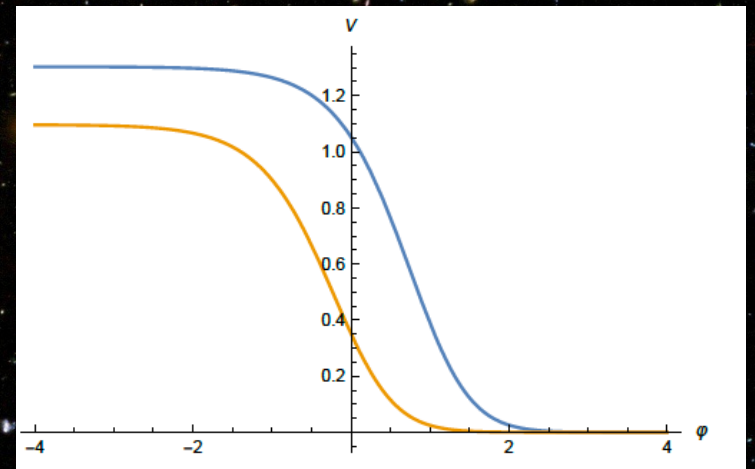
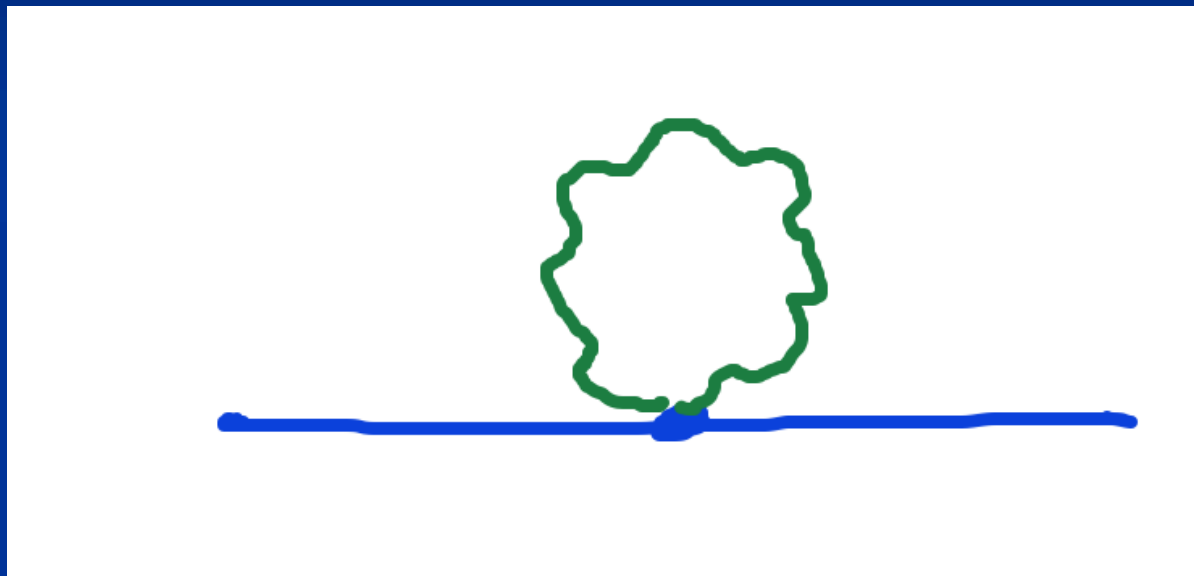


# Quantum gravity connection between inflation and dark energy



# Graviton fluctuations matter



Quantum gravity needs method to take them into account

# Flowing couplings

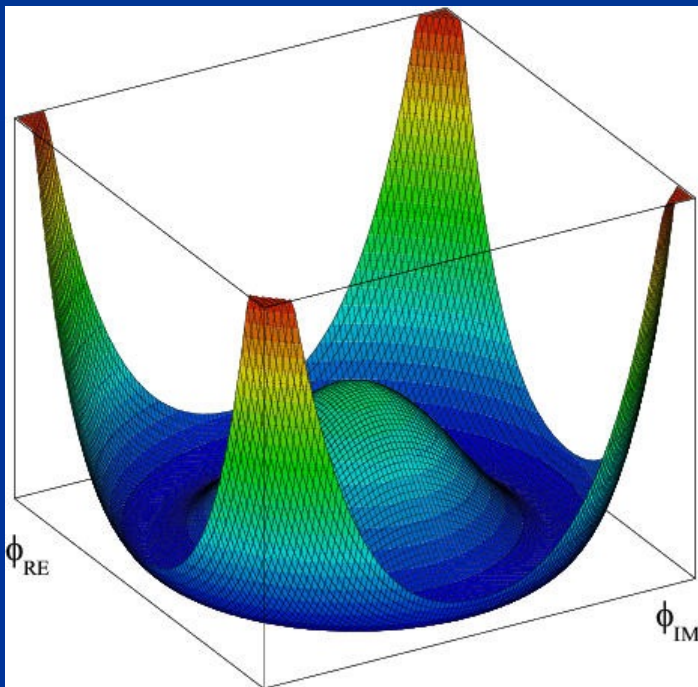
Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale  $k$  : Only fluctuations with momenta larger  $k$  are included. The scale  $k$  can be momenta, geometric quantities, or just be introduced “by hand”.

Flow of  $k$  to zero : all fluctuations included, IR

Flow of  $k$  to infinity : UV

# Flow of quartic Higgs coupling



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2} \lambda (\varphi^\dagger \varphi)^2$$
$$= \frac{1}{2} \lambda (\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$



# Graviton fluctuations erase quartic scalar coupling

Renormalization scale  $k$  : Only fluctuations with momenta larger  $k$  are included.

Consider first only fluctuations of metric or graviton :

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced  
anomalous dimension

$$A > 0$$

for  
constant  $A$  :

$$\lambda(k) = \lambda(\mu) \left( \frac{k}{\mu} \right)^A$$

$$k \rightarrow 0 \Rightarrow \lambda \rightarrow 0$$

*Quantum gravity  
flattens scalar potentials*

# Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left( \frac{k}{\mu} \right)^A$$

The quartic scalar coupling  $\lambda$  has a fixed point at  $\lambda=0$

For  $A>0$  it flows towards the fixed point as  $k$  is lowered: irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

# Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_p^2} \left[ \frac{20}{(1 - v_0)^2} + \frac{1}{(1 - v_0/4)^2} \right]$$

$$\partial_t = k \partial_k$$

running Planck mass :

$$M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2$$

dimensionless  
squared Planck mass

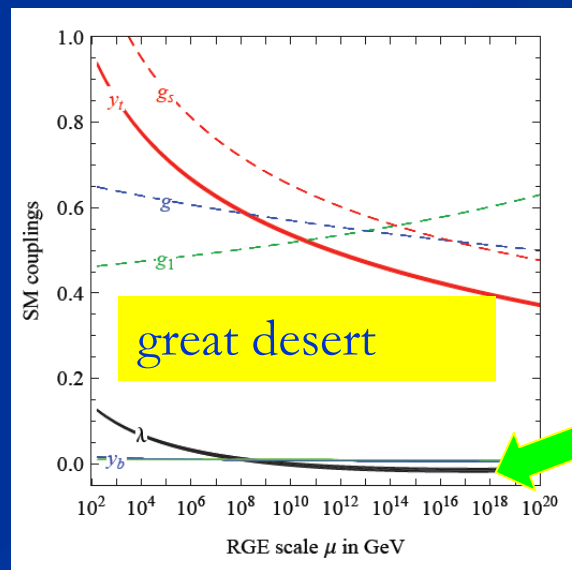
$$\tilde{M}_p^2 = \frac{M_p^2}{k^2}$$

for length scales smaller than the Planck length:  
metric fluctuations dominate, constant  $A$



