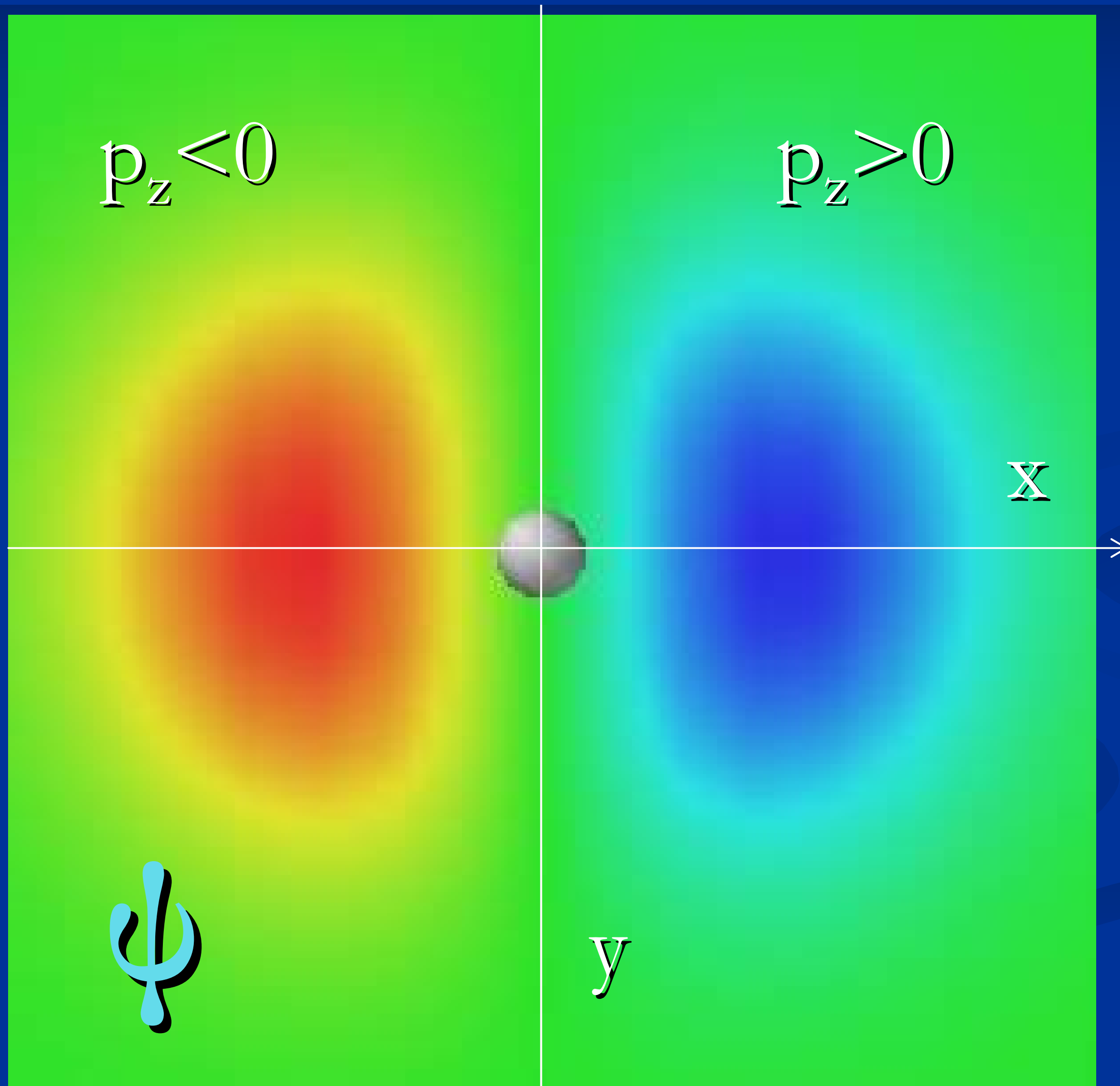
Quantum physics from coarse grained classical probabilities



what is an atom ?

- quantum mechanics : isolated object
- quantum field theory : excitation of complicated vacuum
- classical statistics : sub-system of ensemble with infinitely many degrees of freedom

quantum mechanics can be described by classical statistics !

quantum mechanics from classical statistics

- probability amplitude
- entanglement
- interference
- superposition of states
- fermions and bosons
- unitary time evolution
- transition amplitude
- non-commuting operators
- violation of Bell's inequalities

essence of quantum mechanics

description of appropriate subsystems of classical statistical ensembles

1) equivalence classes of probabilistic observables
2) incomplete statistics
3) correlations between measurements based on conditional probabilities
4) unitary time evolution for isolated subsystems

statistical picture of the world

- basic theory is not deterministic
- basic theory makes only statements about probabilities for sequences of events and establishes correlations
- probabilism is fundamental , not determinism !

quantum mechanics from classical statistics :
not a deterministic hidden variable theory

Probabilistic realism

Physical theories and laws only describe probabilities

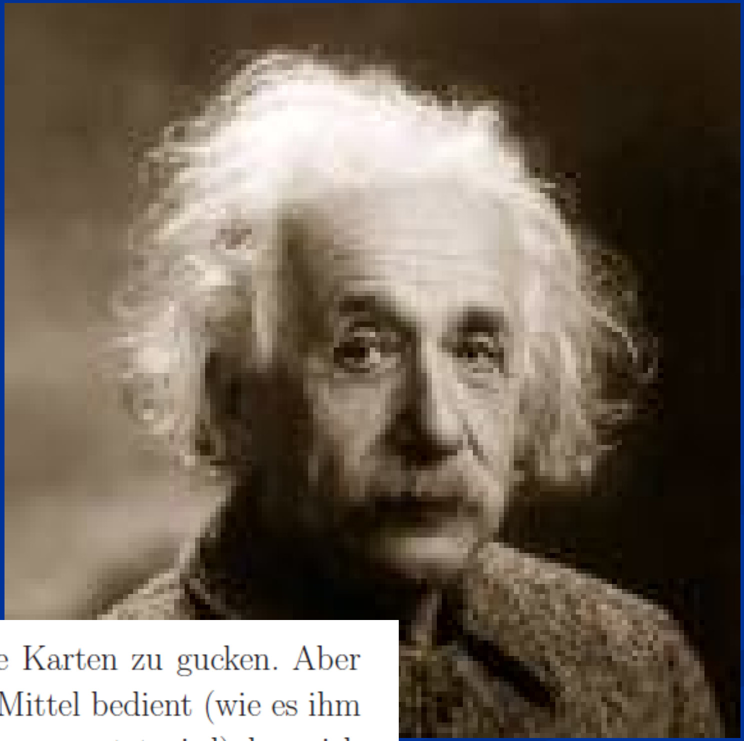
Physics only describes probabilities

Gott würfelt

Physics only describes probabilities

Gott würfelt

Gott würfelt nicht



"Es scheint hart, dem Herrgott in die Karten zu gucken. Aber dass er würfelt und sich telepatischer Mittel bedient (wie es ihm von der gegenwärtigen Quantentheorie zugemutet wird), kann ich keinen Augenblick glauben.."

