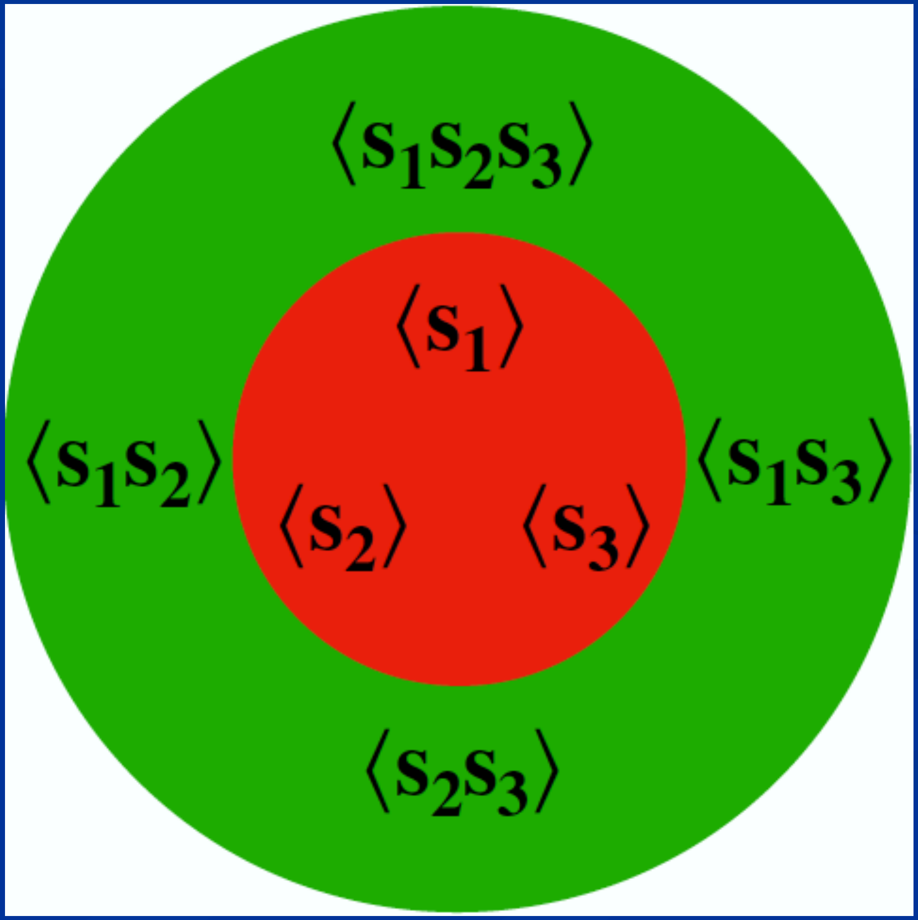
Quantum computing with classical bits



Quantum mechanics : subsystem of classical probabilistic system

- quantum mechanics emerges for

quantum subsystems

- subsystems are genuinely probabilistic
- part of information is lost by focus on subsystem
- ”forgetting” parts of information

Embedding of quantum mechanics in classical statistics

- it works
- quantum mechanics does not need new fundamental concepts beyond classical statistics
- raises interesting new questions

Does our brain use quantum computing ?

Do artificial neural networks employ quantum algorithms ?

Can classical statistical memory materials perform quantum operations ?

Quantum computing

Unitary evolution in quantum mechanics

solution of Schrödinger equation :

$$\psi(t+\epsilon) = U(t+\epsilon, t)\psi(t)$$

$\psi(t)$ wave function

$U(t+\epsilon, t)$ evolution operator

wave function

- complex function
- 2^Q components for Q qubits
- normalization $\psi^\dagger \psi = 1$

Density matrix

density matrix for pure quantum state:

$$\rho_{\alpha\beta}(t) = \psi_\alpha(t)\psi^*_\beta(t)$$

ρ hermitean $n \times n$ matrix $n = 2^Q$

$\rho^\dagger = \rho \ , \ \mathrm{tr}\rho = 1$

$\rho^\dagger_{\alpha\beta} = \rho^*_{\beta\alpha} = \psi^*_\beta\psi_\alpha = \rho_{\alpha\beta}$

$$\mathrm{tr}\rho = \sum_\alpha \rho_{\alpha\alpha} = \sum_\alpha \psi_\alpha\psi^*_\alpha = 1$$

pure and mixed quantum states

pure state : $\rho^2 = \rho$ $\rho_{\alpha\beta}\rho_{\beta\gamma} = \psi_\alpha \psi_\beta^* \psi_\beta \psi_\gamma^* = \psi_\alpha \psi_\gamma^* = \rho_{\alpha\gamma}$

mixed state : density matrix is weighed sum of pure state density matrices $\rho^{(a)}$ each realized with probability p_a

$\rho = \sum_a p_a \rho^{(a)},$ $p_a \geq 0\ , \quad \sum_a p_a = 1$ $\rho^{(a)}$: pure state density matrix

$$\mathrm{tr}\rho = 1 \qquad \rho^\dagger = \rho$$

positivity of density matrix

ρ is a positive matrix :

it has only positive eigenvalues

$\Rightarrow$ diagonal elements are positive

$$\rho_{\alpha\alpha} \geq 0$$

quantum evolution

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

$$\rho_{\alpha\beta}(t) = \psi_\alpha(t)\psi^*_\beta(t)$$

quantum gates

- split continuous evolution into discrete steps

$$t,\ t+\epsilon,\ t+2\epsilon,\ t+3\epsilon \quad \text{etc.}$$

- quantum gate $U(t+\epsilon, t) \to U(t)$

A quantum gate transforms a wave function at t to a new wave function at $t+\epsilon$. This is a "quantum operation".

Quantum computing is a sequence of quantum operations.