# Prediction for quartic Higgs coupling

- great desert
- high scale fixed point
- quartic scalar coupling predicted to be very small at transition scale where gravity decouples



fixed point

# Prediction of mass of Higgs boson

## Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

*Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*

Christof Wetterich

*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany*

12 January 2010

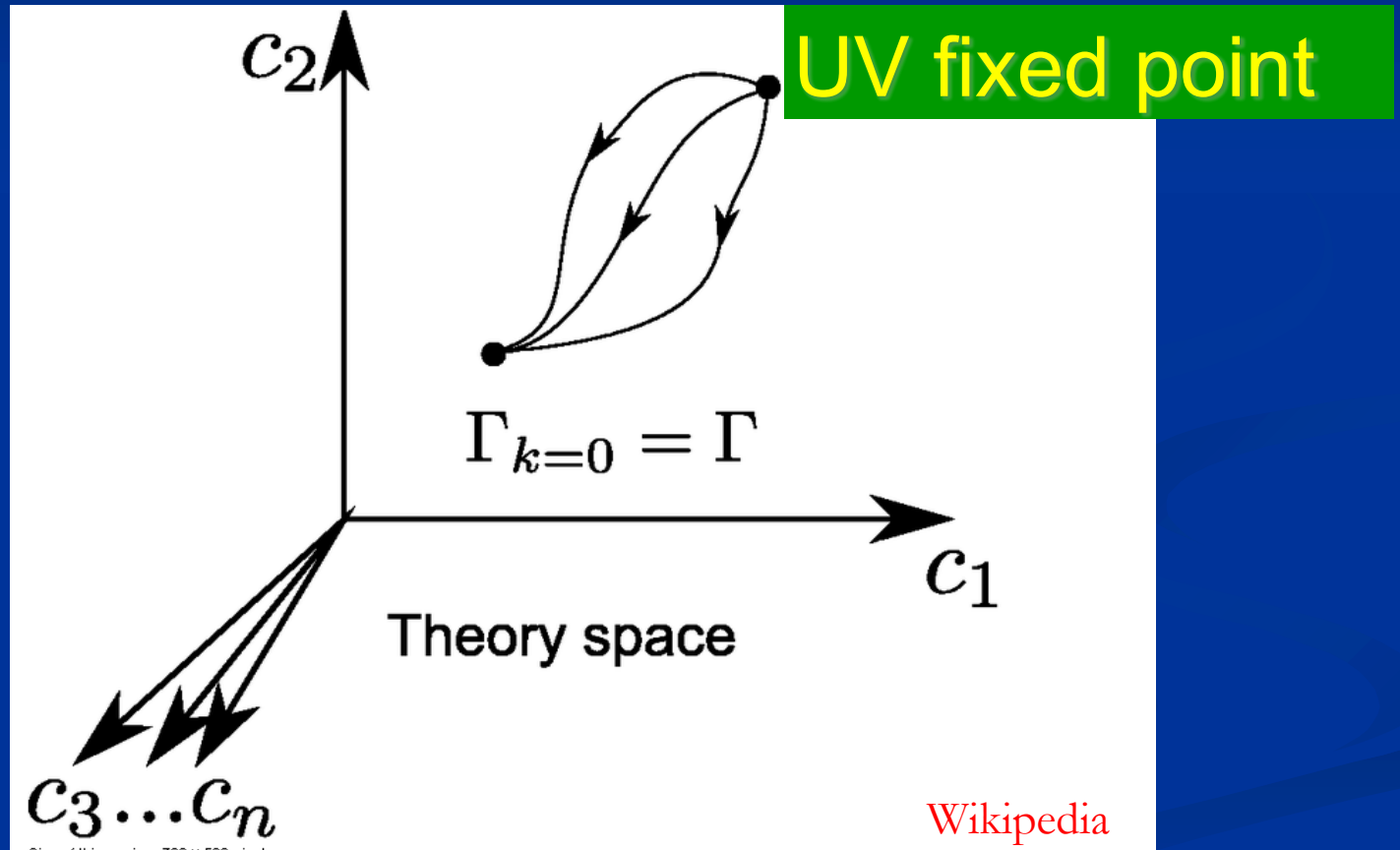
### Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson  $m_H$  can be predicted. For a positive gravity induced anomalous dimension  $A_\lambda > 0$  the running of the quartic scalar self interaction  $\lambda$  at scales beyond the Planck mass is determined by a fixed point at zero. This results in  $m_H = m_{\min} = 126$  GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in  $m_H = m_{\min} = 126$  GeV, with o

# Scaling solutions

# Ultraviolet fixed point





# Renormalizable theories

1) Ultraviolet fixed point :

scaling solution

2) Flow away from fixed point :

relevant parameters

# Scaling solutions

- At fixed point: all ( infinitely many ) dimensionless couplings take fixed values
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

# Scaling solutions are restrictive

- Scaling solutions are particular solutions of non-linear differential equations
- In presence of gravitational fluctuations: scalar effective potential no longer approximated by polynomial

# Scaling solutions and cosmology

- Cosmology involves scalar potentials over large range of field values
- Inflaton potential
- Higgs potential for Higgs inflation
- Cosmon potential for dynamical dark energy or quintessence



*Quantum gravity :  
these potentials are not arbitrary*

# Scaling potential in standard model

$u$  : dimensionless  
scalar potential  
 $u = U/k^4$

$x$  : logarithm of  
scalar field value

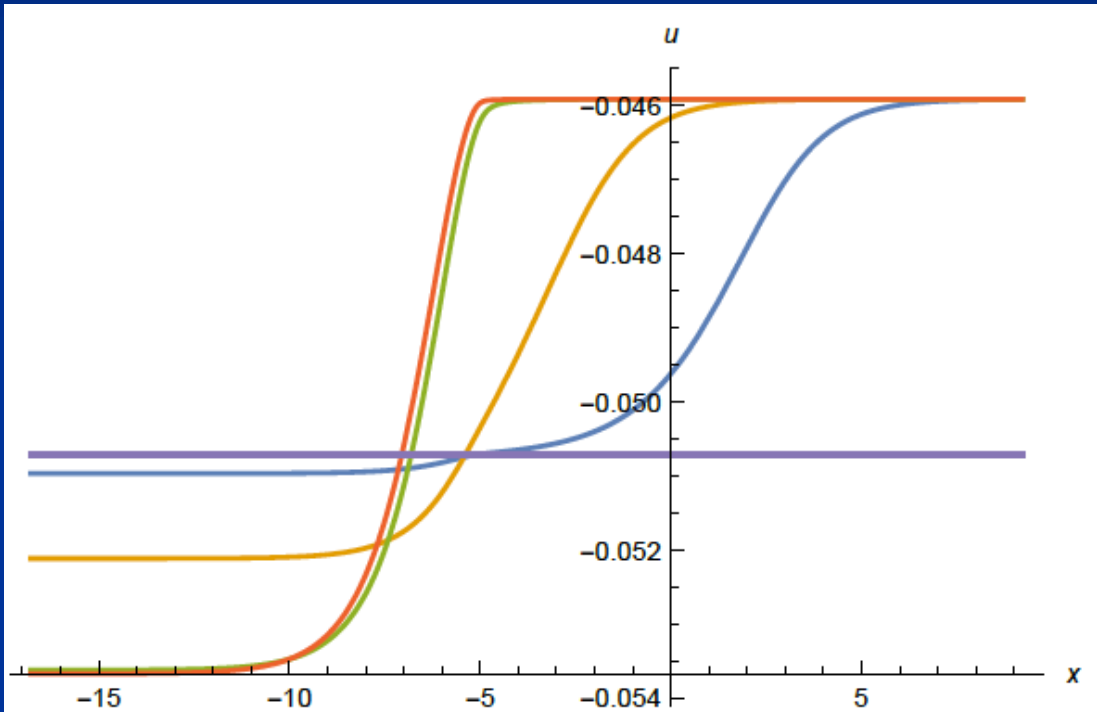


FIG. 19. Effective potential  $u$  as function of  $x = \ln \tilde{\rho}$  for  $\xi_\infty = 0.1$  (blue),  $1.0$  (orange),  $10^3$  (green) and  $10^4$  (red), from right to left in the right part and from top to bottom in the left part. The horizontal line indicates the scaling solution. The particle content is the one of the standard model,  $N_S = 4$ ,  $N_V = 12$ ,  $N_F = 45$ .