Einstein: Brief an Cornelius Lanczos am 21. März 1942

Physics only describes probabilities

Gott würfelt Gott würfelt nicht

humans can only deal with probabilities

probabilistic Physics

- There is one reality
- This can be described only by probabilities

one droplet of water …

- 10^{20} particles
- electromagnetic field
- exponential increase of distance between two neighboring trajectories

probabilistic realism

The basis of Physics are probabilities

for predictions of real events

laws are based on probabilities

determinism as special case :

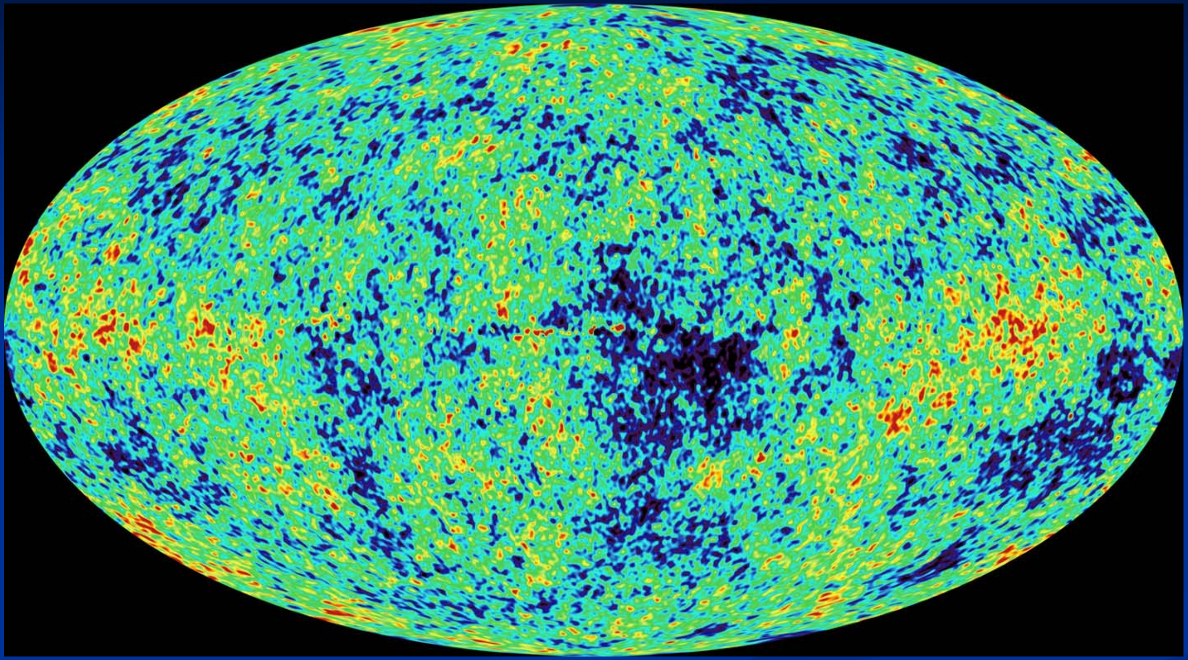
probability for event = 1 or 0

- law of big numbers
- unique ground state …

conditional probability

sequences of events(measurements)
are described by
conditional probabilities

both in classical statistics
and in quantum statistics

$w(t_1)$  :

not very suitable
for statement , if here and now
a pointer falls down

Schrödinger's cat

conditional probability :
if nucleus decays
then cat dead with $w_c = 1$
(reduction of wave function)

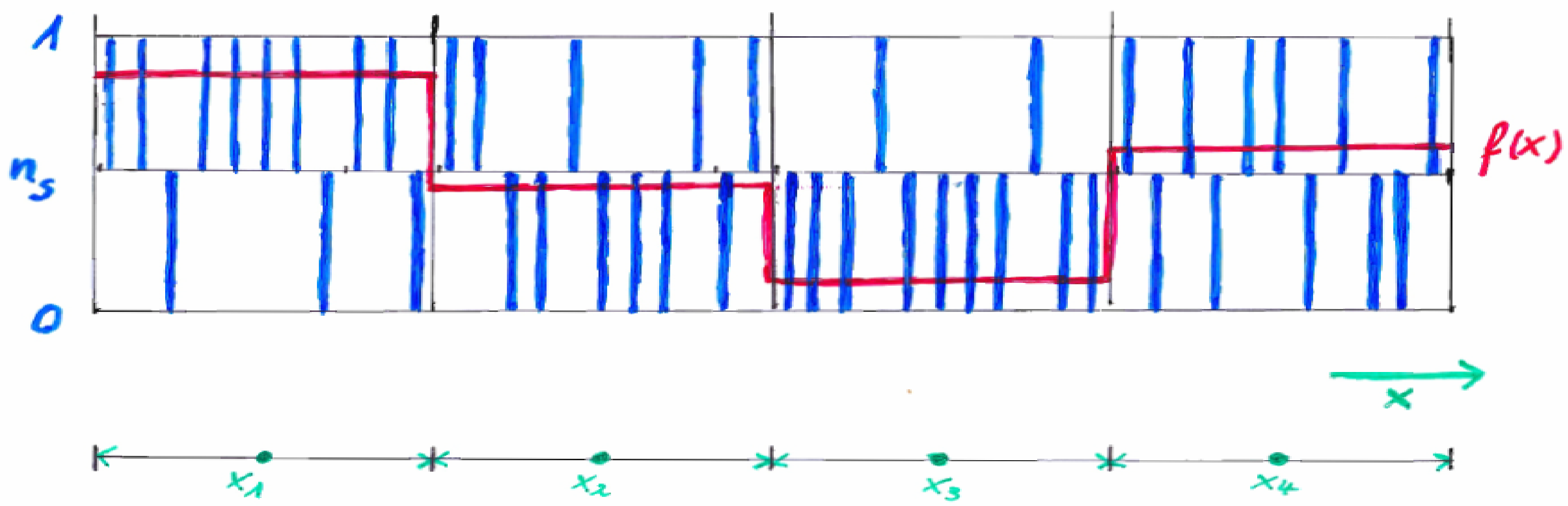
one - particle wave function from coarse graining of microphysical classical statistical ensemble

non – commutativity in classical statistics

microphysical ensemble

- states τ
- labeled by sequences of occupation numbers or bits $n_s = 0$ or 1
- $\tau = [\, n_s \,] = [0,0,1,0,1,1,0,1,0,1,1,1,1,0,\ldots]$ etc.
- probabilities $p_\tau > 0$

function observable



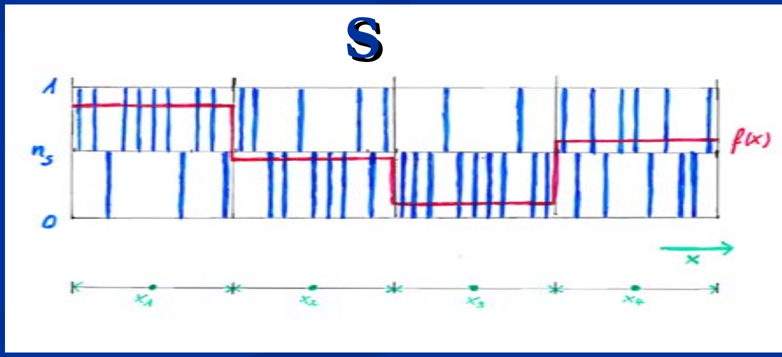
function observable

$$f_{\mathcal{T}}(x_i) = \mathcal{N}^{-\frac{1}{2}} \sum_{s \in I(x_i)} (2n_s - 1)$$

normalized difference between occupied and empty bits in interval

$$\int dx\, f_{\mathcal{T}}^2(x) = \sum_i f_{\mathcal{T}}^2(x_i) = 1$$

$$\mathcal{N} = \sum_{x_i} \Big(\sum_{s \in I(x_i)} (2n_s - 1) \Big)^2$$



$I(x_1)$ $I(x_2)$ $I(x_3)$ $I(x_4)$

generalized function observable

normalization

$$\int dx\, f_\tau^2(x) = 1$$

classical expectation value

$$\langle f(x) \rangle = \sum_\tau p_\tau f_\tau(x)$$

several species α

$$\sum_\alpha \int dx\, f_{\alpha,\tau}^2(x) = 1$$

position

$$X_\tau = \int dx\, x\, f_\tau^2(x)$$

classical observable :

fixed value for every state τ

momentum

- derivative observable

$$P_\tau = \int dx [f_{1,\tau}(x)\partial_x f_{2,\tau}(x) - f_{2,\tau}(x)\partial_x f_{1,\tau}(x)]$$

classical observable :
fixed value for every state τ

complex structure

$$f_\tau(x) = f_{1,\tau}(x) + i f_{2,\tau}(x)$$

$$\int dx\, f^*_\tau(x) f_\tau(x) = 1$$

$$P_\tau = \int dx\, f^*_\tau(x)(-i\partial_x) f_\tau(x)$$

$$X_\tau = \int dx\, f^*_\tau(x)\, x\, f_\tau(x)$$

$$P_\tau = \int dx [f_{1,\tau}(x)\partial_x f_{2,\tau}(x) - f_{2,\tau}(x)\partial_x f_{1,\tau}(x)]$$

classical product of position and momentum observables

$$\langle X \cdot P \rangle_{cl} = \langle P \cdot X \rangle_{cl} = \sum_{\tau} p_{\tau} X_{\tau} P_{\tau}$$

commutes !