- the order matters :
- quantum operations do not commute $U_1 U_2 \neq U_2 U_1$

basis gates for one qubit

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^{\dagger}(t)$$

Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Quantum subsystem

One qubit from three classical Ising spins

Ising spin : macroscopic two-level observable

- Neuron fires above or below certain level
- Particle present or not
- Bit in a computer

Three Ising spins (classical bits)

$$s_k(t) = \pm 1, \ k = 1, 2, 3$$

Arbitrary macroscopic two-level observables

Eight states

$$\tau = 1, \ldots, 8$$

We may number the states by $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$, where 1 denotes spin up and 0 stands for spin down. The expectation value of s_3 is then given by $\langle s_3 \rangle = -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8$, or for $A = s_1 s_2$ one has $\langle s_1 s_2 \rangle = p_1 + p_2 - p_3 - p_4 - p_5 - p_6 + p_7 + p_8$. We denote the expectation values of the three spins by ρ_z, $z = 1, \ldots, 3$,

Classical statistics

$$p_\tau \geq 0 \, , \quad \sum_\tau p_\tau = 1$$

$$\langle A \rangle = \sum_\tau A_\tau p_\tau$$

One qubit from three classical bits

Expectation values : $\rho_z = \langle s_z \rangle \,, \quad -1 \leq \rho_z \leq 1$

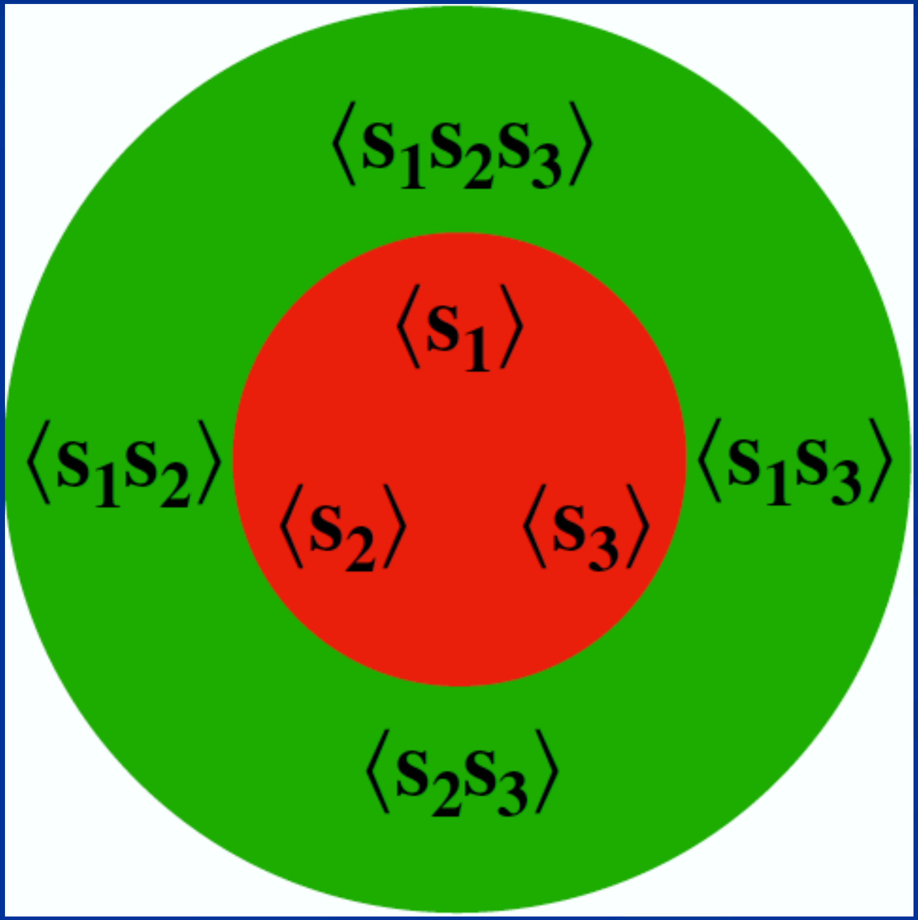
Quantum subsystem defined by density matrix

$$\rho = \frac{1}{2}(1 + \rho_z \tau_z) \,, \quad \rho^\dagger = \rho \,, \quad \mathrm{tr}(\rho) = 1$$

Only part of classical statistical information used for subsystem

$$\rho_1 = -p_1 - p_2 - p_3 - p_4 + p_5 + p_6 + p_7 + p_8$$
$$\rho_2 = -p_1 - p_2 + p_3 + p_4 - p_5 - p_6 + p_7 + p_8$$
$$\rho_3 = -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8.$$

Subsystem in space of correlation functions



Thermalization of pure quantum state

- where does the information go ?
- into n – point functions with extremely high n ! (Avogadro's number)
- initial information is no longer visible in low order correlation functions, which approach thermal equilibrium values
- subsystem : low order correlation functions

Aarts, Bonini, Berges, Borsanyi…

Quantum subsystems

Quantum systems are subsystems of classical statistical systems

They use only part of the available probabilistic information

Incomplete statistics

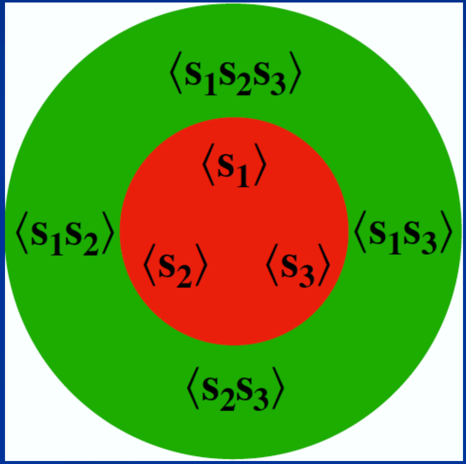
Incomplete statistics

Classical correlation function

between Ising spin 1 and Ising spin 2

cannot be computed

from information in subsystem !