# Coefficient of curvature scalar in standard model

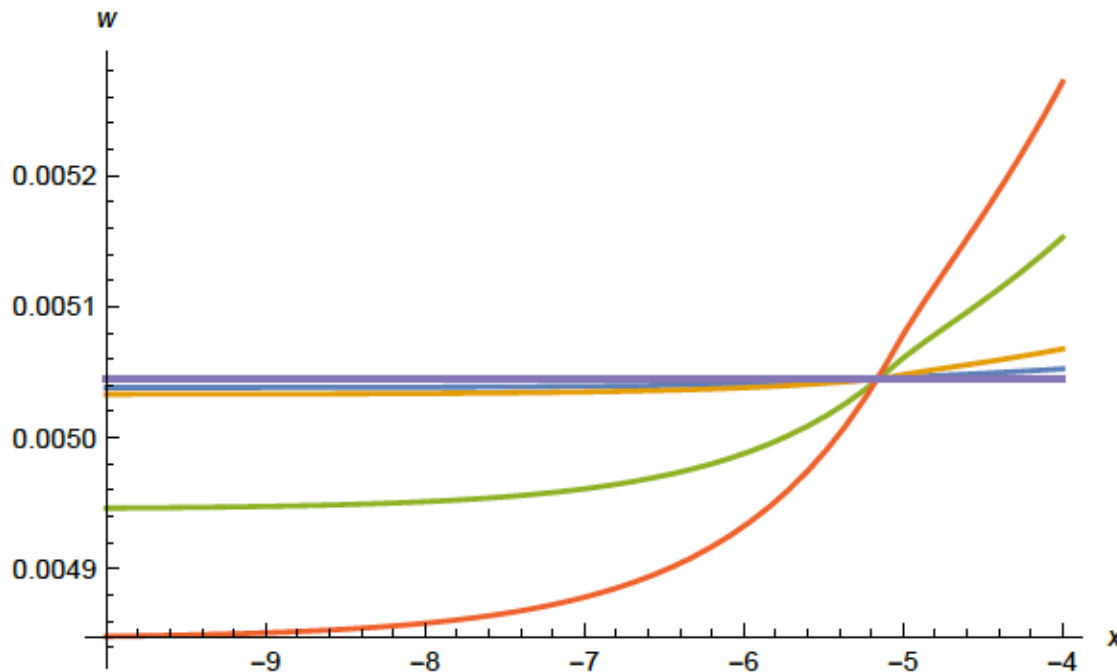


FIG. 21. Dimensionless squared Planck mass  $w$  as function of  $x = \ln \tilde{\rho}$  for  $\xi_\infty = 2 \cdot 10^{-5}$  (blue),  $10^{-4}$  (orange),  $10^{-3}$  (green), 0.003 (red), from top to bottom on the left. The horizontal line denotes the scaling solution which is approached for  $\xi_\infty \rightarrow 0$ . All curves meet in a common point at  $x \approx -5.05$ .

$w$  : dimensionless  
field dependent  
squared Planck  
mass

$$w = 2 M^2 / k^2$$

non-minimal  
coupling of  
scalar field  
to gravity

# Scaling solutions and inflation

- Scaling solutions are interesting for inflation:
- flat potential
- non minimal scalar – gravity coupling
- general properties do not depend on particular representation of scalar field
- scalar singlet = inflaton



# Higgs inflation not compatible with asymptotic safety

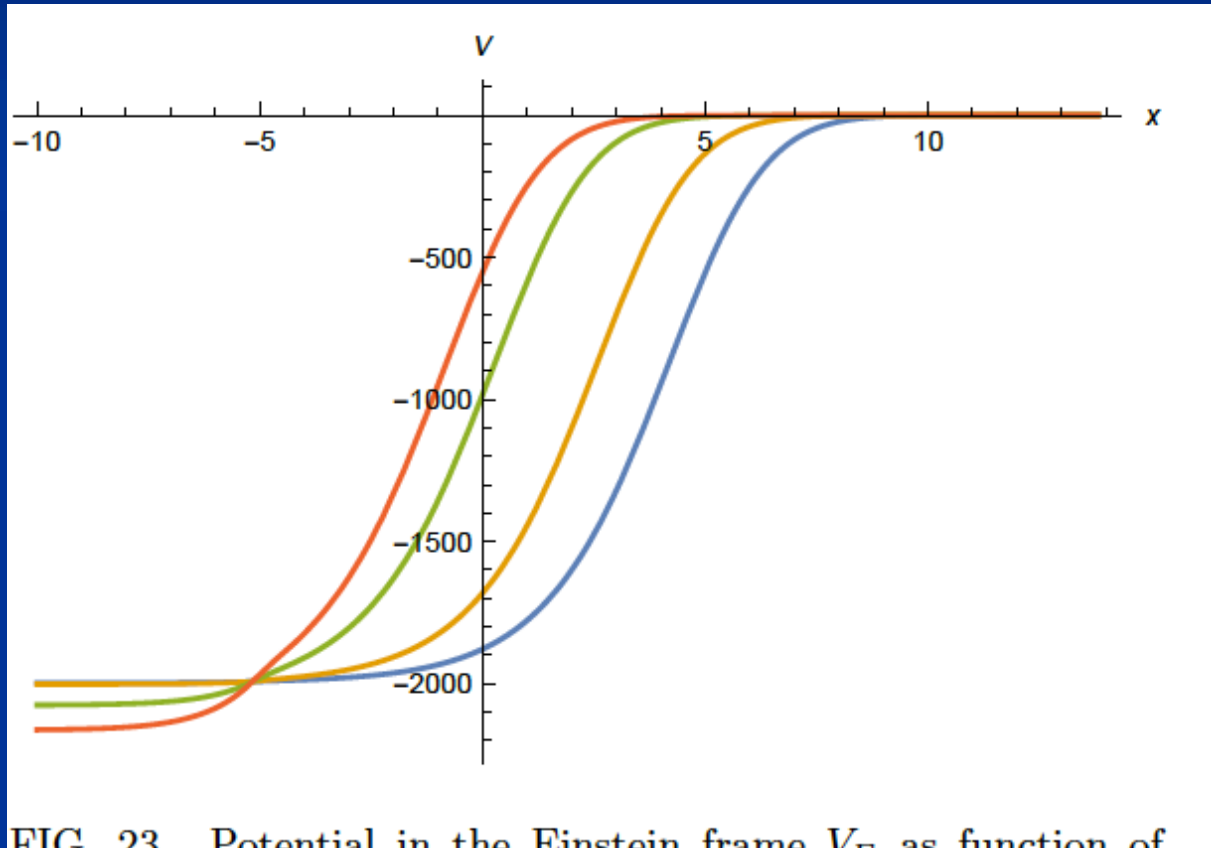


FIG. 23. Potential in the Einstein frame  $V_E$  as function of  $x = \ln \tilde{\rho}$ , for  $\xi_\infty = 2 \cdot 10^{-5}$  (blue),  $10^{-4}$  (orange),  $10^{-3}$  (green), 0.003 (red), from right to left.

# Scaling potential for GUT with non-zero gauge coupling

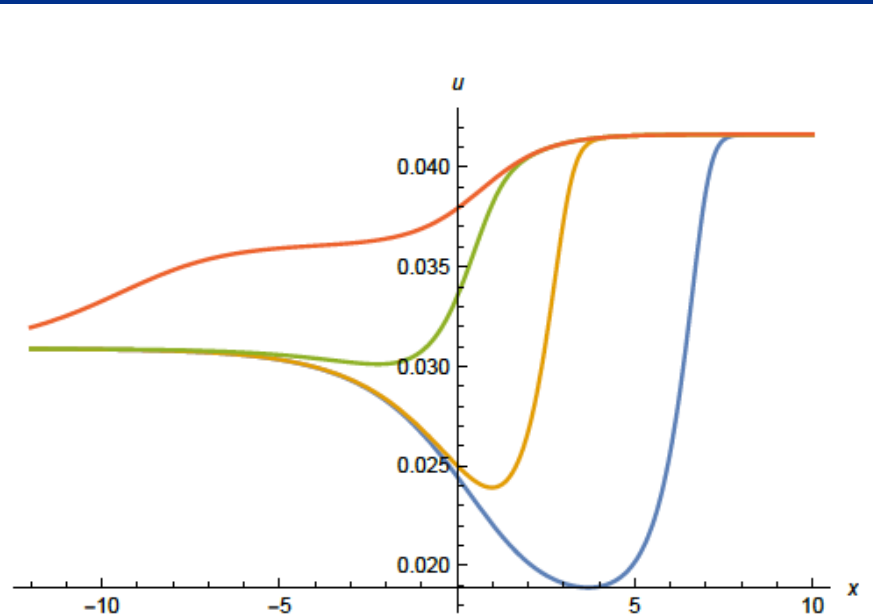


FIG. 9. Effective potential  $u(x)$  as function of  $x = \ln(\tilde{\rho})$ . Parameters are  $N = 20$ ,  $\bar{N}_V = 3$ ,  $c_g = 1$ ,  $w_0 = 0.05$ ,  $\alpha = g^2/4\pi = 1/40$ . The initial conditions for the four curves from up to down are  $u(x=0) = 0.037923$ ,  $u(x=0) = 0.0335$ ,  $u(x=0) = 0.025$  and  $u(x=0) = 0.024447$ . The initial values for the upper and lower curves limit the interval for which a scaling solution is found. For these solutions one has  $A_0 = 2.91$ .

