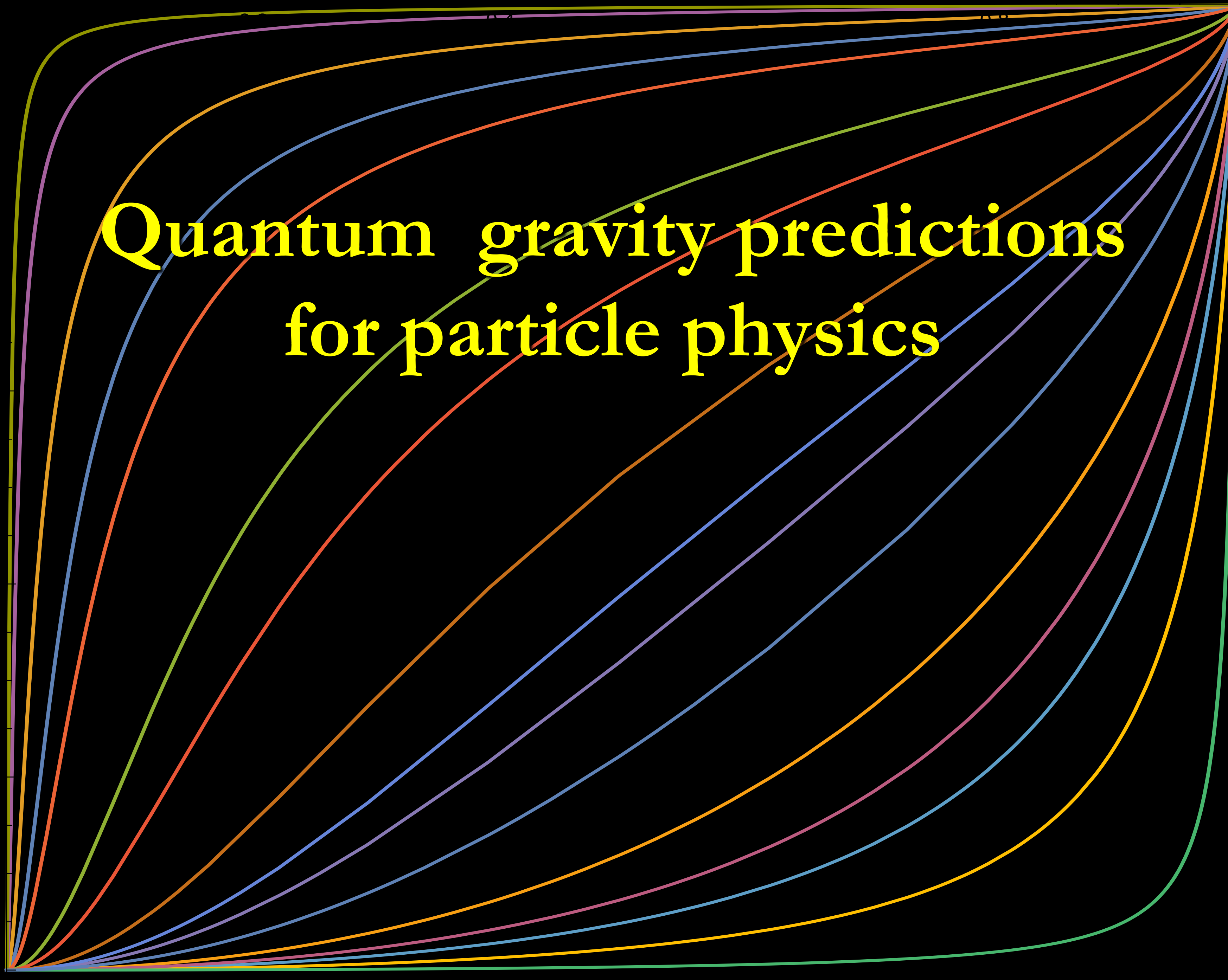
Quantum gravity predictions for particle physics



Prediction of mass of Higgs boson

Asymptotic safety of gravity and the Higgs boson mass

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Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Why can quantum gravity make predictions for particle physics ?

Spontaneous symmetry breaking

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu\varphi^\dagger\partial_\mu\varphi + V(\varphi)$$

$$V(\varphi) = -\mu^2\varphi^\dagger\varphi + \tfrac{1}{2}\lambda(\varphi^\dagger\varphi)^2$$

$$= \tfrac{1}{2}\lambda(\varphi^\dagger\varphi - \varphi_0^2)^2 + \text{const.})$$

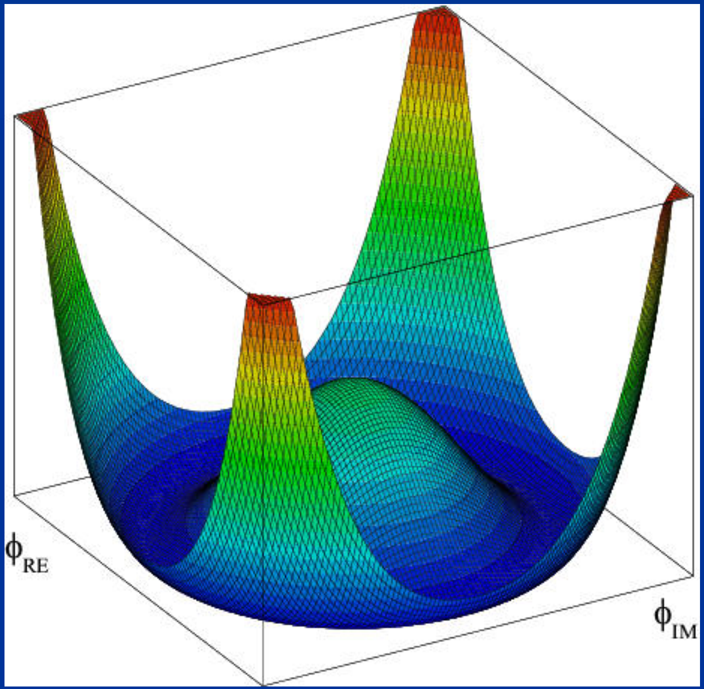
$$\mu^2 > 0$$

$$\varphi_0^2 = \frac{\mu^2}{\lambda}$$

Fermi scale

$$\varphi_0 = 175 \text{ GeV}$$

Scalar potential



$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \frac{1}{2}\lambda(\varphi^\dagger \varphi)^2$$

$$= \frac{1}{2}\lambda(\varphi^\dagger \varphi - \varphi_0^2)^2 + \text{const.})$$

Radial mode and Goldstone mode

expand around minimum of potential

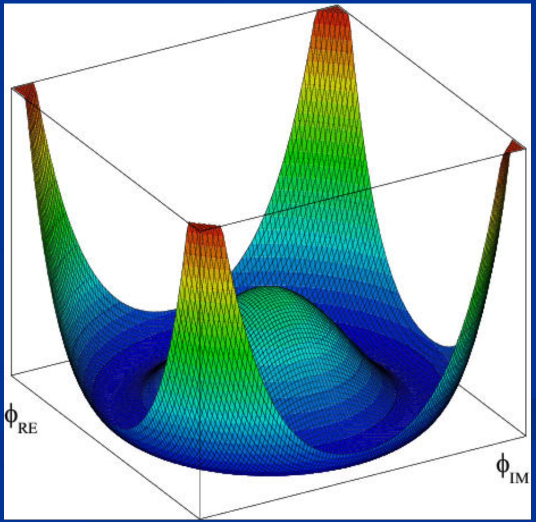
$$\varphi = \varphi_0 + \sigma + i\eta$$

$$\varphi_0, \sigma, \eta : \text{ real}$$

$$-\mathcal{L}_\varphi = \tfrac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \tfrac{1}{2}\partial^\mu \eta \partial_\mu \eta + \tfrac{1}{2} m^2 \sigma^2 + \ldots$$

mass term for radial mode

$$m^2 = 2\lambda\varphi_0^2$$



Quartic scalar coupling

prediction of mass of Higgs boson

=

prediction of value of quartic scalar coupling λ at Fermi scale

$$m^2 = 2\lambda\varphi_0^2$$

Why can quantum gravity make predictions for quartic scalar coupling ?

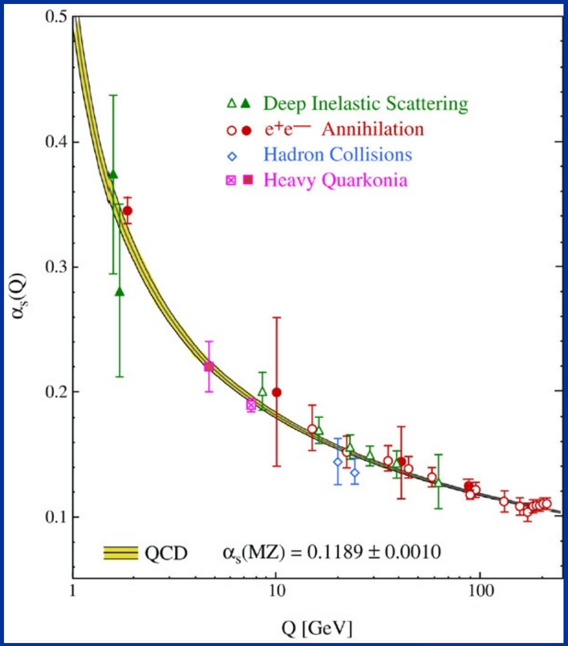
Mass scales

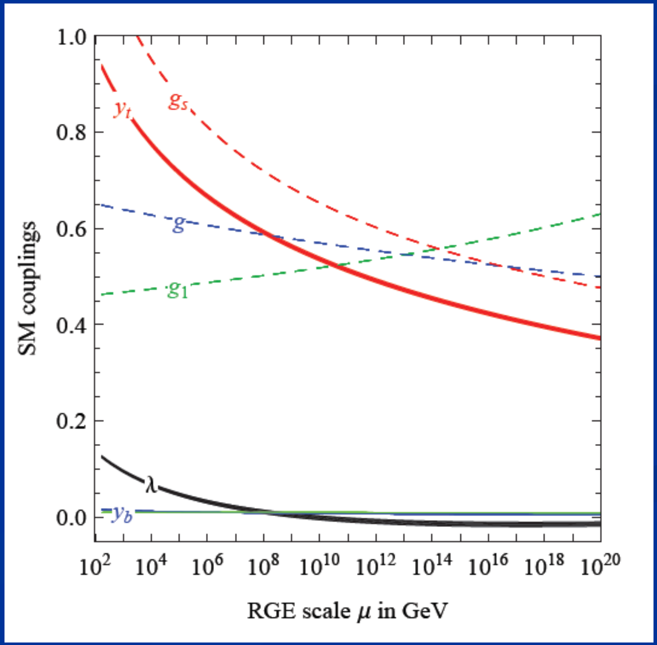
- Fermi scale φ_0 ~ 100 GeV
- Planck mass M ~ 10^{18} GeV
- Gravity at Fermi scale is very weak : How can it influence the effective potential for the Higgs scalar and the mass of the Higgs boson ?

$$\varepsilon = \frac{\varphi_0^2}{M^2} = 5 \cdot 10^{-33}$$

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model





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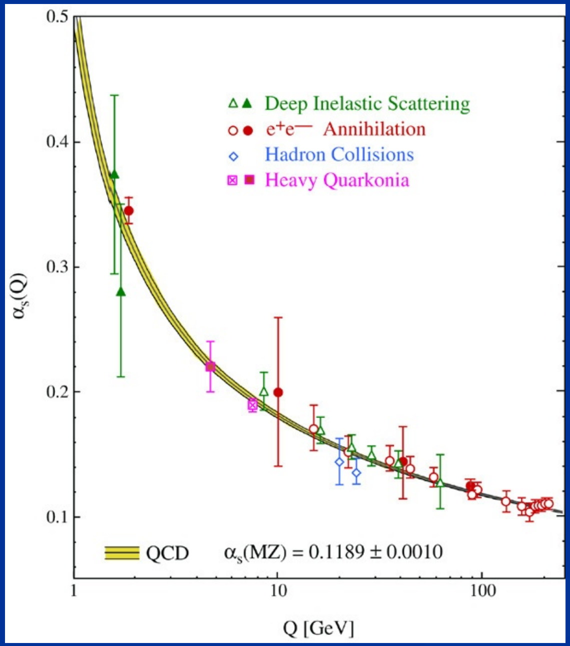
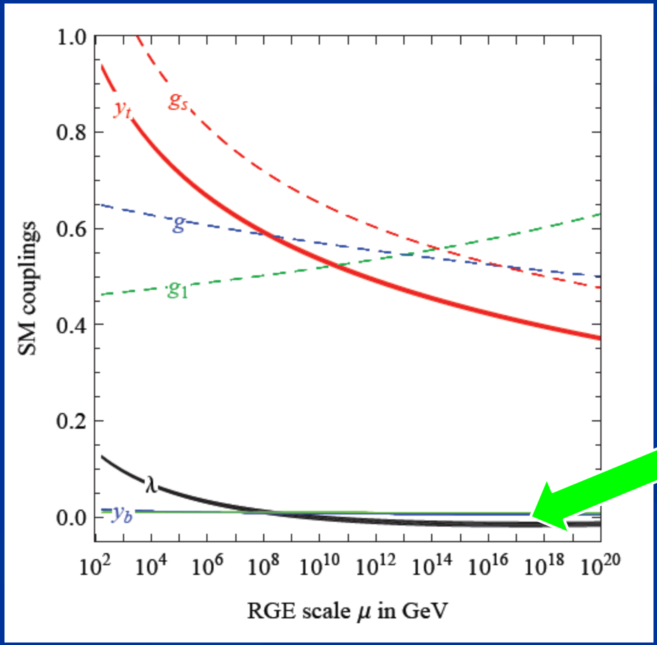
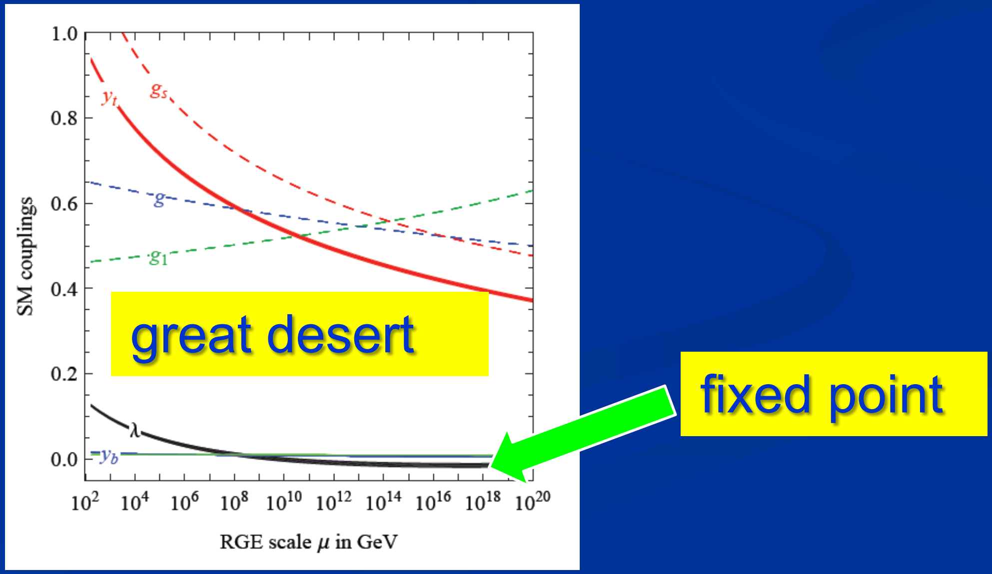


