Quantum computation

with classical bits

Does our brain use quantum computing ?

Do artificial neural networks employ quantum algorithms ?

Can classical statistical memory materials perform quantum operations ?

Quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^{\dagger}(t)$$

Single qubit:
Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Two qubits:
CNOT gate

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}$$

Entanglement

- CNOT - gate transforms product state for two qubits into entangled state

$$\psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\downarrow\rangle\right) = \frac{1}{\sqrt{2}}\left(0,1,0,-1\right) ,$$

$$\psi_f = U_C\psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(0,1,-1,0\right).$$

- All unitary operations for two qubits can be constructed from Hadamard, rotation and CNOT gates.

Probabilistic computing

- Not necessarily quantum computing
- Transforms probabilistic information about state at layer t to probabilistic information at neighboring layer
- Deterministic computing special case with sharp values of bits 1 or 0, or Ising spins 1 or -1
- Probabilistic computing : only expectation values of Ising spins and correlations (probabilistic information)

Can probabilistic computing in classical statistical systems perform quantum operations?

Quantum subsystems

Quantum systems are subsystems of classical statistical systems

They use only part of the available probabilistic information

Incomplete statistics

Example

One qubit from three classical Ising spins

Ising spin : macroscopic two-level observable

Neuron fires above or below certain level

Three Ising spins (classical bits)

$$s_k(t) = \pm 1,\ k = 1, 2, 3$$

Arbitrary macroscopic two-level observables

Eight states

$$\tau = 1, \ldots, 8$$

We may number the states by $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$, where 1 denotes spin up and 0 stands for spin down. The expectation value of s_3 is then given by $\langle s_3 \rangle = -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8$, or for $A = s_1 s_2$ one has $\langle s_1 s_2 \rangle = p_1 + p_2 - p_3 - p_4 - p_5 - p_6 + p_7 + p_8$. We denote the expectation values of the three spins by ρ_z, $z = 1, \ldots, 3$,

Classical statistics

$$p_\tau \geq 0\,, \quad \sum_\tau p_\tau = 1$$

$$\langle A \rangle = \sum_\tau A_\tau p_\tau$$

One qubit from three classical bits

Expectation values : $\rho_z = \langle s_z \rangle \,, \quad -1 \le \rho_z \le 1$

Quantum subsystem defined by density matrix

$$\rho = \frac{1}{2}(1 + \rho_z \tau_z) \,, \quad \rho^\dagger = \rho \,, \quad \mathrm{tr}(\rho) = 1$$

Only part of classical statistical information used for subsystem

$$\rho_1 = -p_1 - p_2 - p_3 - p_4 + p_5 + p_6 + p_7 + p_8$$
$$\rho_2 = -p_1 - p_2 + p_3 + p_4 - p_5 - p_6 + p_7 + p_8$$
$$\rho_3 = -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8.$$

Incomplete statistics

Classical correlation function

between Ising spin 1 and Ising spin 2

cannot be computed

from information in subsystem !

It involves information about environment of subsystem : probabilistic information beyond

$$\rho_z = \langle s_z \rangle \,, \quad -1 \leq \rho_z \leq 1$$

Quantum condition

Positivity of density matrix requires quantum condition :

$$\rho_z \rho_z \leq 1$$

This implies uncertainty relation !

Non-commuting operators

$$L_z = \tau_z$$

Quantum rule for expectation values follows from classical statistical rules :

$$\langle s_z \rangle = \mathrm{tr}\left(L_z \rho\right) = \rho_z$$

$$\rho = \frac{1}{2}\left(1 + \rho_z \tau_z\right)\,, \quad \rho^\dagger = \rho\,, \quad \mathrm{tr}(\rho) = 1$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\rho_1 \to \rho_3 \,, \quad \rho_2 \to -\rho_2 \,, \quad \rho_3 \to \rho_1$$

$$p_1 \leftrightarrow p_3 \,, \quad p_2 \leftrightarrow p_7 \,, \quad p_4 \leftrightarrow p_5 \,, \quad p_6 \leftrightarrow p_8$$

Can be realized by deterministic spin flip

$$s_1 \leftrightarrow s_3 \,, \; s_2 \to -s_2$$

A few one qubit gates

$$s_1 \to s_2\,,\; s_2 \to -s_1 \;:\; \rho_1 \to \rho_2\,,\; \rho_2 \to -\rho_1 \;:\; U_{12} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\,,$$

$$s_3 \to s_1\,,\; s_1 \to -s_3 \;:\; \rho_3 \to \rho_1\,,\; \rho_1 \to -\rho_3 \;:\; U_{31} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\,,$$

$$s_1 \to -s_1\,,\; s_2 \to -s_2 \;:\; \rho_1 \to -\rho_1\,,\; \rho_2 \to -\rho_2 \;:\; U_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,$$

$$s_1 \to -s_1\,,\; s_3 \to -s_3 \;:\; \rho_1 \to -\rho_1\,,\; \rho_3 \to -\rho_3 \;:\; U_Y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,,$$

$$s_2 \to -s_2\,,\; s_3 \to -s_3 \;:\; \rho_2 \to -\rho_2\,,\; \rho_3 \to -\rho_3 \;:\; U_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,.$$