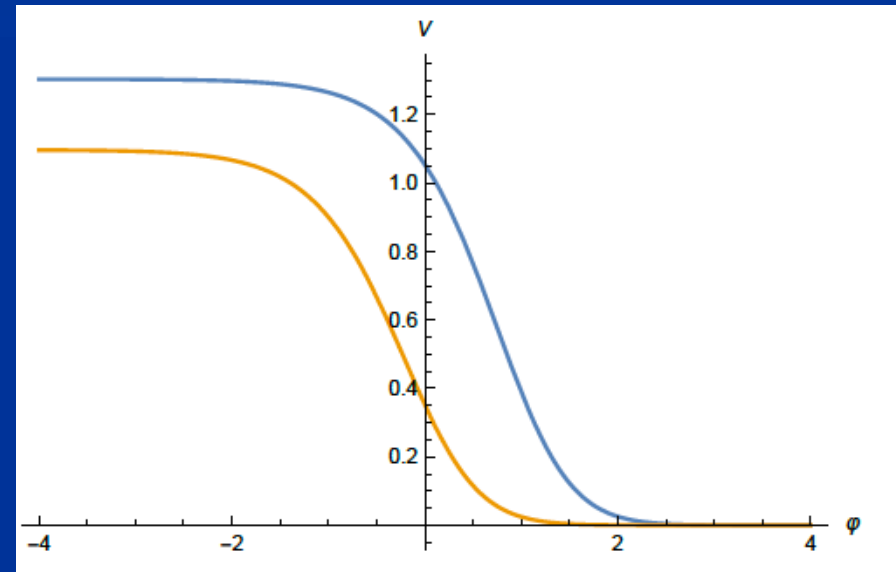
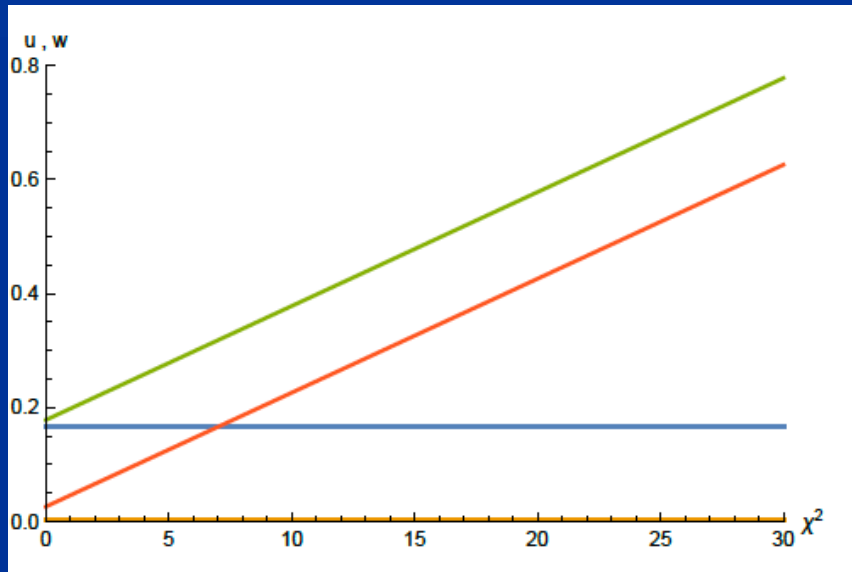
Spontaneous  
symmetry  
breaking by  
scaling solution  
near Planck  
scale

# Dilaton quantum gravity

quantum gravity coupled to a scalar field

Henz, Pawłowski, Rodigast, Yamada, Reichert,  
Eichhorn, Pauly, Laporte, Pereira, Saueressig, Wang...

# Scaling solution in Einstein frame



# Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu} \quad \varphi = 4M \ln(\chi/k)$$

$$\Gamma = \int_{\chi} \sqrt{g} \left\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^{\mu} \chi \partial_{\mu} \chi + U(\chi) \right\}$$

$$\Gamma = \int_{\chi} \sqrt{g} \left\{ -\frac{M^2}{2} R' + \frac{1}{2} Z(\varphi) \partial^{\mu} \varphi \partial_{\mu} \varphi + V(\varphi) \right\}$$

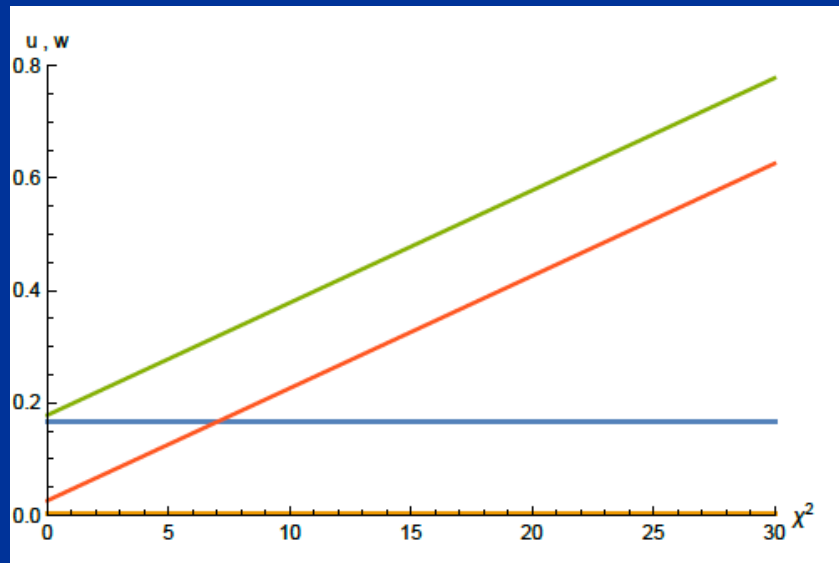
$$V(\varphi) = \frac{UM^4}{F^2}$$

$$Z(\varphi) = \frac{1}{16} \left\{ \frac{\chi^2 K}{F} + \frac{3}{2} \left( \frac{\partial \ln F}{\partial \ln \chi} \right)^2 \right\}$$

# Scaling solution

$$U = u_0 k^4$$

$$F = 2w_0 k^2 + \xi \chi^2$$

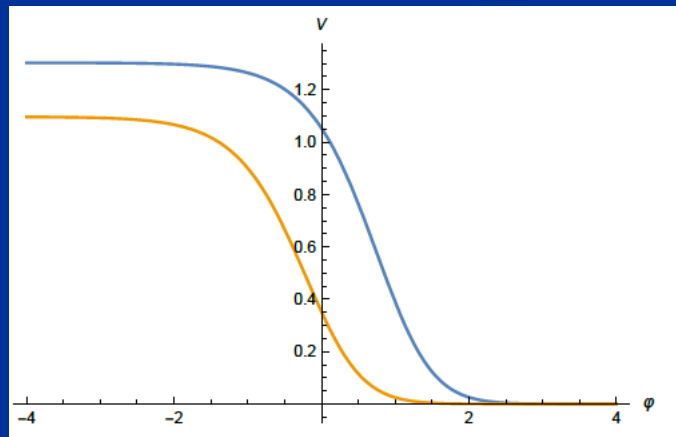


For low energy  
standard model :

$$u_{\infty} = \frac{7}{256\pi^2}$$

# Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right)$$

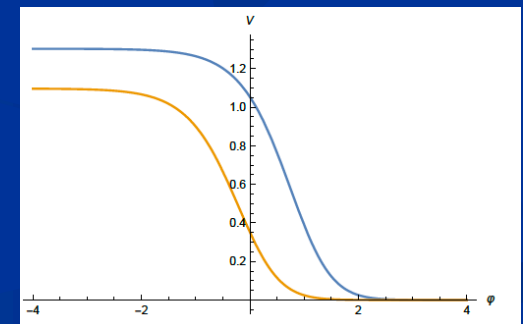




# Mass scales in Einstein frame

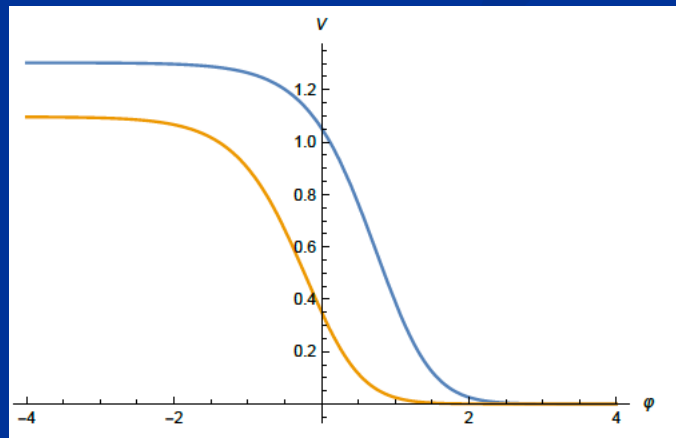
Renormalization scale  $k$  is no longer present  
Planck mass  $M$  not intrinsic: introduced only  
by change of variables