It involves information about environment of subsystem : probabilistic information beyond

$$\rho_z = \langle s_z \rangle$$

incomplete statistics
is origin of
representation of observables by
non-commuting operators

Non-commuting operators

$$L_z = \tau_z$$

Quantum rule for expectation values follows from classical statistical rules :

$$\langle s_z \rangle = \mathrm{tr}\left(L_z \rho\right) = \rho_z$$

$$\rho = \frac{1}{2}\left(1 + \rho_z \tau_z\right) , \quad \rho^\dagger = \rho , \quad \mathrm{tr}(\rho) = 1$$

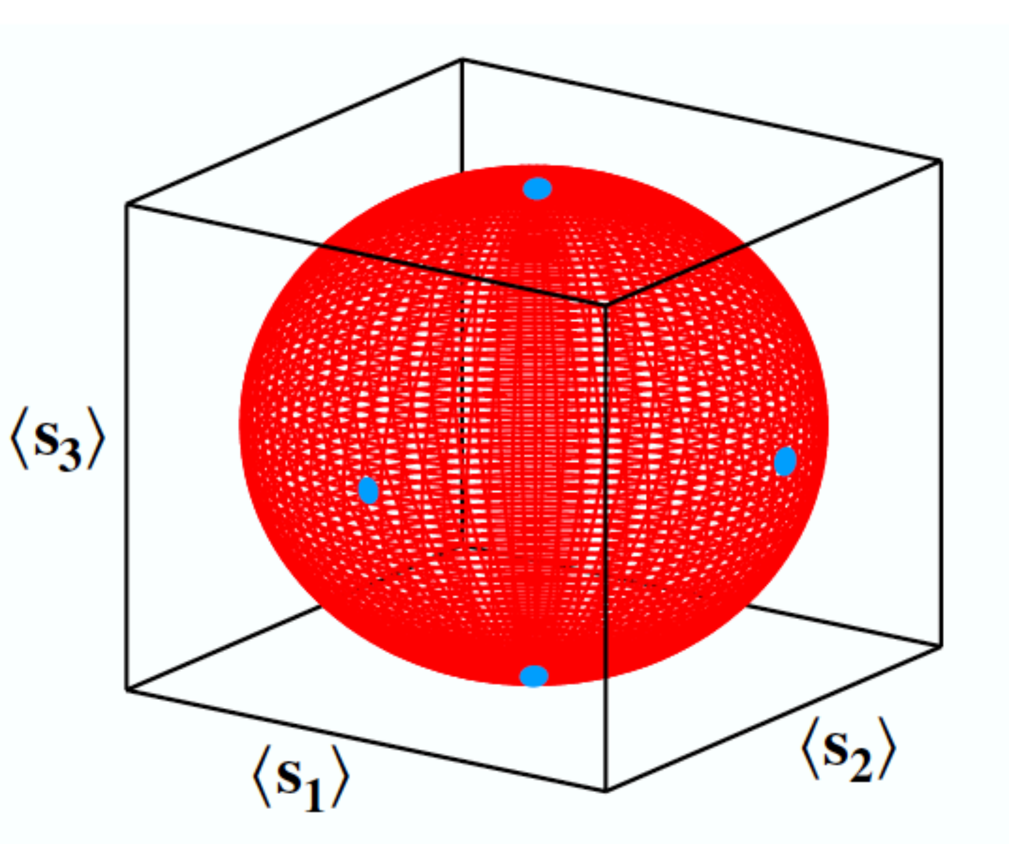
Quantum condition

Positivity of density matrix requires quantum condition :

$$\rho_z \rho_z \leq 1$$

This implies uncertainty relation !

Not every classical probability distribution admits a quantum subsystem



$$\rho_z \rho_z \leq 1$$

Pure quantum states

saturation of quantum condition :

- pure state
- density matrix can be written as bilinear in complex wave function
- located on Bloch sphere

Quantum computing

- quantum computing proceeds by quantum gates
- discrete unitary transformations of density matrix in consecutive time steps

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

- a few basic gates are sufficient

Quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Single qubit:
Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Two qubits:
CNOT gate

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}$$

Realization of quantum gates

- by deterministic manipulations of classical bits or Ising spins

cellular automata

- by changes of classical probability distribution for classical bits

probabilistic computing

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\rho_1 \to \rho_3 \,, \quad \rho_2 \to -\rho_2 \,, \quad \rho_3 \to \rho_1$$

$$p_1 \leftrightarrow p_3 \,, \quad p_2 \leftrightarrow p_7 \,, \quad p_4 \leftrightarrow p_5 \,, \quad p_6 \leftrightarrow p_8$$

Can be realized by deterministic spin flip

$$s_1 \leftrightarrow s_3 , \; s_2 \rightarrow -s_2$$

A few one qubit gates

$$s_1 \to s_2\,,\; s_2 \to -s_1 \;:\; \rho_1 \to \rho_2\,,\; \rho_2 \to -\rho_1 \;:\; U_{12} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\,,$$

$$s_3 \to s_1\,,\; s_1 \to -s_3 \;:\; \rho_3 \to \rho_1\,,\; \rho_1 \to -\rho_3 \;:\; U_{31} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\,,$$

$$s_1 \to -s_1\,,\; s_2 \to -s_2 \;:\; \rho_1 \to -\rho_1\,,\; \rho_2 \to -\rho_2 \;:\; U_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,$$

