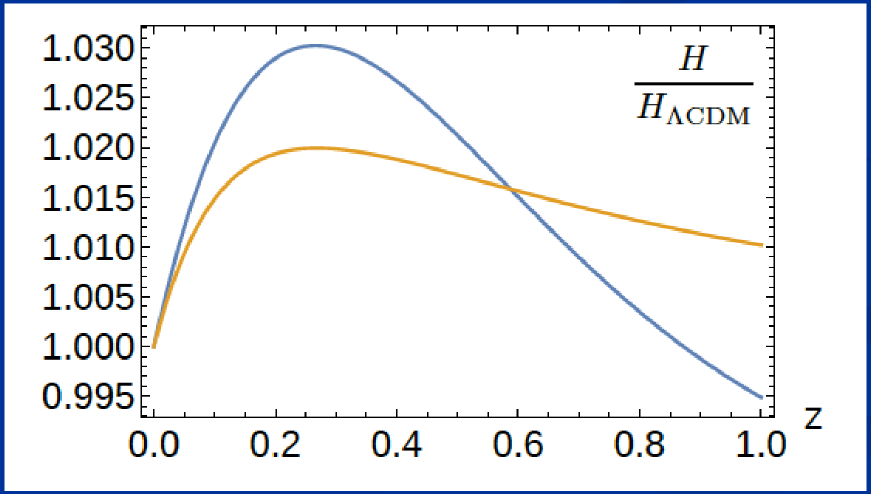
Quantum gravity predictions for dark energy

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

Predictions

- Dynamical solution of cosmological constant problem
- Prediction of dynamical dark energy
- Realistic cosmology?



(1) Largest intrinsic mass scale

- LIMS is the largest mass scale generated by flow of relevant couplings

- Explicit violation of quantum scale symmetry

Example QCD with light quarks: confinement scale

Ultraviolet fixed point

- Flow of couplings stops for k $\rightarrow\infty$
- Self-similarity: no explicit dependence on k
- Theory can be extrapolated to arbitrary short distances
- Completeness
- Renormalizability

Predictivity

- few relevant parameters govern flow away from UV-fixed point
- translate to renormalizable couplings in standard model or extensions
- generate intrinsic mass scales
- largest one : LIMS
- LIMS only sets overall mass scale
- involves no parameter tuning

(2) LIMS in quantum gravity

- Often LIMS is associated to Planck mass
- No need for that!
- Proposal:

LIMS of the order of neutrino masses

or smaller

- Electron mass, Planck mass much larger?
- Particle masses given by field

Metric + scalar field

- Inflation :

  add scalar field (inflaton)
- Dynamical dark energy or quintessence:

  add scalar field (cosmon)

$$\Gamma_k = \int_x \sqrt{g'} \left\{ -\frac{\xi}{2} \chi^2 R' + \right. \ldots$$

(3) Spontaneous breaking of quantum scale symmetry and the scale invariant standard model

- Non-zero cosmological value of scalar field χ(t) breaks quantum scale symmetry spontaneously

- No intrinsic mass parameter larger than LIMS:
- All masses larger than neutrino masses are proportional to χ

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$ $\rightarrow$ $\varphi_0^2 = \epsilon\chi^2$ Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$g(\chi) = \bar{g}$ $\rightarrow$ $\Lambda_{\mathrm{QCD}} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right)$ $b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetry CW'87, Shaposhnikov et al

(4) Scaling solution

- UV – fixed point : scaling solution of functional renormalization

FRG scale k larger than LIMS:

- Scaling solution very good approximation
- Scaling solution is predictive!

Can the scalar potential be predicted by functional renormalization for quantum gravity ?