CNOT gate for two qubits

$$\rho = \frac{1}{4}(1 + \rho_z L_z) = \frac{1}{4}(1 + \rho_{\mu\nu} L_{\mu\nu})$$

$$\rho_z = \rho_{\mu\nu} = \langle \sigma_{\mu\nu} \rangle$$

SU(4)- generators : $$L_z = L_{\mu\nu} = \tau_\mu \otimes \tau_\nu$$

CNOT – gate:

$$\rho_{10} \leftrightarrow \rho_{11}\,, \quad \rho_{20} \leftrightarrow \rho_{21}\,, \quad \rho_{13} \leftrightarrow -\rho_{22}\,,$$
$$\rho_{02} \leftrightarrow \rho_{32}\,, \quad \rho_{03} \leftrightarrow \rho_{33}\,, \quad \rho_{23} \leftrightarrow \rho_{12}\,,$$
$$\rho_{30},\ \rho_{01},\ \rho_{31}\ \ \textit{invariant.}$$

Quantum subsystems using correlations

Scaling for Q qubits

- Independent classical spins for each entry of density matrix: $2^{2Q}-1$ classical bits needed

- Use correlations: $\rho_{k0} = \langle s_k^{(1)} \rangle\,, \quad \rho_{0k} = \langle s_k^{(2)} \rangle\,, \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$

Only 3Q classical bits needed !

Probabilistic computation

If density matrix for quantum subsystem involves correlations:

- CNOT gate cannot be realized by deterministic operations on six classical Ising spins

- All two-qubit gates can be realized if arbitrary changes of probability distribution for six classical Ising spins are allowed

Scaling for many qubits

Scaling for many qubits is theoretically possible if correlations are used

Big question :

How to realize suitable changes of probability distributions in practice ?

Static memory materials

Generalized Ising model:

$$w[s] = Z^{-1} \exp\big(-S[s]\big) b(s_{in}, s_f)$$

$$S = -\frac{\beta}{2} \sum_{x,t} s(t,x) \Big[s(t+1, x+1) + \sigma s(t+1, x-1) \Big]$$

Boundary term :

$$b(s_{in}, s_f) = \bar{f}_f(s_f) f_{in}(s_{in})$$

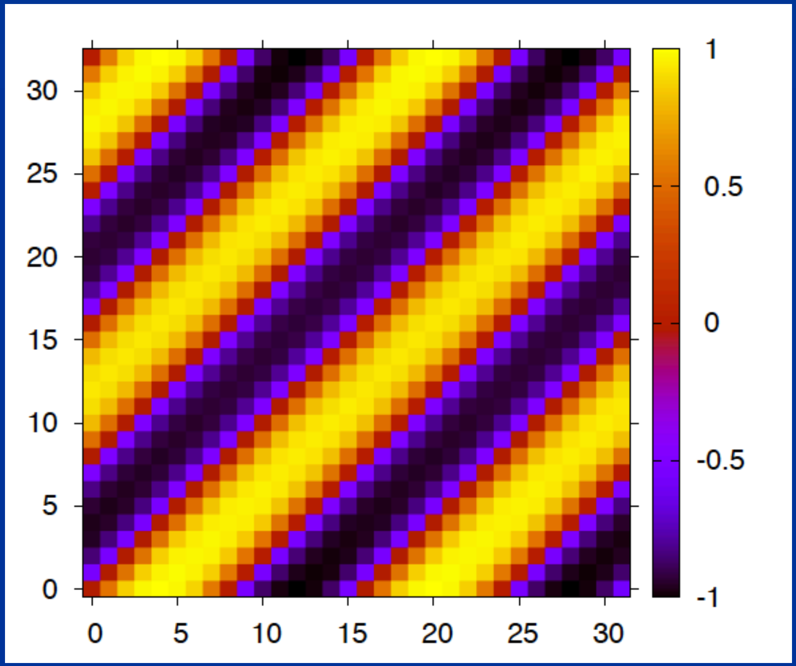
Probabilistic computing with static memory materials ?