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right)$$

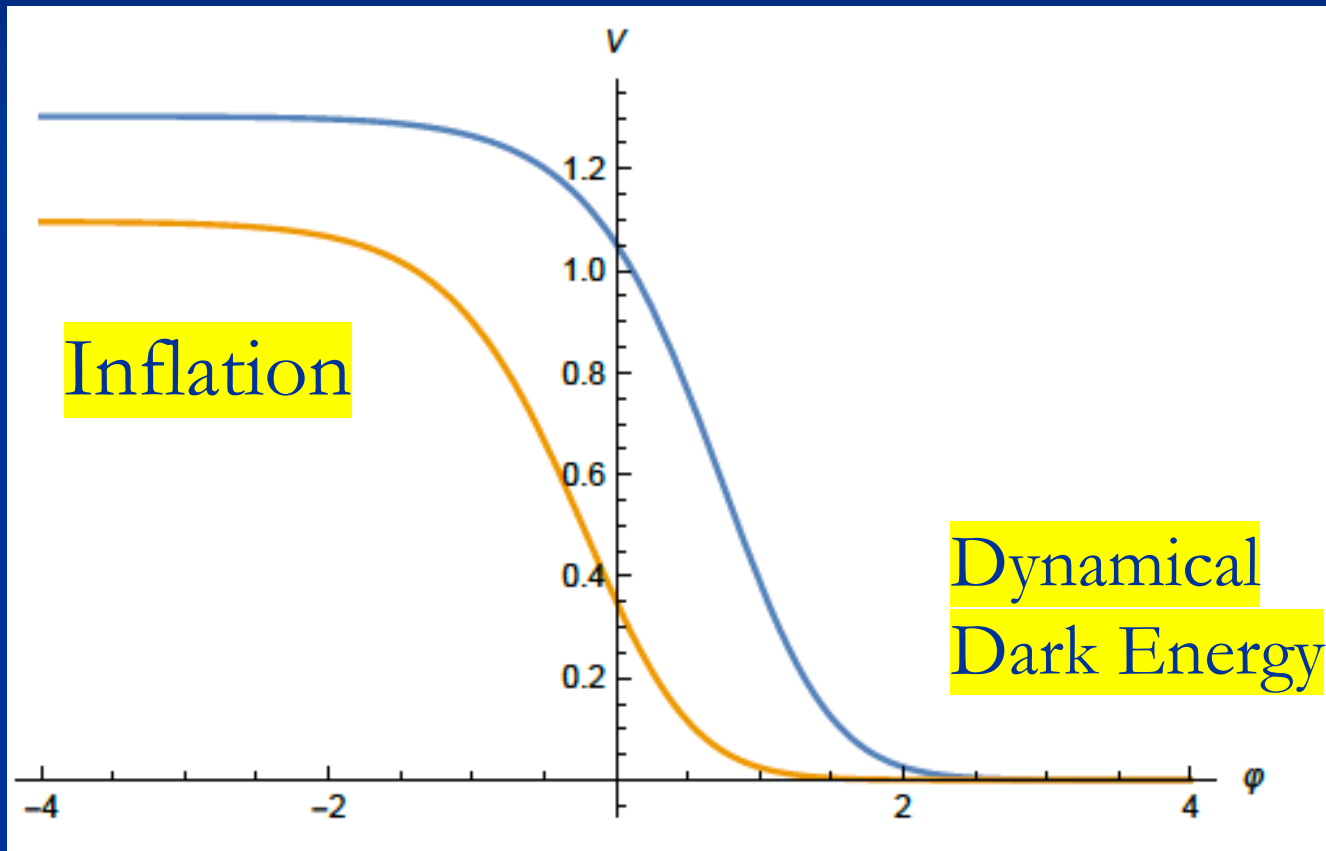


# Asymptotic solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2} \left[1 + \frac{2w_0}{\xi} \exp\left(-\frac{\varphi}{2M}\right)\right]^{-2} \exp\left(-\frac{\varphi}{M}\right)$$



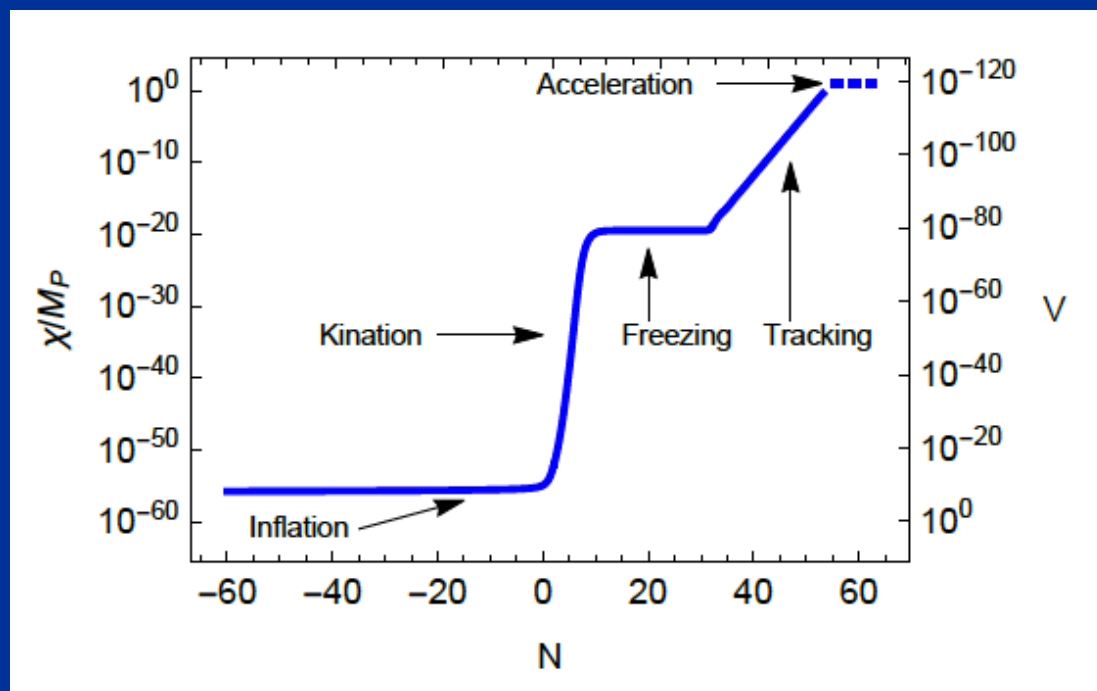
# Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini, Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio...

# Cosmological solution

- scalar field  $\chi$  vanishes in the infinite past
- scalar field  $\chi$  diverges in the infinite future



J.Rubio,...