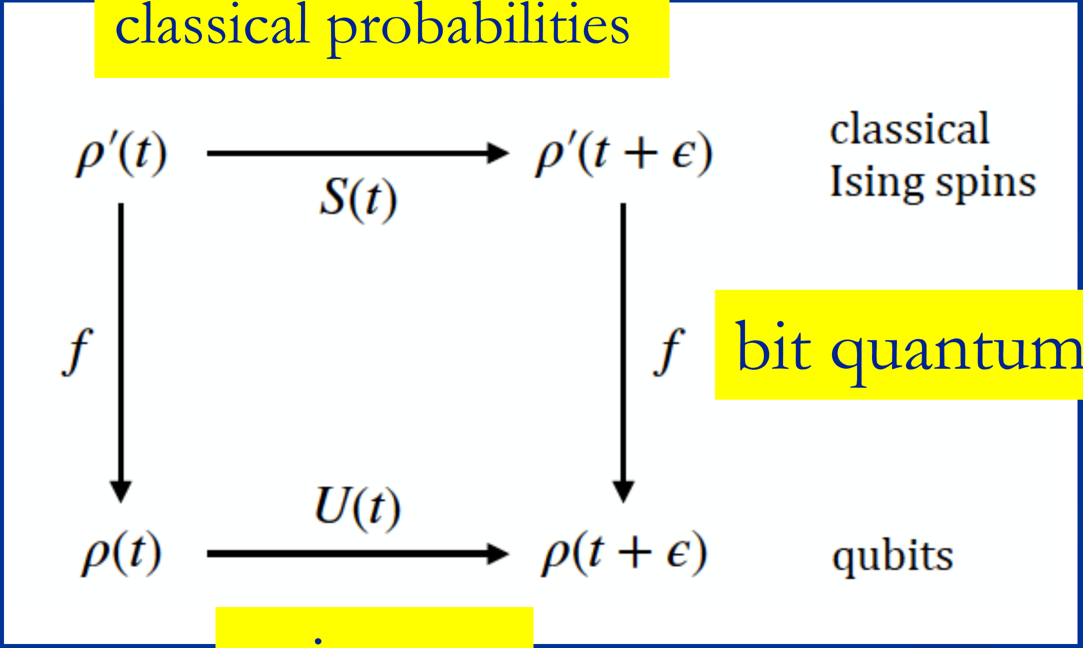
$$s_1 \to -s_1\,,\; s_3 \to -s_3 \;:\; \rho_1 \to -\rho_1\,,\; \rho_3 \to -\rho_3 \;:\; U_Y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,,$$

$$s_2 \to -s_2\,,\; s_3 \to -s_3 \;:\; \rho_2 \to -\rho_2\,,\; \rho_3 \to -\rho_3 \;:\; U_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,.$$

Bit - quantum map

classical
probability
distribution

evolution of
classical probabilities



bit quantum map

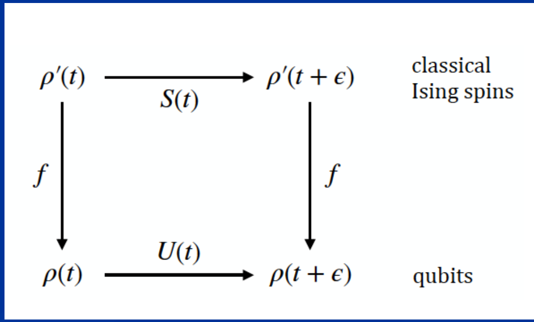
quantum
subsystem

unitary
quantum
evolution

Complete bit – quantum map

- every positive density matrix of the quantum subsystem can be realized by suitable classical probability distribution

- then all quantum operations can be performed by suitable changes of classical probability distribution



Single qubit quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

can be realized by suitable change of classical probability distribution

rotation : not deterministic

T - gate

other names : $\pi/4$-gate (often called $\pi/8$-rotation)

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

$$\rho' = U\rho U^\dagger$$

T - gate is realized by the transformation

$$\rho_1' = \frac{1}{\sqrt{2}}(\rho_1 - \rho_2) \qquad \rho_2' = \frac{1}{\sqrt{2}}(\rho_1 + \rho_2) \qquad \rho_3' = \rho_3$$

needs corresponding change of expectation values of classical Ising spins

Unique jump operation

- Every state τ is mapped to precisely one state ϱ
- This induces a map of probability distributions

$$p_\rho(t+\epsilon) = W_{\rho\tau}(t)\, p_\tau(t)$$

For invertible unique jump operation:

- The matrix W has in each row and column precisely one element one, and zeros otherwise
- Transition probabilities are either one or zero

Deterministic computation

- The operations of deterministic computation are unique jump operations
- cellular automata
- permutations among the states τ
- finite discrete group
- larger than permutations of the classical spins combined with sign changes
- include conditional changes

Conditional change

example :

- If Ising spin 1 has the same sign as Ising spin 2, flip the sign of Ising spin 3
- If the signs of Ising spin 1 and 2 are opposite, leave Ising spin 3 invariant

Quantum operations by cellular automata

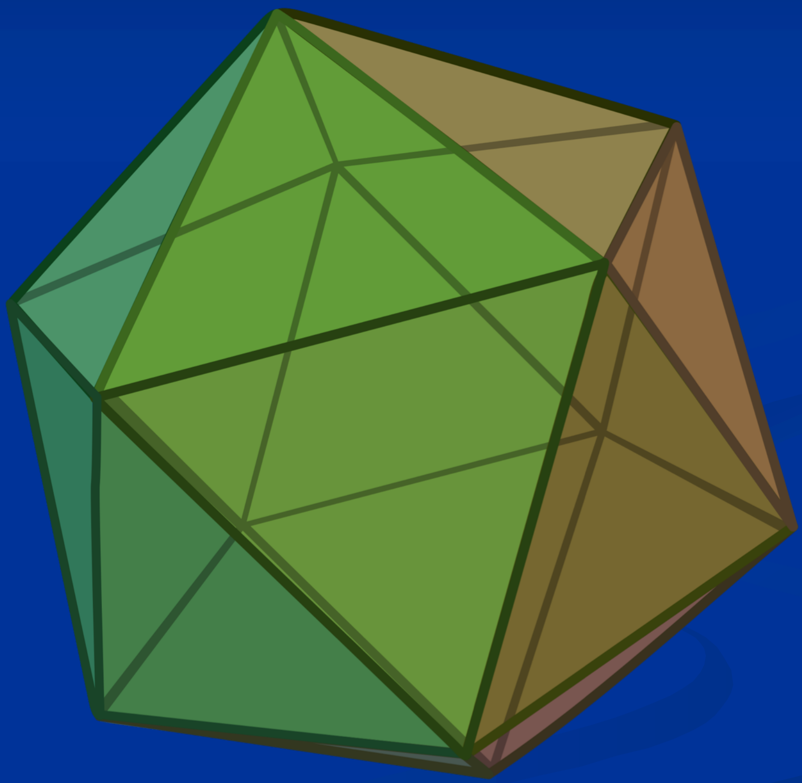
- In order to realize arbitrary unitary transformations for a single qubit one needs infinitely many classical bits
- The continuous group of unitary transformations can be approximated arbitrarily closely by permutations of infinitely many states, while a finite number of states is not sufficient

More classical bits…

could realize more quantum operations by cellular automata?

