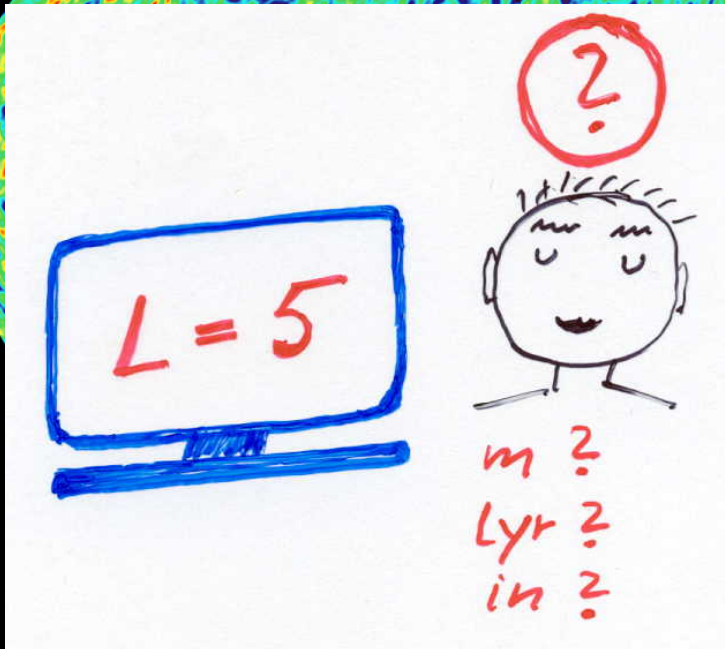


Scale symmetry – a link from quantum gravity to cosmology



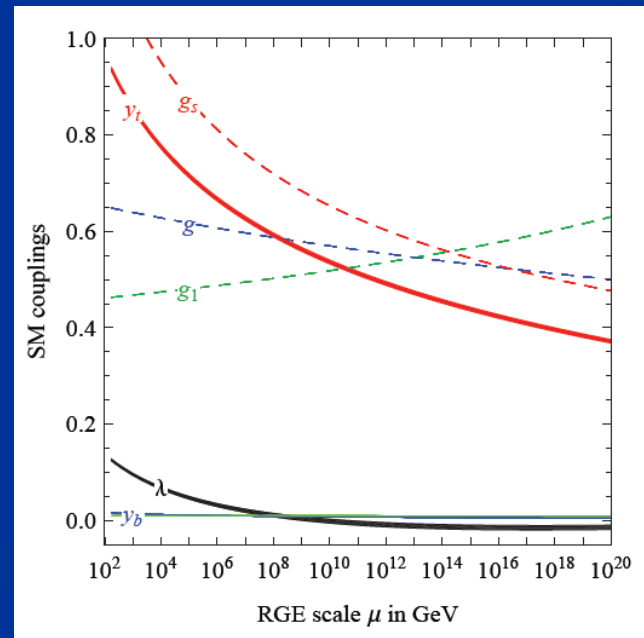
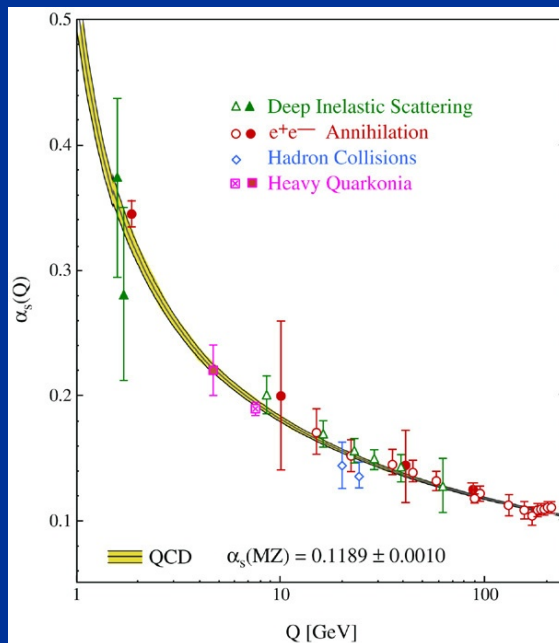
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

scale symmetry

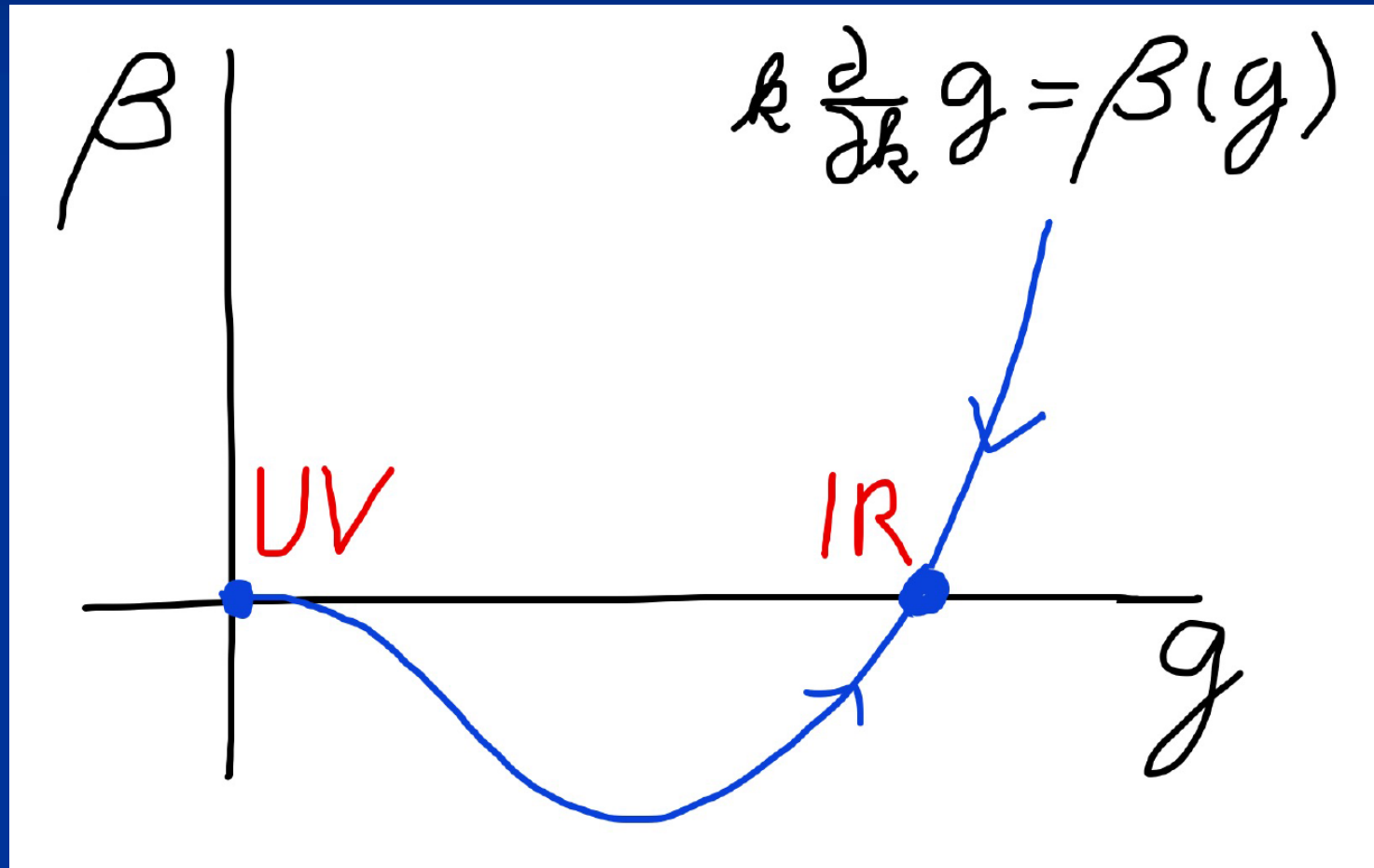
No intrinsic length or mass scale present

Fluctuations induce running couplings

- violation of scale symmetry
- well known in QCD or standard model



Fixed Points



Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !

quantum gravity – the role of scale symmetry

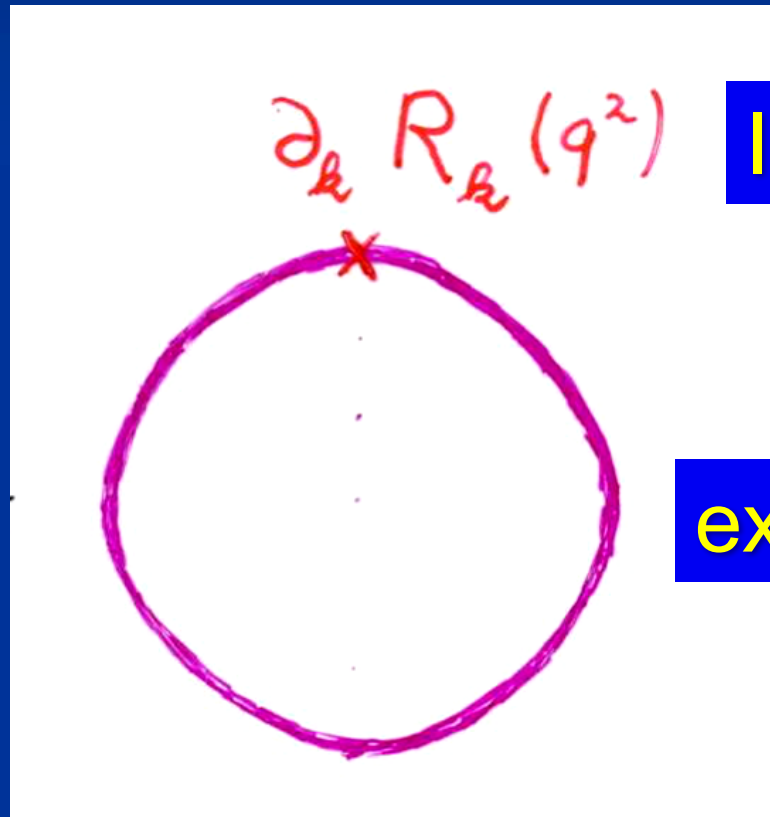
Asymptotic safety

if UV fixed point exists :