Dilaton quantum gravity

functional renormalization for
quantum gravity coupled to a scalar field

$$\Gamma = \int_\chi \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

Henz, Pawlowski, Rodigast, Yamada, Reichert,
Eichhorn, Pauly, Laporte, Pereira, Saueressig,

Wang, Knorr, …
for low order polynomial expansion of potential : Percacci, Narain, …

Functional flow equation for scalar potential

$$\Gamma_k = \int_x \sqrt{g'} \left\{ -\frac{\xi}{2}\chi^2 R' + u(\chi) k^4 + \frac{1}{2} K \partial^\mu \chi \partial_\mu \chi \right\}$$

$$k\partial_k u = -4u + 2\tilde{\rho}\partial_{\tilde{\rho}} u + 4c_U$$

$$\tilde{\rho} = \chi^2/(2k^2)$$

$$c_U = \frac{1}{128\pi^2}\left(\bar{N}_S(\tilde{\rho}) + 2\bar{N}_V(\tilde{\rho}) - 2\bar{N}_F(\tilde{\rho}) + \bar{N}_g(\tilde{\rho})\right)$$

Scaling solution: no explicit dependence of u on k

Generic form of scaling potential

- Interpolates between plateaus
- Scalar potential =

  field dependent "cosmological constant"
- Effectively massless particles contribute to flow
- Different numbers of massless particles in different regions of field space

No quartic coupling $\lambda\,\chi^4$

Scaling solutions are restrictive

- scaling solutions are solutions of non-linear differential equations
- scaling potential needs to extend over whole range of scalar field
- predictivity !

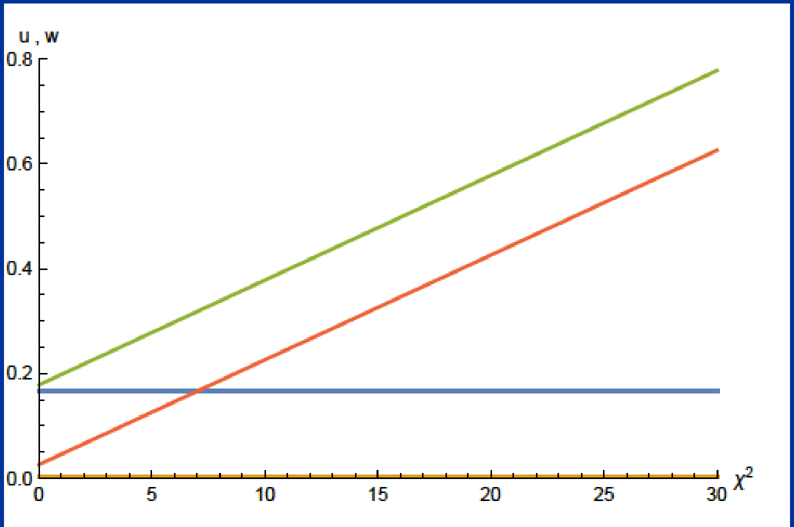
in presence of gravitational fluctuations:

scalar effective potential no longer approximated by polynomial

Coefficient of curvature scalar in standard model

non-minimal coupling of scalar field to gravity:

$$F=\xi\chi^2 R$$



w : dimensionless

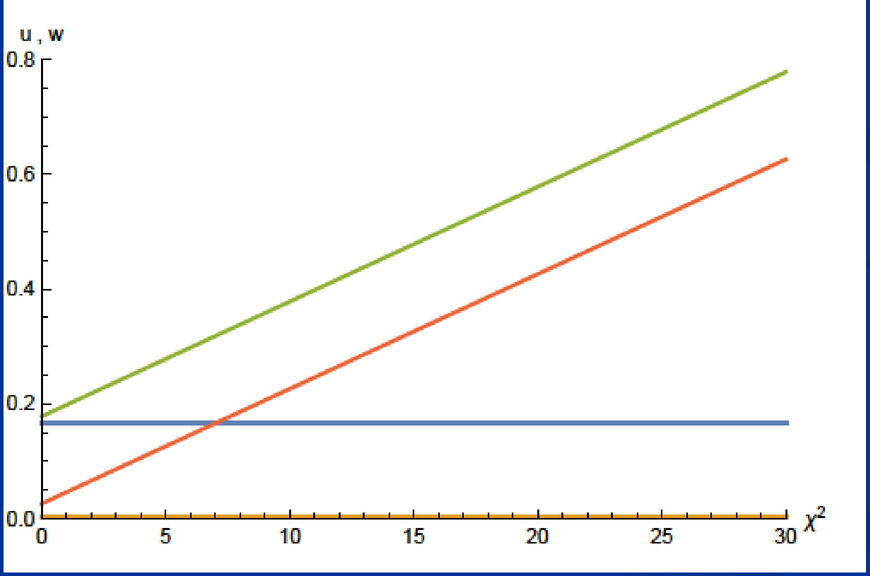
field dependent squared Planck mass

$$u = \frac{U}{k^4}\,, \qquad w = \frac{F}{2k^2}$$

Approximate scaling solution

- flat potential: u constant
- non-minimal scalar- gravity coupling:

  for large scalar field w increases proportional χ^2



squared scalar field value χ^2

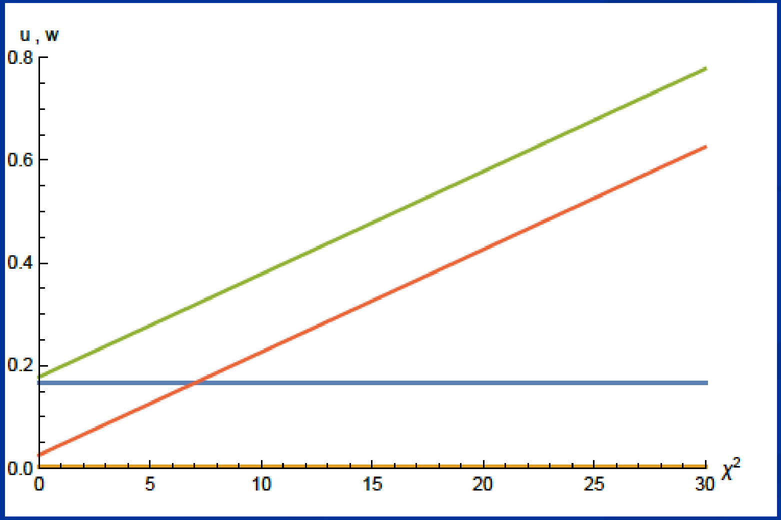
$$U = u_0 k^4$$

$$F = 2w_0 k^2 + \xi\chi^2$$

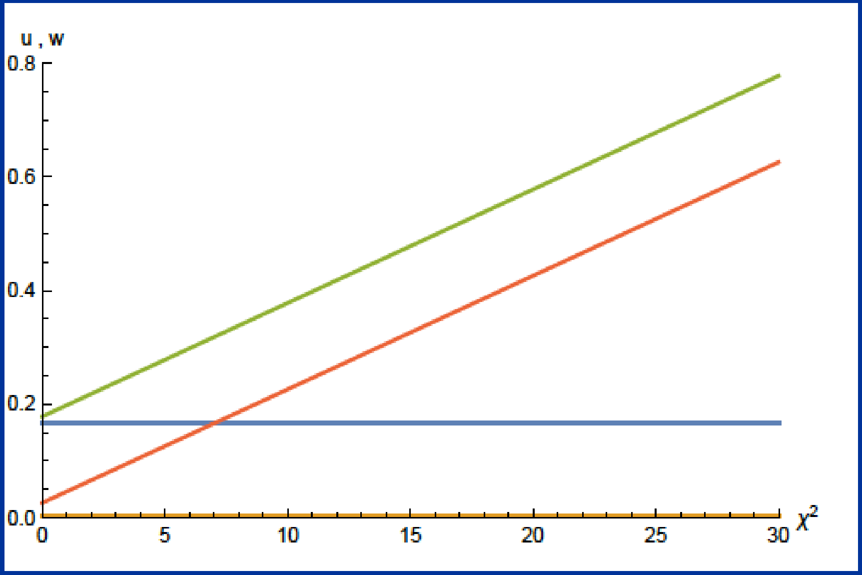
looks natural

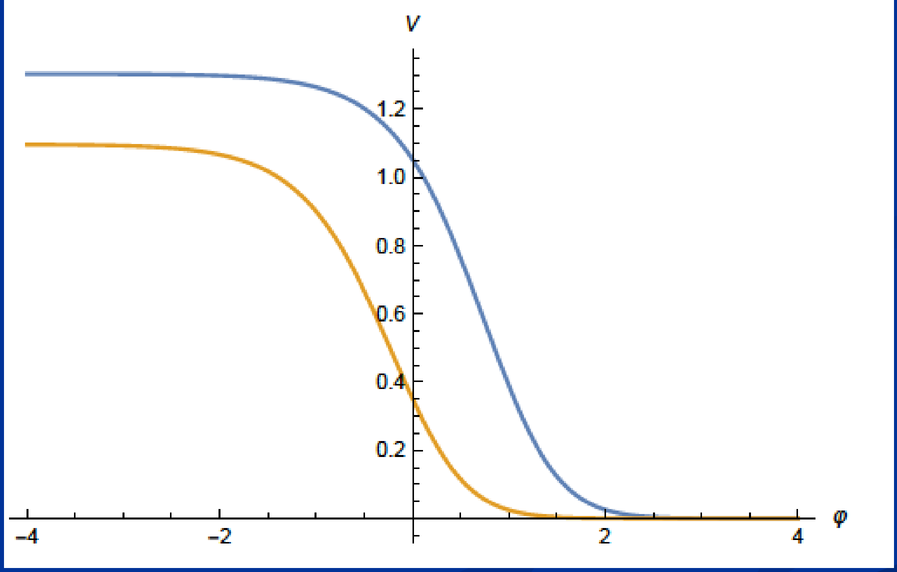
no small parameter

no tuning



Scaling solution in Einstein frame





Weyl transformation for variable gravity

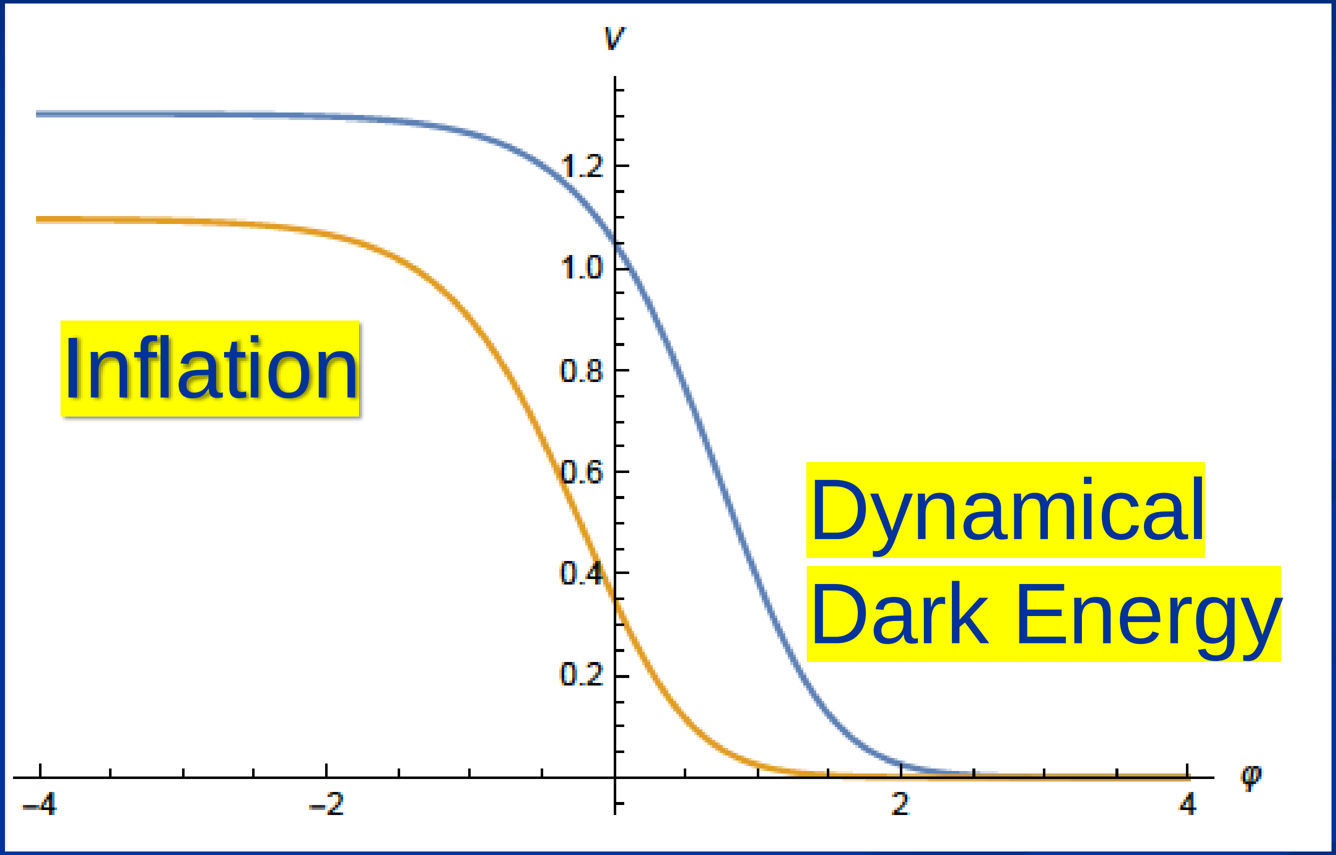
$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu} \qquad \varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_\chi \sqrt{g}\Big\{-\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

