt{g}\Big\{-\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

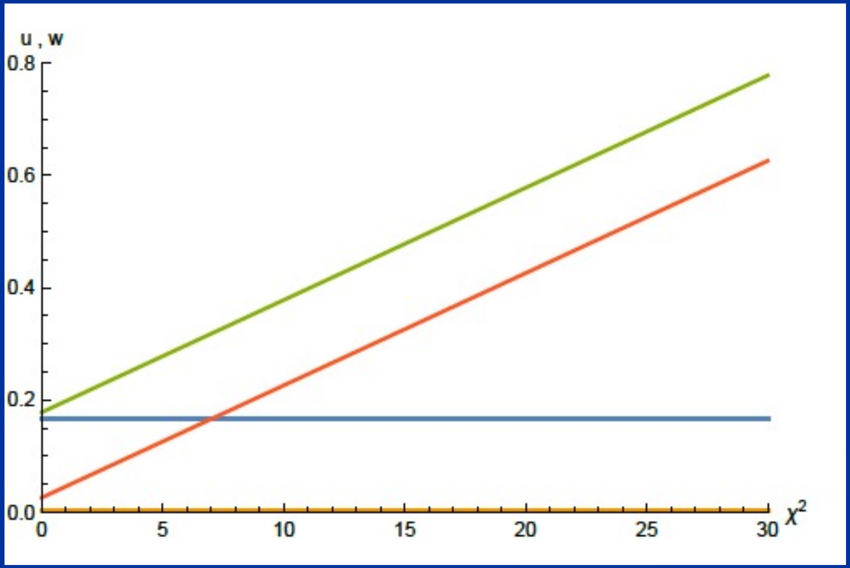
$$\Gamma = \int_\chi \sqrt{g}\Big\{-\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\Big\}$$

$$V(\varphi) = \frac{UM^4}{F^2} \qquad Z(\varphi) = \frac{1}{16}\Big\{\frac{\chi^2K}{F} + \frac{3}{2}\Big(\frac{\partial\ln F}{\partial\ln\chi}\Big)^2\Big\}$$

Scaling solution

$$U = u_0 k^4$$

$$F = 2 w_0 k^2 + \xi \chi^2$$

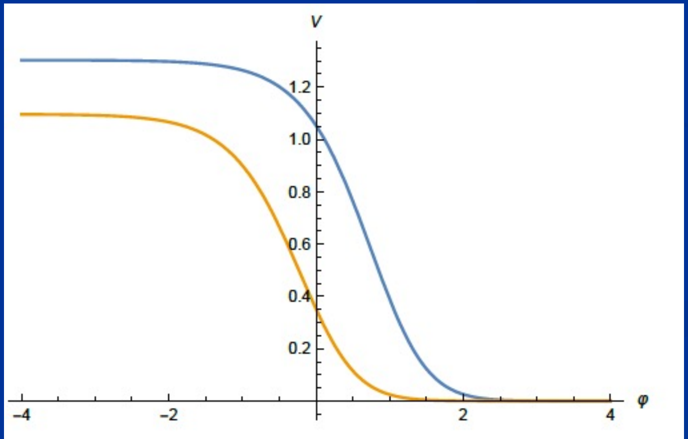


For low energy standard model :

$$u_\infty = \frac{7}{256\pi^2}$$

Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$

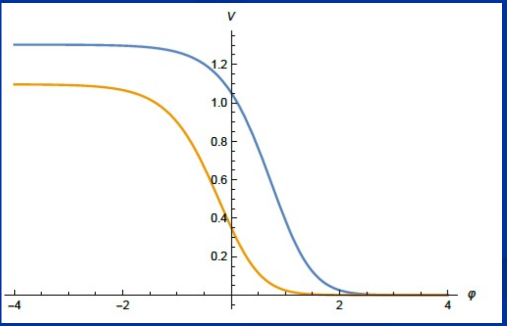


Mass scales in Einstein frame

Renormalization scale k is no longer present

Planck mass M not intrinsic: introduced only by change of variables

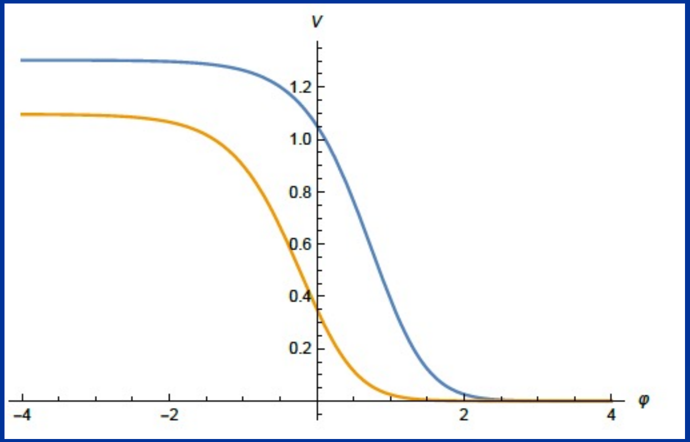
$$
\begin{aligned}
V &= \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} \\
&= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)
\end{aligned}
$$