*quantum gravity is
non-perturbatively renormalizable !*

S. Weinberg , M. Reuter

Functional flow equation for scale dependent effective action



IR cutoff

exact propagator

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \text{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

'92

$$\left(\Gamma_k^{(2)} \right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$$\text{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STr)



From

Microscopic Laws
(Interactions, classical action)

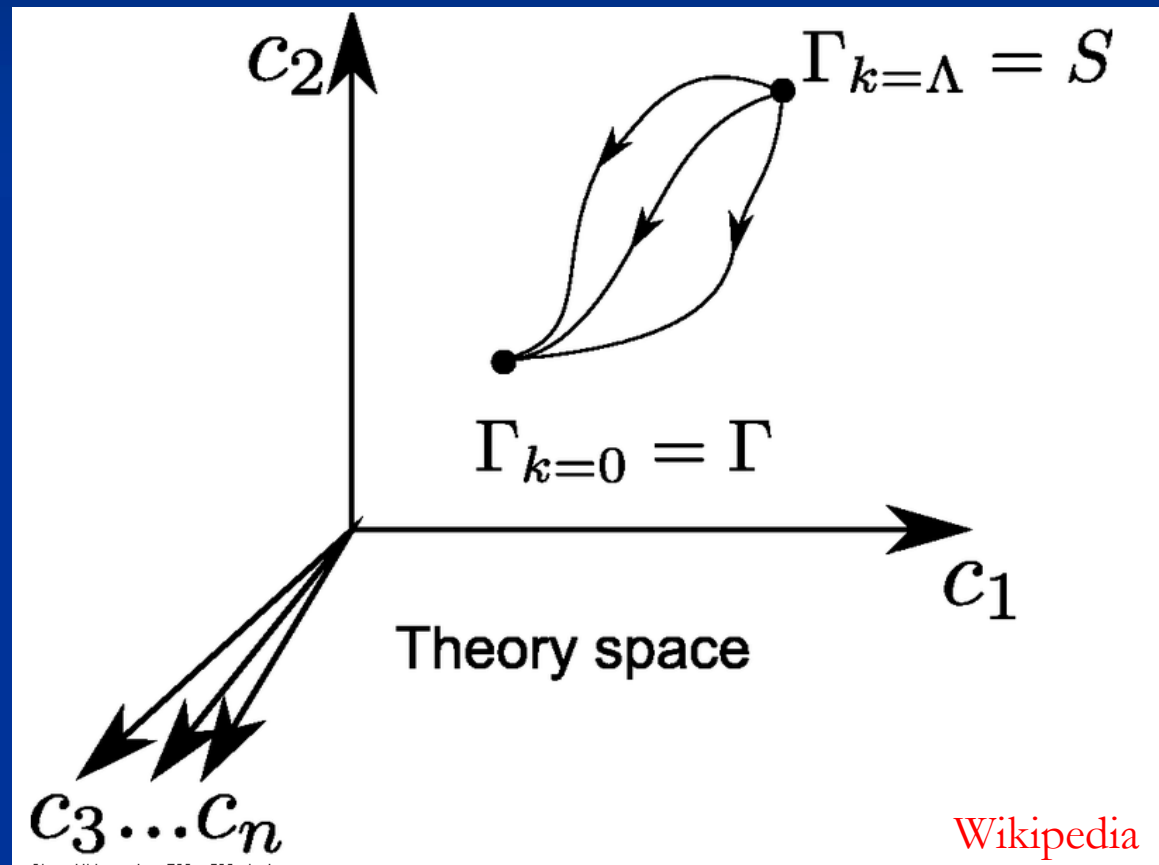
to

Fluctuations!

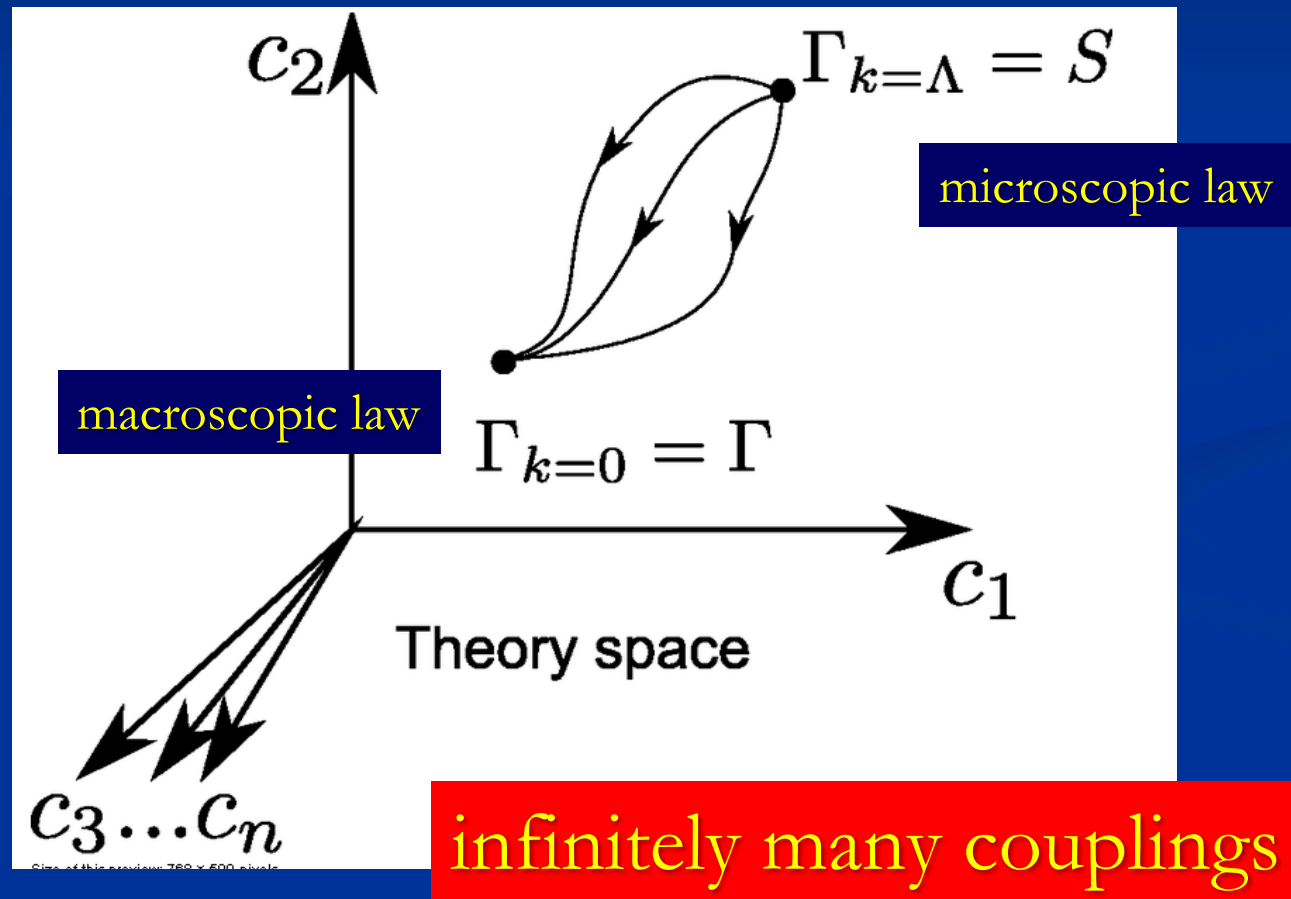


Macroscopic Observation
(Free energy functional,
effective action)

functional renormalization : flowing action



flowing action



flow of functions

Effective potential includes **all** fluctuations

Average potential U_k

\equiv scale dependent effective potential

\equiv coarse grained free energy

Only fluctuations with
momenta $q^2 > k^2$ included

k : infrared cutoff for fluctuations, "average scale"

Λ : characteristic scale for microphysics

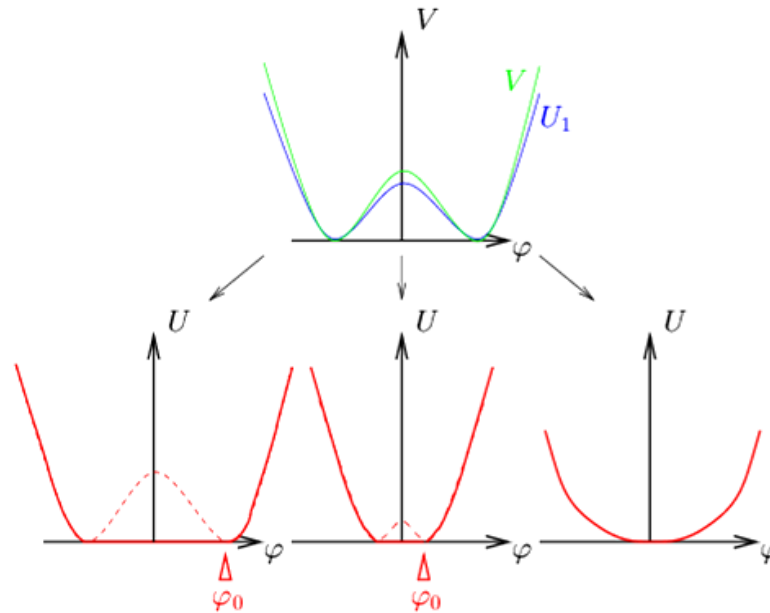
$$U_\Lambda \approx S \rightarrow U_0 \equiv U$$

Scalar field theory

$\varphi_a(x)$: magnetization, density, chemical concentration, Higgs field, meson field, inflaton, cosmon