different products of observables

$$(X^2)_\tau = \int dx f_\tau^*(x) x^2 f_\tau(x)$$

$$\langle X^2 \rangle = \sum_\tau p_\tau (X^2)_\tau$$

differs from classical product

$$\begin{aligned} \langle X \cdot X \rangle &= \sum_\tau p_\tau X_\tau^2 \\ &= \sum_\tau p_\tau \Big(\int dx f_\tau^*(x) x f_\tau(x) \Big)^2 \end{aligned}$$

Which product describes correlations of measurements ?

coarse graining of information for subsystems

density matrix from coarse graining

- position and momentum observables use only small part of the information contained in p_τ ,

- relevant part can be described by density matrix

$$\rho(x,x') = \sum_\tau p_\tau f_\tau(x) f_\tau^*(x')$$

- subsystem described only by information which is contained in density matrix
- coarse graining of information

quantum density matrix

density matrix has the properties of a quantum density matrix

$$\mathrm{Tr}\rho = \int dx \rho(x,x) = 1\,,\ \rho^*(x,x') = \rho(x',x)$$

$$\rho(x,x') = \sum_\tau p_\tau f_\tau(x) f^*_\tau(x')$$

quantum operators

$$\hat{X}(x', x) = \delta(x' - x)x$$
$$\hat{P}(x', x) = -i\delta(x' - x)\frac{\partial}{\partial x}$$

$$\langle X \rangle = \sum_{\tau} p_\tau X_\tau = \mathrm{Tr}(\hat{X}\rho) = \int dx\, x\, \rho(x, x)$$

$$\begin{aligned} \langle P \rangle &= \sum_{\tau} p_\tau P_\tau = \mathrm{Tr}(\hat{P}\rho) \\ &= -i \int dx' dx\, \delta(x' - x)\partial_x \rho(x, x') \end{aligned}$$

quantum product of observables

the product

$$(X^2)_\tau = \int dx f_\tau^*(x) x^2 f_\tau(x)$$

$$\langle X^2 \rangle = \sum_\tau p_\tau (X^2)_\tau$$

is compatible with the coarse graining

$$\langle X^2 \rangle = \int dx x^2 \rho(x, x)$$

and can be represented by operator product

incomplete statistics

classical product

$$\langle X \cdot X \rangle = \sum_\tau p_\tau X_\tau^2$$
$$= \sum_\tau p_\tau \Big(\int dx\, f_\tau^*(x) x f_\tau(x) \Big)^2$$

- is not computable from information which is available for subsystem !
- cannot be used for measurements in the subsystem !

classical and quantum dispersion

$$\Delta_x^2 = \langle X^2 \rangle - \langle X \rangle^2 \,,\, (\Delta_x^{(cl)})^2 = \langle X \cdot X \rangle - \langle X \rangle^2$$

$$\Delta_x^2 - (\Delta_x^{(cl)})^2 = \sum_\tau p_\tau \int dx \, f_\tau^*(x)(x - X_\tau)^2 f_\tau(x) \geq 0$$

$$\begin{aligned} \langle X \cdot X \rangle &= \sum_\tau p_\tau X_\tau^2 \\ &= \sum_\tau p_\tau \Big(\int dx \, f_\tau^*(x) x f_\tau(x) \Big)^2 \end{aligned}$$

$$(X^2)_\tau = \int dx \, f_\tau^*(x) x^2 f_\tau(x)$$

$$\langle X^2 \rangle = \sum_\tau p_\tau (X^2)_\tau$$

subsystem probabilities

$$w(x) = \rho(x, x) = \sum_{\tau} p_\tau |f_\tau(x)|^2$$

$$w(x) \geq 0 \,, \quad \int dx w(x) = 1$$

$$\langle X^n \rangle = \int dx x^n w(x)$$

in contrast :

$$\langle X \cdot X \rangle = \int dx dy \; xy \; w_{cl}(x, y)$$

$$w_{cl}(x, y) = \sum_{\tau} p_\tau |f_\tau|^2(x) |f_\tau|^2(y)$$

squared momentum

$$
\begin{aligned}
(P^2)_\tau &= \int dx\, f_\tau^*(x)(-\partial_x^2) f_\tau(x) \\
&= \int dx |\partial_x f_\tau(x)|^2
\end{aligned}
$$

$$
\begin{aligned}
\langle P^2 \rangle &= \sum_\tau p_\tau (P^2)_\tau = \mathrm{tr}(\hat{P}^2 \rho) \\
&= \int dx dx' \delta(x' - x)(-\partial_x^2)\rho(x, x')
\end{aligned}
$$

$$
\begin{aligned}
\langle P \cdot P \rangle &= \sum_\tau p_\tau P_\tau^2 \\
&= -\sum_\tau p_\tau \left(\int dx\, f_\tau^*(x) \partial_x f_\tau(x) \right)^2
\end{aligned}
$$

quantum product between classical observables : maps to product of quantum operators

non – commutativity in classical statistics

$$(XP)_\tau = \int dx f_\tau^*(x) x(-i\partial_x) f_\tau(x)$$

$$(PX)_\tau = \int dx f_\tau^*(x)(-i\partial_x) x f_\tau(x)$$

$$\langle XP \rangle = \mathrm{tr}(\hat{X}\hat{P}\rho) \ , \ \langle PX \rangle = \mathrm{tr}(\hat{P}\hat{X}\rho)$$

$$XP - PX = i$$

commutator depends on choice of product !

measurement correlation

- correlation between measurements of positon and momentum is given by quantum product
- this correlation is compatible with information contained in subsystem

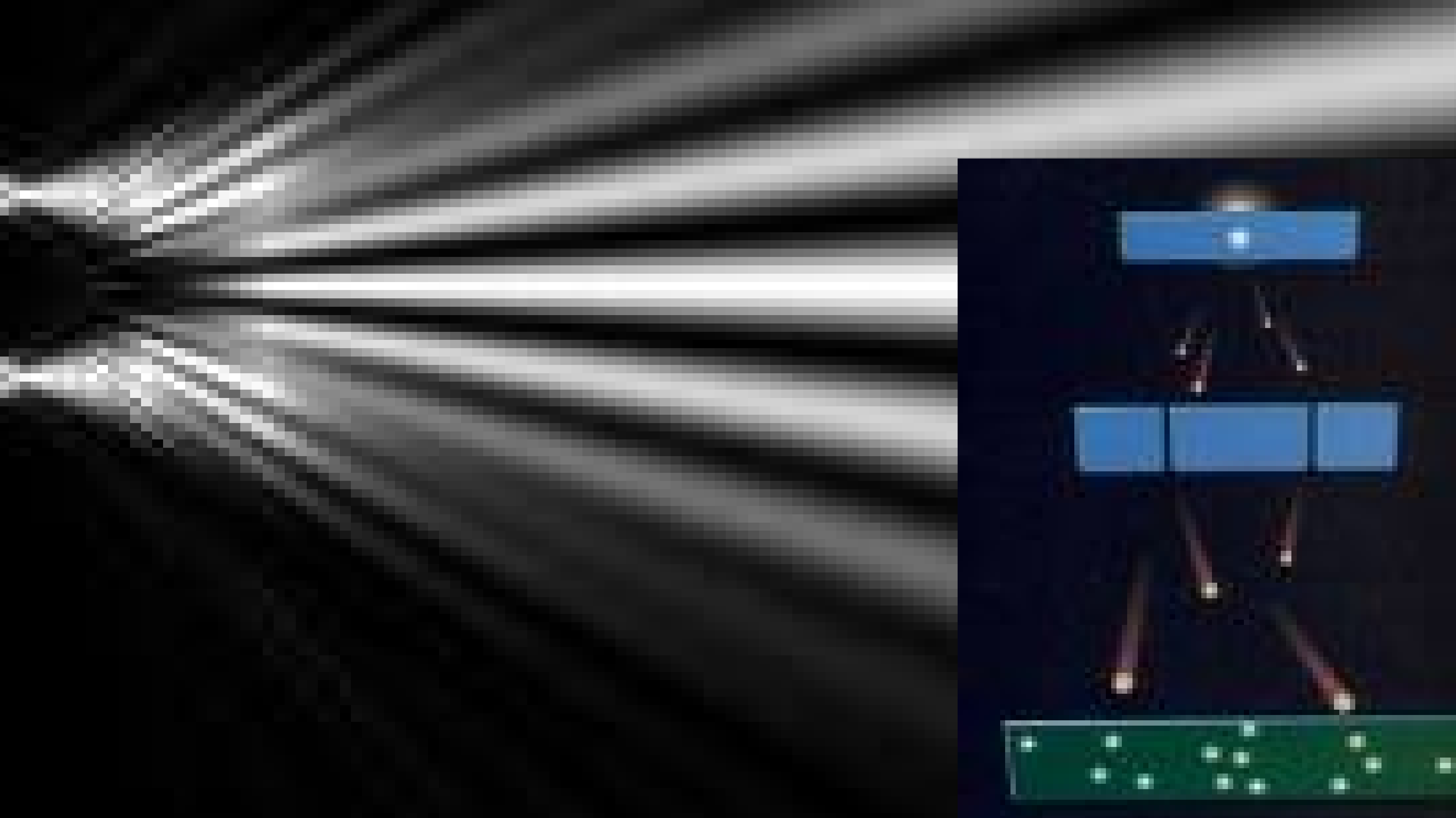
$$\langle XP\rangle_m = \frac{1}{2}(\langle XP\rangle + \langle PX\rangle)$$