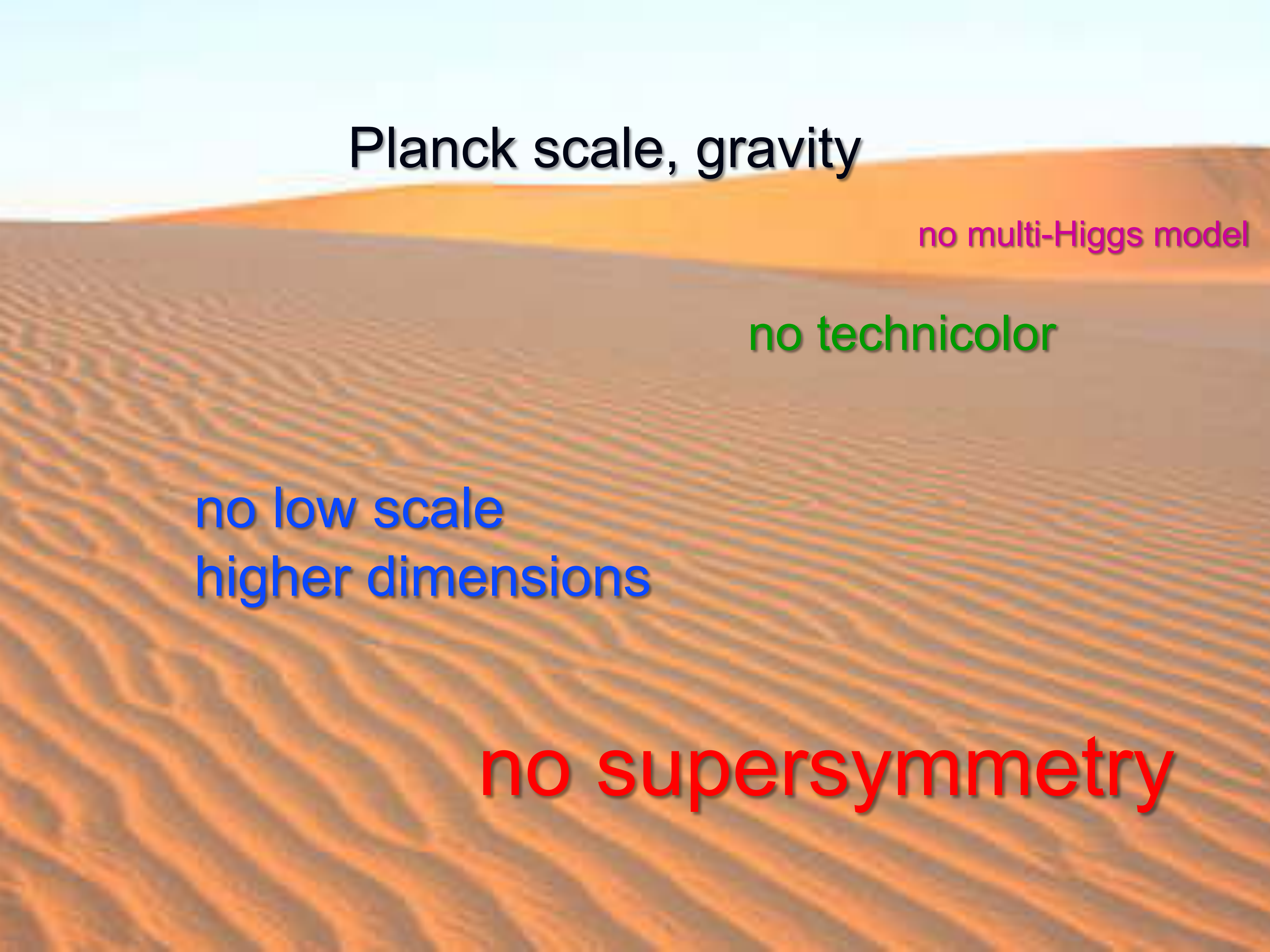


The mass of the Higgs boson, the great desert, and asymptotic safety of gravity

key points

- great desert
- solution of hierarchy problem at high scale
- high scale fixed point
- vanishing scalar coupling at fixed point

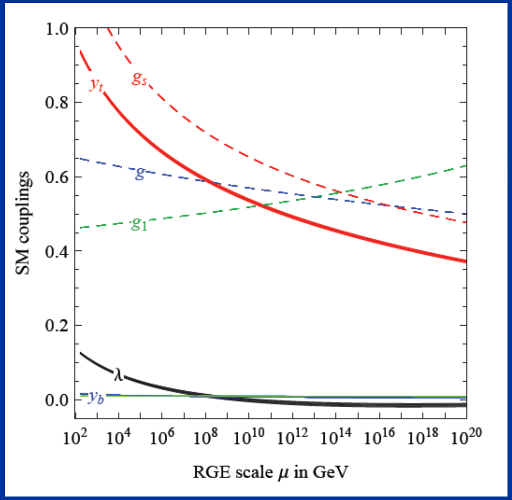



Planck scale, gravity
no multi-Higgs model
no technicolor
no low scale
higher dimensions
no supersymmetry

Essential point for prediction of Higgs boson mass:

Initial value of quartic scalar coupling near Planck mass is predicted by quantum gravity

Extrapolate perturbatively to Fermi scale



Results in prediction for ratio Higgs boson mass over W- boson mass, or Higgs boson mass over top quark mass

Near Planck mass gravity is not weak !

Predictive power !

Flowing couplings

Couplings change with momentum scale due to quantum fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included. The scale k can be momenta, geometric quantities, or just be introduced "by hand".

Flow of k to zero : all fluctuations included, IR
Flow of k to infinity : UV

Renormalization group

How do couplings or physical laws change with scale k ?

Graviton fluctuations erase quartic scalar coupling

Renormalization scale k : Only fluctuations with momenta larger k are included

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

gravity induced anomalous dimension

A > 0

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu}\right)^{A}$$

$$k \to 0 \Rightarrow \lambda \to 0$$

Fixed point

$$k \frac{\partial \lambda}{\partial k} = A \lambda$$

$$\lambda(k) = \lambda(\mu) \left(\frac{k}{\mu} \right)^A$$

The quartic scalar coupling λ has a

fixed point at λ=0

It flows towards the fixed point as k is lowered :

irrelevant coupling

For a UV – complete theory it is predicted to assume the fixed point value

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_\mathrm{p}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t = k\partial_k$$

running Planck mass : $M_\mathrm{p}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$

dimensionless squared Planck mass

$$\tilde{M}_\mathrm{p}^2 = \frac{M_\mathrm{p}^2}{k^2}$$

Strength of gravity

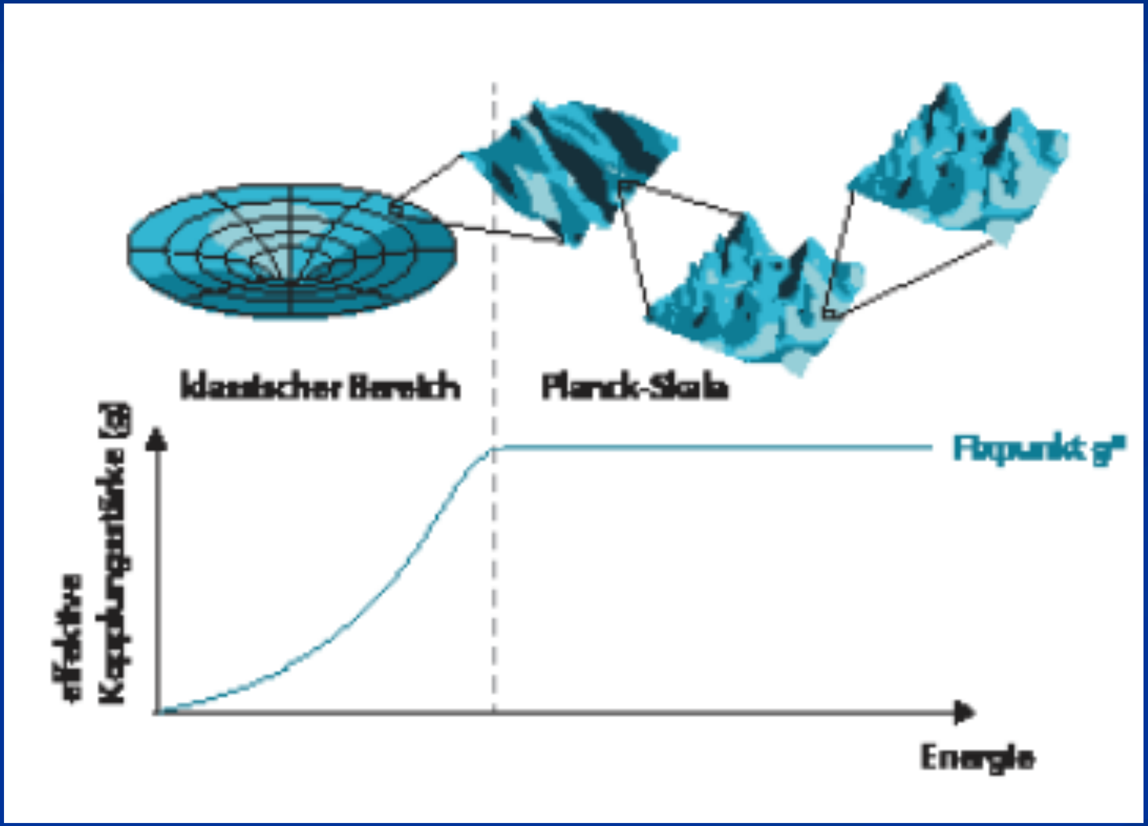
$$g_{grav} = \frac{\ell_p^2}{2\ell^2} = \frac{k^2}{2M^2}$$

l_p : Planck length
M : Planck mass

running gravitational coupling

$$g_{grav} = \frac{k^2}{2M^2(k)} = w^{-1}(k)$$

Strength of gravity



A.Eichhorn, CW, Spektrum der Wissenschaft

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running Planck mass :

$$M_{\rm p}^2(k) = M^2 + \tilde{M}_{\rm p*}^2 k^2$$

dimensionless squared Planck mass

$$\tilde{M}_{\rm p}^2 = \frac{M_{\rm p}^2}{k^2} = \tilde{M}_{\rm p*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{\rm p*}^2}$$

large k : constant A

small k: $A \sim k^2 / M^2$

transition at

$k_t \sim 10^{19}$ GeV

Flowing dimensionless Planck mass

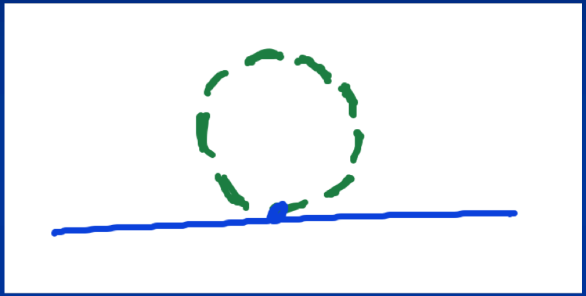
- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

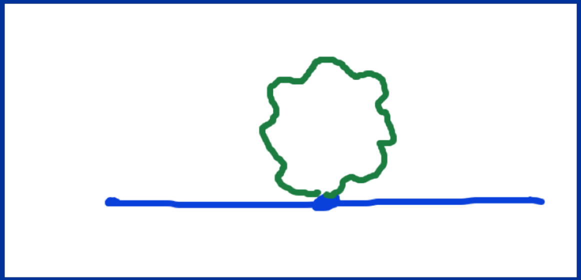
$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