Maximal discrete subgroup of SO(3)

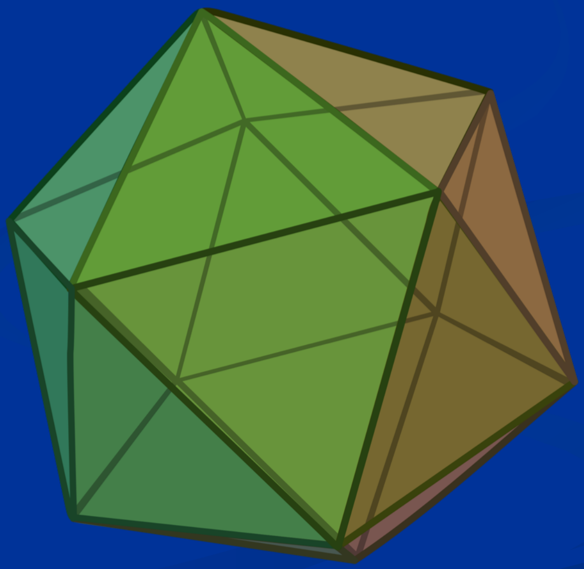
Icosahedron



Maximal discrete subgroup of SO(3)

- Can be realized with six classical Ising spins
- Classical spins “situated” at coins of iscosahedron
- Quantum subsystem defined by

$$\langle s_{1\pm} \rangle = a\rho_1 \pm b\rho_3$$
$$\langle s_{2\pm} \rangle = a\rho_2 \pm b\rho_1$$
$$\langle s_{3\pm} \rangle = a\rho_3 \pm b\rho_2$$



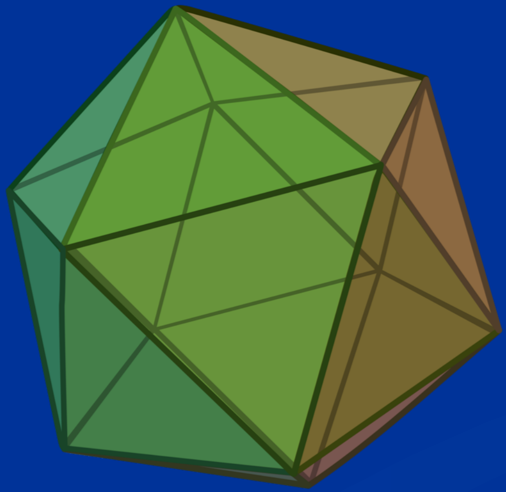
$$a = \left(\frac{1+\sqrt{5}}{2\sqrt{5}}\right)^{1/2} \qquad b = \left(\frac{2}{5+\sqrt{5}}\right)^{1/2} \qquad b = \frac{2a}{1+\sqrt{5}} \qquad a^2 + b^2 = 1$$

Quantum spins in six directions

$$(a, 0, \pm b),\ (\pm b, a, 0) \text{ and } (0, \pm b, a)$$

$$S_{k\pm} = a\tau_k \pm b\tilde{\tau}_k$$

$$\tilde{\tau}_3 = \tau_2,\ \tilde{\tau}_2 = \tau_1,\ \tilde{\tau}_1 = \tau_3$$



Quantum constraints

- Six expectation values are given by three numbers

$$\langle s_{1\pm}\rangle = a\rho_1 \pm b\rho_3$$
$$\langle s_{2\pm}\rangle = a\rho_2 \pm b\rho_1$$
$$\langle s_{3\pm}\rangle = a\rho_3 \pm b\rho_2$$

- resulting in constraints of the type

$$\rho_1 = \frac{1}{2a}\left(\langle s_{1+}\rangle + \langle s_{1-}\rangle\right) = \frac{1}{2b}\left(\langle s_{2+}\rangle - \langle s_{2-}\rangle\right)$$

$$\langle s_{2+}\rangle - \langle s_{2-}\rangle = \frac{2}{1+\sqrt{5}}\left(\langle s_{1+}\rangle + \langle s_{1-}\rangle\right)$$

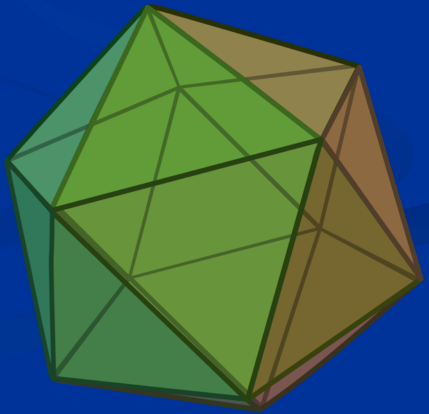
Quantum operations

- Reflections realized by

$$U_Y : s_{1\pm} \to -s_{1\pm} , \quad s_{2+} \leftrightarrow s_{2-} , \quad s_{3+} \leftrightarrow s_{3-}$$

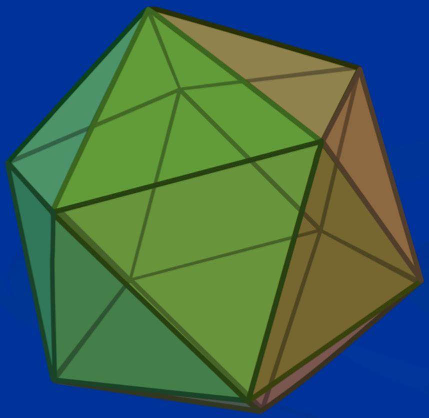
- Rotations by $\pi/4$ or $\pi/2$ not realized,
- U_T and U_H not realized

- Replaced by $2\pi/5$-rotations



Quantum operations

- Denser set of quantum operations can be performed by unique jump operations or deterministic computing