$O(N)$ -symmetry:

$$S = \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi_a + V(\rho) \right\}; \quad \rho = \frac{1}{2} \varphi_a \varphi_a$$



Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}_{k,i}^2(\varphi)}$$

cutoff

**propagator
with cutoff**

$$\bar{M}_{k,ab}^2 = \frac{\partial^2 U_k}{\partial \varphi_a \partial \varphi_b} \quad : \quad \text{Mass matrix}$$

$$\bar{M}_{k,i}^2 \quad : \quad \text{Eigenvalues of mass matrix}$$

R_k : IR-cutoff

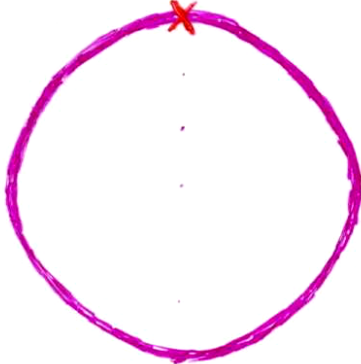
e.g. $R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$

or $R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$ (Litim)

$$\lim_{k \rightarrow 0} R_k = 0$$

$$\lim_{k \rightarrow \infty} R_k \rightarrow \infty$$

Simple one loop structure –
nevertheless (almost) exact

$$\partial_k U_k = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + M_k^2 + R_k(q^2)}$$


$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}_{k,i}^2(\varphi)}$$

Flowing action for dilaton quantum gravity

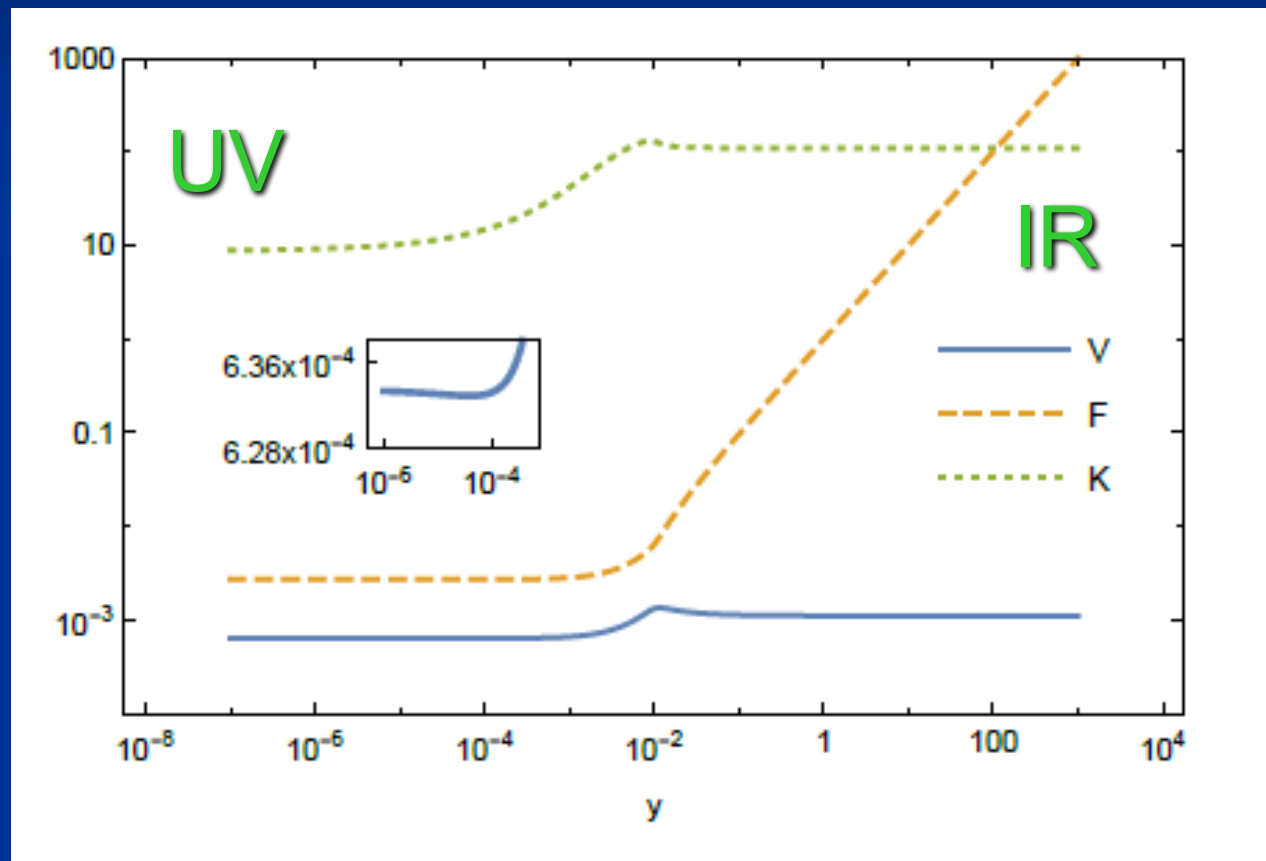
second order derivative expansion

$$\Gamma = \int_x \sqrt{g} \left(V(\chi^2) - \frac{1}{2} F(\chi^2) R + \frac{1}{2} K(\chi^2) g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

variable gravity
metric and scalar field

Scaling solutions for Dilaton Quantum Gravity

T. Henz, J. M. Pawłowski, and C. Wetterich

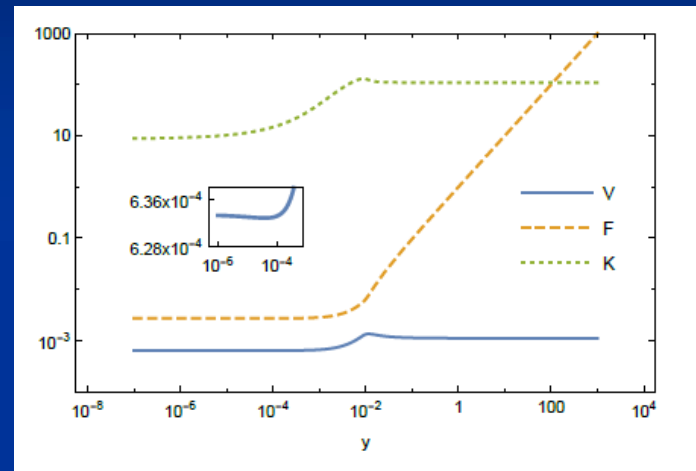


$$\Gamma = \int_x \sqrt{g} \left(V(\chi^2) - \frac{1}{2} F(\chi^2) R + \frac{1}{2} K(\chi^2) g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right)$$

$$y \equiv \chi^2 / k^2.$$

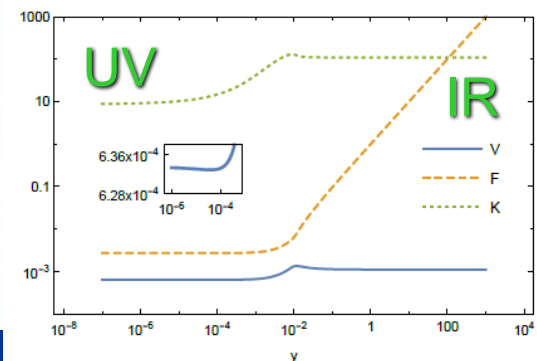
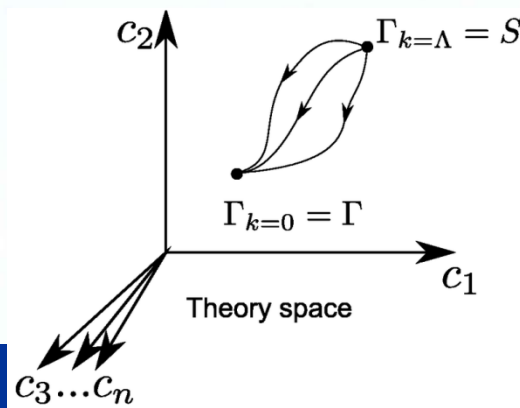
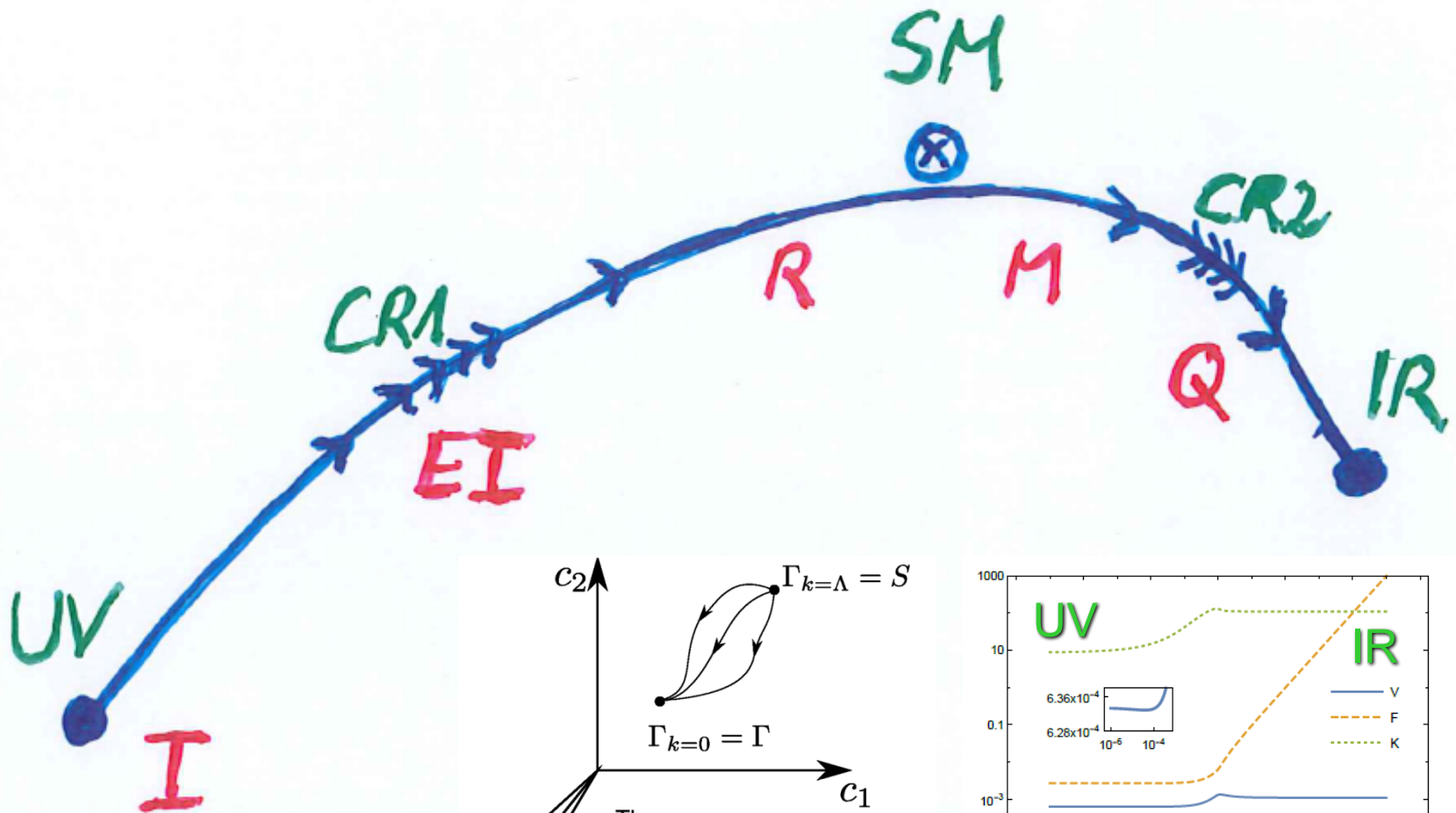
open points