$$\Gamma = \int_\chi \sqrt{g}\Big\{-\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\Big\}$$

$$V(\varphi) = \frac{UM^4}{F^2} \qquad Z(\varphi) = \frac{1}{16}\Big\{\frac{\chi^2 K}{F} + \frac{3}{2}\Big(\frac{\partial\ln F}{\partial\ln\chi}\Big)^2\Big\}$$

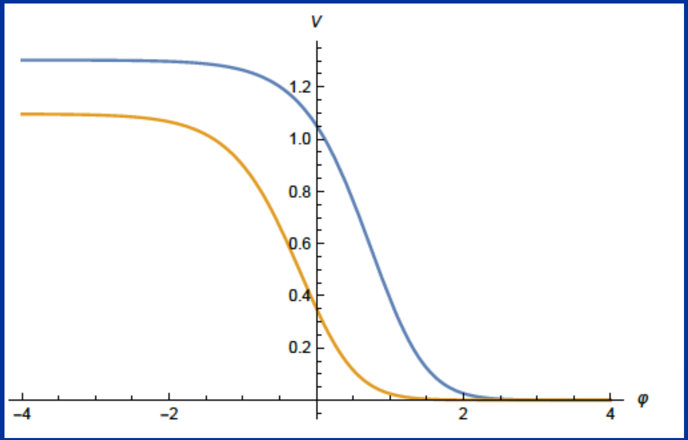
Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini,
Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

(5) Dynamical solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !

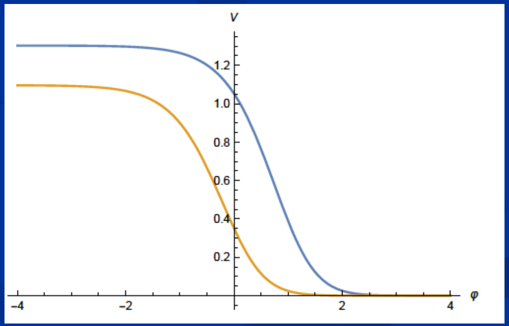
$$V(\varphi) = \frac{UM^4}{F^2}$$

Mass scales in Einstein frame

Renormalization scale k is no longer present

Planck mass M not intrinsic: introduced only by change of variables

$$
\begin{aligned}
V =& \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} \\
=& \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)
\end{aligned}
$$



(6) Dynamical dark energy

prediction of (approximate) quantum scale symmetry:

dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich, Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles, B.Ratra, ApJ.Lett.325(1988)L17, 20.10.87

Cosmon

- Spontaneously broken scale symmetry induces a Goldstone boson
- Massless dilaton

- Intrinsic mass scale from LIMS :
  Pseudo-Goldstone boson acquires a tiny mass
- Cosmon

Naturally very light scalar particle !

(7) Scaling potential for standard model

- Mass thresholds matter

$$\tilde{\rho}\partial_{\tilde{\rho}}u = 2(u - c_U)$$

$$c_U = \frac{1}{128\pi^2}\left(\bar{N}_S(\tilde{\rho}) + 2\bar{N}_V(\tilde{\rho}) - 2\bar{N}_F(\tilde{\rho}) + \bar{N}_g(\tilde{\rho})\right)$$

$$\bar{N}_F = \sum_f \left(1 + \tilde{m}_f^2\right)^{-1}$$

$$\tilde{m}_f^2 = \frac{m_f^2(\tilde{\rho})}{k^2}$$

$$m_f = h_f\chi\ , \quad \tilde{m}_f^2(\tilde{\rho}) = 2h_f^2(\tilde{\rho})\tilde{\rho}$$

$$u = \frac{5}{128\pi^2} - \frac{1}{64\pi^2}\sum_f t_u(\tilde{m}_f^2)$$

$$\tilde{\rho}\partial_{\tilde{\rho}}t_u = 2t_u - \frac{2}{1 + \tilde{m}_f^2}$$

Quantum scale symmetry violation in neutrino sector

- Neutrino masses involve beyond standard model physics

$$m_\nu = b_\nu \frac{\varphi_0^2(\chi)}{m_{\mathrm{B-L}}(\chi)} = \frac{b_\nu \varepsilon \chi^2}{g_{\mathrm{B-L}}(\chi)\chi}$$

$$h_\nu(\tilde{\rho}) = \frac{b_\nu \varepsilon}{g_{\mathrm{B-L}}(\tilde{\rho})}$$

$$g_{\mathrm{B-L}}(\tilde{\rho}) = \bar{g}_{\mathrm{B-L}} - c_{\mathrm{B-L}} \ln\left(\frac{\chi}{k}\right)$$

- Effective neutrino Yukawa coupling in Einstein frame

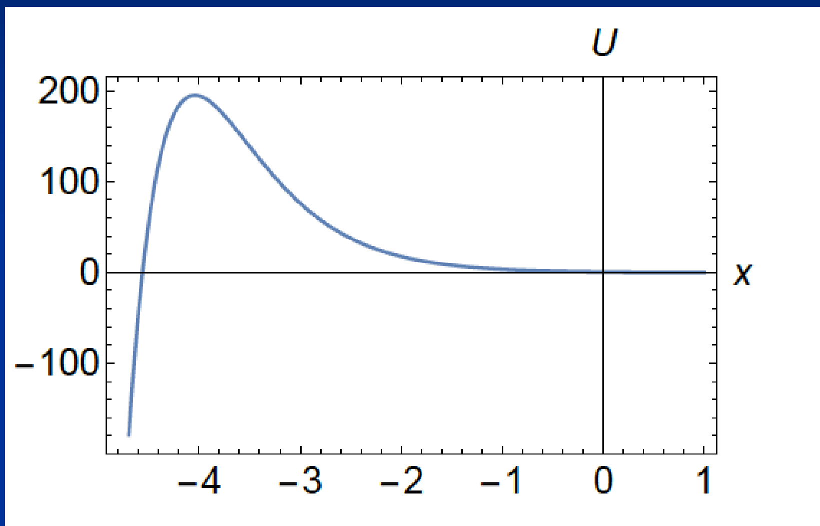
$$h_\nu(\varphi) = \frac{4 b_\nu \varepsilon M}{4\bar{g}_{\mathrm{B-L}} M - c_{\mathrm{B-L}}\varphi}$$