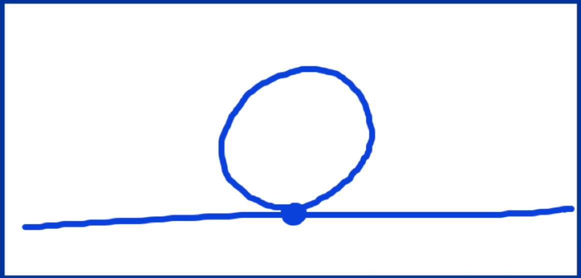
Universality of gravity



scalar loop,
fermion loop

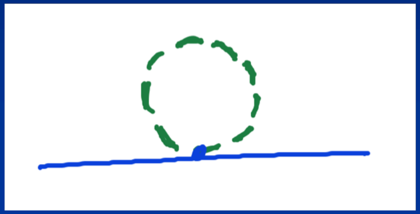


gauge boson
loop

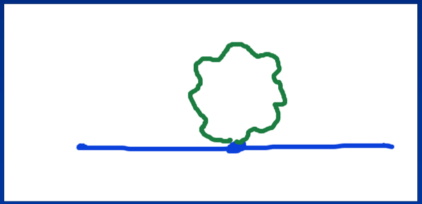


graviton
loop

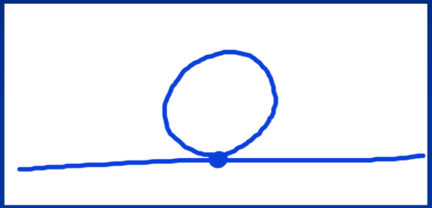
Universality of gravity



scalar loop,
fermion loop



gauge boson
loop



graviton
loop

c is independent of coupling constants

$$\partial_t M^2 = 4ck^2$$

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

matter contribution

$$c_M = \frac{\mathcal{N}_M}{192\pi^2}.$$

$$\mathcal{N}_M = 4\,N_V - N_S - N_F$$

with graviton contribution

$$c_M = \frac{1}{192\pi^2}\left(\mathcal{N}_M + \frac{43}{6} + \frac{75(1-\eta_g/6)}{2(1-v)}\right)$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

solution :

$$M^2(k) = M^2 + 2c_M k^2$$

Flowing dimensionless Planck mass

Flowing
Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

Dimensionless
squared Planck mass

$$w = \frac{M^2}{2k^2}$$

$$\partial_t w = -2w + 2c$$

solution :

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Fixed point and flow away from fixed point

UV - fixed point $w_* = c$ $\partial_t w = -2w + 2c$

approached for k ⟶ ∞ $w = c + \frac{\bar{M}^2}{2k^2}$ $M^2(k) = M^2 + 2c_M k^2$

Near UV – fixed point : M ~ k $\tilde{M}^2_{p*} = 2c$

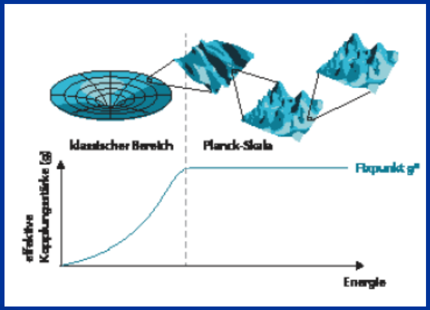
Transition to constant M for small k, gravity gets weak, w^{-1} decreases to zero

Weak and constant gravity

$$M_{\mathrm{p}}^2(k) = \begin{cases} \tilde{M}_{\mathrm{p}*}^2\, k^2 & \text{for } k > k_t \\ M^2 & \text{for } k < k_t \end{cases}$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{\mathrm{p}*}^2}$$

$$M_{\mathrm{p}}^2(k) = M^2 + \tilde{M}_{\mathrm{p}*}^2 k^2$$



Two regimes for the (inverse) strength of gravity

$$\tilde{M}_{\mathrm{p}}^2 = \frac{M_{\mathrm{p}}^2}{k^2} = \tilde{M}_{\mathrm{p}*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

Gravitational contribution to running quartic coupling

$$\partial_t \lambda = A_\lambda \lambda$$

$$A = \frac{1}{48\pi^2 \tilde{M}_p^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$$

$$\partial_t = k \partial_k$$

Running Planck mass : $$M_p^2(k) = M^2 + \tilde{M}_{p*}^2 k^2$$

$$\tilde{M}_p^2 = \frac{M_p^2}{k^2} = \tilde{M}_{p*}^2 \left[\left(\frac{k_t}{k} \right)^2 + 1 \right]$$

$$k_t^2 = \frac{M^2}{\tilde{M}_{p*}^2}$$

large k : constant A

small k: $A \sim k^2 / M^2$

transition at

$k_t \sim 10^{19}$ GeV

UV – fixed point for quartic coupling

Flow equation for λ : $\partial_t \lambda_H = A\lambda_H - C_H$

$$C_H^{(p)} = -\beta_\lambda^{(SM)}$$

$$\approx -\frac{1}{16\pi^2}\left\{12\lambda_H^2 + 12y_t^2\lambda_H - 12y_t^4 + \frac{9}{4}g_2^2 + \frac{9}{10}g_2^2 g_1^2 + \frac{27}{100}g_1^4 - \left(9g_2^2 + \frac{9}{5}g_1^2\right)\lambda_H\right\}$$

Fixed point : λ = C / A

$$\lambda(k_{tr}) \approx 0\ ,\ \beta_\lambda(k_{tr}) \approx 0$$

Quantum Gravity

Quantum Gravity is a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

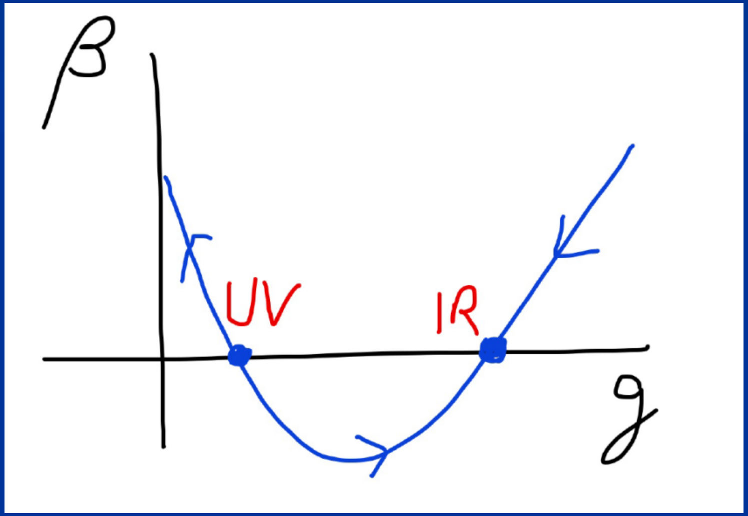
if UV fixed point exists :

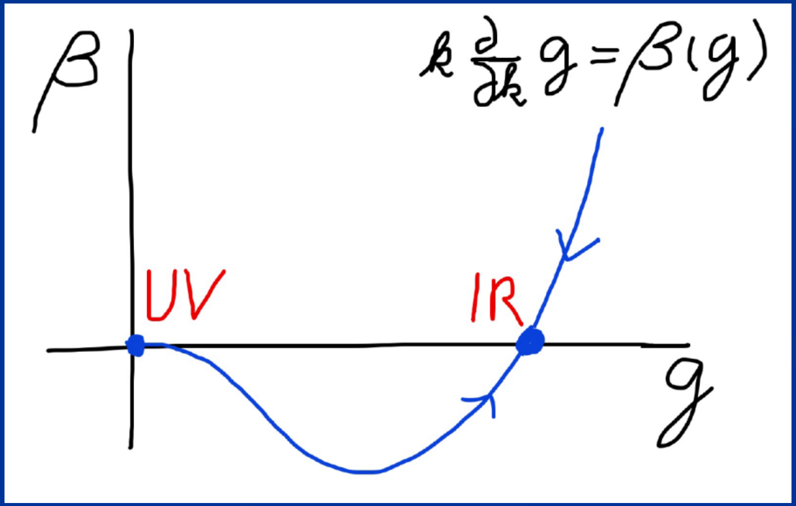
quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