Asymptotic solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$

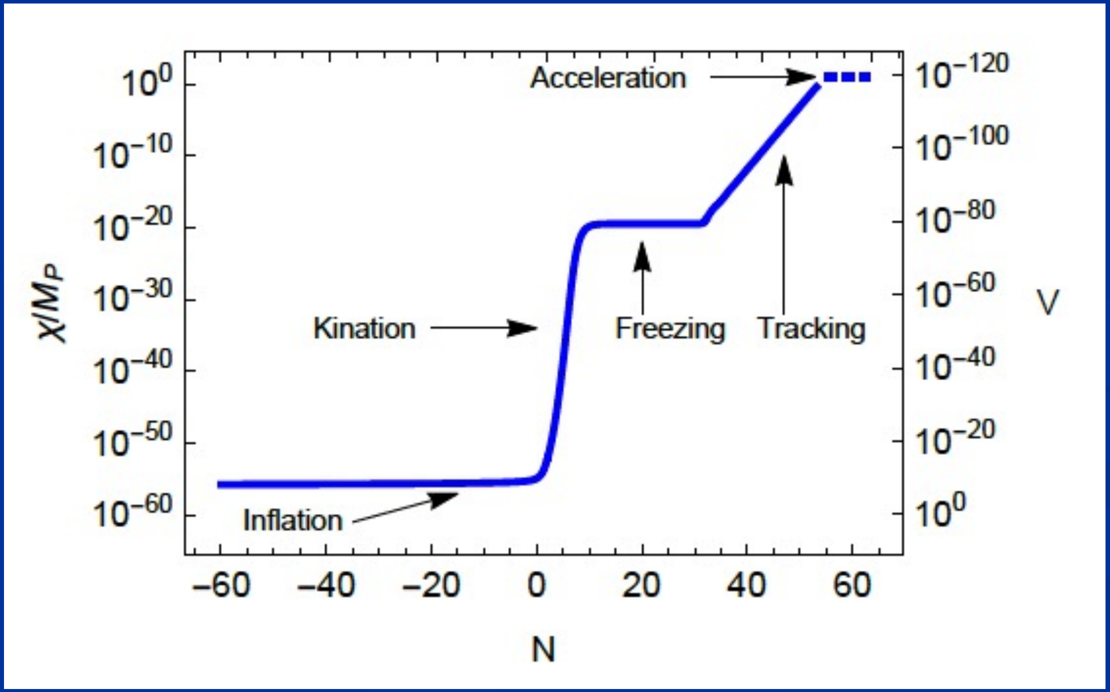
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !

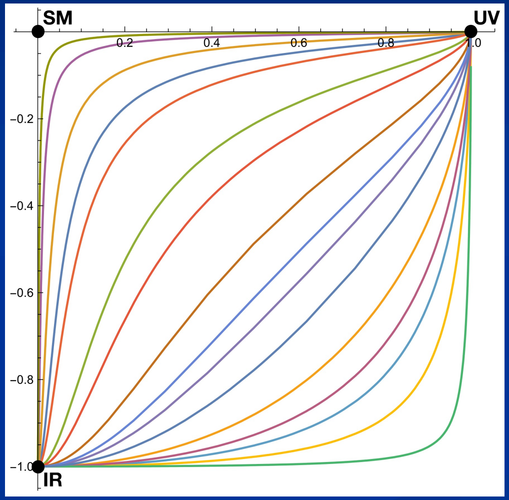
Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Quantum scale symmetry



Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.
EVEN NOT k !

Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Spontaneous breaking of scale symmetry

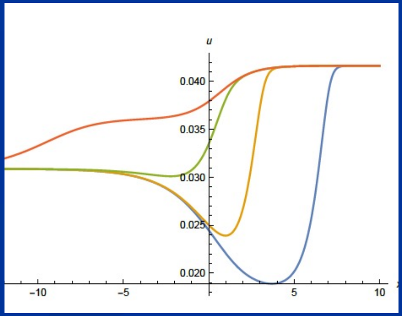
- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Dynamical dark energy

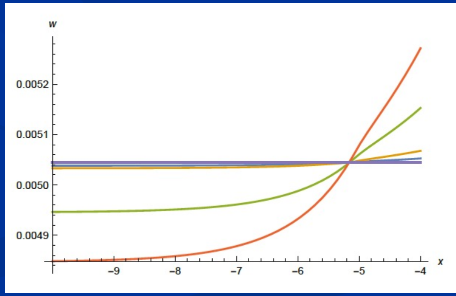
Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \to \frac{u_\infty}{\xi^2 \tilde{\rho}^2} \to \frac{4u_\infty k^4}{\xi^2 \chi^4}$$



vanishes for $\chi \to \infty$!