- Cosmon – neutrino coupling

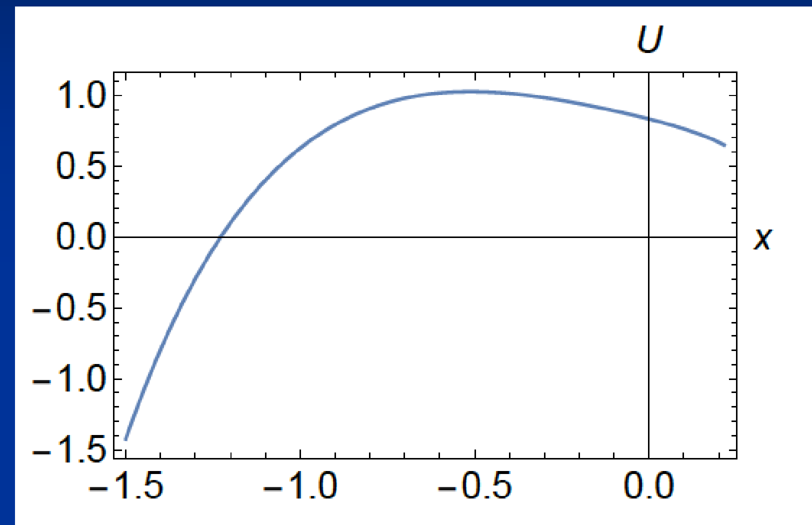
$$\beta = -\frac{\partial \ln m_\nu}{\partial \varphi} M = -\frac{M}{\varphi_c - \varphi}$$