solution with
vanishing
anomalous
dimension



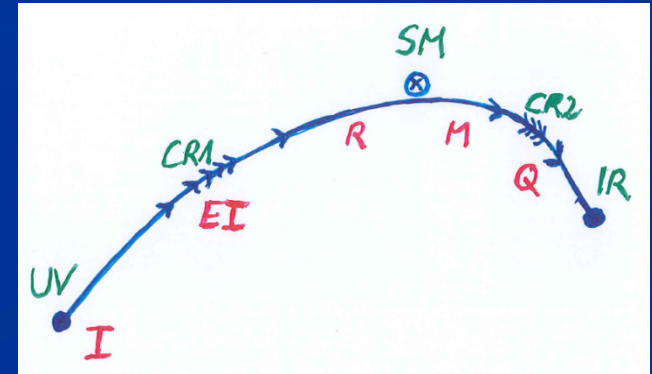
candidates with anomalous dimension
more interesting
also matter and gauge fields need to be
included, higher order curvature terms...

Crossover in quantum gravity



Origin of mass

- UV fixed point : scale symmetry unbroken
all particles are massless
- IR fixed point :
scale symmetry spontaneously broken,
massive particles , massless dilaton
- crossover : explicit mass scale μ important
- approximate SM fixed point : approximate scale symmetry
spontaneously broken, massive particles , almost massless
cosmon, tiny cosmon potential



Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- all mass scales proportional to scalar field χ : electron mass, proton mass, Planck mass
- in presence of massive particles : sign of exact scale symmetry is exactly **massless Goldstone boson** – the dilaton

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

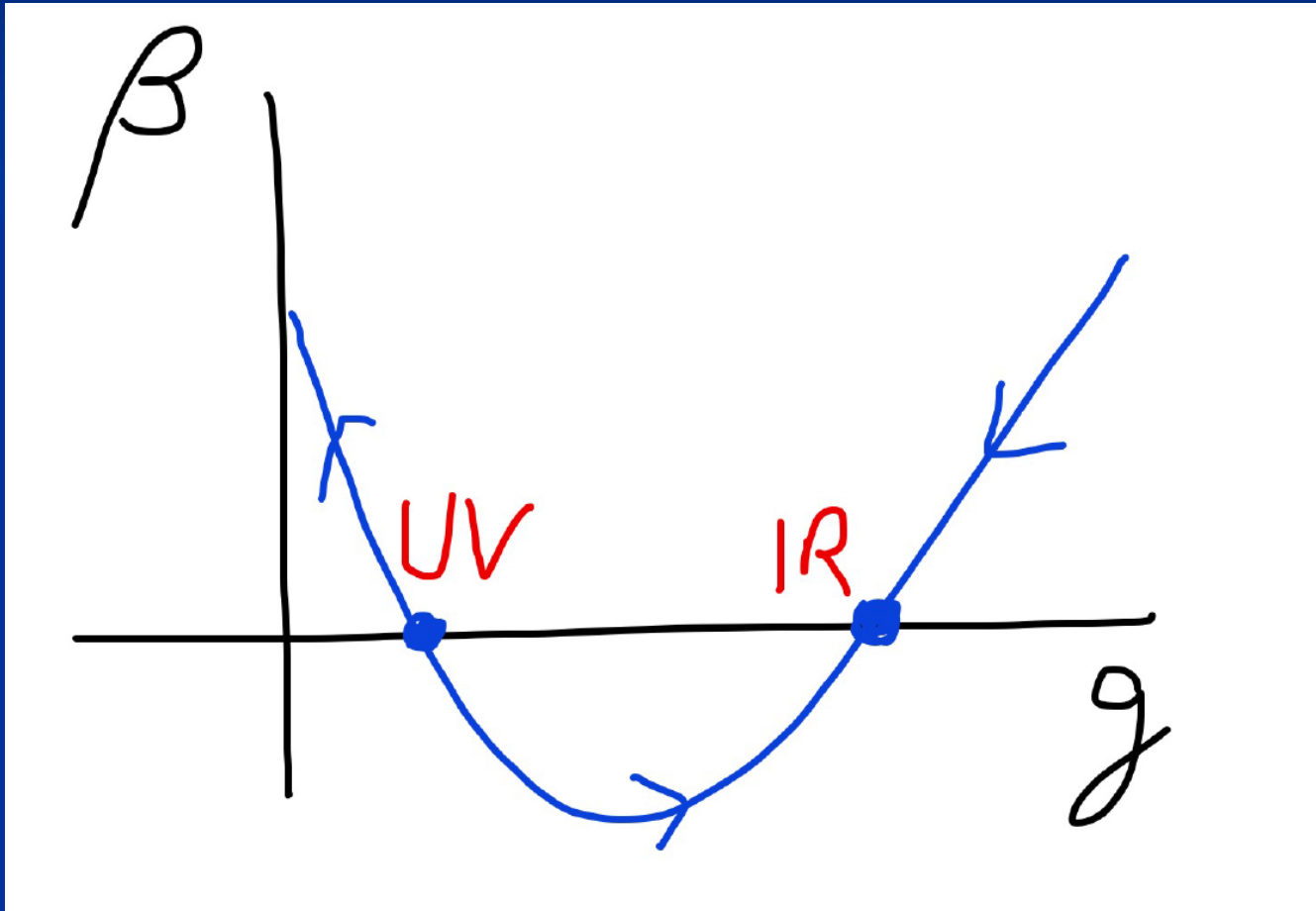
Asymptotic safety

if UV fixed point exists :

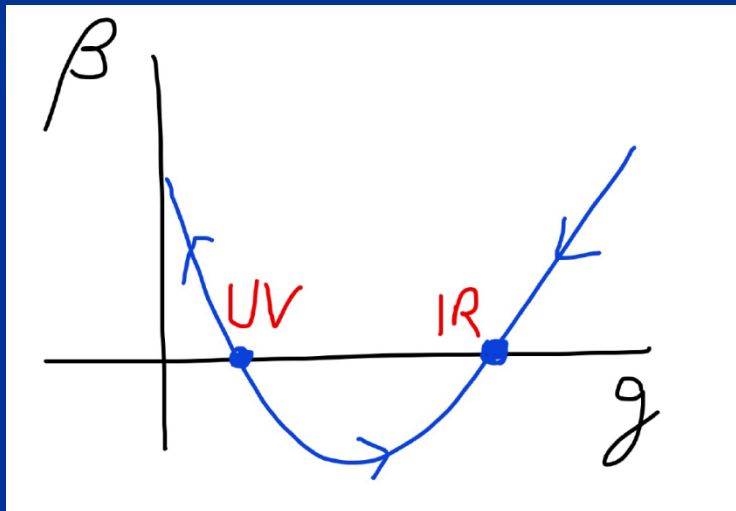
*quantum gravity is
non-perturbatively renormalizable !*

S. Weinberg , M. Reuter

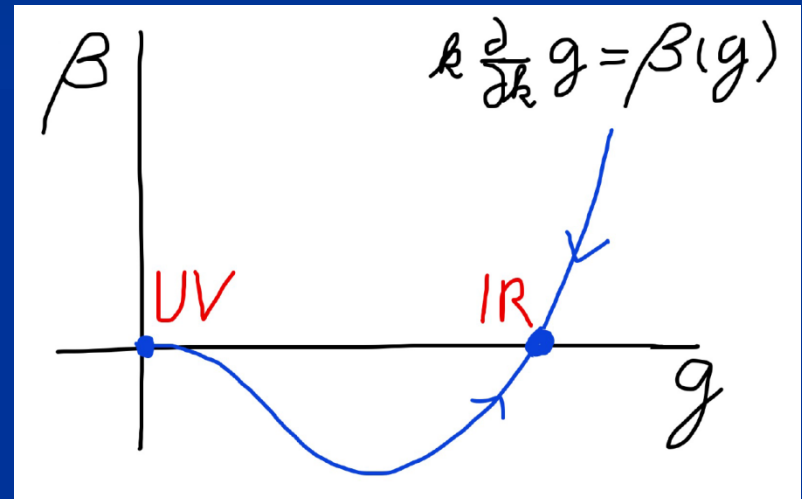
Asymptotic safety



Asymptotic safety



Asymptotic freedom



relevant parameters yield undetermined couplings

a prediction...

Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

Christof Wetterich

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010

Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

and a possibility

Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

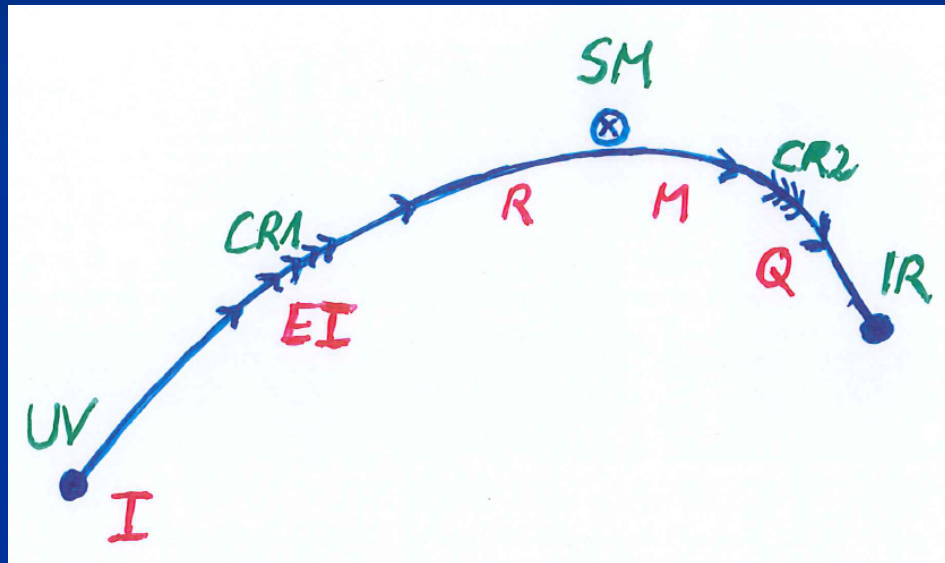
Christof Wetterich¹ and Masatoshi Yamada¹

¹*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

The gauge hierarchy problem could find a solution within the scenario of asymptotic safety for quantum gravity. We discuss a “resurgence mechanism” where the running dimensionless coupling responsible for the Higgs scalar mass first decreases in the ultraviolet regime and subsequently increases in the infrared regime. A gravity induced large anomalous dimension plays a crucial role for the required “self-tuned criticality” in the ultraviolet regime beyond the Planck scale.

arXiv:1612.03069v1 [hep-th] 9 Dec 2016

Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy
with single scalar field

Variable Gravity

Ansatz , not computed !

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

intrinsic scale μ replaces k

quantum effective action,
variation yields field equations,
solve for cosmology

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

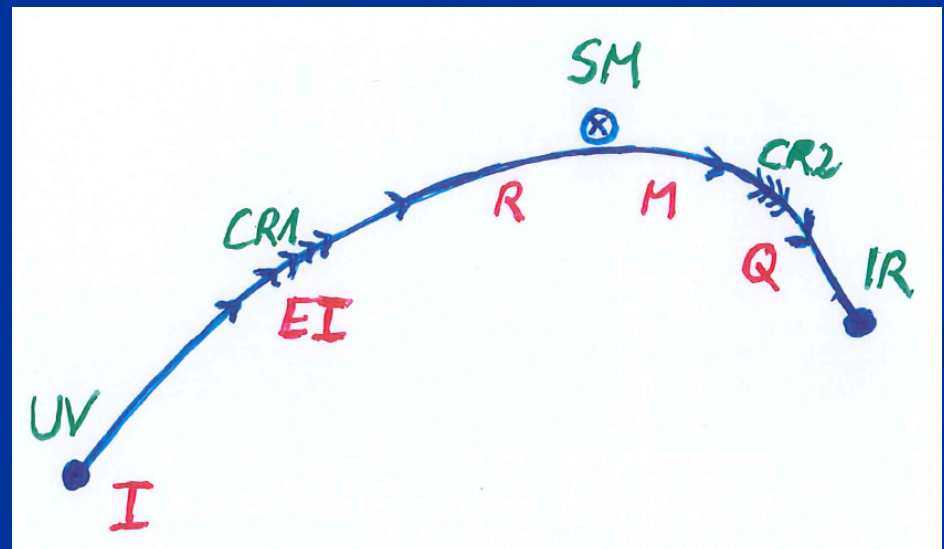
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}(B(\chi/\mu) - 6)\partial^\mu\chi\partial_\mu\chi \right\}$$

Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \rightarrow 0$, $\chi \rightarrow \infty$
- UV: $\mu \rightarrow \infty$, $\chi \rightarrow 0$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

Cosmology makes
crossover between
fixed points by
variation of χ .



renormalization flow and cosmological evolution

- renormalization flow as function of μ

is mapped by dimensionless functions to

- field dependence of effective action on scalar field χ

translates by solution of field equation to

- dependence of cosmology on time t or η

Simplicity

simple description of **all** cosmological epochs

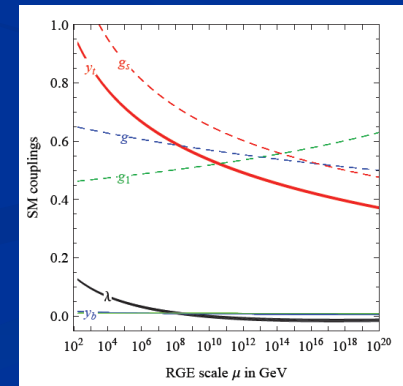
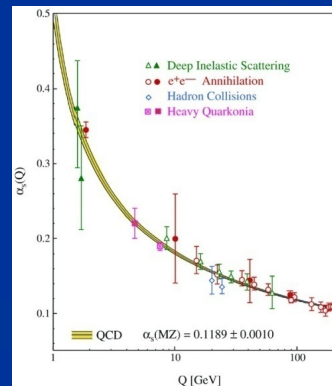
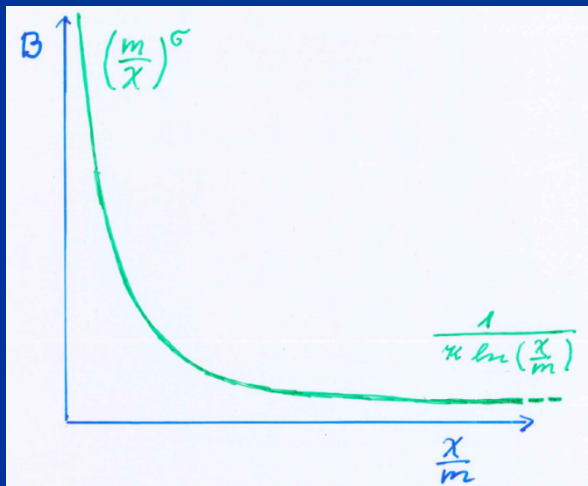
natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

running coupling B

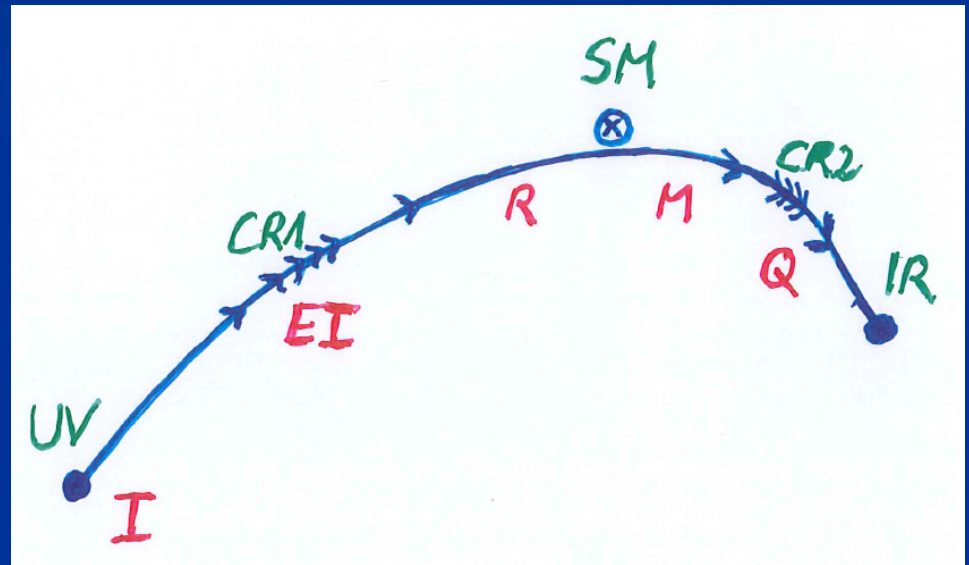
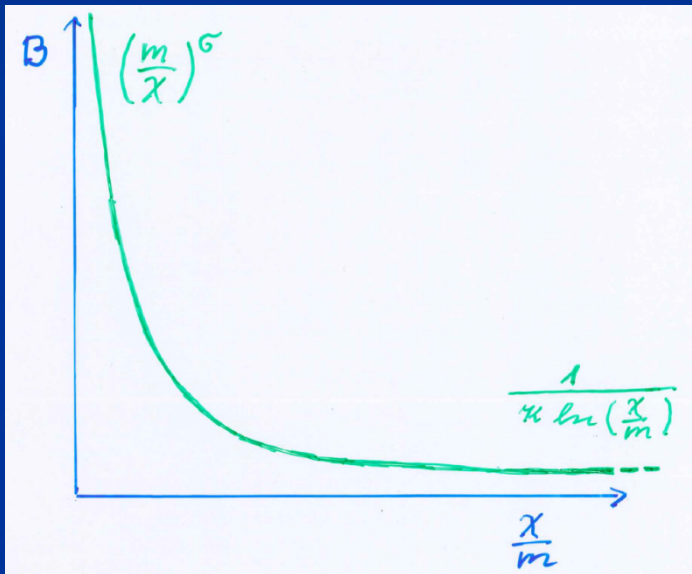
$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