$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + c k^4 \right\}$$

k= 2 • 10^{-3} eV

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87

P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

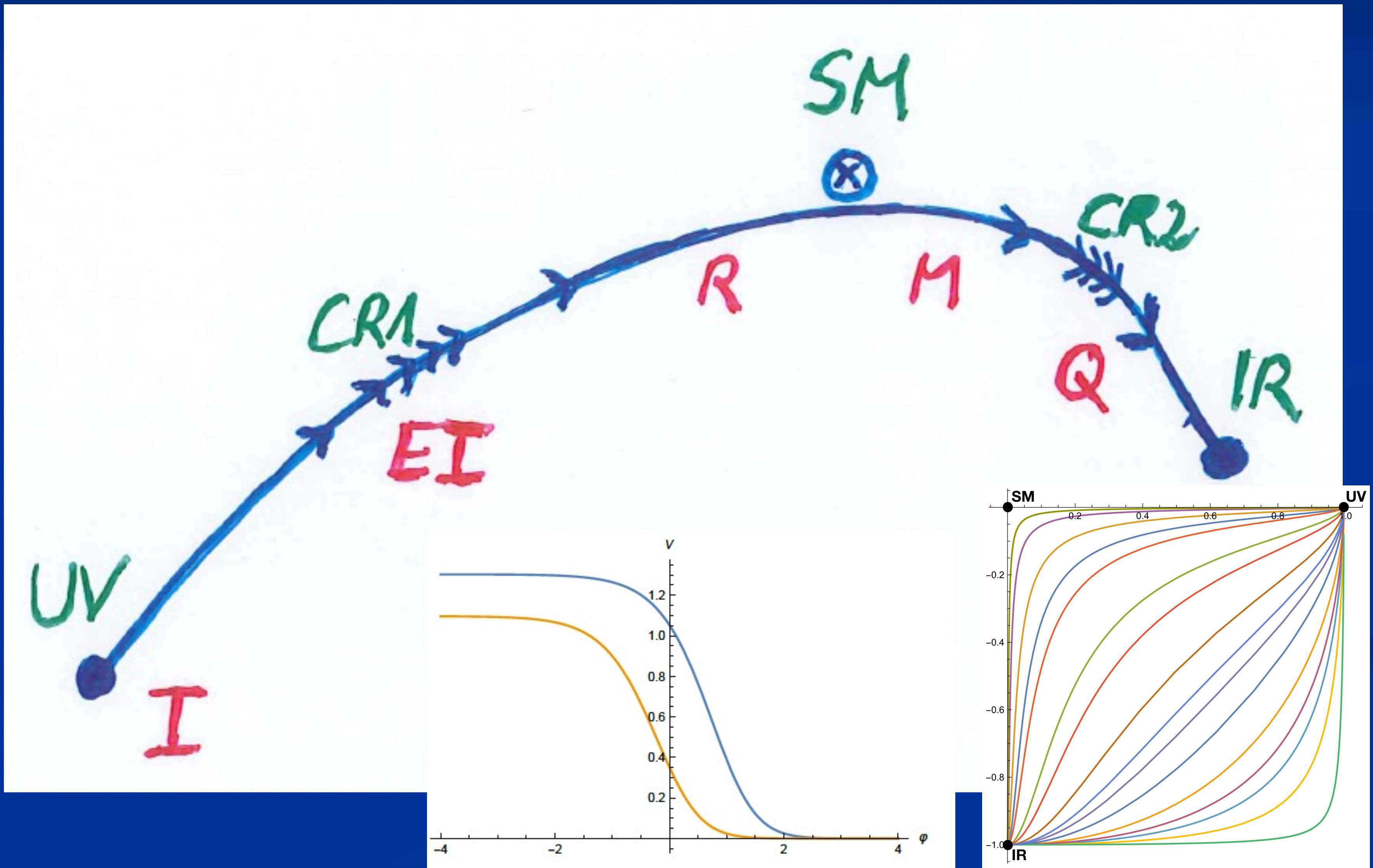
- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Crossover in quantum gravity



Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

Metric frames

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale k disappears !
- Standard gravity coupled to scalar field.

- Exact equivalence of different frames !

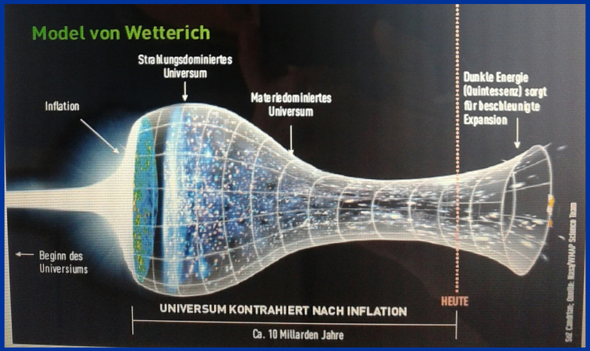
“ different pictures”