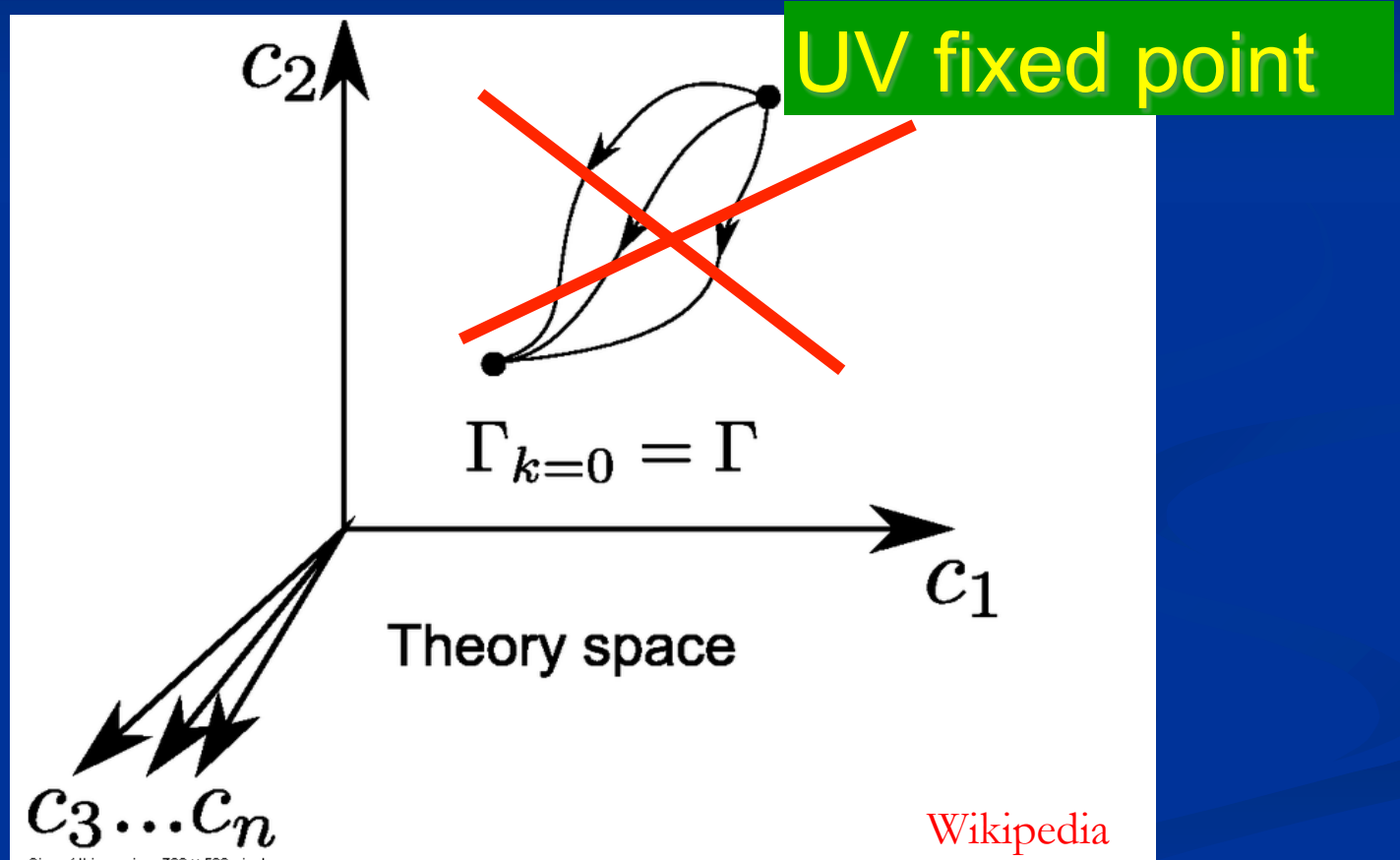
# Fundamental scale invariance

# Fundamental scale invariance

Scaling solution

that's it

# Ultraviolet fixed point



# Fundamental scale invariance

- Scaling solution is exact
- All relevant parameters vanish



# Predictivity

- Theories with fundamental scale symmetry are very predictive
- Absence of relevant parameters
- New criterion for fundamental theories
- Stronger than renormalizability

# Fundamental scale invariance and cosmology

- Scaling solution maps dependence on renormalization scale  $k$  to dependence on cosmon field
- Solution of field equations maps this to time dependence in cosmology
- Direct predictivity for cosmology

# Fundamental theory without scale

Fundamental fields are dimensionless

$$\tilde{\psi}$$

Length scale can be introduced for distances,  
mass=inverse length for derivatives

$$\hbar = c = 1$$

Metric can appear as composite object

$$\tilde{g}_{\mu\nu} \sim f(\tilde{\psi}) \partial_\mu \tilde{\psi} \partial_\nu \tilde{\psi} \quad \text{dimension: mass squared}$$

# Effective action for scale invariant fields

$$\exp(-\Gamma[\tilde{\varphi}]) = \int D\tilde{\chi} \exp \left\{ -S[\tilde{\varphi} + \tilde{\chi}] + \int_x \frac{\partial \Gamma}{\partial \tilde{\varphi}} \tilde{\chi} \right\}$$

$$\tilde{J}_i(x) = \frac{\partial \Gamma}{\partial \tilde{\varphi}_i(x)}, \quad \tilde{J} = \frac{\partial \Gamma}{\partial \tilde{\varphi}}$$

- Assume that continuum limit exists for this effective action
- No intrinsic scale

# Canonical fields

- Canonical metric is dimensionless

- Introduce renormalisation scale  $k$

$$g_{\mu\nu} = k^{-2} \tilde{g}_{\mu\nu}$$

- Canonical scalar fields; dimension mass

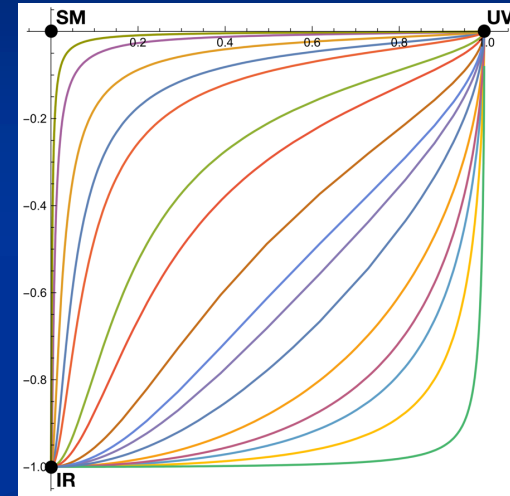
$$\chi = k \tilde{\chi}$$

- General renormalized fields

$$\varphi_{\text{R},i}(x) = k^{d_i} f_i(k) \tilde{\varphi}_i(x)$$

# Quantum scale symmetry

Exactly on fixed point:  
No parameter with dimension of  
length or mass is present in the  
quantum effective action.  
EVEN NOT  $k$  !



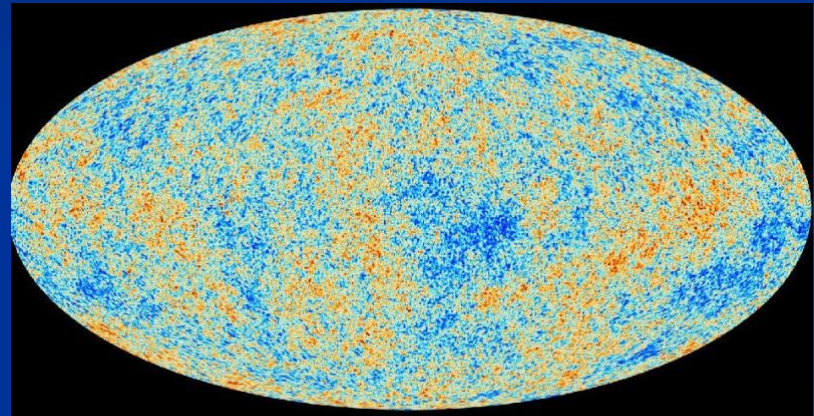
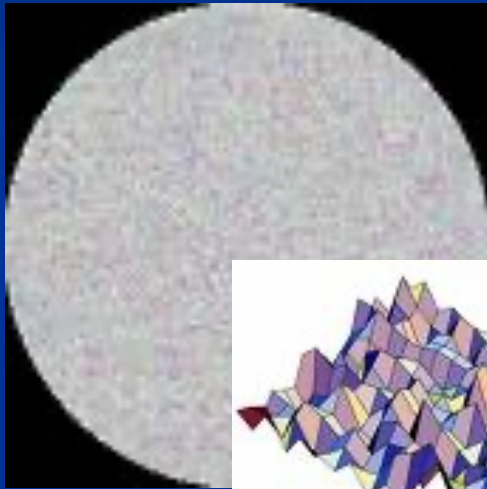
Then invariance under  
dilations or global scale transformations  
is realized as a quantum symmetry.

Continuous global symmetry

# Approximate scale symmetry near fixed points

UV : approximate scale invariance of  
primordial fluctuation spectrum from inflation

# Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe





# Scale symmetry near UV- fixed point

kinetial :  $\frac{K}{16\xi} = \kappa \left( \frac{k}{\chi} \right)^\sigma + \frac{1}{\alpha^2} - \frac{3}{8}$   $Z(\varphi) = \kappa \exp \left( -\frac{\sigma\varphi}{4M} \right) + \frac{1}{\alpha^2(\varphi)}$

scalar anomalous dimension  $\sigma$  , take  $\sigma = 2$

$$\Gamma = \int_x \sqrt{g} \left\{ -w_0 k^2 R + \frac{8\xi \kappa k^2}{\chi^2} \partial^\mu \chi \partial_\mu \chi + u_0 k^4 \right\}$$

Weyl  
scaling

$$g'_{\mu\nu} = \frac{k^2}{\chi^2} g_{\mu\nu}$$

$$\Gamma = \int_x \sqrt{g'} \left\{ -w \chi^2 R' + u \chi^4 + \frac{1}{2} \left( \frac{\chi^2 K}{k^2} - 12w - 12 \frac{\partial w}{\partial \ln \chi} \right) \partial^\mu \chi \partial_\mu \chi \right\}$$

# Approximate scale symmetry near fixed points

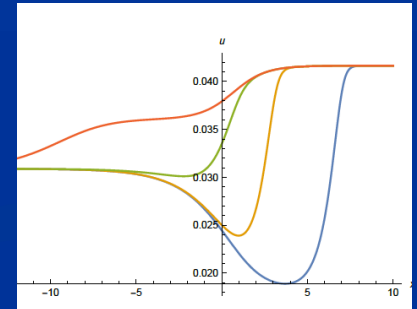
- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

# Dynamical dark energy

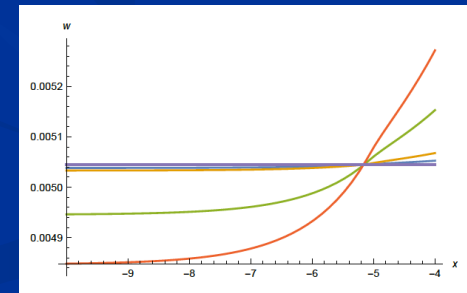
# Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\lambda = \frac{U}{F^2} = \frac{u}{4w^2} \rightarrow \frac{u_\infty}{\xi^2 \tilde{\rho}^2} \rightarrow \frac{4u_\infty k^4}{\xi^2 \chi^4}$$



vanishes for  $\chi \rightarrow \infty$  !