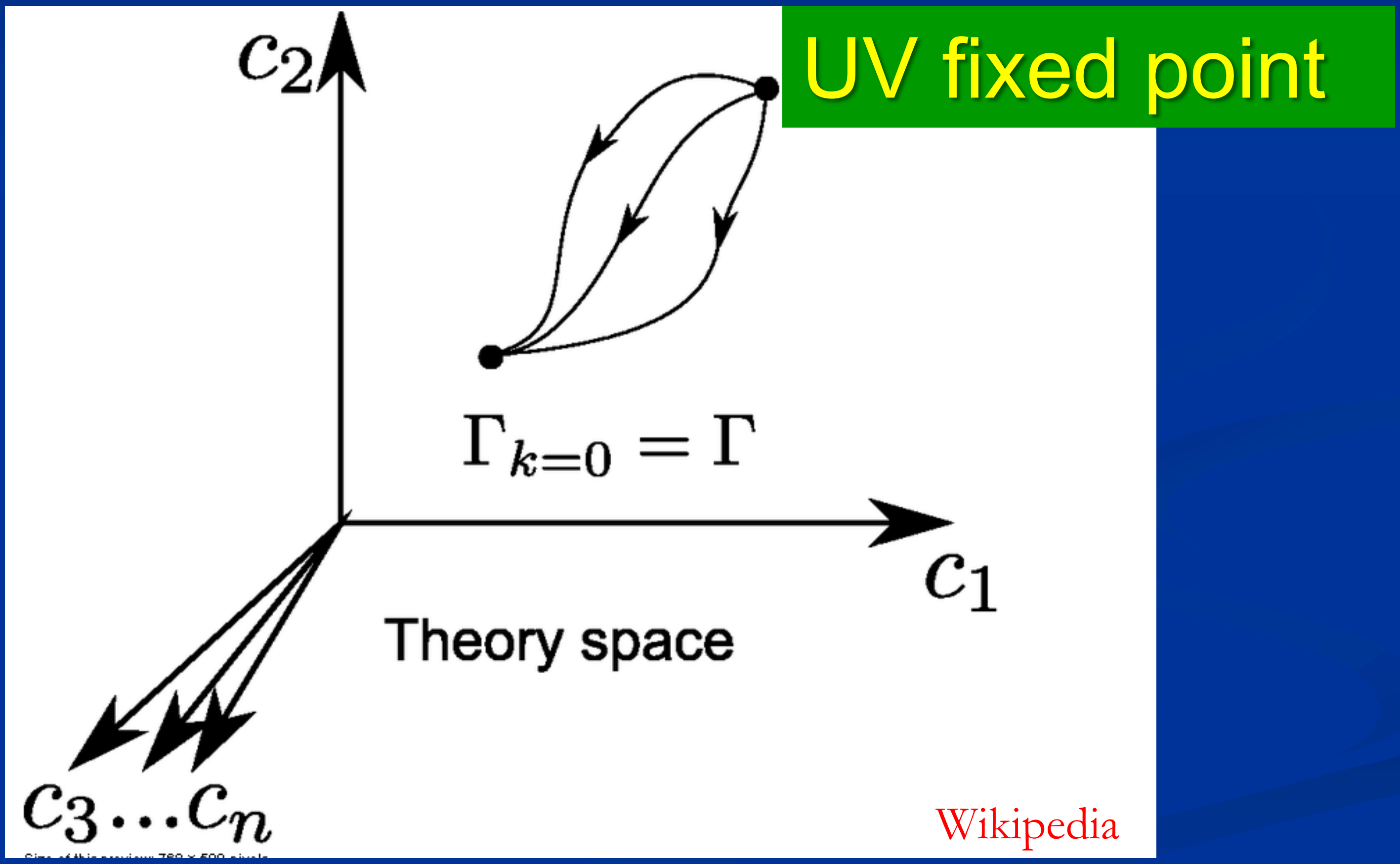
Asymptotic safety   Asymptotic freedom





Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

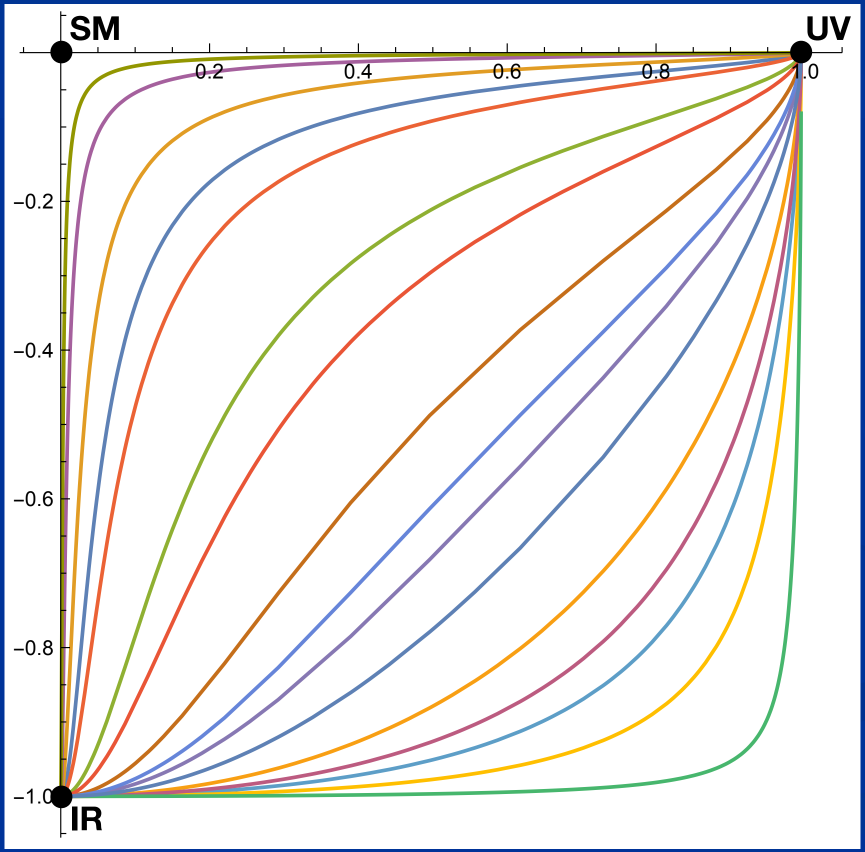
Ultraviolet fixed point



Scale symmetry and fixed points

Relative strength of gravity

Particle scale symmetry

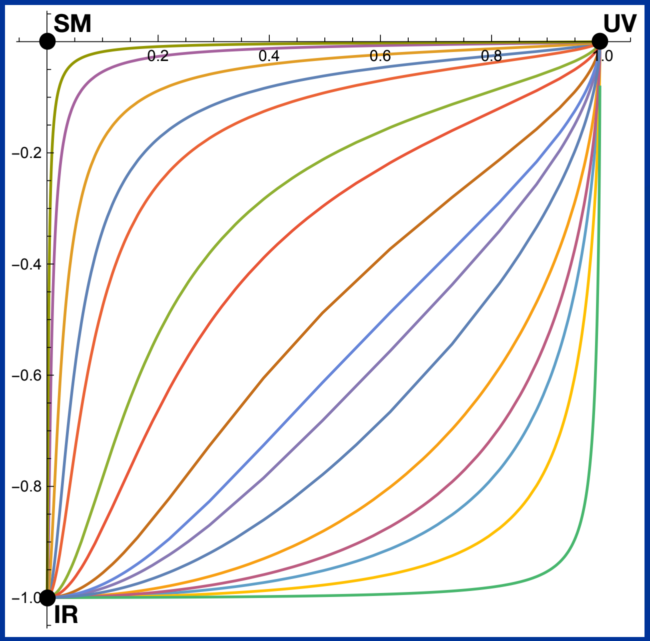


Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

Gravity scale symmetry



At UV – fixed point: gravity scale symmetry is exact quantum scale symmetry

No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized.

Continuous global symmetry

Quantum scale symmetry

- quantum fluctuations can violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !
- quantum fluctuations can generate scale symmetry !

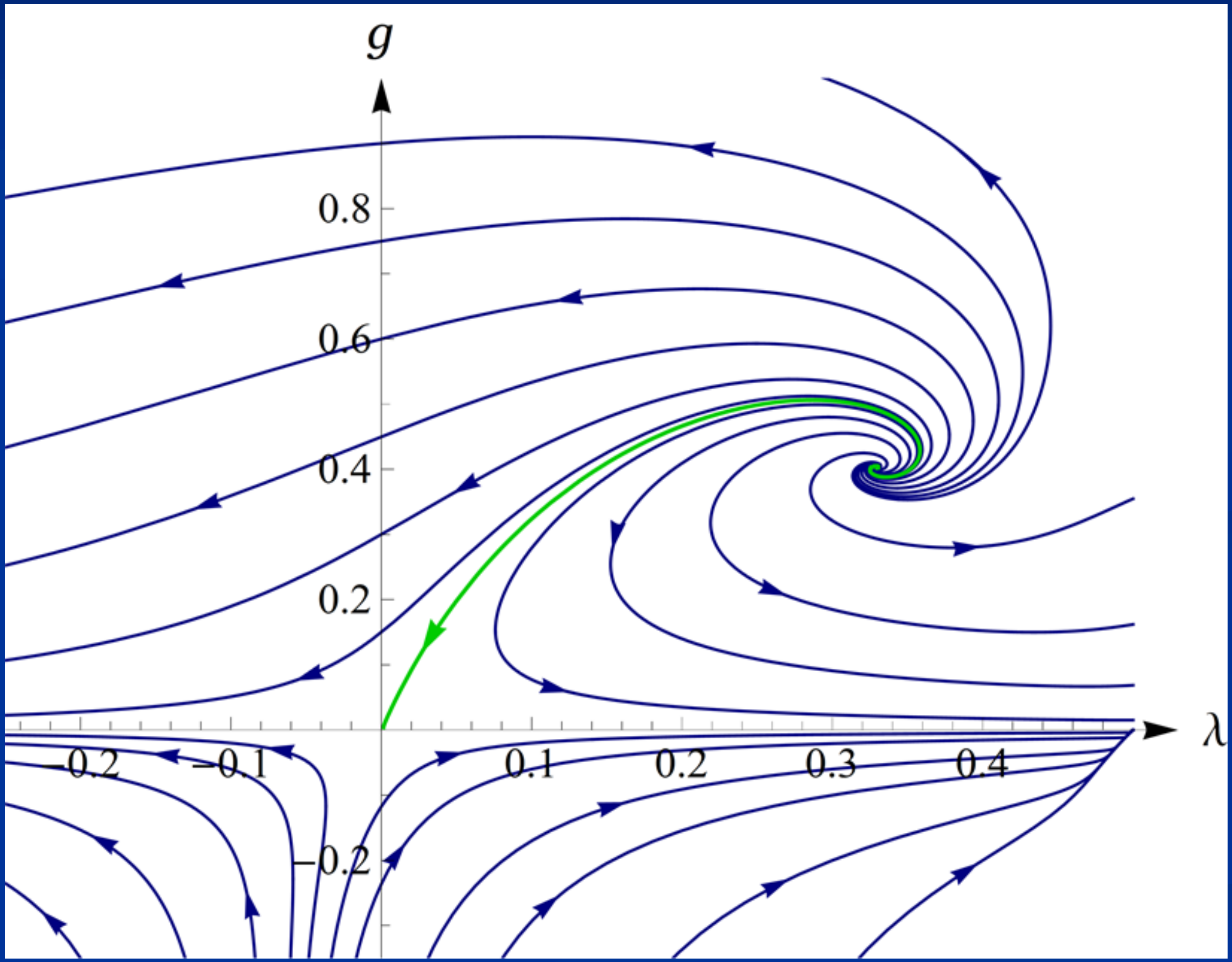
Ultraviolet fixed point for quantum gravity

quantum gravity is

non-perturbatively renormalizable !

S. Weinberg , M. Reuter

UV- fixed point for quantum gravity



Wikipedia

Necessity of UV – fixed point

- If gravity and particle physics can be described by a common quantum field theory which is valid for length scales smaller than the Planck length:
- UV – fixed point is necessary !
- This holds even if the most fundamental formulation of gravity involves different degrees of freedom. Metric may be composite or collective degree of freedom.

Enhanced predictivity for UV – fixed point

- Free parameters of a theory correspond to relevant parameters for small deviations from fixed point.
- If the number of relevant parameters at the UV-fixed point is smaller than the number of free parameters (renormalizable couplings) in the standard model:
- Relations between standard model parameters become predictable !

Fixed points

$$g = \{g_1, \ldots, g_i, \ldots\}$$

couplings

$$\tilde{g}_i = g_i k^{-d_i}$$

dimensionless

Flow equation $\partial_t \tilde{g}_i = \beta_i(\tilde{g})$

Fixed points: zeros of beta-function

No running → No scale

Quantum scale symmetry

Stability matrix

$$g = \{g_1, \ldots, g_i, \ldots\}$$

$$\tilde{g}_i = g_i k^{-d_i}$$

couplings

dimensionless

Flow equation

$$\partial_t \tilde{g}_i = \beta_i(\tilde{g}) = -d_i \tilde{g}_i + f_i(\tilde{g})$$

Expand in vicinity of fixed point

$$\partial_t \tilde{g}_i = \sum_j \left.\frac{\partial \beta_i}{\partial \tilde{g}_j}\right|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

T : stability matrix

Critical exponents

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j}\bigg|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

θ_l : Eigenvalues of stability matrix T
= Critical exponents

Linearized solution

$$\tilde{g}_i = \tilde{g}_{i*} + \sum_l C_l V_i^l \left(\frac{k}{\mu}\right)^{-\theta_l}$$

Irrelevant parameters: eigenvectors in coupling constant space with $\theta_l < 0$

flow towards fixed point values as k is lowered

Irrelevant parameters

- “Forget” information about initial values
- Central ingredient for

  predictivity of quantum field theories
- For UV – complete theories : irrelevant parameters have to take precisely the fixed point values
- **Relevant parameters** flow away from fixed point as k is lowered – they are the only free parameters

Relevant and irrelevant parameters

$$\partial_t \tilde{g}_i = \sum_j \frac{\partial \beta_i}{\partial \tilde{g}_j}\bigg|_{\tilde{g}=\tilde{g}_*} (\tilde{g}_j - \tilde{g}_{j*}) = -T_{ij}(\tilde{g}_j - \tilde{g}_{j*})$$

θ_l : Eigenvalues of stability matrix T
= Critical exponents

Linearized solution

$$\tilde{g}_i = \tilde{g}_{i*} + \sum_l C_l V_i^l \left(\frac{k}{\mu}\right)^{-\theta_l}$$

Irrelevant parameters $\theta_l < 0$ → $C_l = 0$

Relevant parameters for positive
Critical exponents have free C_l

Predictivity at fixed point

- Irrelevant parameters are predicted to take fixed point values
- Only relevant parameters are free
- Number of free parameters of a renormalizable quantum field theory = number of relevant parameters at the fixed point

a prediction…

Asymptotic safety of gravity and the Higgs [illegible] mass

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12 Ja[illegible]

Abstract

There are indications that gravity is [illegible]afe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that th[illegible] case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we [illegible]stion of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalo[illegible]$_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined [illegible] at zero. This results in $m_H = m_{\text{min}} = 126$ GeV, with only a few GeV uncertainty. This prediction is inde[illegible] details of the short distance running and holds for a wide class of extensions of the SM as well.

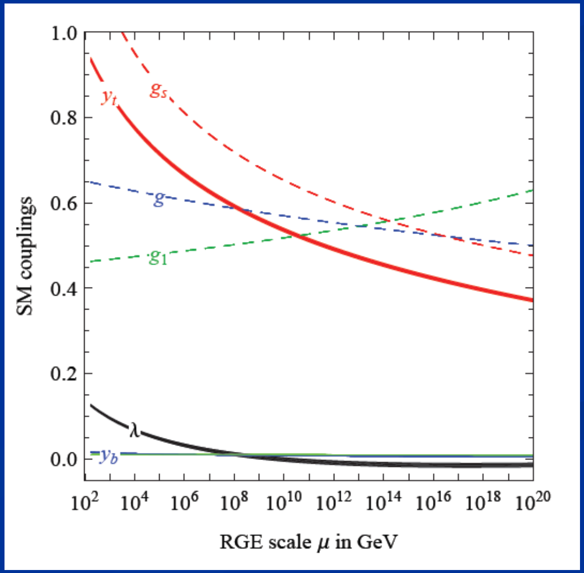
Quartic scalar coupling is irrelevant coupling

s in $m_H = m_{\text{min}} = 126$ GeV, with o

Prediction of Higgs boson mass:

- Value of quartic scalar coupling near Planck mass is predicted by UV- fixed point
- Gravity decouples below Planck mass , resulting in perturbative flow

Extrapolate perturbatively to Fermi scale :



How to compute non-perturbative quantum gravity effects ?

Quantum gravity computation by functional renormalization

Introduce infrared cutoff with scale k, such that only fluctuations with (covariant) momenta larger than k are included.

Then lower k towards zero

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

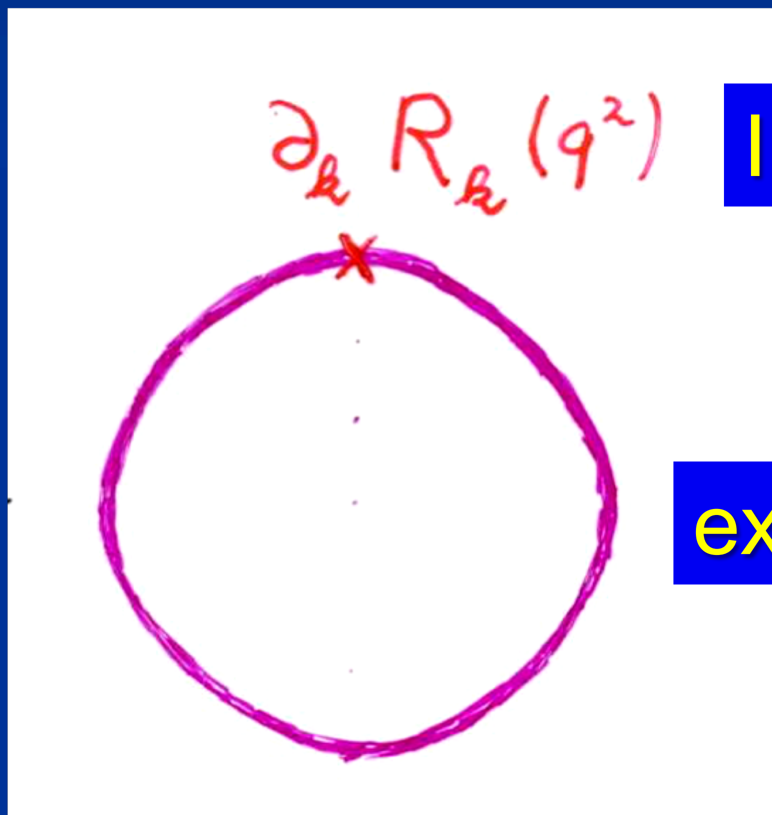
'92

$$\left(\Gamma_k^{(2)} \right)_{ab} (q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$$\mathrm{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)

Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

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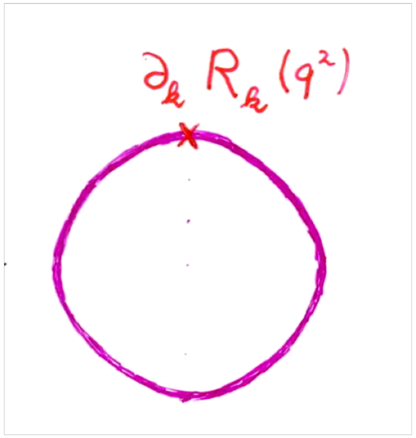
(fermions : STr)



From

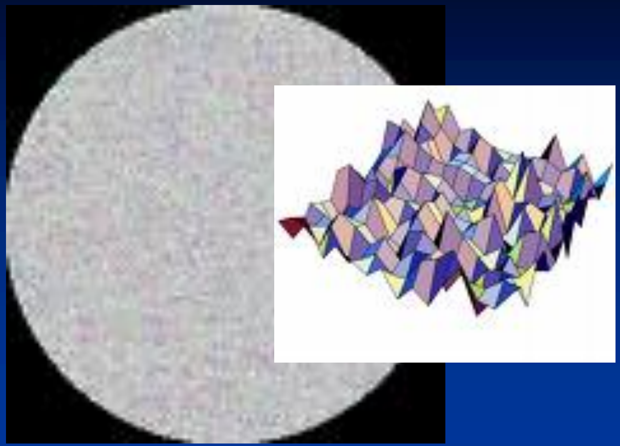
Microscopic Laws

(Interactions, classical action)

to ↓ Fluctuations!