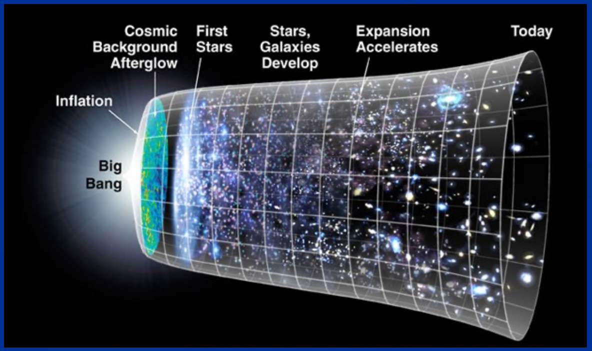
Field relativity

Weyl scaling :

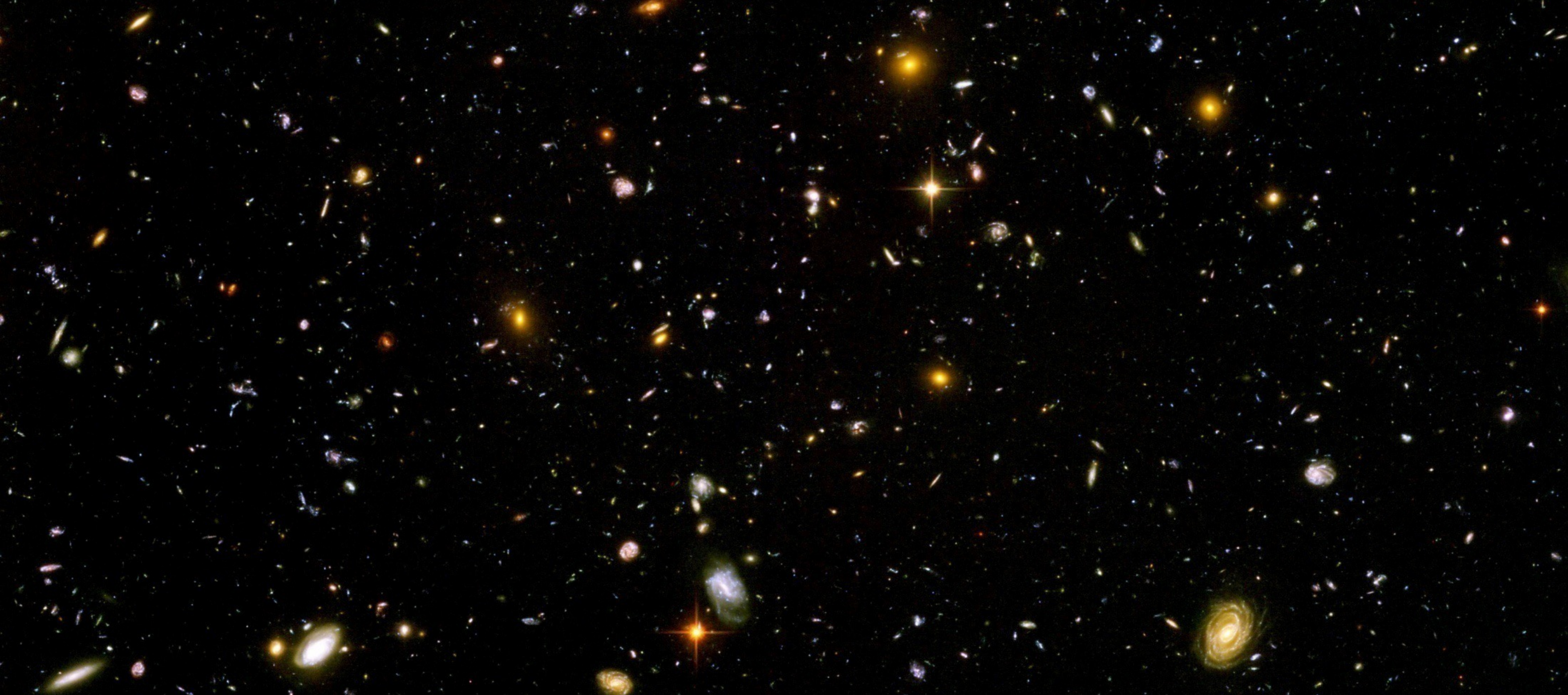
$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,
not a coordinate transformation





Quantum gravity
and
the beginning of the Universe



Beginning of Universe

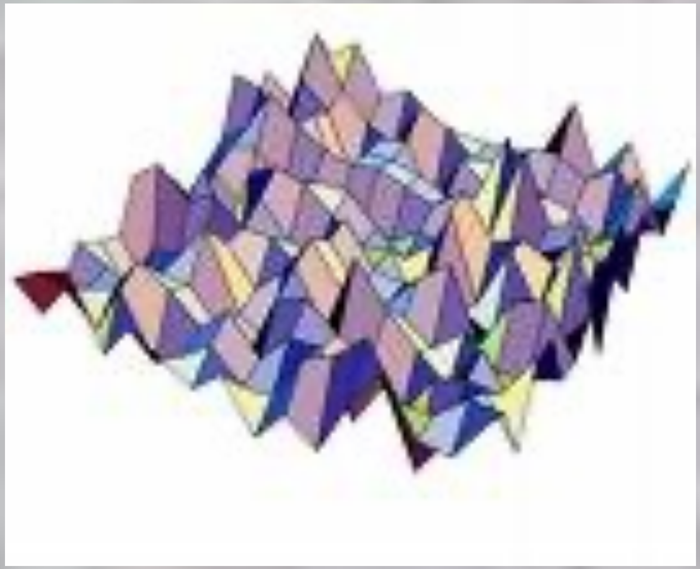
Zu Anfang war die Welt öd und leer und währte ewig.