- B varies if intrinsic scale μ changes
- similar to QCD or standard model

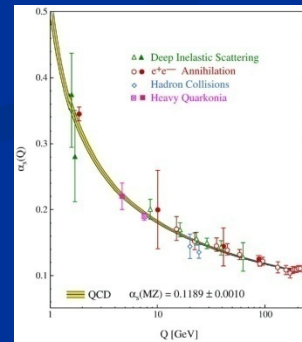


Kinetic B :

Crossover between two fixed points

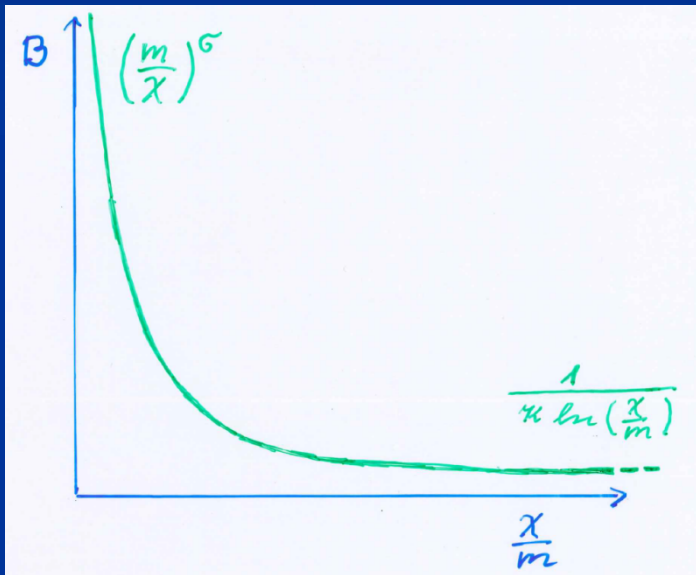


$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$



Kinetic B :

Crossover between two fixed points



running
coupling obeys
flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover

can be exponentially larger than intrinsic scale μ

Cosmological solution

- derive field equation from effective action of variable gravity
- solve them for homogenous and isotropic metric and scalar field
- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}(B(\chi/\mu) - 6)\partial^\mu \chi \partial_\mu \chi \right\}$$

No tiny dimensionless parameters (except matter sector, e.g. gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33} \text{ eV}$
- one time scale $\mu^{-1} = 10^{10} \text{ yr}$
- Planck mass does not appear as parameter
- Planck mass grows large dynamically

Particle masses change with time

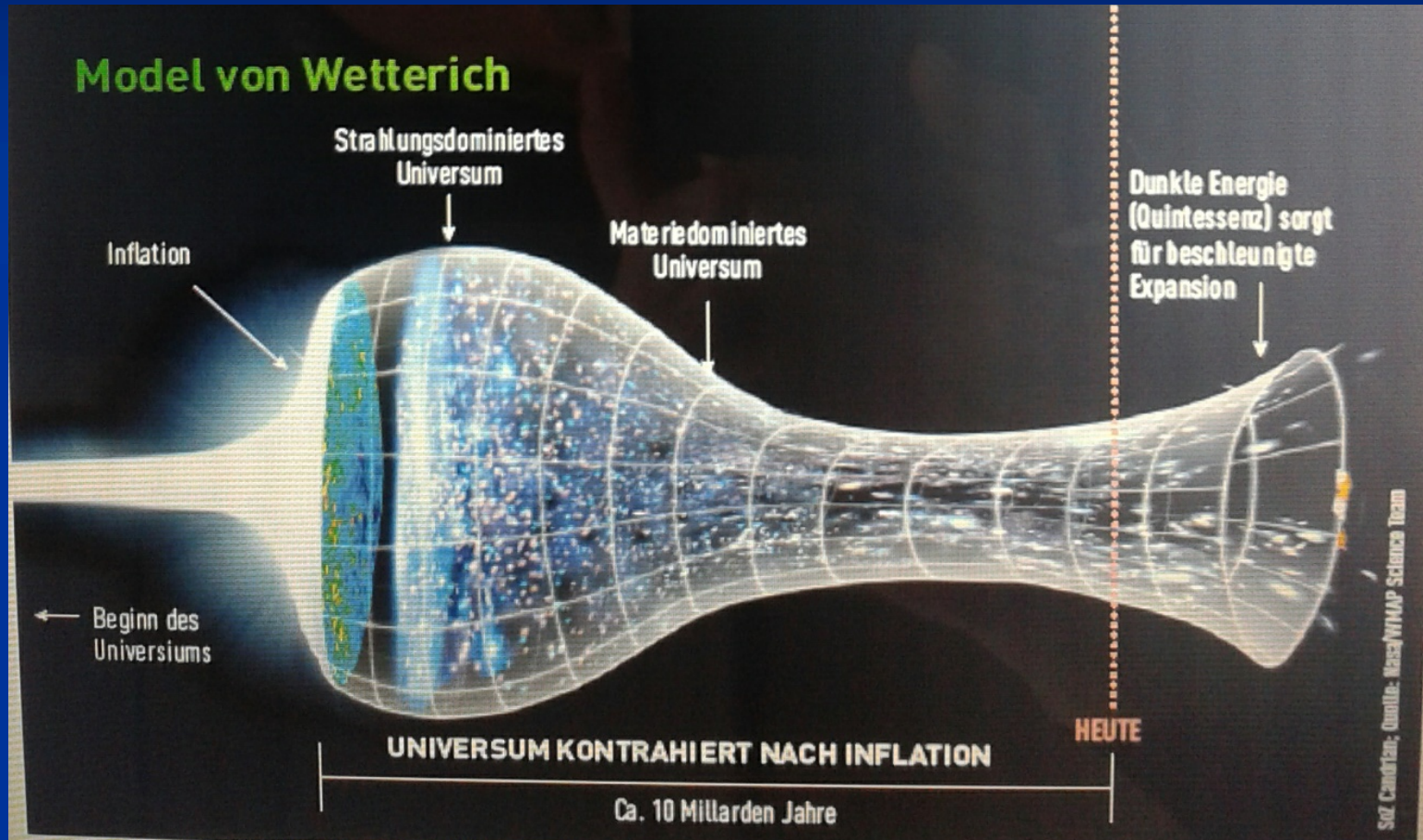
At SM fixed point :

- All particle masses (except for neutrinos) are proportional to scalar field χ .
- Scalar field varies with time – so do particle masses.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

Four-parameter model

- model has four dimensionless parameters
- three in kinetic B :
 - $\sigma \sim 2.5$
 - $\kappa \sim 0.5$
 - $c_t \sim 14$ (or m/μ)
- one parameter for present growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than Λ CDM

Strange evolution of Universe



Sonntagszeitung Zürich , Laukenmann

Slow Universe

Asymptotic solution :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33} \text{ eV}$$

Expansion or shrinking always slow ,
characteristic time scale of the order of the age of the
Universe : $t_{\text{ch}} \sim \mu^{-1} \sim 10 \text{ billion years} !$

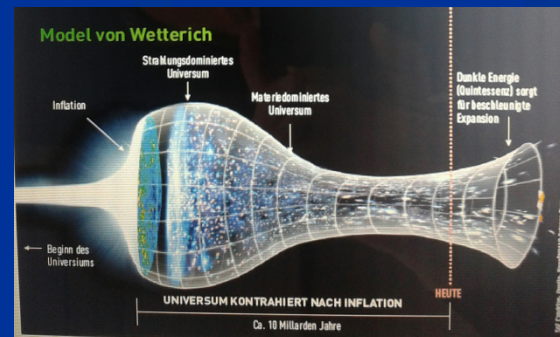
Hubble parameter of the order of **present** Hubble
parameter for all times , including inflation and big bang !
Slow increase of particle masses !

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite



asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \rightarrow \infty$!

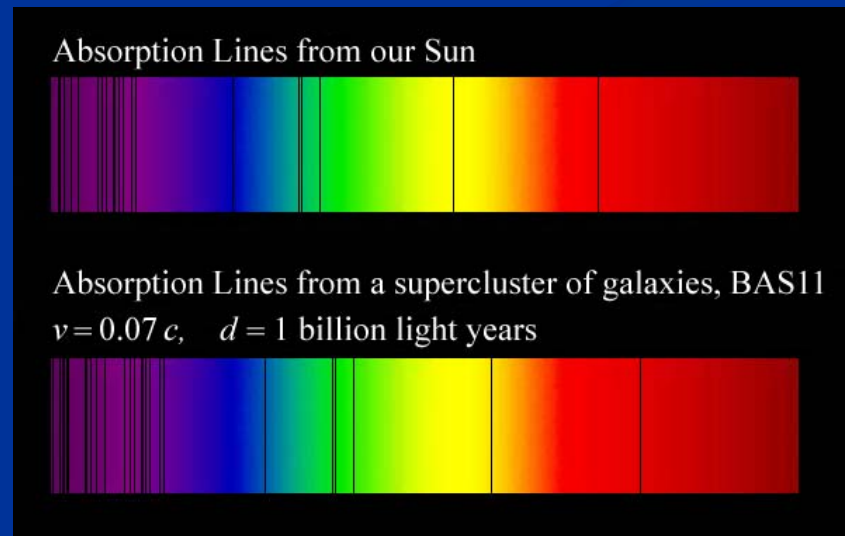
small dimensionless number ?

- needs two intrinsic mass scales
- V and M (cosmological constant and Planck mass)
- variable Planck mass moving to infinity , with fixed V : **ratio vanishes asymptotically !**

Do we know that the Universe expands ?

instead of redshift due to expansion :

smaller frequencies have been emitted in the past,
because electron mass was smaller !



What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

Hot plasma ?

- Temperature in radiation dominated Universe :
 $T \sim \chi^{1/2}$ **smaller** than today
- Ratio temperature / particle mass :
 $T / m_p \sim \chi^{-1/2}$ **larger** than today
- T/m_p counts ! This ratio decreases with time.
- Nucleosynthesis , CMB emission as in standard cosmology !