Macroscopic Observation

(Free energy functional, effective action)



From

Microscopic Laws

(Interactions, classical action)

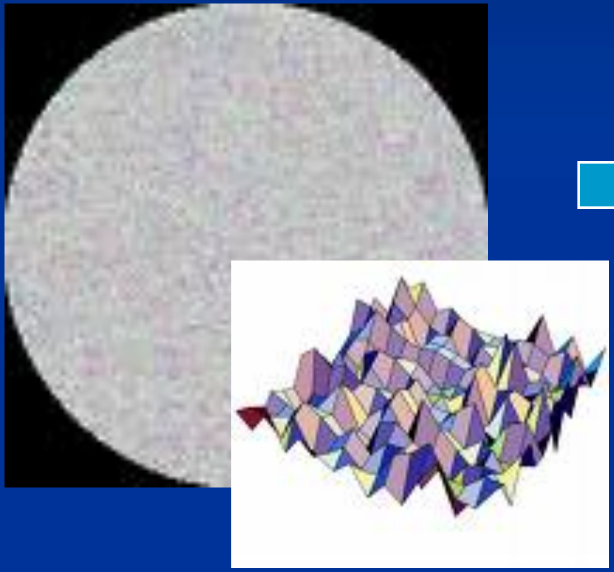
↓

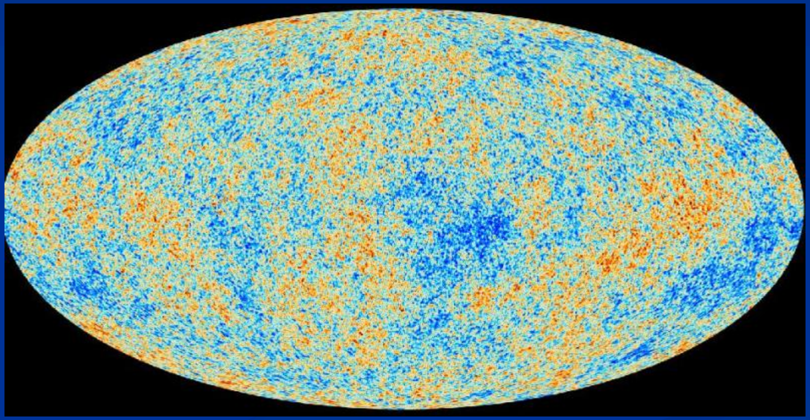
to ↓ Fluctuations!

Macroscopic Observation

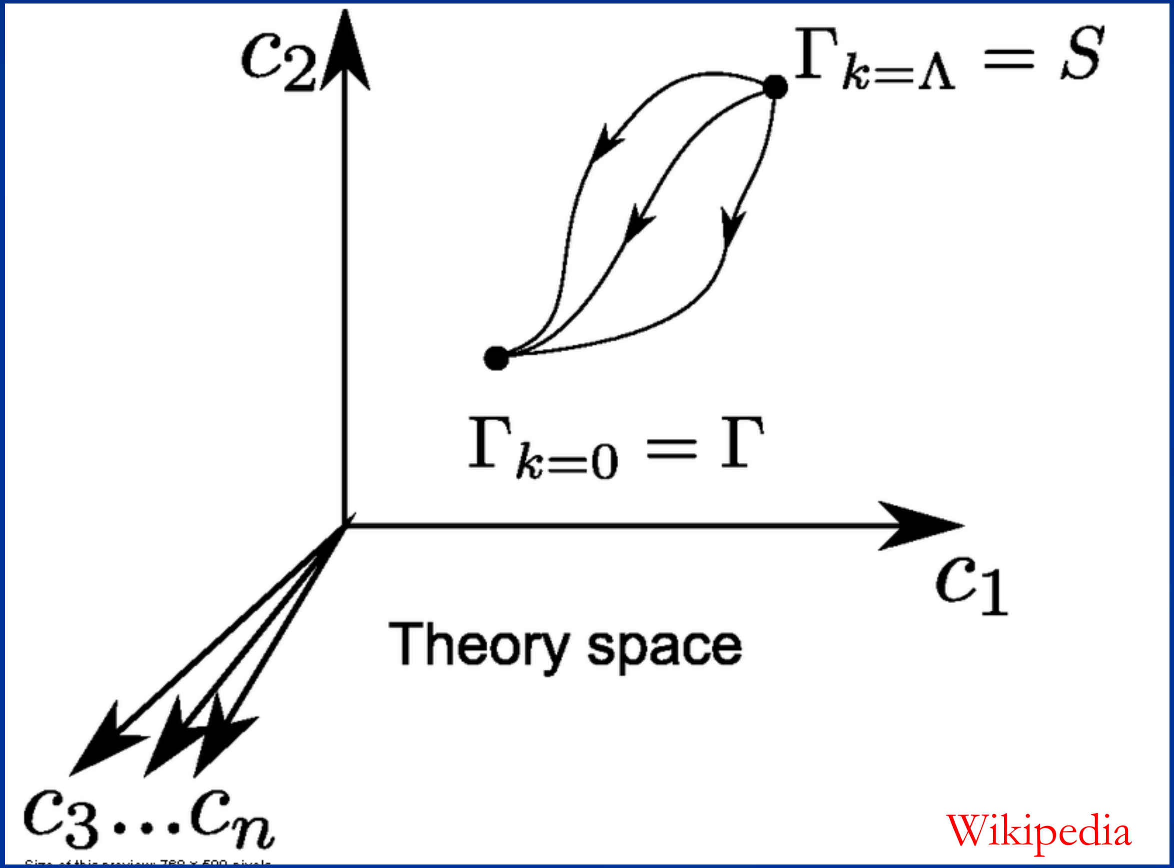
(Free energy functional, effective action)

Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe

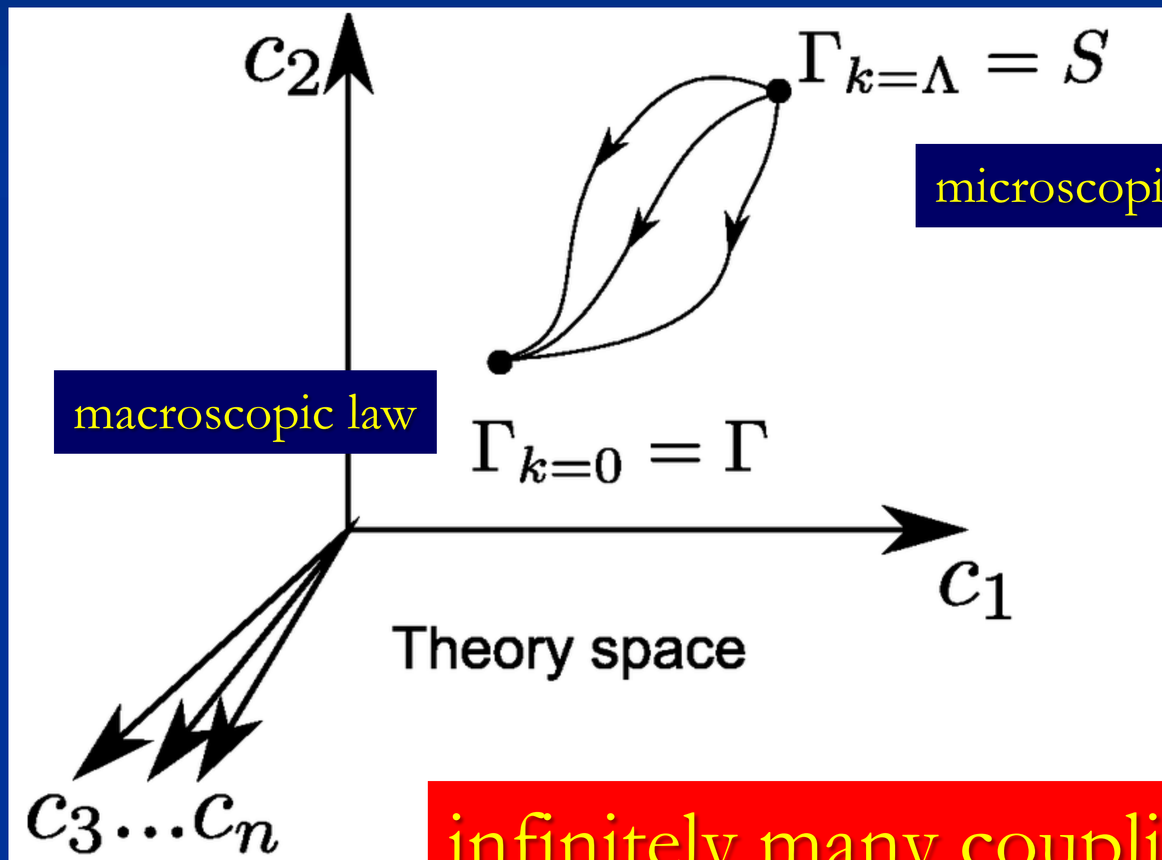




functional renormalization : flowing action



flowing action

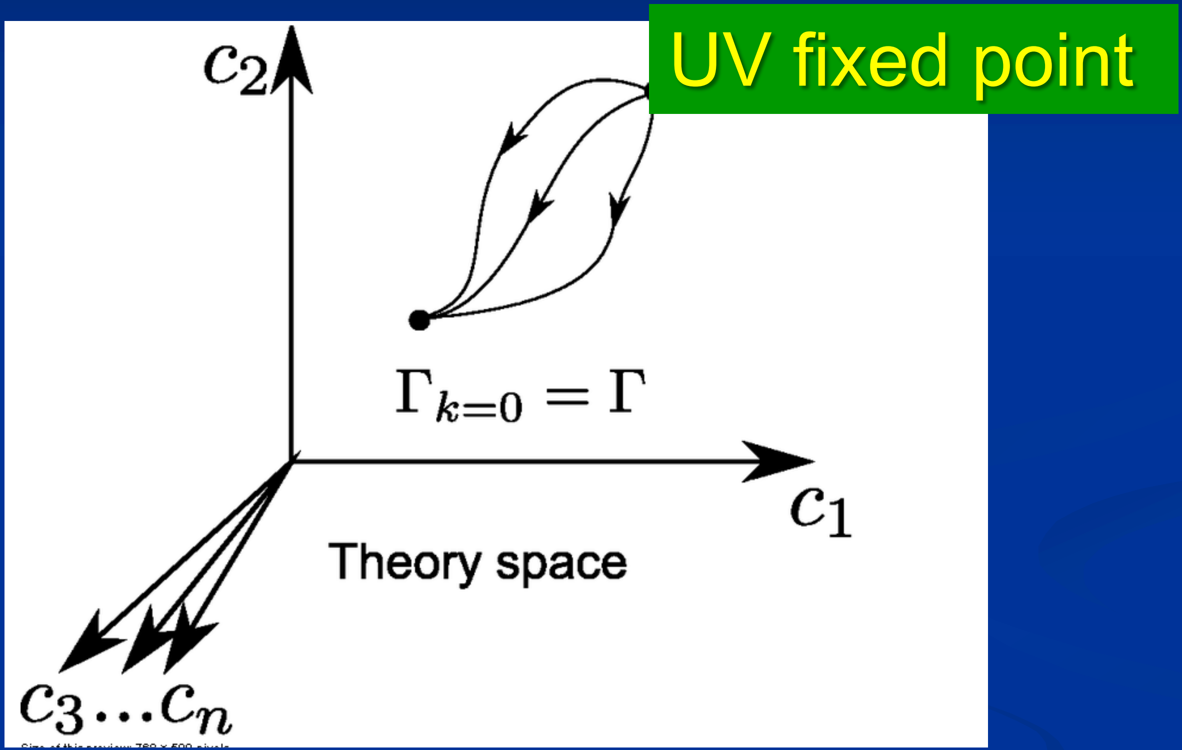


microscopic law

macroscopic law

infinitely many couplings

Ultraviolet fixed point



Extrapolation of microscopic law to infinitely short distances is possible.