In the beginning the Universe was empty and lasted since ever.

Eternal light-vacuum

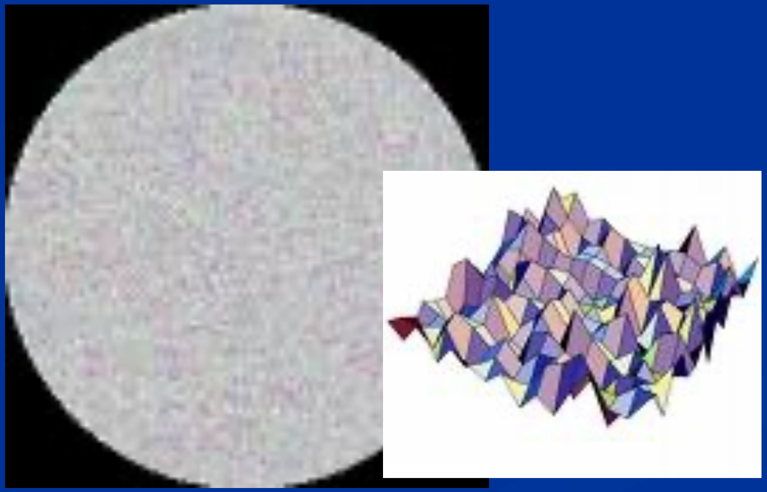
Everywhere almost nothing
only fields and their fluctuations

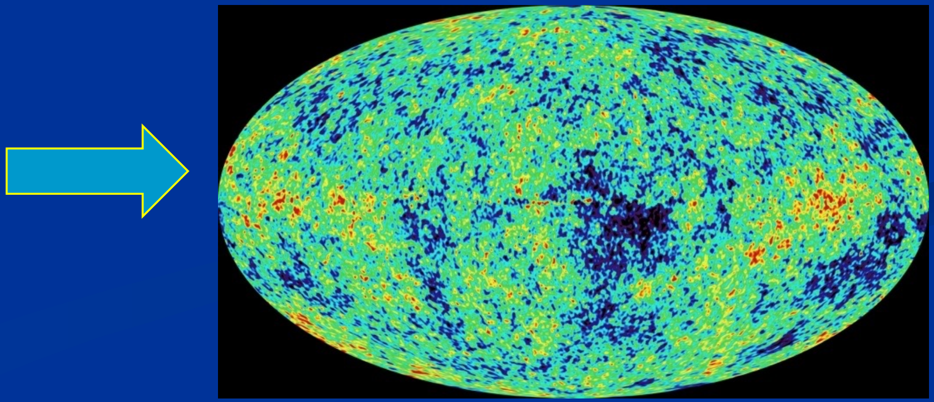
All particles move
with light velocity,
similar to photons



Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.



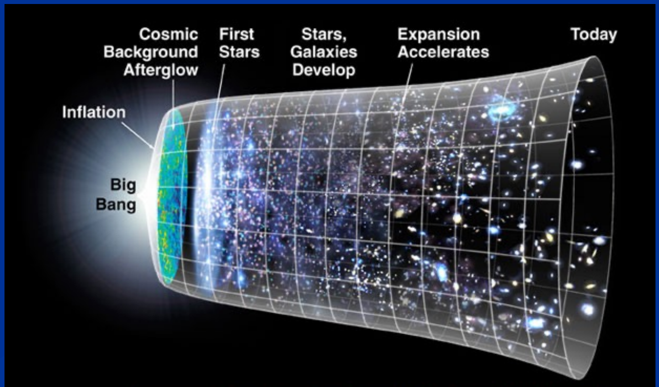


The great emptiness story

In the beginning was light-like emptiness.