$$\frac{\varphi_c}{M} = \frac{4\bar{g}_{\mathrm{B-L}}}{c_{\mathrm{B-L}}}$$

Scaling potential



$$x = \frac{1}{2M}(\varphi - \bar{\varphi}) = \ln \tilde{\rho} - c_\rho$$



$$x = \frac{1}{2M}(\varphi - \bar{\varphi}) = \ln \tilde{\rho} - c_\rho$$

Quantum scale symmetry in neutrino sector, $c_{B\text{-}L}=0$

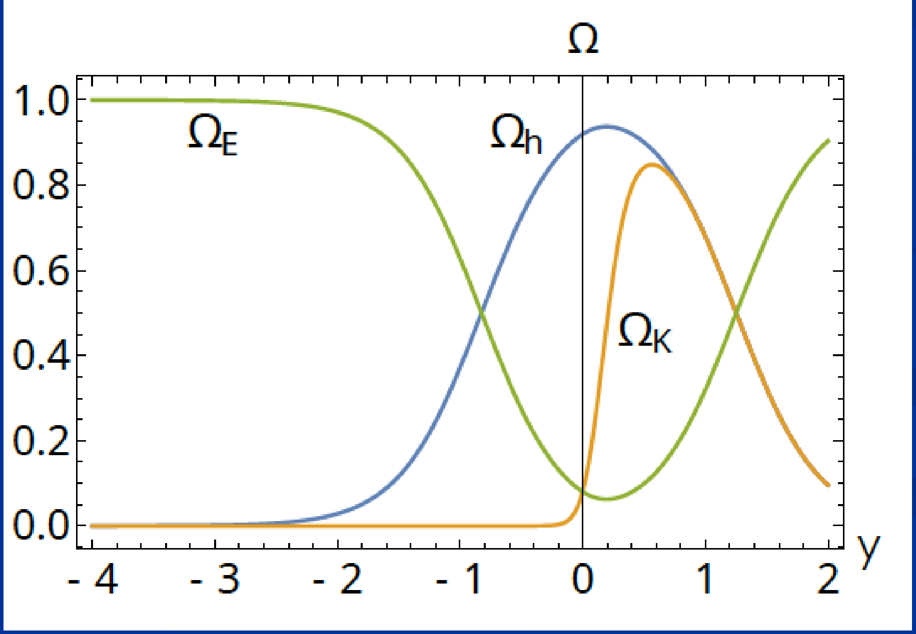
Field dependent neutrino masses

U in units of $U_0 = (2.229 \cdot 10^{-3}\,\mathrm{eV})^4$

(8) Cosmology

For quantum scale symmetry in neutrino sector :

- dynamical solution of cosmological constant problem, but time evolution of dynamical dark energy is not realistic

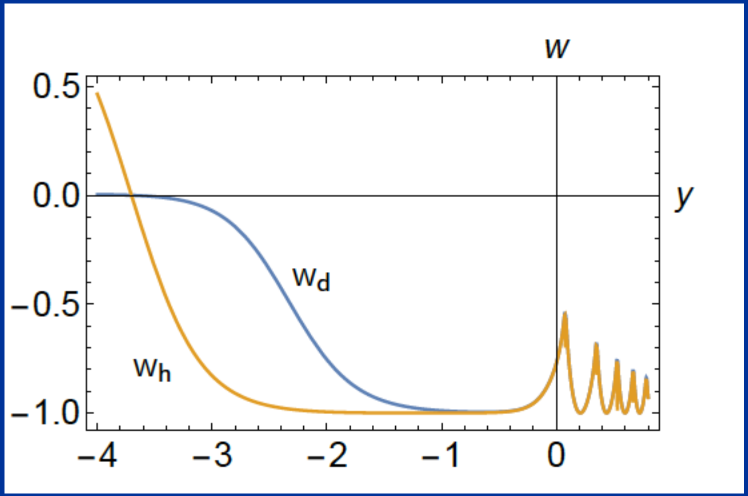


y=ln(a)

Field dependent neutrino mass

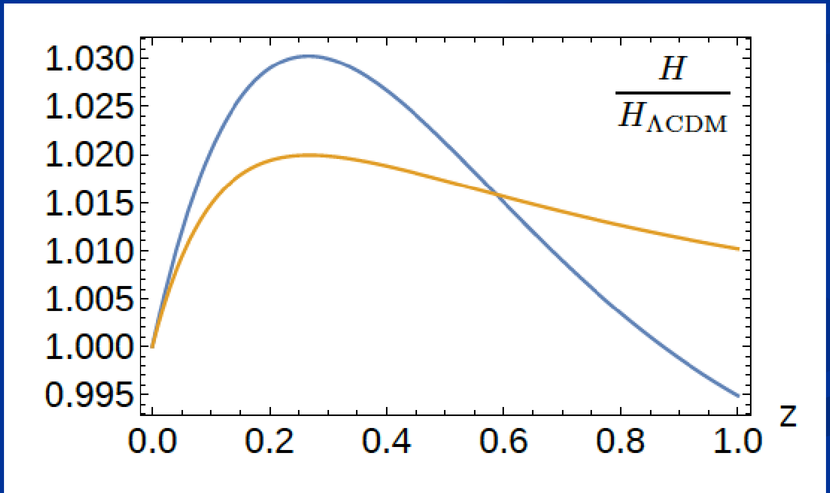
realistic cosmology?

equation of state w



y=ln(a)

Hubble parameter



Conclusion

Fixed point of quantum gravity

with associated quantum scale symmetry,

scaling solutions and relevant parameters

is crucial for understanding the

evolution of our Universe