Complete theory

flow of functions

Effective potential includes all fluctuations

Average potential U_k

$\equiv$ scale dependent effective potential
$\equiv$ coarse grained free energy

Only fluctuations with momenta $q^2 > k^2$ included

k: infrared cutoff for fluctuations, "average scale"

Λ: characteristic scale for microphysics

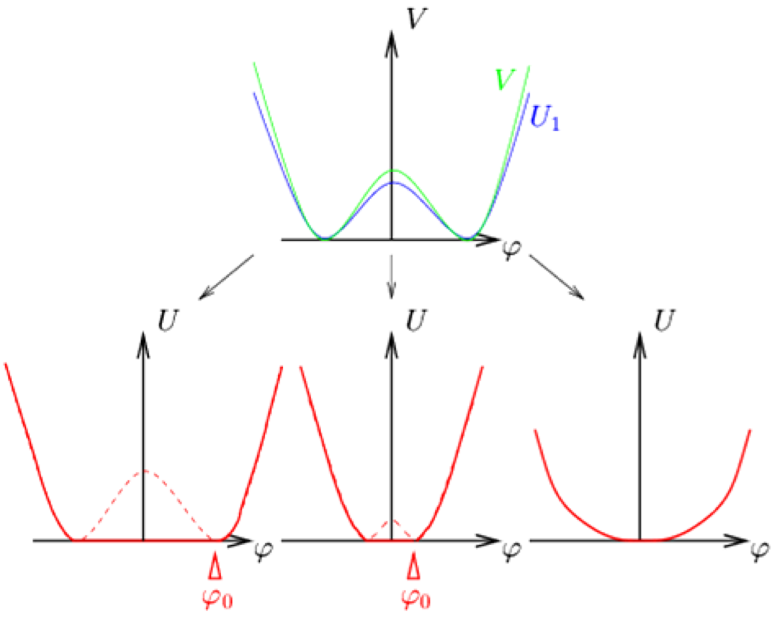
$$U_\Lambda \approx S \rightarrow U_0 \equiv U$$

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$-symmetry:

$$S = \int d^dx \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\} ; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Flow equation for average potential

cutoff

$$\partial_k U_k(\varphi) = \frac{1}{2}\sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

propagator with cutoff

$\bar{M}^2_{k,ab} = \frac{\partial^2 U_k}{\partial\varphi_a \partial\varphi_b}$: Mass matrix

$\bar{M}^2_{k,i}$: Eigenvalues of mass matrix

R_k : IR-cutoff

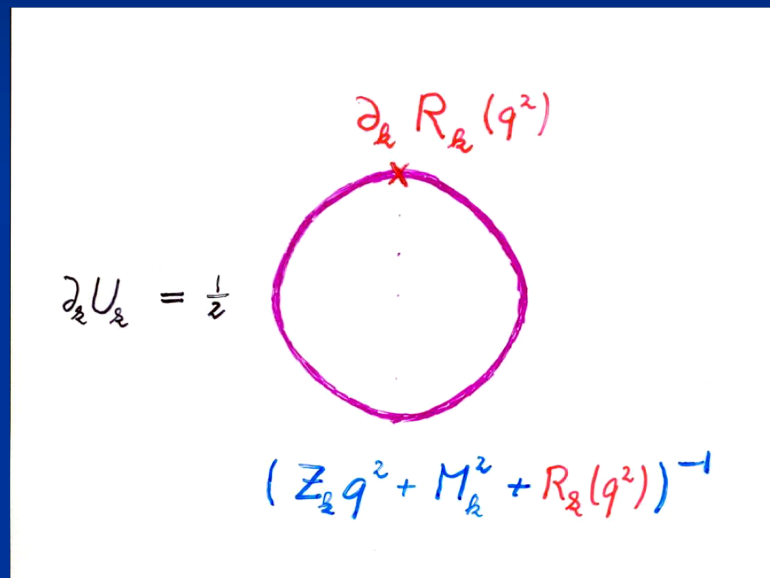
e.g $R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $R_k = Z_k (k^2 - q^2)\Theta(k^2 - q^2)$ (Litim)

$\lim_{k\to 0} R_k = 0$

$\lim_{k\to\infty} R_k \to \infty$

Simple one loop structure – nevertheless (almost) exact



$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}^2_{k,i}(\varphi)}$$

Well tested for non-perturbative phenomena:
critical exponents, Kosterlitz-Thouless phase transition, etc

Critical exponents , d=3

N	ν		η	
0	0.590	0.5878	0.039	0.0292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.980	0.0030	0.003

FRG world FRG world

FRG : first order derivative expansion

"average" of other methods
(typically $\pm(0.0010 - 0.0020)$)

Computation of graviton contribution

traceless transverse metric fluctuations

Graviton approximation

Graviton propagator G

effective action

$$\Gamma = \int_x \sqrt{g}\left(-\frac{M^2}{2}R + V\right)$$

flat space:

$$G^{-1} = \frac{M^2 q^2}{4} - \frac{V}{2}$$

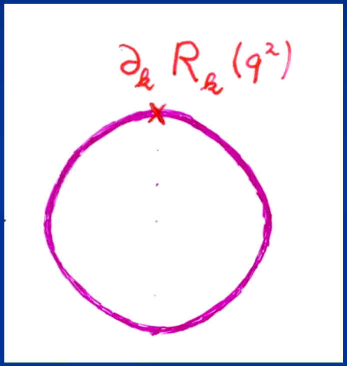
for V>0 : ”tachyonic mass term” $-\frac{2V}{M^2}$

curved space:

$$G^{-1} = \sqrt{g}\left\{\frac{M^2}{4}\left(-D^2 + \frac{2R}{3}\right) - \frac{V}{2}\right\}$$

Graviton contribution to flow of cosmological constant V

$$\partial_t V = k\,\partial_k V = 5 I_k\left(-\tfrac{2V}{M^2}\right)$$



$$I_k(m^2) = \frac{1}{2}\int_q \left(q^2 + R_k(q) + m^2\right)^{-1} \partial_t R_k(q)$$

Litim cutoff: $$R_k(q) = (k^2 - q^2)\,\theta(k^2 - q^2)$$

$$I_k(m^2) = \frac{1}{32\pi^2}\frac{k^6}{k^2 + m^2}$$

$$m^2 = -\frac{2V}{M^2}$$

Valid either for constant M or M = χ

Graviton contribution to flow of effective scalar potential :

Replace V by effective potential U
add scalar fluctuation :

$$\partial_t U = \frac{k^6}{32\pi^2}\left(\frac{5}{k^2 - 2U/M^2} + \frac{1}{k^2 + \partial^2 U/\partial^2\chi}\right)$$

Graviton contribution to flow of quartic scalar coupling :

- positive and substantial anomalous dimension A

$$\partial_t \lambda = A_\lambda \lambda + \frac{9\lambda^2}{16\pi^2},$$

Gravity decouples below Planck mass

$$\partial_t \lambda = \frac{9\lambda^2}{16\pi^2} + \frac{5\lambda k^2}{16\pi^2 M^2}$$

Graviton contributions to effective potential for Higgs field h

Graviton contribution for $M^2 = f\,k^2 + \chi^2$:

$$\partial_t U_g = \frac{5k^6}{32\pi^2}\left(k^2 - \frac{2U}{\chi^2 + fk^2}\right)^{-1}$$

Take derivatives with respect to $\rho = h^2$ (primes)

mass term

$$\partial_t U'_g = A^{(g)} U'$$

quartic coupling

$$\partial_t U''_g\big|_{U'=0} = A^{(g)} U''$$

Gravity induced anomalous dimension

$$A^{(g)} = \frac{5k^6}{16\pi^2(\chi^2 + fk^2)}\left(k^2 - \frac{2U}{\chi^2 + fk^2}\right)^{-2}$$

Graviton contributions to quartic Higgs coupling

Gravity induced anomalous dimension is universal for all scalar fields

$$\partial_t \lambda = A_\lambda \lambda$$

$$A^{(g)} = \frac{5k^6}{16\pi^2(\chi^2 + fk^2)}\left(k^2 - \frac{2U}{\chi^2 + fk^2}\right)^{-2}$$

For large k :

$$A^{(g)} = \frac{5}{12\pi^2 f\left(1 - v_*\right)^2}$$

$$v_* = \frac{2u}{f}$$

f = 2c dimensionless squared Planck mass , u = U/k^4

Gravity contributions to running couplings in standard model

$$k\frac{dx_j}{dk} = \beta_j^{SM} + \beta_j^{grav}$$

$$\beta_j^{grav} = \frac{a_j}{8\pi}\frac{k^2}{M_P^2(k)}x_j$$

Running Planck mass : $M_P^2(k) = M_P^2 + 2\xi_0 k^2$ $\quad k_{tr} = \frac{M_P}{\sqrt{2\xi_0}} \approx 10^{19}\ \text{GeV}$

Large k : $x_j(k) \sim k^{A_j}$ $\quad A_j = \frac{a_j}{16\pi\xi_0}$

For quartic scalar coupling (R.Percacci et al)

$$a_\lambda \approx 3.1\ ,\ A_\lambda \simeq 2.6$$

Add fermions +…

$$\beta_\lambda = \frac{a_\lambda}{16\pi\xi_0}\lambda + \frac{1}{16\pi^2}(24\lambda^2 + 12\lambda h^2 - 6h^4)$$

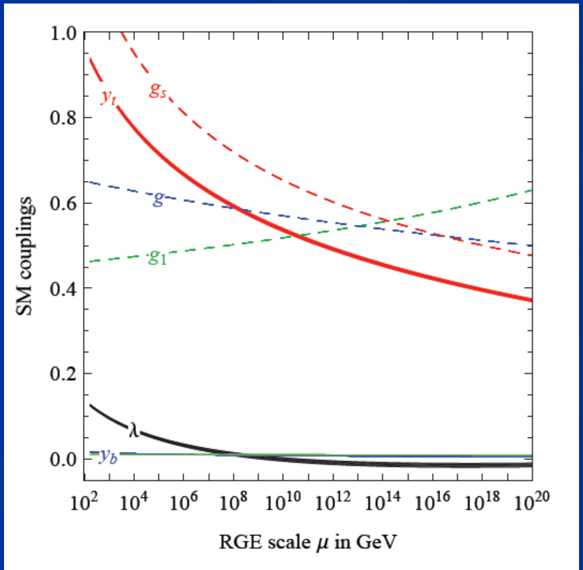
$$24\lambda_*^2 + 12\lambda h_*^2 - 6h_*^4 + \frac{\pi a_\lambda \lambda_*}{\xi_0} = 0$$

$$\lambda(k_{tr}) \approx 0\ ,\ \beta_\lambda(k_{tr}) \approx 0$$

Essential point for prediction of Higgs boson mass:

- More precisely : ratio Higgs boson mass over W-boson mass, or Higgs boson mass over top quark mass
- Initial value of quartic scalar coupling near Planck mass is predicted by UV- fixed point

Extrapolate perturbatively to Fermi scale :



Relevant and irrelevant dimensionless couplings for Higgs potential

J.Pawlowski, M.Reichert, M.Yamada,…

Effective scalar potential depends on $\rho = h^2$
Dimensionless quantities :

$$\tilde{\rho} = Z_\phi \rho / k^2$$

$$\tilde{U}(\tilde{\rho}) = U(\rho) / k^4$$

$$M_{\mathrm{p}}^2(k) = \tilde{M}_{\mathrm{p}*}^2 \, k^2$$

$$v(\rho) = \frac{2U(\rho)}{M_{\mathrm{p}}^2 k^2} = \frac{2\tilde{U}(\rho)}{\tilde{M}_{\mathrm{p}}^2}$$

Effective potential near origin

$$\tilde{U} = \tilde{V} + \tilde{m}_H^2 \tilde{\rho} + \frac{\tilde{\lambda}_H}{2} \tilde{\rho}^2 + \cdots$$