The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around 10^{-40} seconds
- start with singularity : our whole observable Universe evolves from one point



Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

- different metrics related by Weyl transformation, which depends on scalar field (inflaton)

Field - singularity

- Big Bang is field - singularity
- similar (but not identical with) coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

Conclusions

- Fluctuations of the metric matter
- They influence the behavior of scalar potentials for all field values
- Quantum gravity relevant for early cosmology and late cosmology
- Quantum scale symmetry is central ingredient for understanding cosmology
- Understanding of quantum gravity fluctuations still in beginning stage

end

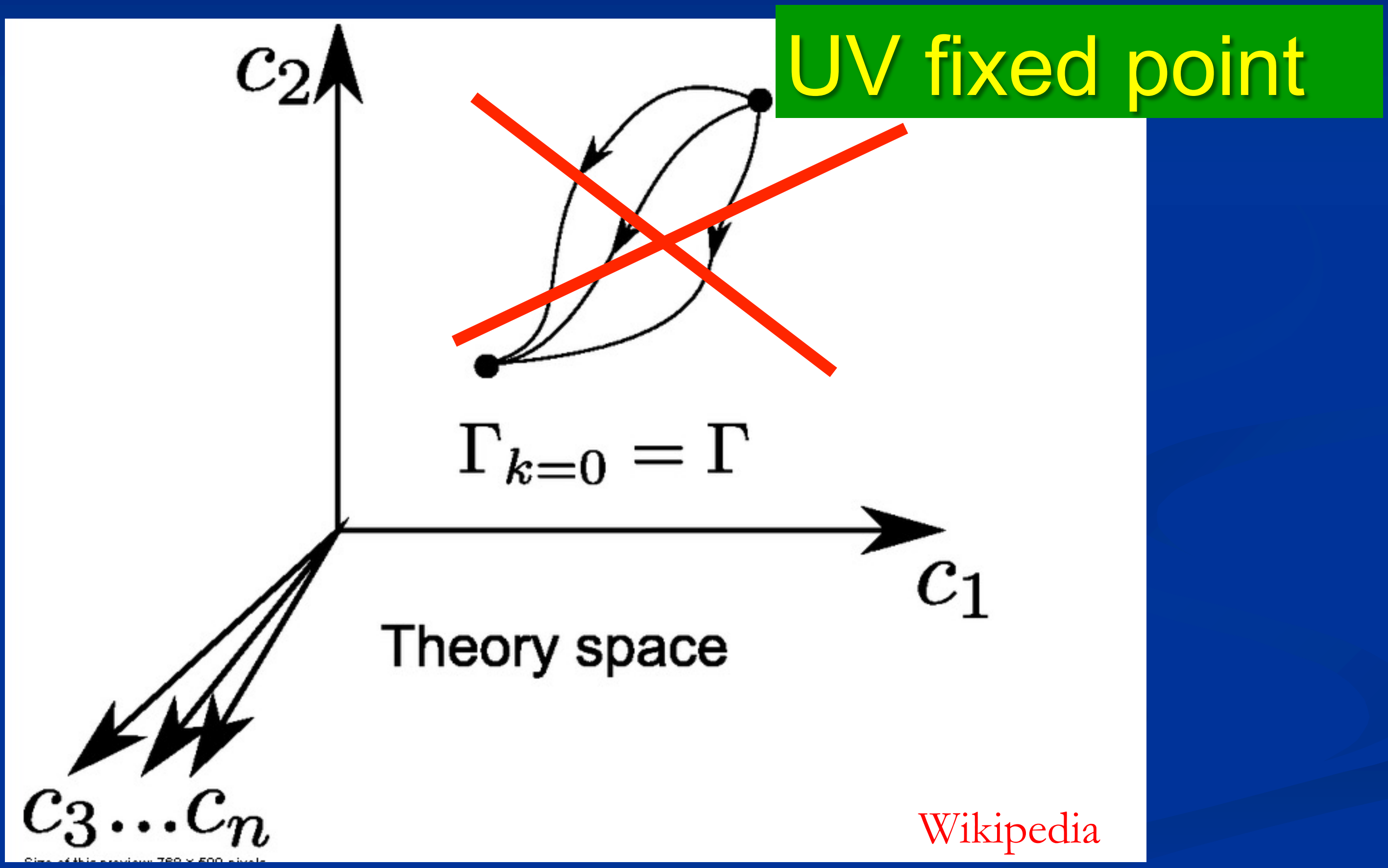
Fundamental scale invariance

Fundamental scale invariance

Scaling solution

that's it

Ultraviolet fixed point



Fundamental scale invariance

- Scaling solution is exact
- All relevant parameters vanish

Predictivity

- Theories with fundamental scale symmetry are very predictive
- Absence of relevant parameters
- New criterion for fundamental theories
- Stronger than renormalizability

Fundamental scale invariance and cosmology

- Scaling solution maps dependence on renormalization scale k to dependence on cosmon field
- Solution of field equations maps this to time dependence in cosmology
- Direct predictivity for cosmology

Fundamental theory without scale

Fundamental fields are dimensionless $\tilde{\psi}$

Length scale can be introduced for distances, mass=inverse length for derivatives $\hbar = c = 1$

Metric can appear as composite object

$\tilde{g}_{\mu\nu} \sim f(\tilde{\psi})\partial_\mu\tilde{\psi}\partial_\nu\tilde{\psi}$ dimension: mass squared