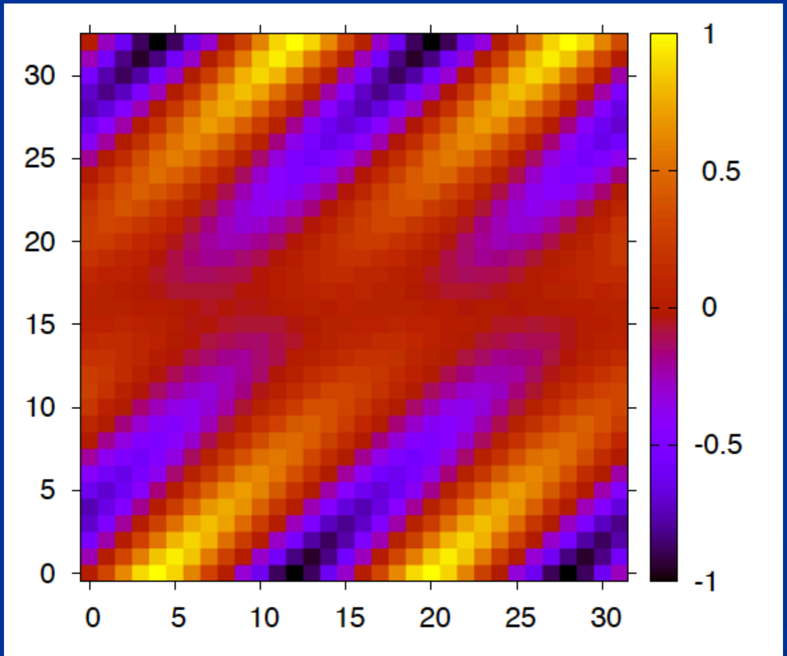
- Let general equilibrium classical statistics transport information from one layer to the next

- Simulation, with D. Sexty

Classical interference

Depending on boundary conditions :





Positive interference

Negative interference

Quantum formalism for classical statistics

- Formalism for information transport from one hypersurface to the next:
- Classical wave functions and density matrix
- Transfer matrix formalism : Heisenberg picture
- Wave functions : Schroedinger picture
- Non commuting operators for observables
- Quantum rules from classical statistical rules

Artificial neural networks

- Can neural networks learn to perform quantum operations ?

Arbitrary quantum operations

- Arbitrary quantum gates for an arbitrary number of qubits can be realized by suitable changes of probability distributions
- Infinite number of classical bits or continuous Ising spins needed
- Similar to description of rotations by classical bits

Quantum Field Theories

- Continuous classical observables or fields always involve an infinite number of bits
- Bits: yes/no decisions
- Possible measurement values 1 or 0

  or 1 or -1

Discrete spectrum of observables

Conclusion

- Quantum operations can be performed by classical statistical systems
- Very low temperature or

  well isolated systems of microscopic qubits

  not needed !

end

quantum mechanics can be described by classical statistics !

statistical picture of the world

- basic theory is not deterministic
- basic theory makes only statements about probabilities for sequences of events and establishes correlations
- probabilism is fundamental , not determinism !

quantum mechanics from classical statistics :
not a deterministic hidden variable theory

Probabilistic realism

Physical theories and laws only describe probabilities

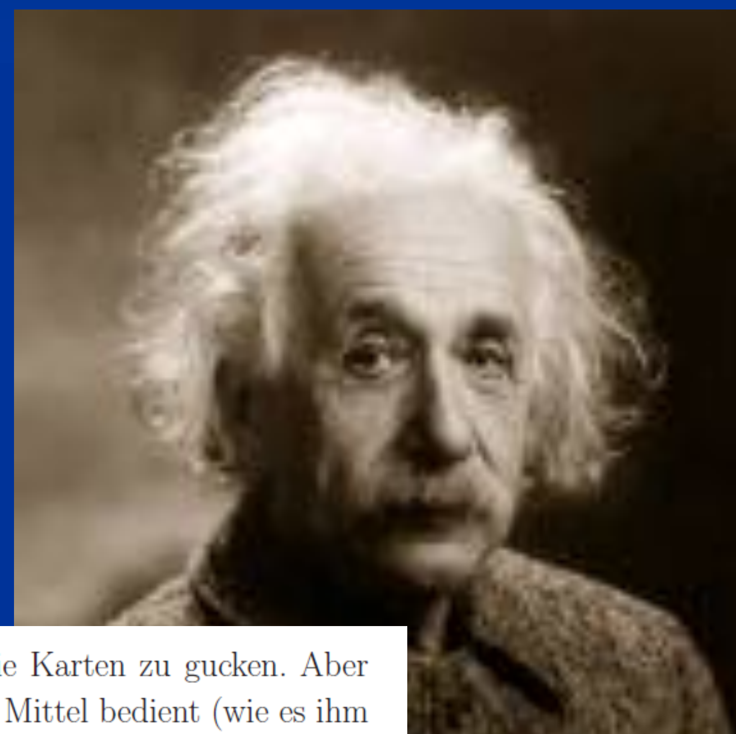
Physics only describes probabilities

Gott würfelt

Physics only describes probabilities

Gott würfelt

Gott würfelt nicht



"Es scheint hart, dem Herrgott in die Karten zu gucken. Aber dass er würfelt und sich telepatischer Mittel bedient (wie es ihm von der gegenwärtigen Quantentheorie zugemutet wird), kann ich keinen Augenblick glauben.."