coarse graining

from fundamental fermions at the Planck scale $p([n_s])$

to atoms at the Bohr scale $\rho(x\,,\,x')$

quantum particle from classical probabilities in phase space



quantum particle

and

classical particle

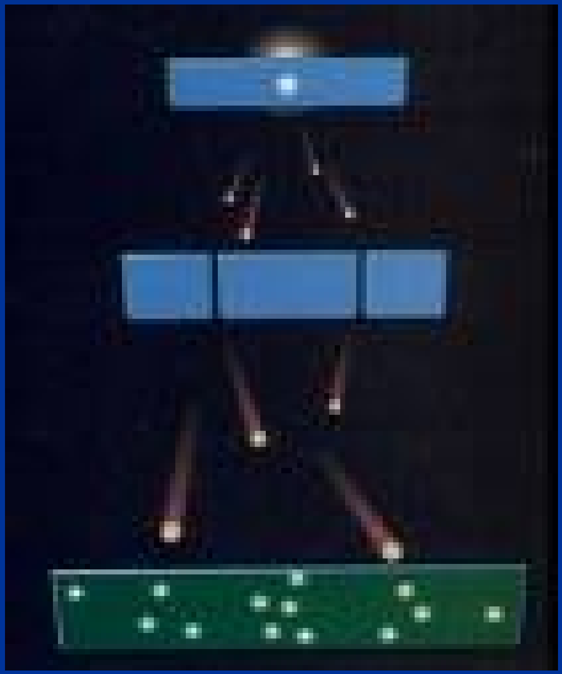
quantum particle

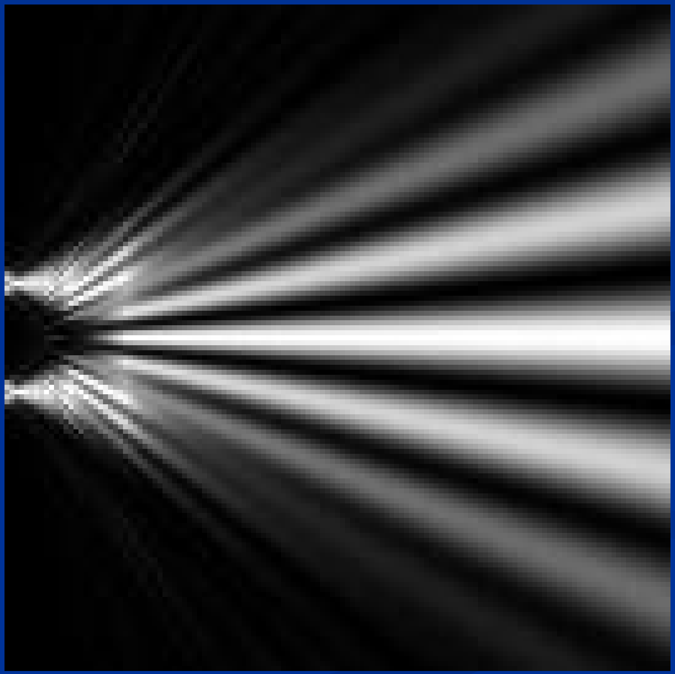
- particle-wave duality
- uncertainty
- no trajectories
- tunneling
- interference for double slit

classical particle

- particles
- sharp position and momentum
- classical trajectories
- maximal energy limits motion
- only through one slit