- Generalization to more qubits may allow for new algorithms and computing structures

Continuous classical variables

- Take x $\in \mathbb{R}^3$, e : unit vector
- Define infinitely many classical Ising spins

$$s(\boldsymbol{e};x) = \begin{cases} 1 & \text{for } \boldsymbol{x}\cdot\boldsymbol{e} > 0 \\ -1 & \text{for } \boldsymbol{x}\cdot\boldsymbol{e} < 0 \end{cases}$$

$$s(\boldsymbol{e};x) = \theta(\boldsymbol{x}\cdot\boldsymbol{e}) - \theta(-\boldsymbol{x}\cdot\boldsymbol{e})$$

- Consider family of probability distributions,

$p(\boldsymbol{\rho}, x)$ characterized by a vector $\boldsymbol{\rho}$

- given by $p(\boldsymbol{\rho}, x) = \tilde{p}(r)\,(\boldsymbol{\rho}\cdot\boldsymbol{f})\,\theta(\boldsymbol{\rho}\cdot\boldsymbol{f})$ $\boldsymbol{x} = r\boldsymbol{f}\,, \quad |\boldsymbol{f}| = 1$, f : unit vector

- Expectation values obey $\langle s(\boldsymbol{e})\rangle = \int \mathrm{d}^3x\, p(\boldsymbol{\rho}, x) s(\boldsymbol{e}, x) = \boldsymbol{\rho}\cdot\boldsymbol{e}$

One qubit quantum system

Quantum subsystem (density matrix) characterized by ϱ

In particular, the three spins s_1, s_2, s_3 corresponding to $\boldsymbol{e_1} = (1,0,0)$, $\boldsymbol{e_2} = (0,1,0)$, $\boldsymbol{e_3} = (0,0,1)$ obey $\langle s_k \rangle = \rho_k$. They define the density matrix for the quantum subsystem by eq. (6). We require the quantum condition $\sum_k \rho_k^2 \leq 1$.

Expectation values of quantum spins in arbitrary directions equal the classical expectation values of corresponding Ising spins.

Arbitrary unitary transformations can be achieved by deterministic rotations of x.

Two qubits

$$\rho = \frac{1}{4}(1 + \rho_z L_z) = \frac{1}{4}(1 + \rho_{\mu\nu} L_{\mu\nu})$$

$$\rho_z = \rho_{\mu\nu} = \langle \sigma_{\mu\nu} \rangle$$

SU(4)- generators : $L_z = L_{\mu\nu} = \tau_\mu \otimes \tau_\nu$

use fifteen classical bits

or use six classical bits and correlations

$$\rho_{k0} = \langle s_k^{(1)} \rangle \,, \quad \rho_{0k} = \langle s_k^{(2)} \rangle \,, \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$$

different bit - quantum maps

Correlation map

- Completeness of correlation map not yet established

$$\rho_{k0} = \langle s_k^{(1)} \rangle \,, \quad \rho_{0k} = \langle s_k^{(2)} \rangle \,, \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$$

Quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Single qubit:
Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Two qubits:
CNOT gate

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}$$

CNOT gate for two qubits

$$\rho = \frac{1}{4}(1 + \rho_z L_z) = \frac{1}{4}(1 + \rho_{\mu\nu} L_{\mu\nu})$$ $$\rho_z = \rho_{\mu\nu} = \langle \sigma_{\mu\nu} \rangle$$

SU(4)- generators : $L_z = L_{\mu\nu} = \tau_\mu \otimes \tau_\nu$

CNOT – gate:

$$\rho_{10} \leftrightarrow \rho_{11}\,, \quad \rho_{20} \leftrightarrow \rho_{21}\,, \quad \rho_{13} \leftrightarrow -\rho_{22}\,,$$
$$\rho_{02} \leftrightarrow \rho_{32}\,, \quad \rho_{03} \leftrightarrow \rho_{33}\,, \quad \rho_{23} \leftrightarrow \rho_{12}\,,$$
$$\rho_{30},\ \rho_{01},\ \rho_{31} \ \textit{invariant}.$$

to be achieved by
suitable change of probability distribution
for fifteen classical bits : deterministic

Entanglement

- CNOT - gate transforms product state for two qubits into entangled state

$$\psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\downarrow\rangle\right)$$
$$= \frac{1}{\sqrt{2}}(0,1,0,-1) \ ,$$

$$\psi_f = U_C \psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}(0,1,-1,0).$$

- All unitary operations for two qubits can be constructed from Hadamard, rotation and CNOT gates.

Classical probability distribution for maximally entangled state

the eight probabilities p_τ for which $s_k^{(1)}$ and $s_k^{(2)}$ are opposite for all k take all the value 1/8. The other 56 probabilities, for which at least for one k the spins $s_k^{(1)}$ and $s_k^{(2)}$ take the same value, vanish.

Deterministic computing for two qubits

- Determine non-abelian discrete subgroups D of SU(4)
- Look for subgroups $D_1 x D_2$, where D_1 and D_2 each act on single qubit
- CNOT- gate may or may not be realized by D

Scaling for many qubits

- Independent classical spins for each entry of density matrix: $2^{2Q}-1$ classical bits needed
- Use correlations: $\rho_{k0} = \langle s_k^{(1)} \rangle , \quad \rho_{0k} = \langle s_k^{(2)} \rangle , \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$

Only 3Q classical bits needed !

Q qubits

- SU(N) – generators

$$L_{\mu_1\mu_2\ldots\mu_Q} = \tau_{\mu_1} \otimes \tau_{\mu_2} \otimes \ldots \otimes \tau_{\mu_Q}$$

- density matrix

$$\rho = 2^{-Q}\left(1 + \rho_{\mu_1\ldots\mu_Q} L_{\mu_1\ldots\mu_Q}\right)$$

$$\rho_{\mu_1\ldots\mu_Q} = \langle \sigma_{\mu_1\ldots\mu_Q} \rangle$$

- correlation map

$$\rho = \frac{1}{2^Q}\left(1 + \langle s^{(1)}_{\mu_1} s^{(2)}_{\mu_2} \ldots s^{(Q)}_{\mu_Q} \rangle L_{\mu_1\mu_2\ldots\mu_Q}\right)$$

$$s_0^{(m)} = 1$$

Probabilistic computing with static memory materials ?