$$\tilde{V} = U(\rho = 0)/k^4 \qquad v_0 = 2\tilde{V}/\tilde{M}_p^2$$

dimensionless mass term

$$\tilde{m}_H^2 = m_H^2/(Z_\phi k^2)$$

quartic coupling

$$\tilde{\lambda}_H = \lambda_H / Z_\phi^2$$

Flow equations

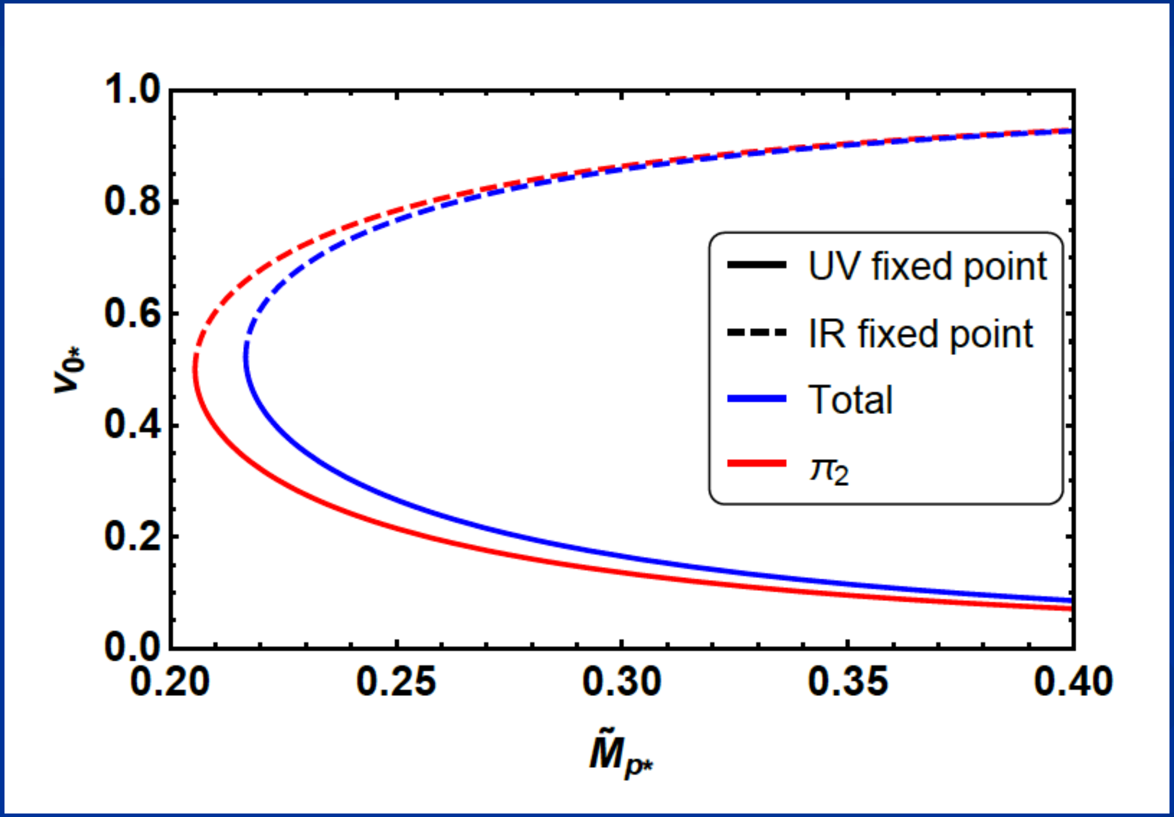
$$\partial_t v_0 = -4v_0 + \frac{\Delta N}{16\pi^2 \tilde{M}_{\mathrm{p}}^2} + \frac{1}{12\pi^2 \tilde{M}_{\mathrm{p}}^2}\left[\frac{5}{1-v_0} + \frac{1}{1-v_0/4}\right] - \frac{\tilde{m}_H^2}{4\pi^2 \tilde{M}_{\mathrm{p}}^2}$$

$$\Delta N = \Delta N_S + 2N_B - 2N_F$$

$$\partial_t \tilde{m}_H^2 = -2\tilde{m}_H^2 - \frac{3\tilde{\lambda}_H}{16\pi^2} + \frac{\tilde{m}_H^2}{48\pi^2 \tilde{M}_{\mathrm{p}}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

$$\partial_t \tilde{\lambda}_H = \frac{\tilde{\lambda}_H}{48\pi^2 \tilde{M}_{\mathrm{p}}^2}\left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2}\right]$$

Fixed point for v_0



Stability matrix

For fixed point at $\tilde{m}^2_{H*} = \tilde{\lambda}_{H*} = 0$

$$T = \begin{pmatrix} 4-A & \frac{1}{4\pi^2 \tilde{M}_{\rm p}^2} & 0 \\ 0 & 2-A & \frac{3}{16\pi^2} \\ 0 & 0 & -A \end{pmatrix}$$

$$A = \frac{1}{48\pi^2 \tilde{M}_{\rm p}^2} \left[\frac{20}{(1-v_0)^2} + \frac{1}{(1-v_0/4)^2} \right]$$

Prediction of mass of Higgs boson ?

needs $\theta_l < 0$ or A>0

Asymptotic safety of gravity and the Higgs boson mass

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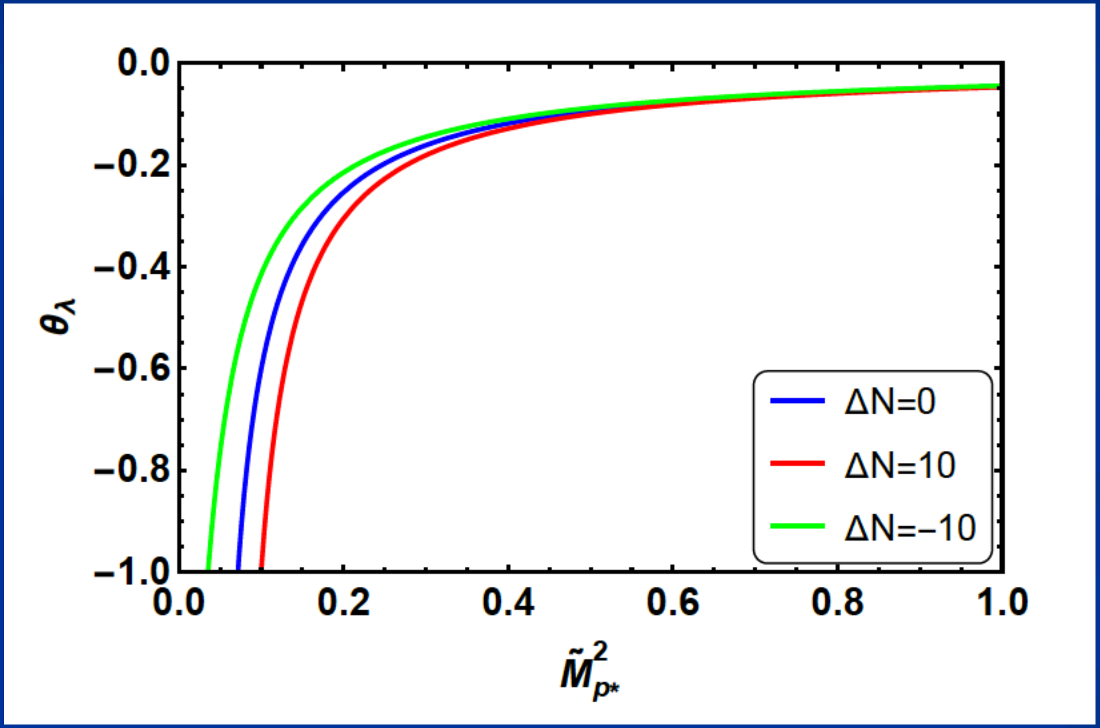
12 January 2010

s in $m_H = m_{\text{min}} = 126$ GeV, with o

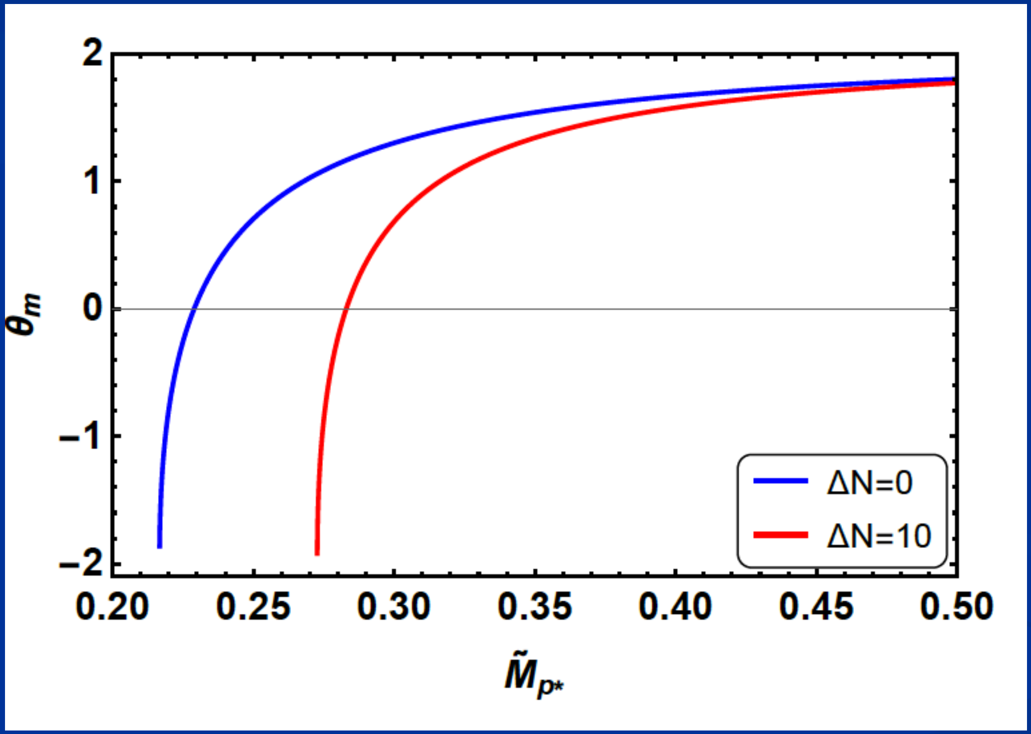
Quartic scalar coupling is irrelevant parameter



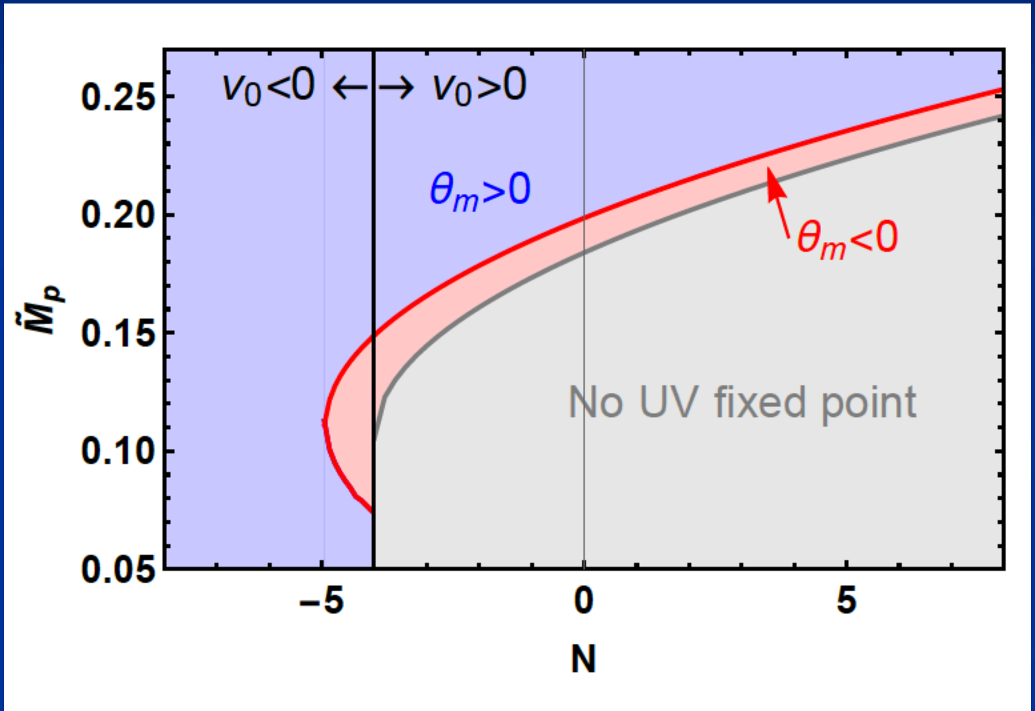
Can be predicted !

Predictivity for Fermi scale ?

Higgs mass term is irrelevant for strong enough gravity



Critical exponent for Higgs mass term



For suitable particle content of model:

Higgs mass term is an irrelevant parameter

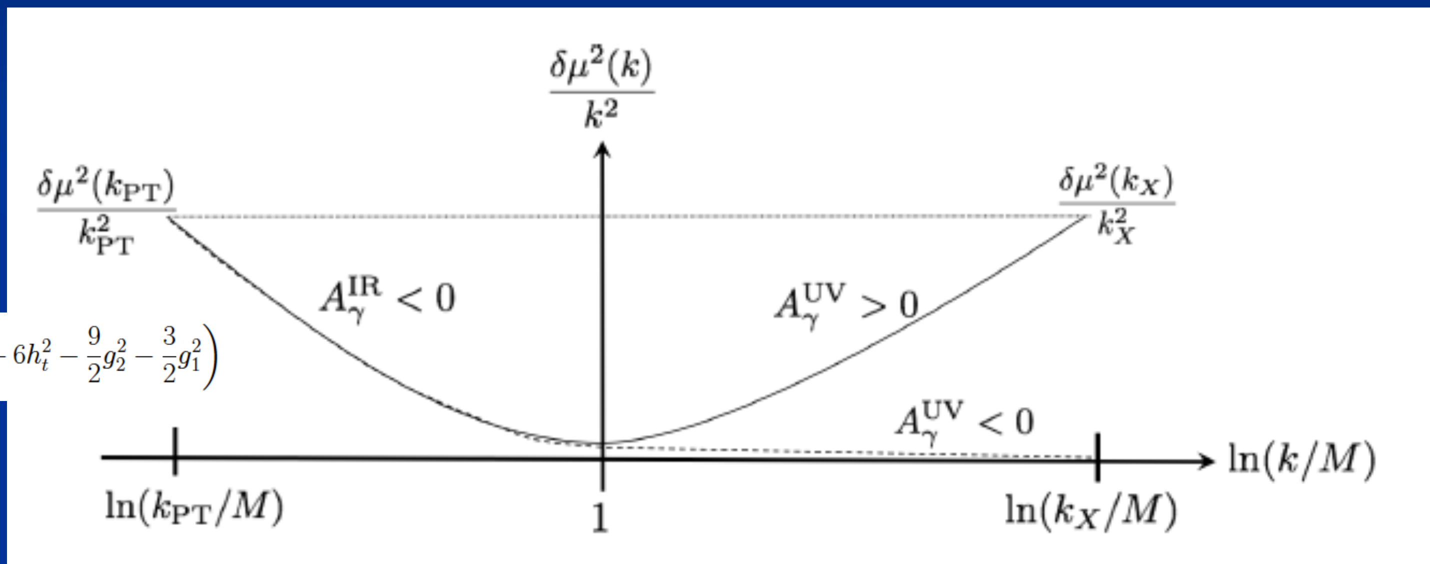
Gauge hierarchy

- Possible explanation of small parameter : distance from second order vacuum electroweak phase transition is **irrelevant parameter** at UV – fixed point

Possible explanation of gauge hierarchy

$$A_\gamma^{\mathrm{IR}} = -2 + A,$$

$$A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$



Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

Christof Wetterich[1] and Masatoshi Yamada[1]

Phys.Lett. B770 (2017) 268-271

Prediction of Fermi scale

- If scalar mass term is irrelevant and vacuum electroweak phase transition would be precisely second order:
- The Fermi scale would be predicted to be zero !
- Running gauge and Yukawa couplings in standard model imply that vacuum electroweak phase transition is not precisely second order. Small effect.
- Small Fermi scale and huge gauge hierarchy expected.
- May be a couple of orders too small as compared to observation ? Not known definitely.