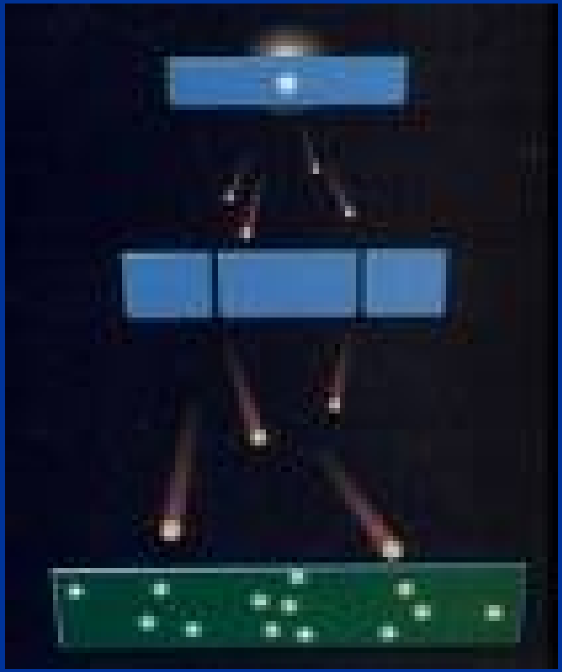
double slit experiment

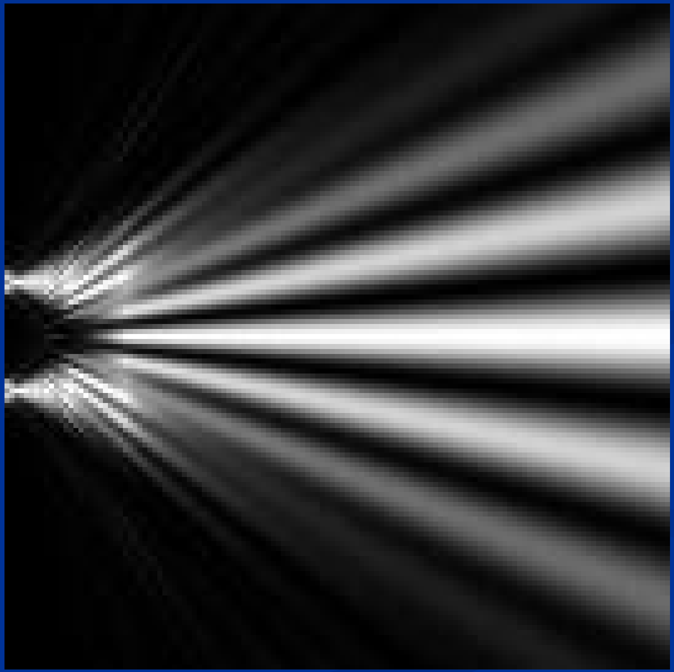




double slit experiment

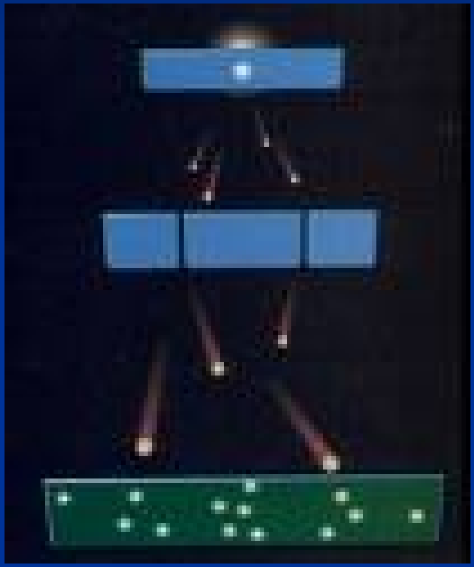
probability –
distribution

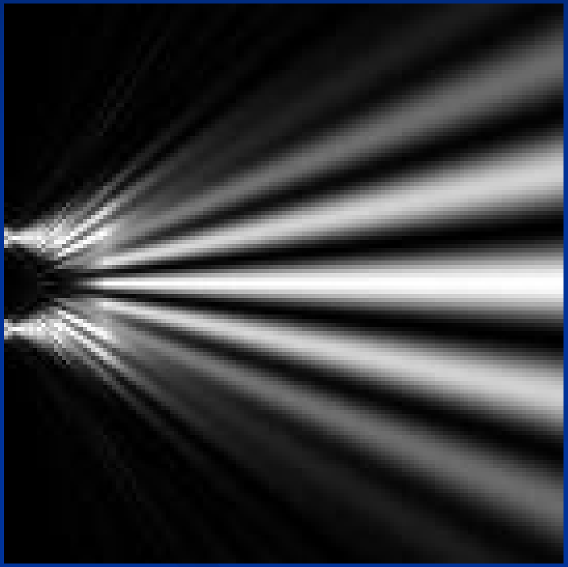




one isolated particle !
no interaction between atoms passing through slits

double slit experiment





Is there a classical probability distribution in phase space , and a suitable time evolution , which can describe interference pattern ?

quantum particle from classical probabilities in phase space

- probability distribution in phase space for

one particle

w(x,p)

$$w(x,p) \geq 0$$

$$\int_{x,p} w(x,p) = 1$$

as for classical particle !

- observables different from classical observables
- time evolution of probability distribution different from the one for classical particle

quantum mechanics can be described by classical statistics !

quantum mechanics from classical statistics

- probability amplitude
- entanglement
- interference
- superposition of states
- fermions and bosons
- unitary time evolution
- transition amplitude
- non-commuting operators
- violation of Bell's inequalities

quantum physics

wave function

probability

phase

Can quantum physics be described by classical probabilities ?

“ No go “ theorems

Bell , Clauser , Horne , Shimony , Holt

implicit assumption : use of classical correlation function for correlation between measurements

Kochen , Specker

assumption : unique map from quantum operators to classical observables

zwitters

- no different concepts for classical and quantum particles
- continuous interpolation between quantum particles and classical particles is possible

  (not only in classical limit)

classical particle without classical trajectory

quantum particle

- particle-wave duality
- uncertainty
- no trajectories
- tunneling
- interference for double slit

classical particle

- particle **– wave duality**
- sharp position and momentum
- ~~classical trajectories~~
- maximal energy limits motion
- only through one slit

no classical trajectories

also for classical particles in microphysics :

trajectories with sharp position
and momentum for each moment
in time are inadequate idealization !

still possible formally as limiting case

quantum particle

- quantum - probability - amplitude ψ(x)
- Schrödinger - equation

$$i\hbar\frac{\partial}{\partial t}\psi_Q(x) = -\frac{\hbar^2}{2m}\Delta\psi_Q(x) + V(x)\psi_Q(x)$$

classical particle

- classical probability in phase space w(x,p)
- Liouville - equation for w(x,p) (corresponds to Newton eq. for trajectories)

$$\frac{\partial}{\partial t}w = -Lw$$

$$L = \frac{p}{m}\frac{\partial}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

quantum formalism for classical particle

probability distribution for **one** classical particle

classical probability distribution in phase space

$$w(x, p; t)$$

wave function for classical particle

classical probability distribution in phase space

$$w(x, p; t)$$

wave function for classical particle

$$\psi_c(x, p; t)$$

depends on position and momentum !

$$w = \psi_c^2$$

wave function for one classical particle

$$\psi_C(x, p; t)$$

$$w = \psi_C^2$$

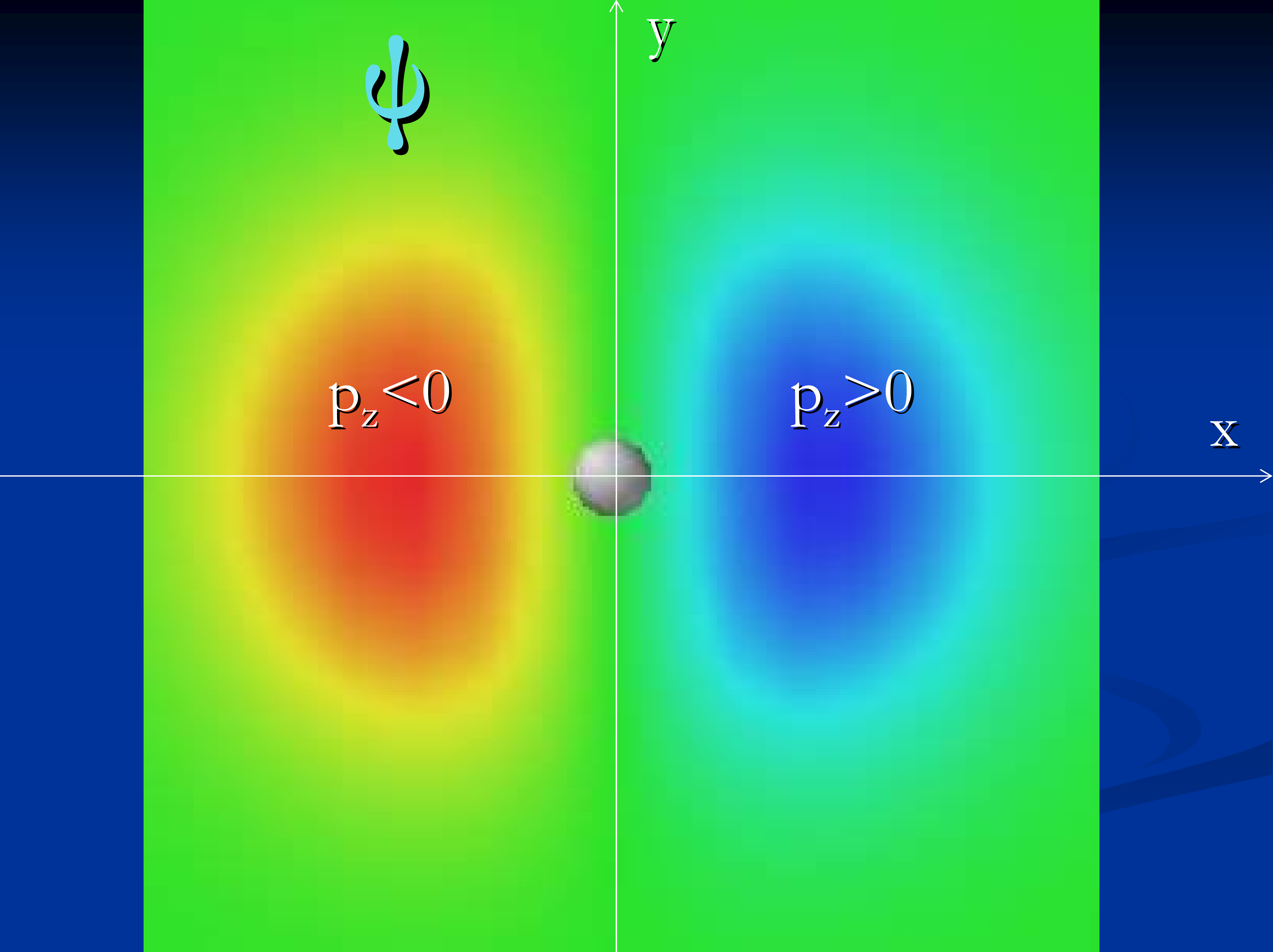
- real
- depends on position and momentum
- square yields probability