Einstein: Brief an Cornelius Lanczos am 21. März 1942

Physics only describes probabilities

Gott würfelt   Gott würfelt nicht

humans can only deal with probabilities

probabilistic Physics

- There is one reality
- This can be described only by probabilities

one droplet of water …

- 10^{20} particles
- electromagnetic field
- exponential increase of distance between two neighboring trajectories

probabilistic realism

The basis of Physics are probabilities
for predictions of real events

laws are based on probabilities

determinism as special case :

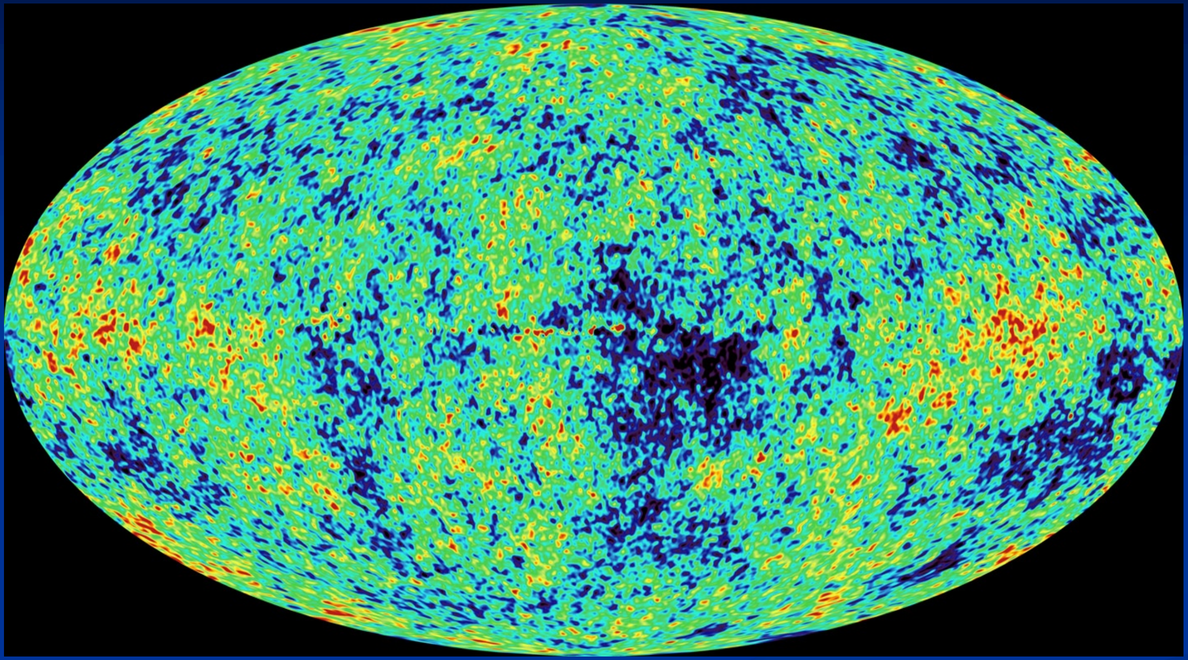
probability for event = 1 or 0

- law of big numbers
- unique ground state …

conditional probability

sequences of events(measurements)
are described by
conditional probabilities

both in classical statistics
and in quantum statistics

$w(t_1)$  :

not very suitable
for statement , if here and now
a pointer falls down

Schrödinger's cat

conditional probability :
if nucleus decays
then cat dead with $w_c = 1$
(reduction of wave function)

structural elements of quantum mechanics

unitary time evolution

$$\nu$$

h

Simple conversion factor for units

i

presence of complex structure

$$[A,B]=C$$

non-commuting observables

- classical statistical systems admit many product structures of observables
- many different definitions of correlation functions possible , not only classical correlation !
- type of measurement determines correct selection of correlation function !

conclusion

- quantum statistics emerges from classical statistics

  quantum state, superposition, interference, entanglement, probability amplitude
- unitary time evolution of quantum mechanics can be described by suitable time evolution of classical probabilities
- memory materials are quantum simulators
- conditional correlations for measurements both in quantum and classical statistics

Can quantum physics be described by classical probabilities ?

“ No go “ theorems

Bell , Clauser , Horne , Shimony , Holt

implicit assumption : use of classical correlation function for correlation between measurements

Kochen , Specker

assumption : unique map from quantum operators to classical observables end