Effective action for scale invariant fields

$$\exp(-\Gamma[\tilde{\varphi}]) = \int D\tilde{\chi} \exp\left\{-S[\tilde{\varphi} + \tilde{\chi}] + \int_x \frac{\partial \Gamma}{\partial \tilde{\varphi}} \tilde{\chi}\right\}$$

$$\tilde{J}_i(x) = \frac{\partial \Gamma}{\partial \tilde{\varphi}_i(x)}, \quad \tilde{J} = \frac{\partial \Gamma}{\partial \tilde{\varphi}}$$

- Assume that continuum limit exists for this effective action
- No intrinsic scale

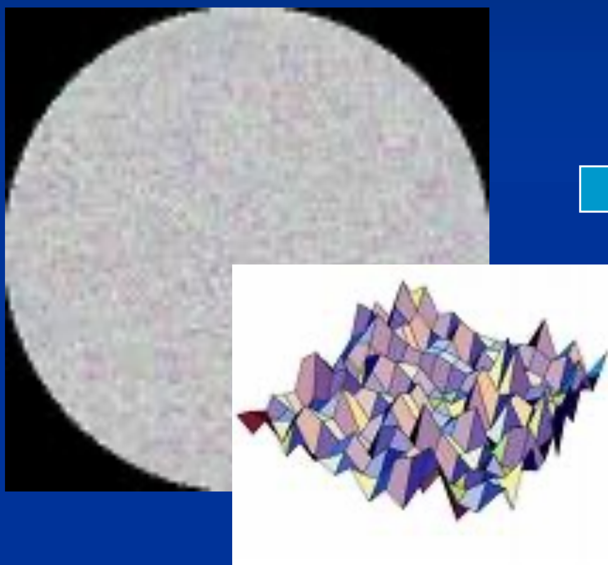
Canonical fields

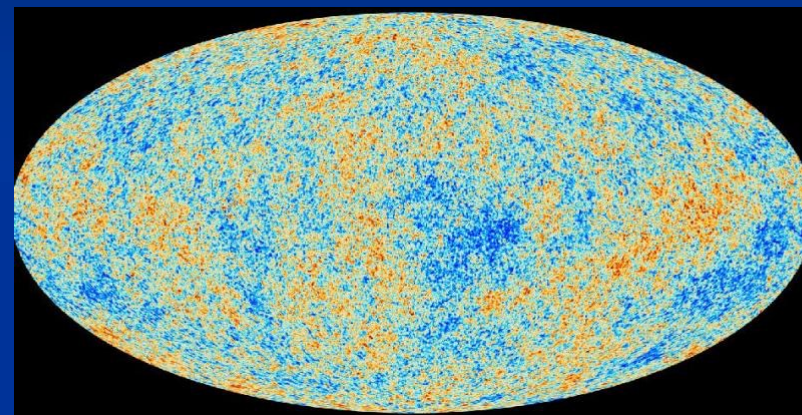
- Canonical metric is dimensionless
- Introduce renormalisation scale k $$g_{\mu\nu} = k^{-2}\tilde{g}_{\mu\nu}$$
- Canonical scalar fields; dimension mass $$\chi = k\tilde{\chi}$$
- General renormalized fields $$\varphi_{\mathrm{R},i}(x) = k^{d_i} f_i(k)\tilde{\varphi}_i(x)$$

Approximate scale symmetry near fixed points

UV : approximate scale invariance of primordial fluctuation spectrum from inflation

Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe





Scale symmetry near UV- fixed point

kinetial : $$\frac{K}{16\xi} = \kappa\left(\frac{k}{\chi}\right)^{\sigma} + \frac{1}{\alpha^2} - \frac{3}{8}$$ $$Z(\varphi) = \kappa \exp\left(-\frac{\sigma\varphi}{4M}\right) + \frac{1}{\alpha^2(\varphi)}$$

scalar anomalous dimension σ , take σ =2

$$\Gamma = \int_x \sqrt{g}\left\{ - w_0 k^2 R + \frac{8\xi\kappa k^2}{\chi^2}\partial^\mu\chi\partial_\mu\chi + u_0 k^4 \right\}$$

Weyl scaling

$$g'_{\mu\nu} = \frac{k^2}{\chi^2} g_{\mu\nu}$$

$$\Gamma = \int_x \sqrt{g'}\left\{ - w\chi^2 R' + u\chi^4 + \frac{1}{2}\left(\frac{\chi^2 K}{k^2} - 12w - 12\frac{\partial w}{\partial \ln\chi}\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