similarity to Hilbert space for classical mechanics
by Koopman and von Neumann
in our case : **real** wave function permits computation
of wave function from probability distribution
(up to some irrelevant signs)

quantum laws for observables

$$\langle x^2 \rangle = \int_{x,p} \psi^*_{\mathbf{C}}(x,p)\, x^2\, \psi_{\mathbf{C}}(x,p)$$

$$\langle x^2 \rangle = \int_{x,p} x^2 w(x,p)$$


ψ
y
x
p_z<0
p_z>0

time evolution of classical wave function

Liouville - equation

$$\frac{\partial}{\partial t} w = -Lw$$

$$L = \frac{p}{m}\frac{\partial}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

describes classical time evolution of classical probability distribution for one particle in potential V(x)

time evolution of classical wave function

$$\frac{\partial}{\partial t} w = -Lw \qquad w = \psi_C^2$$

$$\Rightarrow$$

$$\frac{\partial}{\partial t} \psi_C = -L\psi_C$$

$$\partial_t \psi^2 = 2\psi \partial_t \psi = -2\psi L \psi = -L\psi^2$$

wave equation

$$\frac{\partial}{\partial t}\psi = -L\psi$$

$$i\hbar\frac{\partial}{\partial t}\psi_{\mathbf{C}} = H_L\psi_{\mathbf{C}}$$

$$H_L = -i\hbar L = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + i\hbar\frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

modified Schrödinger - equation

wave equation

$$i\hbar\frac{\partial}{\partial t}\psi_C = H_L\psi_C$$

$$H_L = -i\hbar L = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + i\hbar\frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

fundamenal equation for classical particle in potential V(x)

replaces Newton's equations

particle - wave duality

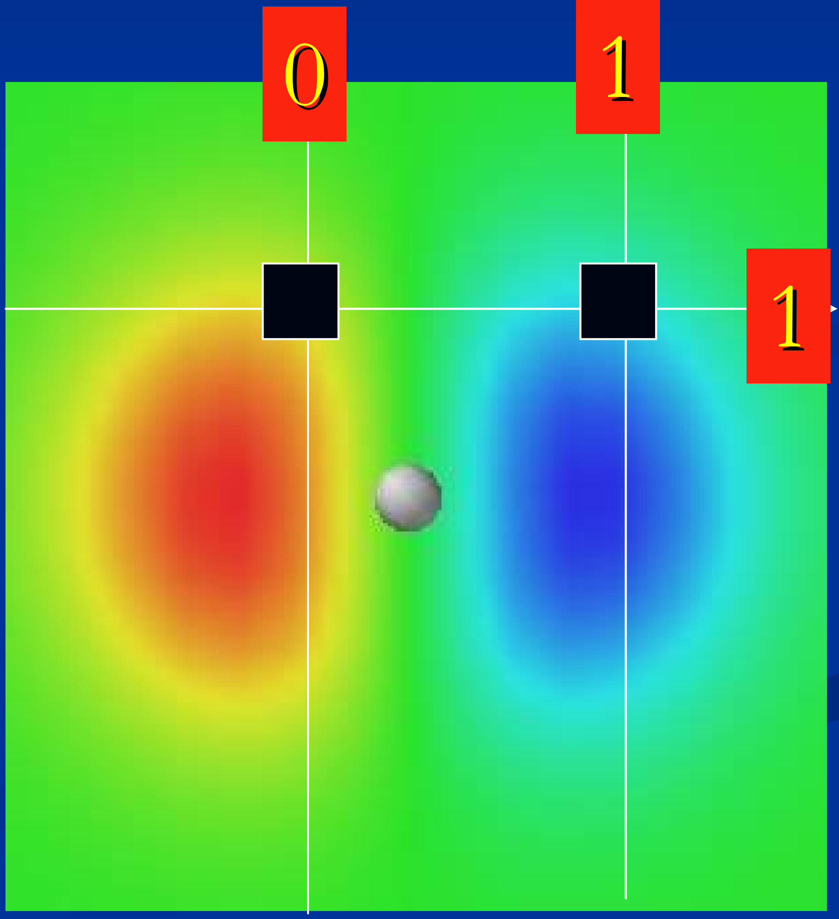
wave properties of particles :

continuous probability distribution

particle – wave duality

experiment if particle at position x – yes or no :
discrete alternative

probability distribution
for finding
particle at position x :
continuous



particle – wave duality

All statistical properties of classical *particles*

can be described in quantum formalism !

no quantum particles yet !

modification of Liouville equation

evolution equation

- time evolution of probability has to be specified as fundamental law
- not known a priori
- Newton's equations with trajectories should follow only in limiting case

zwitters

- *same formalism for quantum and classical particles*
- *different time evolution of probability distribution*

- **zwitters :**

between quantum and classical particle –

continuous interpolation of time evolution equation

quantum time evolution

modification of evolution for classical probability distribution

$$i\hbar\frac{\partial}{\partial t}\psi_C = H_L\psi_C$$

$$H_L = -i\hbar L = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + i\hbar\frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

$$H_L \longrightarrow H_W$$

$$H_W = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + V\left(x + \frac{i\hbar}{2}\frac{\partial}{\partial p}\right) - V\left(x - \frac{i\hbar}{2}\frac{\partial}{\partial p}\right)$$

quantum particle

evolution equation

$$
\begin{aligned}
\partial_t \psi_C(x,p) &= -\frac{p}{m}\partial_x \psi_C(x,p) + K(x,\partial_p)\psi_C(x,p), \\
K &= -i\left[V\left(x+\frac{i}{2}\partial_p\right) - V\left(x-\frac{i}{2}\partial_p\right)\right]
\end{aligned}
$$

fundamental equation for quantum particle in potential V

replaces Newton's equations

modified observables

Which observables should be chosen ?

- momentum: p or $-i\hbar\frac{\partial}{\partial x}$?
- position : x or $i\hbar\frac{\partial}{\partial p}$?
- Different possibilities , should be adapted to specific setting of measurements

position - observable

- different observables according to experimental situation
- suitable observable for microphysis has to be found
- classical position observable : idealization of infinitely precise resolution
- quantum – observable : remains computable with coarse grained information

quantum - observables

observables for classical position and momentum

$$X_{cl} = x \ , \ P_{cl} = p \ , \ [X_{cl}, P_{cl}] = 0$$

observables for quantum - position and momentum

$$X_Q = x + \frac{i\hbar}{2}\frac{\partial}{\partial p}$$

$$P_Q = p - \frac{i\hbar}{2}\frac{\partial}{\partial x}$$

… *do not commute !*

$$[X_Q, P_Q] = i\hbar$$

uncertainty

$$[X_Q, P_Q] = i\hbar$$

Heisenberg's uncertainty relation

$$\langle P_Q^2 \rangle = \langle P_{cl}^2 \rangle + \frac{1}{16} \langle (\partial_x \ln w)^2 \rangle$$

quantum – observables contain

statistical part

(similar to entropy , temperature)

Use quantum observables
for description of measurements of
position and momentum of particles !

quantum particle

with evolution equation

$$
\begin{aligned}
\partial_t \psi_C(x,p) &= -\frac{p}{m}\partial_x \psi_C(x,p) + K(x,\partial_p)\psi_C(x,p), \\
K &= -i\left[V\left(x+\frac{i}{2}\partial_p\right) - V\left(x-\frac{i}{2}\partial_p\right)\right]
\end{aligned}
$$

all expectation values and correlations for quantum – observables , as computed from classical probability distribution , coincide for all times precisely with predictions of quantum mechanics for particle in potential V

quantum particle from classical probabilities in phase space !