- Let general equilibrium classical statistics transport information from one layer to the next

- Simulation, with D. Sexty

Static memory materials

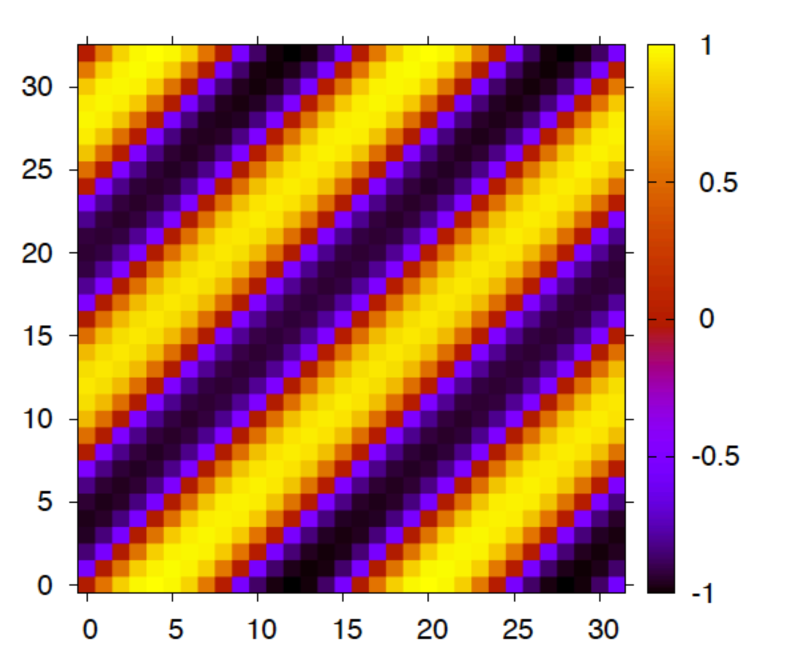
Generalized Ising model: $w[s] = Z^{-1} \exp\left(-S[s]\right) b(s_{in}, s_f)$

$$S = -\frac{\beta}{2} \sum_{x,t} s(t,x) \left[s(t+1, x+1) + \sigma s(t+1, x-1) \right]$$

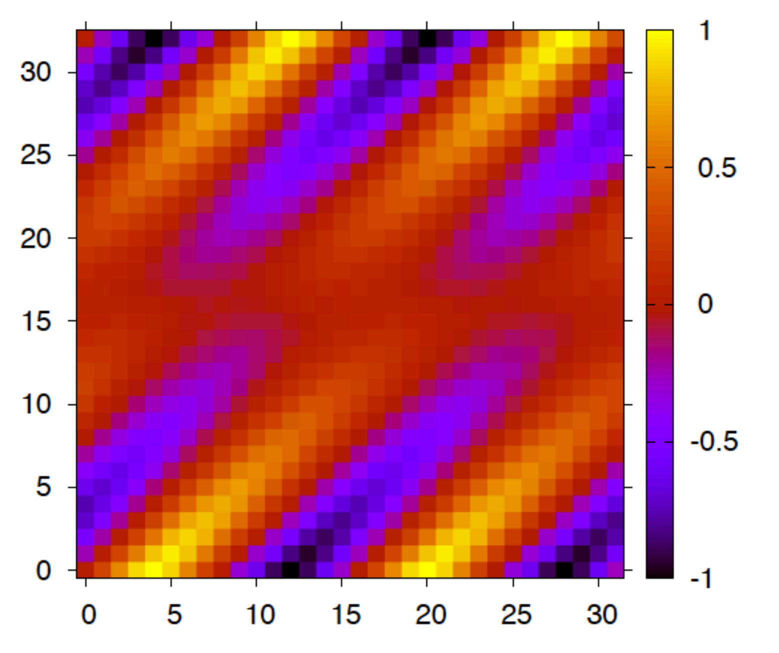
Boundary term : $b(s_{in}, s_f) = \bar{f}_f(s_f) f_{in}(s_{in})$

Classical interference

Depending on boundary conditions :



Positive interference



Negative interference

Probabilistic formalism for cellular automata

Generalized Ising model:

$$w[s] = Z^{-1} \exp\left(-S[s]\right) b(s_{in}, s_f)$$

$$S = -\frac{\beta}{2} \sum_{x,t} s(t,x) \left[s(t+1,x+1) + \sigma s(t+1,x-1) \right]$$

limit : beta to infinity , sigma to zero :
only one possibility for change , unique jump

probabilistic aspects only in boundary term :

$$b(s_{in}, s_f) = \bar{f}_f(s_f) f_{in}(s_{in})$$

Static memory material for two dimensional Ising spins on Euclidean square lattice can describe propagation of Weyl fermion in two- dimensional Minkowski space

Quantum formalism for classical statistics

- Formalism for information transport from one hypersurface to the next:
- Classical wave functions and density matrix
- Transfer matrix formalism : Heisenberg picture
- Wave functions : Schroedinger picture
- Non commuting operators for observables
- Quantum rules from classical statistical rules

Arbitrary quantum operations

- Arbitrary quantum gates for an arbitrary number of qubits can be realized by suitable changes of probability distributions
- Infinite number of classical bits or continuous Ising spins needed
- Similar to description of rotations by classical bits

Quantum Field Theories

- Continuous classical observables or fields always involve an infinite number of bits
- Bits: yes/no decisions
- Possible measurement values 1 or 0

  or 1 or -1

Discrete spectrum of observables

Artificial neural networks

- Can neural networks learn to perform quantum operations ?