Higgs inflation not compatible with asymptotic safety

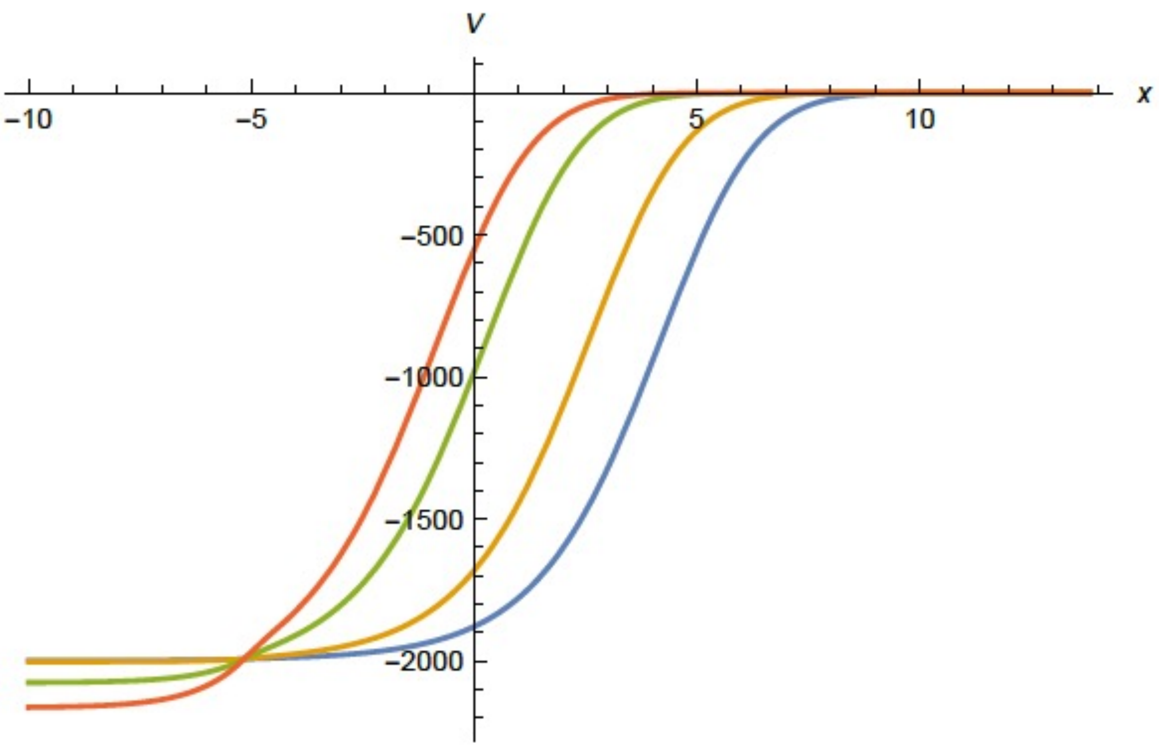


FIG. 23. Potential in the Einstein frame V_E as function of $x = \ln \tilde{\rho}$, for $\xi_\infty = 2 \cdot 10^{-5}$ (blue), 10^{-4} (orange), 10^{-3} (green), 0.003 (red), from right to left.

Scaling potential for GUT with non-zero gauge coupling

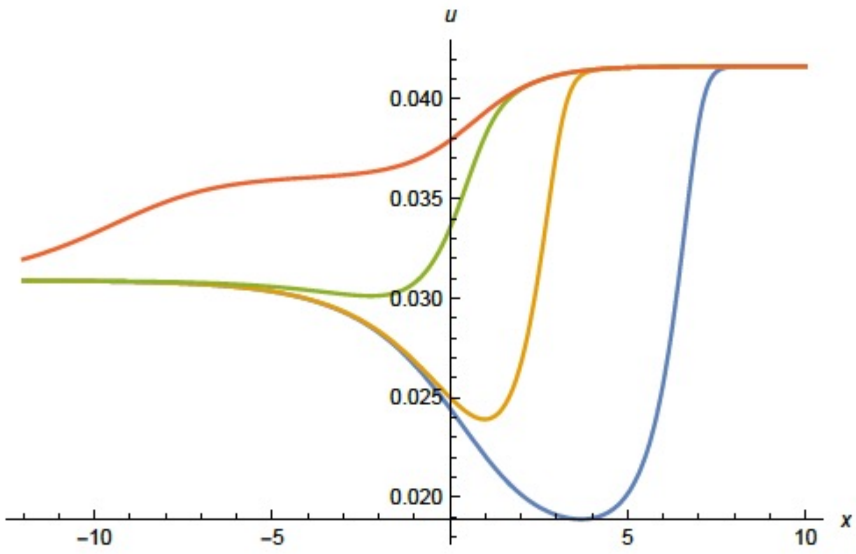


FIG. 9. Effective potential $u(x)$ as function of $x = \ln(\tilde{\rho})$. Parameters are $N = 20$, $\bar{N}_V = 3$, $c_g = 1$, $w_0 = 0.05$, $\alpha = g^2/4\pi = 1/40$. The initial conditions for the four curves from up to down are $u(x=0) = 0.037923$, $u(x=0) = 0.0335$, $u(x=0) = 0.25$ and $u(x=0) = 0.24447$. The inital values for the upper and lower curves limit the interval for which a scaling solution is found. For these solutions one has $A_0 = 2.91$.

Spontaneous symmetry breaking by scaling solution near Planck scale

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \rightarrow \varphi_0^2 = \epsilon\chi^2$$

Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \rightarrow \Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetry

CW'87

For $\chi_0 \neq 0$: massless Goldstone boson