classical probabilities –

not a deterministic classical theory

quantum formalism
from classical probabilities

pure state

described by quantum wave function

$$\psi_Q(x)$$

realized by classical probabilities in the form

$$w(x,p) = \int_{r,r'} e^{ip(r'-r)}$$
$$\psi_Q^*\left(x+\frac{r'}{2}\right)\psi_Q\left(x-\frac{r'}{2}\right)\psi_Q^*\left(x-\frac{r}{2}\right)\psi_Q\left(x+\frac{r}{2}\right)$$

time evolution described by Schrödinger – equation

$$i\hbar\frac{\partial}{\partial t}\psi_Q(x) = -\frac{\hbar^2}{2m}\Delta\psi_Q(x) + V(x)\psi_Q(x)$$

density matrix and Wigner-transform

Wigner – transformed density matrix in quantum mechanics

$$\bar{\rho}_w$$

permits simple computation of expectation values of quantum mechanical observables

$$\langle F(X_Q, P_Q)\rangle = \int_{x,p} F(x,p)\,\bar{\rho}_w(x,p)$$

can be constructed from classical wave function !

$$\bar{\rho}_w(x,p) = \int_{r,r',s,s'} \psi_C\!\left(x+\frac{r}{2}, p+s\right)\psi_C\!\left(x+\frac{r'}{2}, p+s'\right)\cos(s'r - sr')$$

quantum observables and classical observables

$$\langle F(X_{cl}, P_{cl}) \rangle = \int_{x,p} F(x,p) w(x,p)$$

$$\langle F(X_Q, P_Q) \rangle = \int_{x,p} F(x,p) \bar{\rho}_w(x,p)$$

zwitter

difference between quantum and classical particles only through different time evolution

$$H_L = -i\hbar L = -i\hbar \frac{p}{m}\frac{\partial}{\partial x} + i\hbar \frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

CL

continuous interpolation

$$H_W = -i\hbar \frac{p}{m}\frac{\partial}{\partial x} + V\left(x + \frac{i\hbar}{2}\frac{\partial}{\partial p}\right) - V\left(x - \frac{i\hbar}{2}\frac{\partial}{\partial p}\right)$$

QM

zwitter - Hamiltonian

$$H_\gamma = \cos^2\gamma H_W + \sin^2\gamma H_L$$

- $\gamma=0$: quantum – particle
- $\gamma=\pi/2$: classical particle

other interpolating Hamiltonians possible !

How good is quantum mechanics ?

small parameter γ can be tested experimentally

$$H_\gamma = \cos^2\gamma H_W + \sin^2\gamma H_L$$

zwitter : no conserved microscopic energy

static state : $H_\gamma \psi_c = 0$ or $[H_\gamma, \rho_Q] = 0$

ground state for zwitter

static state with lowest $\langle H_Q \rangle$

$$H_Q = \frac{1}{2m} P_Q^2 + V(X_Q)$$

quantum - energy

eigenstate for quantum energy $H_Q \psi_n = E_n \psi_n$

zwitter –ground state has admixture of excited levels of quantum energy

$$\psi_Q = \psi_0 + c_1 \sin^2 \gamma \psi_1 + \ldots$$

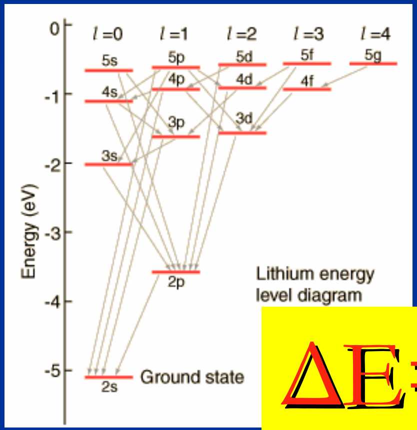
energy uncertainty of zwitter ground state

$$\Delta E = (\langle H_Q^2 \rangle - \langle H_Q \rangle^2)^{1/2} = f_1 \sin^2 \gamma |E_0|$$

also small shift of energy

$$\langle H_Q \rangle = E_0 + \delta E^{(\gamma)} \, , \, \delta E^{(\gamma)} = f_2 \sin^2 \gamma \Delta E$$

experiments for determination or limits on zwitter – parameter γ ?



$\Delta E \neq 0$

almost degenerate energy levels…?

lifetime of nuclear spin states > 60 h

$\gamma < 10^{-14}$ (Heil et al.)

experiments for determination or limits on zwitter – parameter γ ?

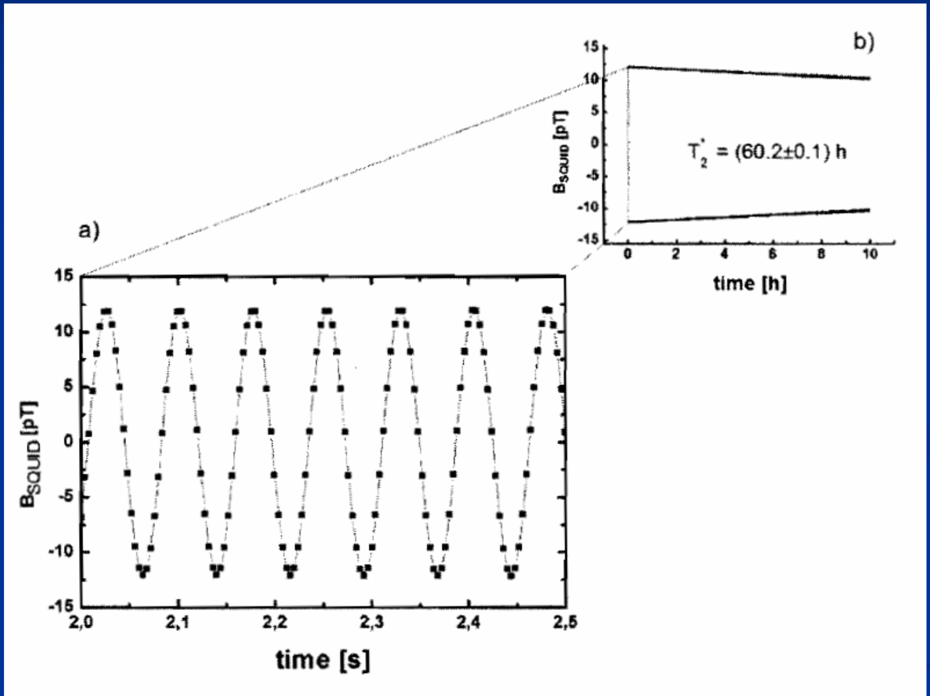


Fig.3 : a) Free spin-precession signal of a polarized ^3He sample cell recorded by means of a low-T_c SQUID (sampling rate: 250 Hz). b) Envelope of the decaying signal amplitude. From an exponential fit to the data, a transverse relaxation time of $T_2^* = (60.2 \pm 0.1)[h]$ can be deduced.

lifetime of nuclear spin states > 60 h (Heil et al.) : $\gamma < 10^{-14}$

sharpened observables – between quantum and classical

$$X_\beta = \cos^2\beta X_Q + \sin^2\beta X_{cl}$$
$$P_\beta = \cos^2\beta P_Q + \sin^2\beta P_{cl}$$

ß=0 : quantum observables, ß=1 :classical observables

$$[P_\beta, X_\beta] = -i\hbar\cos^2\beta$$

weakening of uncertainty relation

$$\langle X_\beta^2 \rangle = \frac{\hbar}{4m\omega}(1+\cos^4\beta)$$

$$\langle P_\beta^2 \rangle = \frac{m\omega\hbar}{4}(1+\cos^4\beta)$$

$$\Delta X_\beta \Delta P_\beta = \frac{\hbar}{4}(1+\cos^4\beta)$$

experiment ?

quantum particles and classical statistics

- common concepts and formalism for quantum and classical particles :

  classical probability distribution and wave function
- different time evolution , different Hamiltonian
- quantum particle from “ coarse grained “ classical probabilities (only information contained in Wigner function is needed)
- continuous interpolation between quantum and classical particle : zwitter

conclusion

- quantum statistics emerges from classical statistics

  quantum state, superposition, interference, entanglement, probability amplitude
- unitary time evolution of quantum mechanics can be described by suitable time evolution of classical probabilities
- conditional correlations for measurements both in quantum and classical statistics

experimental challenge

- quantitative tests , how accurate the predictions of quantum mechanics are obeyed
- zwitter
- sharpened observables
- small parameter :