Quantum gravity prediction for the cosmological “constant” ?

Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,
variation yields field equations

Asymptotically vanishing cosmological „constant"

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \rightarrow \infty$!

small dimensionless number ?

- needs two intrinsic mass scales
- standard approach :V and M (cosmological constant and Planck mass)
- variable gravity : Planck mass moving to infinity , with fixed V ➡ ratio vanishes asymptotically !

Variable Gravity in scaling frame

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\big(B(\chi/\mu) - 6\big)\partial^\mu\chi\partial_\mu\chi \right\}$$

Variable gravity in Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} \ , \ \ \varphi = \frac{2M}{\alpha} \ln\left(\frac{\chi}{\mu}\right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp\left(-\frac{\alpha\varphi}{M}\right)$$

$$k^2 = \frac{\alpha^2 B}{4}$$

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations

…. modifications

(different growth of neutrino mass)

Quantum gravity restricts the increase of scalar potential for large fields

Quantum gravity with scalar field

M^2 and V depend on scalar field χ

$$M^2 = c_1 + c_2\,\chi^2 \qquad V = d_1 + d_2\,\chi^2 + d_3\,\chi^4$$

question : behavior of V for $\chi \to \infty$

- $d_3 \neq 0$ excluded!
- $d_3 < 0$ unstable potential
- $d_3 > 0$ instability of graviton propagator

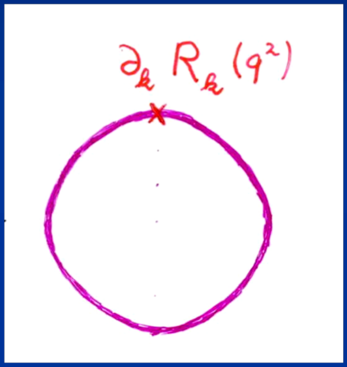
Infrared flow in gravity

for k much smaller than M

Graviton contribution to flow of scalar potential

$$\partial_t V = k\,\partial_k V = 5 I_k\left(-\frac{2V}{M^2}\right)$$

$$I_k(m^2) = \frac{1}{2}\int_q \left(q^2 + R_k(q) + m^2\right)^{-1} \partial_t R_k(q)$$



$$I_k(m^2) = \frac{1}{32\pi^2}\frac{k^6}{k^2 + m^2}$$

$$m^2 = -\frac{2V}{M^2}$$

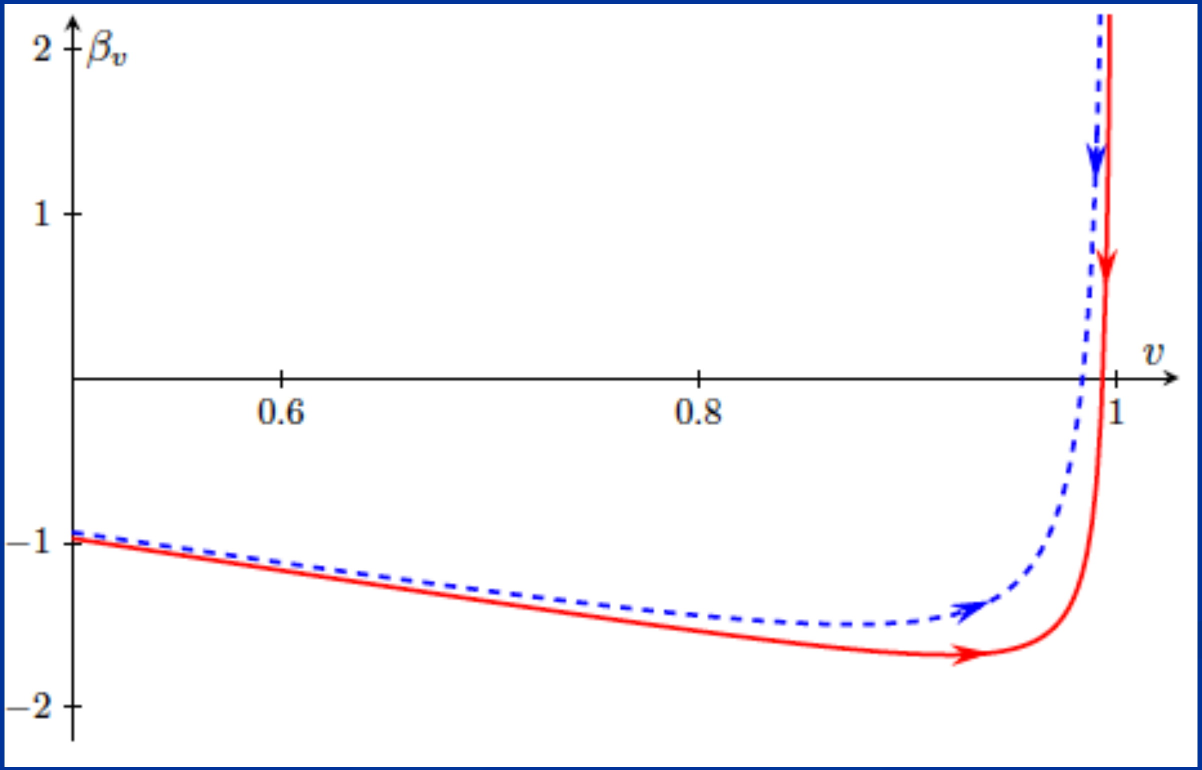
crucial dimensionless quantity

$$v = \frac{2V}{M^2 k^2}$$

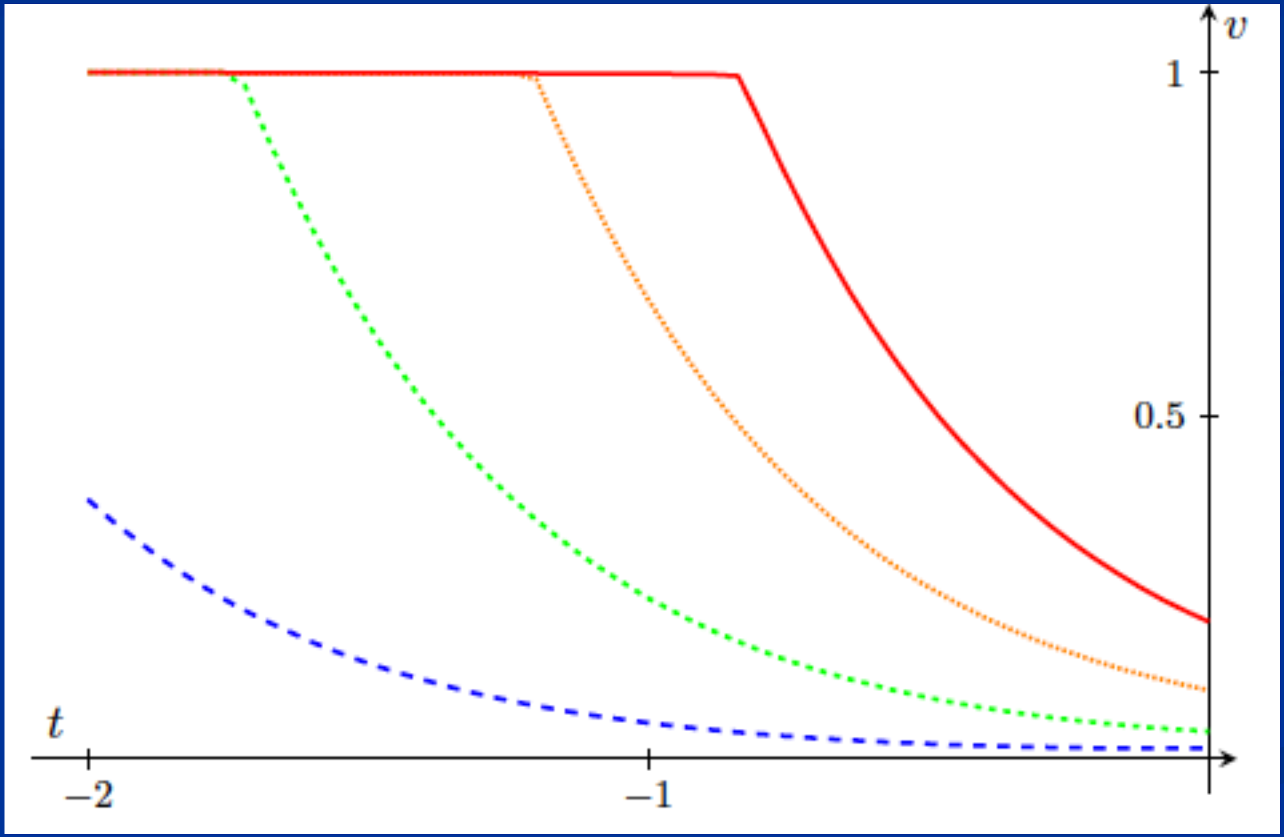
Flow equation for v

$$\partial_t v = \beta_v = -2v + \frac{5k^2}{16\pi^2 M^2}(1-v)^{-1}.$$

$$v = \frac{2V}{M^2 k^2}$$



Flow of v for different initial conditions



Infrared value of effective scalar potential for $k/\chi \rightarrow 0$

v=1

$$U = \frac{\bar{k}^2}{2} M^2(\chi)$$

graviton barrier !

Graviton fluctuations erase the cosmological constant

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ, and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

Normalization of scalar field

If M increases monotonically with χ :

choose normalization of scalar M= χ

V cannot increase stronger than χ^2 !

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

In quantum gravity,
the graviton fluctuations can
play an important role on
distances as large as the
size of the Universe

- for long range scalar fields and dynamical dark energy
- not for all quantities

Reason : instability of graviton propagator in flat space for V>0

effective action $$\Gamma = \int_x \sqrt{g}\left(-\frac{M^2}{2}R + V\right)$$

flat space: $$G^{-1} = \frac{M^2 q^2}{4} - \frac{V}{2}$$

Instability for V>0 : ”tachyonic mass term” $-\frac{2V}{M^2}$

Strong enhancement of quantum fluctuations

Quantum fluctuations avoid instability

Graviton barrier

Quantum gravity computation :

For $\chi \to \infty$

V cannot increase stronger than M^2 !

Instability of graviton propagator is avoided

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ, and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

$$\mathbf{M} = \chi \quad : \quad \mathbf{V} = \mu^2 \chi^2$$

Asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2\chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \to \infty$!

Predictions of quantum gravity ?

Simple approximation for graviton contribution to scalar potential:

- Predicts mass of Higgs scalar
- Solves Gauge Hierarchy problem ?
- Solves cosmological constant problem

Conclusions

- Quantum gravity is a renormalizable quantum field theory, realized by UV - fixed point of running couplings or flowing effective action

- Quantum gravity is predictive :

  Mass of the Higgs boson (and more …?)

  Properties of inflation

  Properties of dark energy

end

Gauge invariant flow equation involves projection on physical fluctuations

$$k\partial_k\bar{\Gamma} = \zeta_k = \pi_k + \delta_k - \epsilon_k$$

$$\pi_k = \frac{1}{2}Str(k\partial_k\bar{R}_P G_P)$$

G_P : propagator for physical fluctuations

$$PG_P = G_P P^T = G_P$$

$$\delta_\xi \bar{g} = (1-P)\delta_\xi \bar{g}$$

measure contributions $\delta_k - \epsilon_k$ do not depend on effective action

Closed flow equation

projection on physical fluctuations makes second functional derivative invertible

$$\bar{\Gamma}_P^{(2)} = P^T \bar{\Gamma}^{(2)} P$$

$$\bar{\Gamma}^{(2)ij} = \frac{\partial^2 \bar{\Gamma}}{\partial \bar{g}_i \partial \bar{g}_j}$$

$$\left(\bar{\Gamma}_P^{(2)} + \bar{R}_P\right) G_P = P^T$$

On shell graviton propagator

$$G^{-1} = \sqrt{g}\left\{\frac{M^2}{4}\left(-D^2 + \frac{2R}{3}\right) - \frac{V}{2}\right\}$$

on shell :
(for solution of field equations)

$$R = \frac{4V}{M^2}$$

homogenous isotropic metric,
conformal time

$$g_{\mu\nu} = a^2(\eta)\,\delta_{\mu\nu}$$

$$\mathcal{H} = \frac{\partial \ln a}{\partial \eta}$$

inverse graviton
propagator in
de Sitter space

$$a^{-2}\left(-D^2 + \frac{R}{6}\right)a^2 = \frac{1}{a^2}\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + \vec{q}^{\,2}\right)$$

milder instability, not tachyonic, absent for
cosmologies close to de Sitter space

IR – instability for graviton fluctuations

problem solved ?

- yes for primordial cosmic fluctuations (on shell)
- no for quantum gravity (off shell)
- Computation of effective action is an off-shell problem.
- example : one needs the effective potential for the Higgs field in the vicinity of its minimum (off shell), not only at the minimum (on shell)