Model is compatible with present observations

Together with variation of neutrino mass over
electron mass in present cosmological epoch :
model is compatible with all present observations

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
- Standard gravity coupled to scalar field.
- Only neutrino masses are growing.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp \left(-\frac{\alpha \varphi}{M} \right)$$

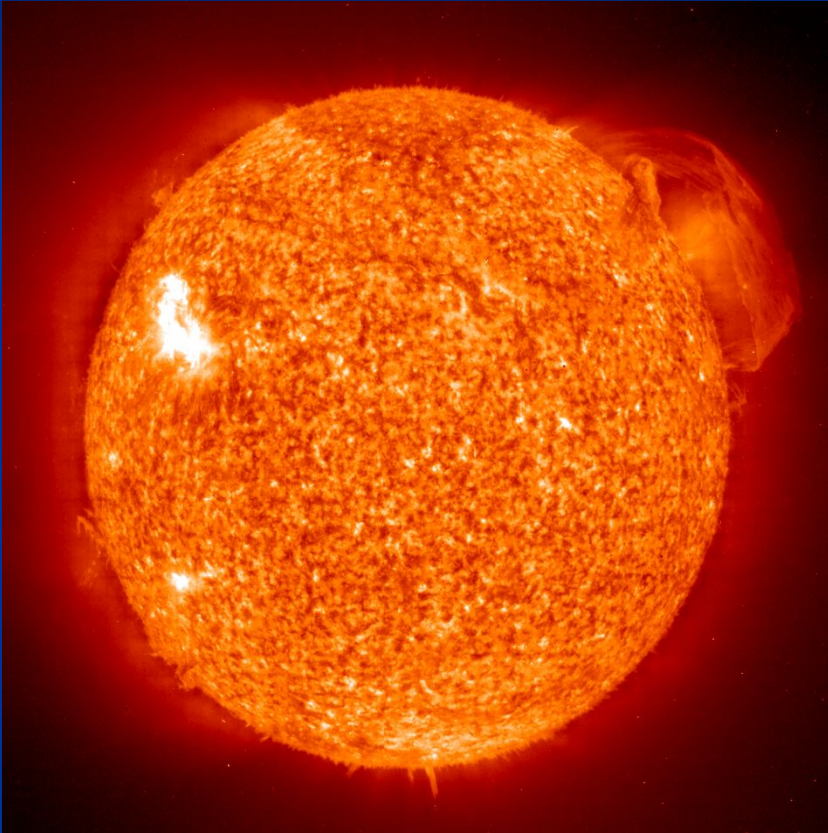
$$k^2 = \frac{\alpha^2 B}{4}$$

Field relativity :

different pictures of cosmology

- same physical content can be described by different pictures
- related by field – redefinitions ,
e.g. Weyl scaling , conformal scaling of metric
- which picture is usefull ?

Big bang or freeze ?



The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies and stars against a black background. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. Some are bright and clear, while others are faint and distant. The stars appear as small, bright points of light, some with prominent diffraction spikes. The overall scene is a dense, colorful mosaic of cosmic objects.

Big bang or freeze ?

just two ways of looking at same physics

Infinite past : slow inflation

$\sigma = 2$: field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2} \frac{\dot{\chi}}{\chi} \right) \dot{\chi} = \frac{2\mu^2 \chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

approximate
solution

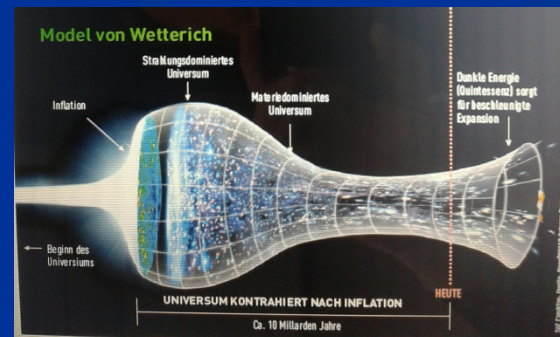
$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

Eternal Universe

Asymptotic solution in freeze frame :

$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite



Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \quad \left\{ \partial_\eta^2 + k^2 + a^2 \left(m^2 - \frac{R}{6} \right) \right\} \tilde{\varphi}_k = 0$$

determine **physical time** by counting number of oscillations

$$\tilde{t}_p = n_k$$

$$n_k = \frac{k\eta}{\pi}$$

(m=0)

*Big bang singularity
in Einstein frame is
field singularity !*

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

conclusions

Fixed points and scale symmetry crucial

Big bang singularity is artefact
of inappropriate choice of field variables –
no physical singularity

conclusions (2)

- crossover in quantum gravity is reflected in crossover in cosmology
- quantum gravity becomes testable by cosmology
- quantum gravity plays a role not only for primordial cosmology
- crossover scenario explains different cosmological epochs
- simple model is compatible with present observations
- no more parameters than Λ CDM : tests possible

end

Quintessence

Dynamical dark energy ,
generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

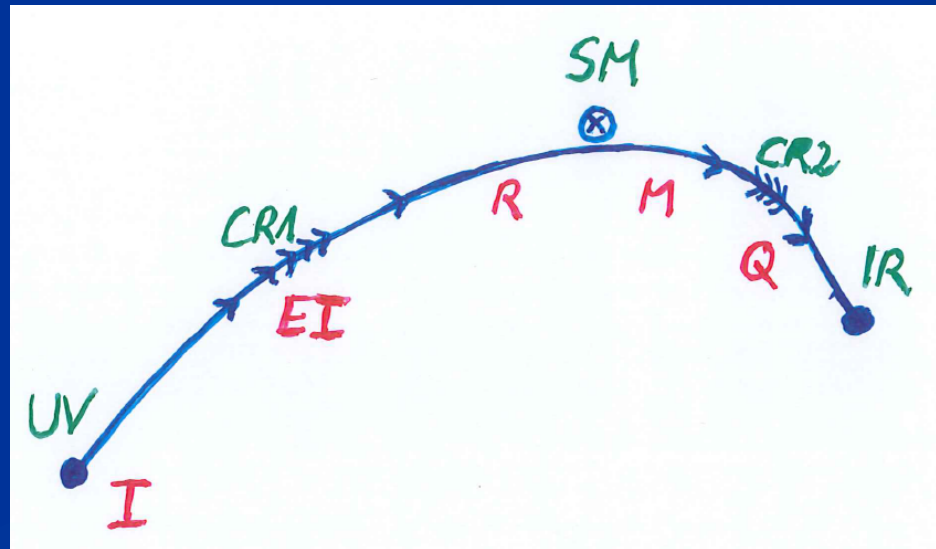
**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... modifications
(different growth of neutrino mass)

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- All particle masses **except for neutrinos** are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that **ratio neutrino mass over electron mass grows**.

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

L.Amendola,
M.Baldi, ...

present dark energy density given by neutrino mass

present equation
of state given by
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

Inflation

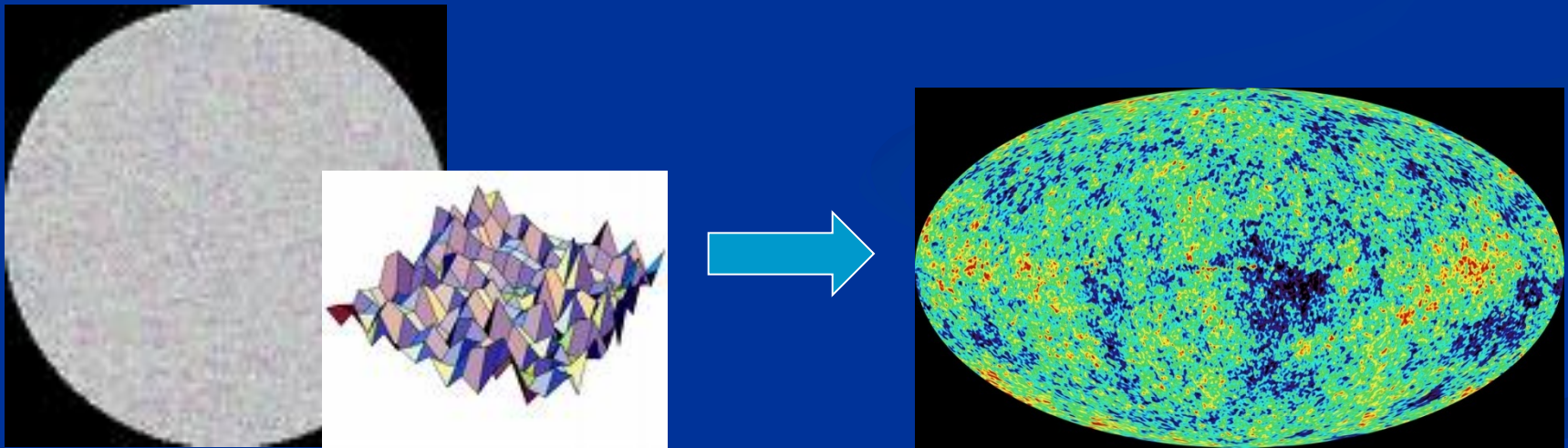
solution for small χ : inflationary epoch

kinetial characterized by
anomalous dimension σ

$$B = b \left(\frac{\mu}{\chi} \right)^{\sigma} = \left(\frac{m}{\chi} \right)^{\sigma}$$

Primordial fluctuations

- inflaton field : χ
- primordial fluctuations of inflaton become observable in cosmic microwave background