$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + c k^4 \right\}$$

$$k = 2 \cdot 10^{-3} \text{ eV}$$

# Quintessence

Dynamical dark energy ,  
generated by scalar field (cosmon )

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87  
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

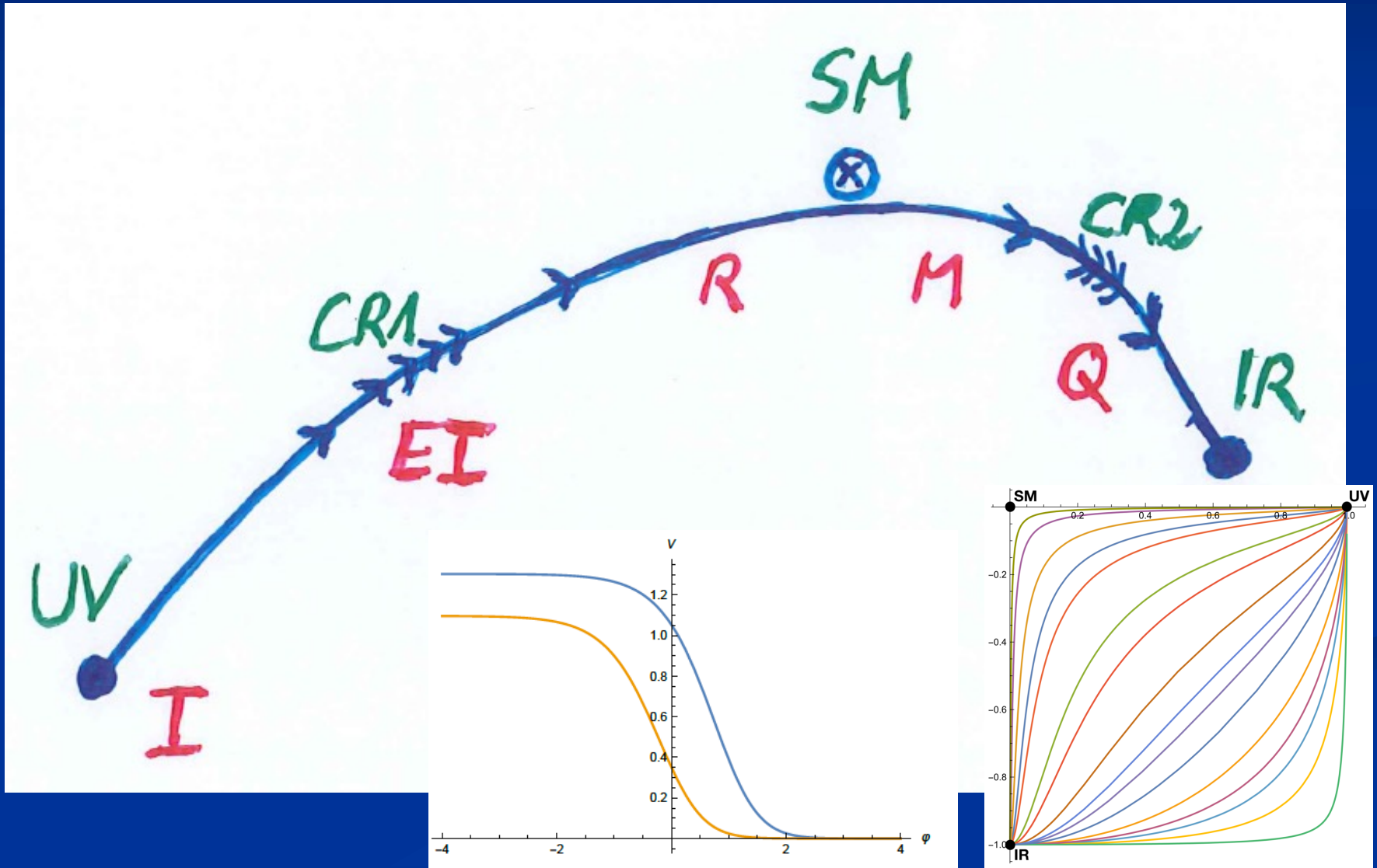
Prediction :

homogeneous dark energy  
influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations  
.... modifications  
( different growth of neutrino mass )

# Crossover in quantum gravity



# Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton



# Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

# Simplicity

simple description of all cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

# Einstein frame

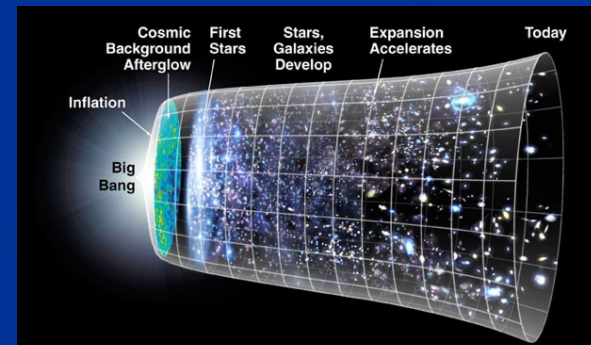
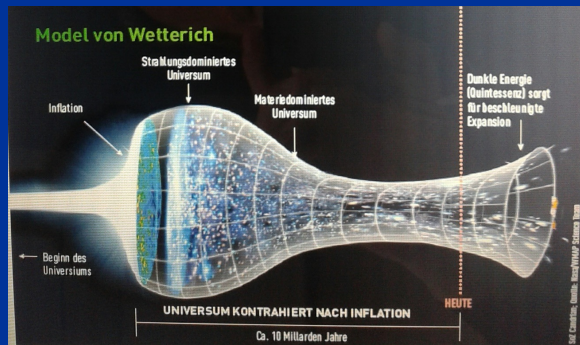
- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- For scaling solutions: scale  $k$  disappears !
- Standard gravity coupled to scalar field.
- Exact equivalence of different frames !  
“different pictures”