“ almost quantum mechanics “

end

Quantenmechanik aus klassischen Wahrscheinlichkeiten

klassische Wahrscheinlichkeitsverteilung kann explizit angegeben werden für :

- quantenmechanisches Zwei-Zustands-System Quantencomputer : Hadamard gate
- Vier-Zustands-System (CNOT gate)
- verschränkte Quantenzustände
- Interferenz

Bell'sche Ungleichungen

werden verletzt durch bedingte Korrelationen

Bedingte Korrelationen für zwei Ereignisse oder Messungen reflektieren bedingte Wahrscheinlichkeiten

Unterschied zu klassischen Korrelationen

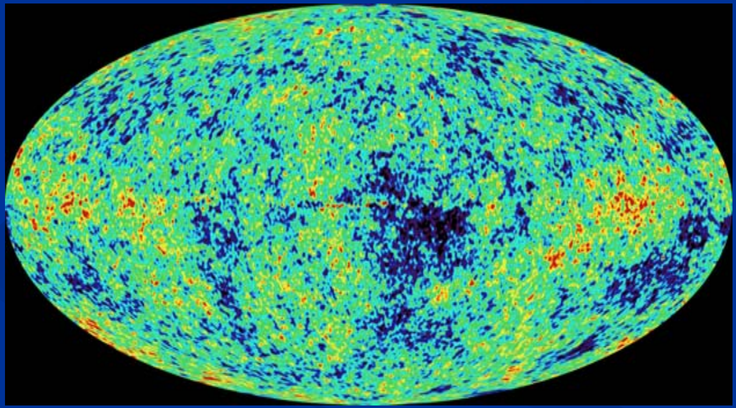
(Klassische Korrelationen werden implizit zur Herleitung der Bell'schen Ungleichungen verwandt.)

Bedingte Dreipunkt- Korrelation nicht kommutativ

Realität

- Korrelationen sind physikalische Realität , nicht nur Erwartungswerte oder Messwerte einzelner Observablen

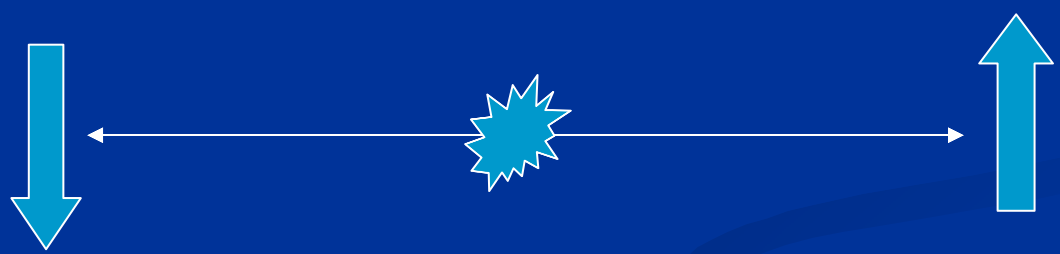
- Korrelationen können nicht-lokal sein (auch in klassischer Statistik) ; kausale Prozesse zur Herstellung nicht-lokaler Korrelationen erforderlich



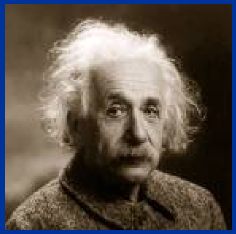
- Korrelierte Untersysteme sind nicht separabel in unabhängige Teilsysteme – Ganzes mehr als Summe der Teile

EPR - Paradoxon

Korrelation zwischen zwei Spins wird bei Teilchenzerfall hergestellt



Kein Widerspruch zu Kausalität oder Realismus wenn Korrelationen als Teil der Realität verstanden werden

( hat mal nicht Recht)

Untersystem und Umgebung: unvollständige Statistik

typische Quantensysteme sind Untersysteme
von klassischen Ensembles mit unendlich vielen
Freiheitsgraden (Umgebung)

probabilistische Observablen für Untersysteme :
Wahrscheinlichkeitsverteilung für Messwerte
in Quantenzustand

conditional correlations

example :

two - level observables ,

A , B can take values ± 1

conditional probability

$$w^{AB}_{+,\alpha}$$ probability to find value +1 for product of measurements of A and B

$$\begin{aligned} w^{AB}_{+,\alpha} &= (w^A_+)^B_+ w^B_{+,\alpha} + (w^A_-)^B_- w^B_{-,\alpha} \\ w^{AB}_{-,\alpha} &= (w^A_+)^B_- w^B_{-,\alpha} + (w^A_-)^B_+ w^B_{+,\alpha} \end{aligned}$$

$$(w^A_+)^B_+$$ probability to find A=1 after measurement of B=1

… can be expressed in terms of expectation value of A in eigenstate of B

$$\begin{aligned} (w^A_\pm)^B_+ &= \frac{1}{2}(1 \pm \langle A \rangle_{+B}) \\ (w^A_\pm)^B_- &= \frac{1}{2}(1 \pm \langle A \rangle_{-B}) \end{aligned}$$

measurement correlation

$$
\begin{aligned}
\langle BA\rangle_m &= (w_+^B)_+^A w_{+,s}^A - (w_-^B)_+^A w_{+,s}^A \\
&\quad -(w_+^B)_-^A w_{-,s}^A + (w_-^B)_-^A w_{-,s}^A
\end{aligned}
$$

After measurement A=+1 the system must be in eigenstate with this eigenvalue. Otherwise repetition of measurement could give a different result ! ρ_{A+}

$$
(w_+^B)_+^A - (w_-^B)_+^A = \mathrm{tr}(\hat{B}\rho_{A+})
$$

measurement changes state
in all statistical systems !

quantum and classical

eliminates possibilities that are not realized

physics makes statements
about possible
sequences of events
and their probabilities

unique eigenstates for M=2

$$M = 2 :$$

$$\rho_{A+} = \frac{1}{2}(1 + \hat{A})$$

$$(w_{\pm}^{B})_{+}^{A} = \frac{1}{2} \pm \frac{1}{4}\mathrm{tr}(\hat{B}\hat{A}) \; , \; (w_{\pm}^{B})_{-}^{A} = \frac{1}{2} \mp \frac{1}{4}\mathrm{tr}(\hat{B}\hat{A})$$

eigenstates with A = 1

$$\rho_{A+} = \frac{1}{M}(1 + \hat{A} + X)\ ,\ \mathrm{tr}(\hat{A}X) = 0\ ,\ \mathrm{tr}X = 0$$

$$P = M\mathrm{tr}(\rho_{A+}^2) = 1 + \frac{1}{M}\mathrm{tr}X^2$$

$$\rho_{A+}^2 - \rho_{A+} = \frac{1}{M^2}(X^2 + \{\hat{A}, X\}) - \left(1 - \frac{2}{M}\right)\rho_{A+}$$

measurement preserves pure states if projection

$$\rho_{A+} = \frac{1}{2(1 + \langle A \rangle)}(1 + \hat{A})\rho(1 + \hat{A})$$

measurement correlation equals quantum correlation

$$\langle BA \rangle_m = \frac{1}{2}\mathrm{tr}(\{\hat{A}, \hat{B}\}\rho)$$

probability to measure A=1 and B=1 :

$$w_{++} = \frac{1}{4}(1 + \langle A \rangle + \langle B \rangle + \langle AB \rangle_m)$$

sequence of three measurements and quantum commutator

$$\begin{aligned}
\langle ABC\rangle_m - \langle ACB\rangle_m &= \frac{1}{4}\mathrm{tr}\Big([\hat{A},[\hat{B},\hat{C}]]\,\rho\Big),\\
\langle ABC\rangle_m - \langle CBA\rangle_m &= \frac{1}{4}\mathrm{tr}\Big([\hat{B},[\hat{A},\hat{C}]]\,\rho\Big),\\
\langle ABC\rangle_m - \langle BAC\rangle_m &= 0
\end{aligned}$$

two measurements commute , not three