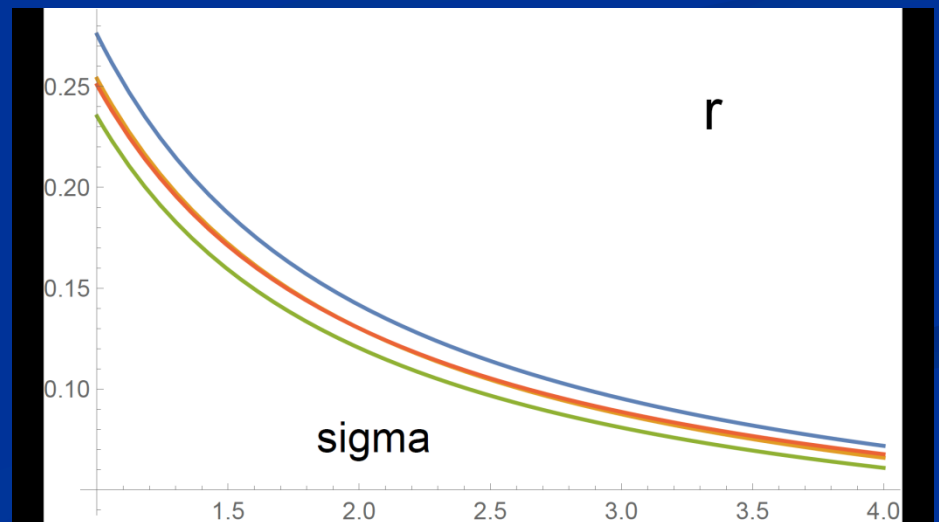
Anomalous dimension determines spectrum of primordial fluctuations

$$r = \frac{0.26}{\sigma}$$

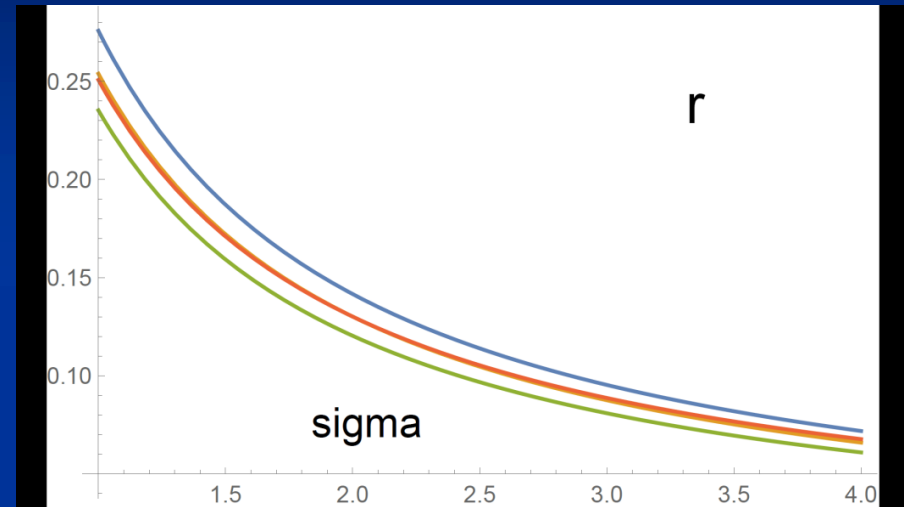
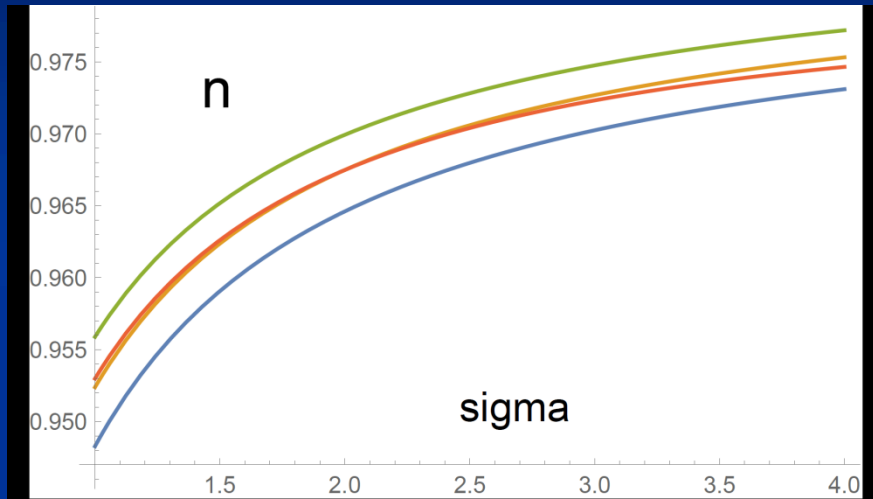
$$n = 1 - \frac{0.065}{\sigma} \cdot \left(1 + \frac{\sigma - 2}{4}\right)$$

spectral index n

tensor amplitude r



relation between n and r



$$r = 8.19 (1 - n) - 0.1365$$

Amplitude of density fluctuations

small because of logarithmic running
near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln \left(\frac{m}{\mu} \right) = 14.1. \quad \sigma=1$$

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60} \right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

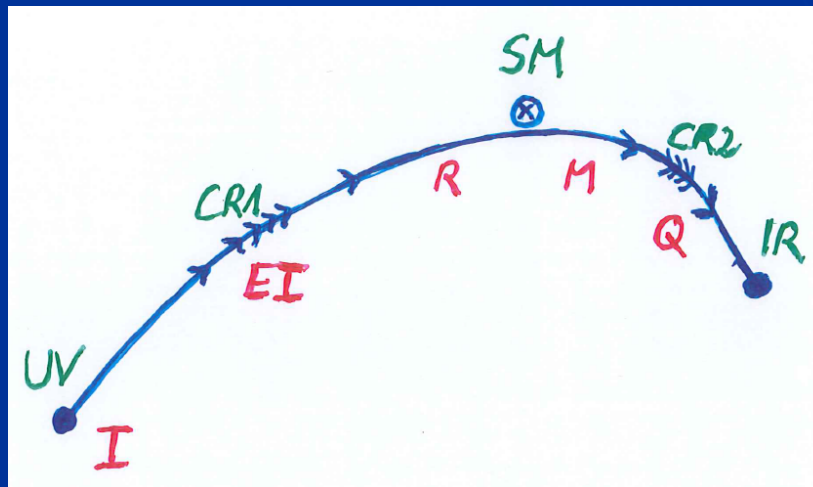
N : number of e – foldings at horizon crossing

First step of crossover ends inflation

- induced by crossover in B

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

- after crossover B changes only very slowly



Scaling solution

- Heating of the Universe after inflation
- Scaling solution with almost fixed fraction of Early Dark Energy

Cosmon inflation

Unified picture of inflation and
dynamical dark energy

Cosmon and inflaton are the same
scalar field

Compatibility with observations and possible tests

- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as Λ CDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Simplicity

simple description of **all** cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch

conclusions (3)

- Variable gravity cosmologies can give a simple and realistic description of the Universe
- Compatible with tests of equivalence principle and bounds on variation of fundamental couplings if nucleon and electron masses are proportional to variable Planck mass
- Cosmon dependence of ratio neutrino mass/ electron mass can explain why Universe makes a transition to Dark Energy domination **now**
- **characteristic signal : neutrino lumps**

Infrared fixed point

■ $\mu \rightarrow 0$

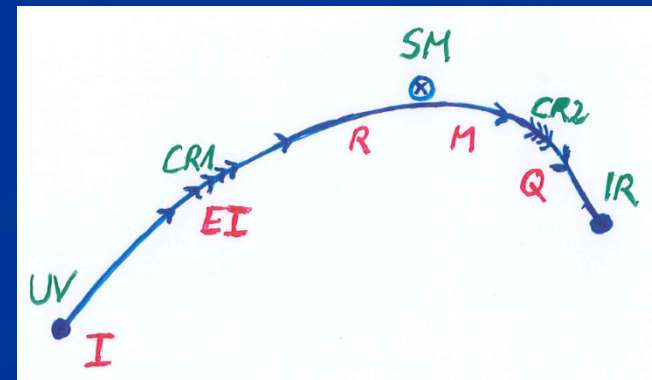
■ $B \rightarrow 0$

$$\mu \partial_\mu B = \kappa B^2 \quad \text{for} \quad B \rightarrow 0$$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

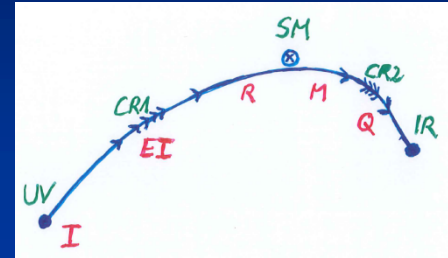
■ no intrinsic mass scale

■ scale symmetry



Ultraviolet fixed point

■ $\mu \rightarrow \infty$



■ kinetic diverges

$$B = b \left(\frac{\mu}{\chi} \right)^{\sigma} = \left(\frac{m}{\chi} \right)^{\sigma}$$

■ scale symmetry with anomalous dimension σ

$$g_{\mu\nu} \rightarrow \alpha^2 g_{\mu\nu} , \quad \chi \rightarrow \alpha^{-\frac{2}{2-\sigma}} \chi$$

Renormalized field at UV fixed point

$$\chi_R = b^{\frac{1}{2}} \left(1 - \frac{\sigma}{2}\right)^{-1} \mu^{\frac{\sigma}{2}} \chi^{1-\frac{\sigma}{2}}$$

$$1 < \sigma$$

$$\Gamma_{UV} = \int_x \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \chi_R \partial_\mu \chi_R - \frac{1}{2} C R^2 + D R^{\mu\nu} R_{\mu\nu} \right\}$$

no mass
scale

$$\Delta\Gamma_{UV} = \int_x \sqrt{g} E \left(\mu^2 - \frac{R}{2} \right) \mu^{-\frac{2\sigma}{2-\sigma}} \chi_R^{\frac{4}{2-\sigma}},$$

deviation from
fixed point
vanishes for

$$E = b^{-\frac{2}{2-\sigma}} \left(1 - \frac{\sigma}{2}\right)^{\frac{4}{2-\sigma}}$$

$$\mu \rightarrow \infty$$