# Field relativity

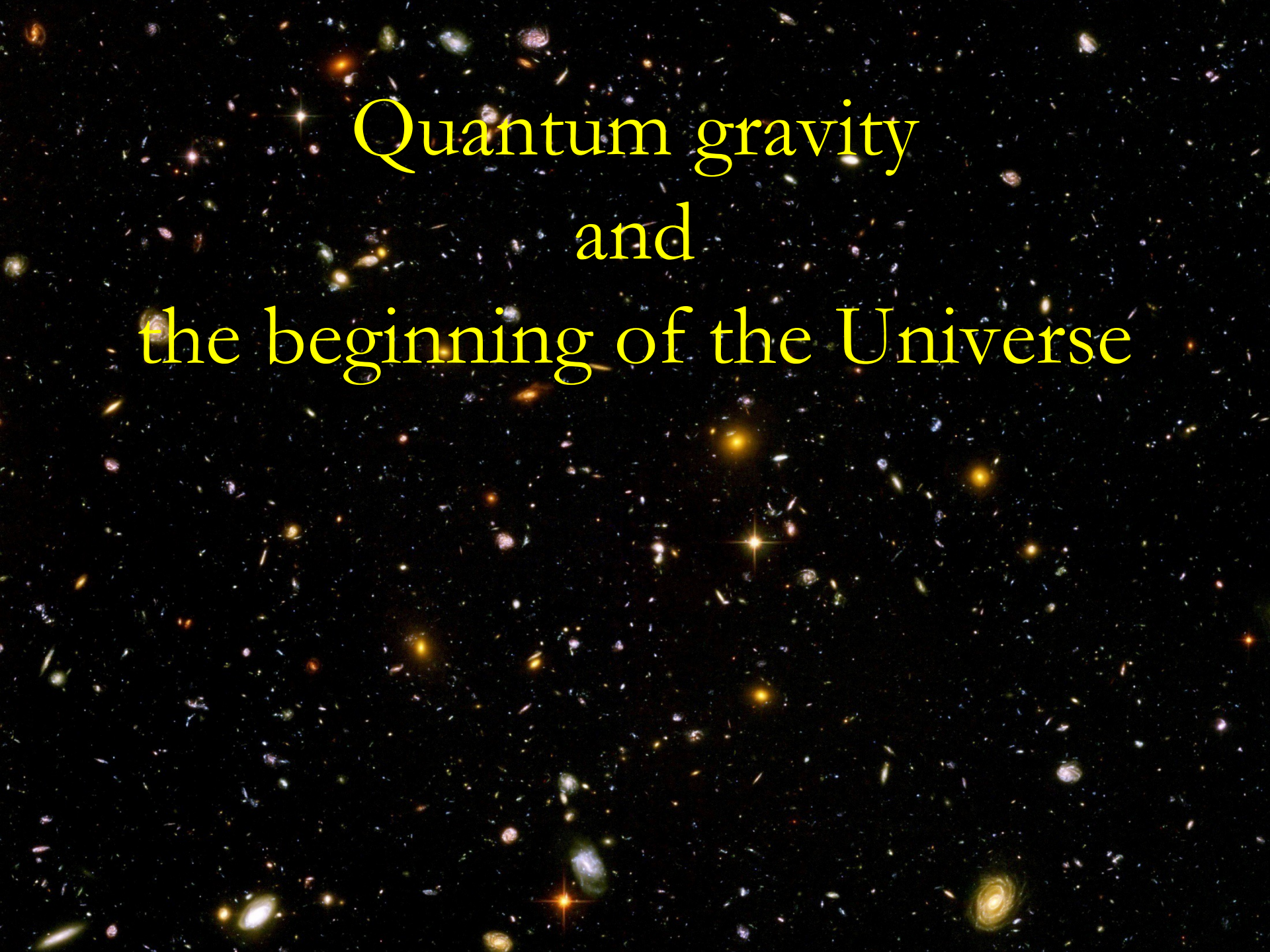
Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,  
not a coordinate transformation





The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies and stars against a black background. The galaxies are of various shapes and sizes, some appearing as bright, diffuse clouds, others as more compact, elongated structures. The stars are represented by numerous small, bright points of light, some with prominent diffraction spikes. The overall scene is a dense, colorful mosaic of cosmic objects.

# Quantum gravity and the beginning of the Universe

# Beginning of Universe

*Zu Anfang war die Welt öd und leer und währte ewig.*

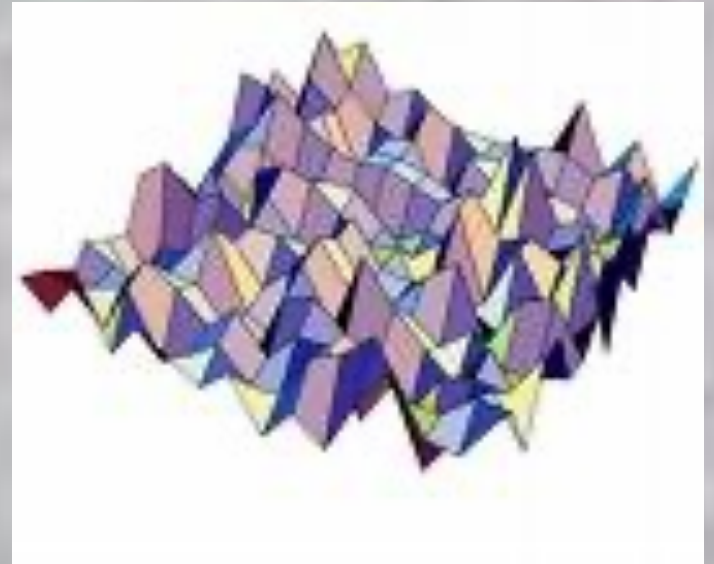
*In the beginning the Universe was empty and lasted since ever.*



# Eternal light-vacuum

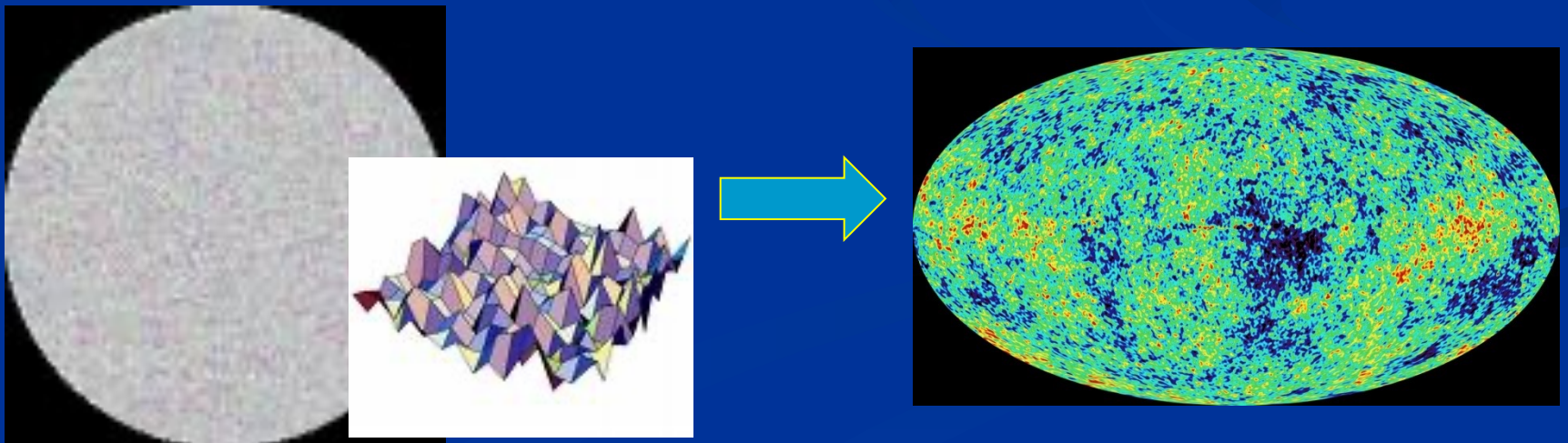
Everywhere almost nothing  
only fields and their fluctuations

All particles move  
with light velocity,  
similar to photons



# Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.



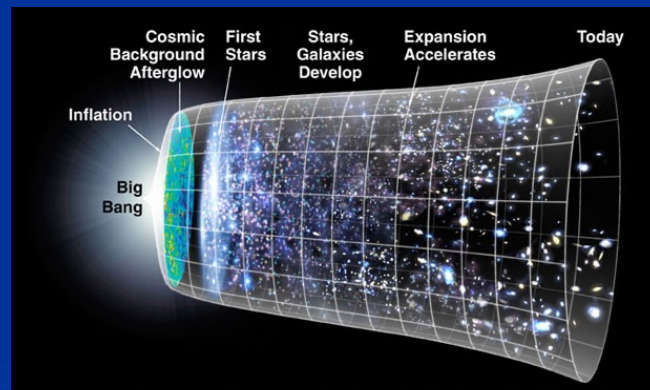


# The great emptiness story

*In the beginning was light-like emptiness.*

# The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around  $10^{-40}$  seconds
- start with singularity : our whole observable Universe evolves from one point



# Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

- different metrics related by Weyl transformation, which depends on scalar field ( inflaton )

# Conclusions

- Fluctuations of the metric matter
- They influence the behavior of scalar potentials for all field values
- Quantum gravity relevant for early cosmology and late cosmology
- Quantum scale symmetry is central ingredient for understanding cosmology
- Understanding of quantum gravity fluctuations still in beginning stage

end

# Scale symmetric standard model

- Replace all mass scales by scalar field  $\chi$

(1) Higgs potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \quad \longrightarrow \quad \varphi_0^2 = \epsilon\chi^2 \quad \text{Fujii, Zee, CW}$$

(2) Strong gauge coupling, normalized at  $\mu = \chi$ , is independent of  $\chi$

$$g(\chi) = \bar{g} \quad \longrightarrow \quad \Lambda_{\text{QCD}} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2} \left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

*Quantum effective action for standard model does  
not involve intrinsic mass or length*

Quantum scale symmetry

CW'87

For  $\chi_0 \neq 0$  : massless Goldstone boson