Emulating quantum computation with artificial neural networks

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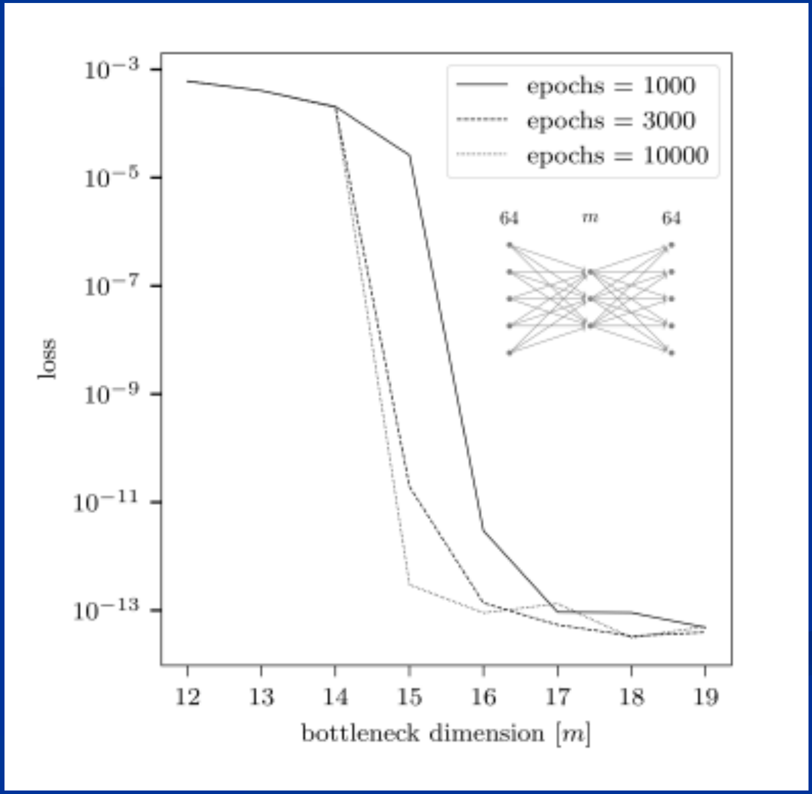
We demonstrate, that artificial neural networks (ANN) can be trained to emulate single or multiple basic quantum operations. In order to realize a quantum state, we implement a novel "quantumness gate" that maps an arbitrary matrix to the real representation of a positive hermitean normalized density matrix. We train the CNOT gate, the Hadamard gate and a rotation in Hilbert space as basic building blocks for processing the quantum density matrices of two entangled qubits. During the training process the neural networks learn to represent the complex structure, the hermiticity, the normalization and the positivity of the output matrix. The requirement of successful training

Learning unitary operations

step 1: quantumness gate
learns to map input information
on two – qubit density matrix

step 2 : learns basic
quantum gates for
unitary operations individually



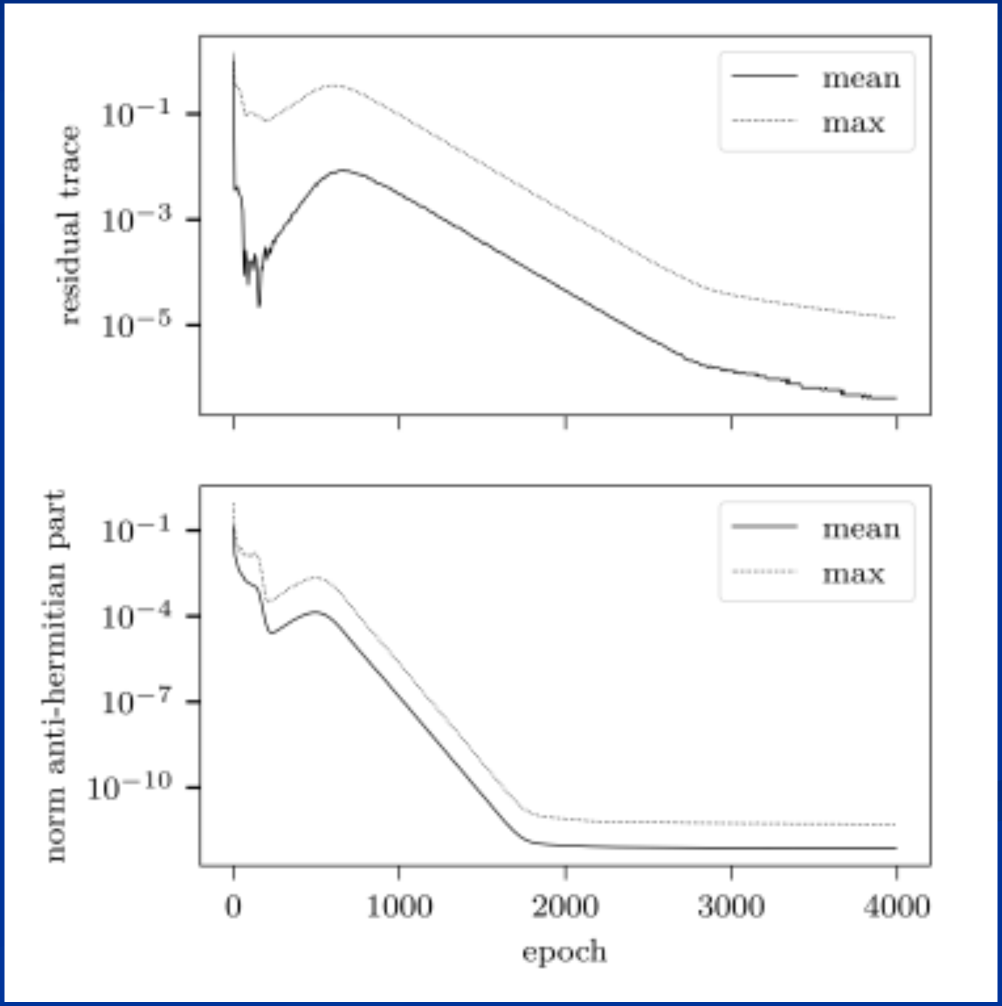
Learning unitary operations

learns :

normalization

hermiticity

of output matrix



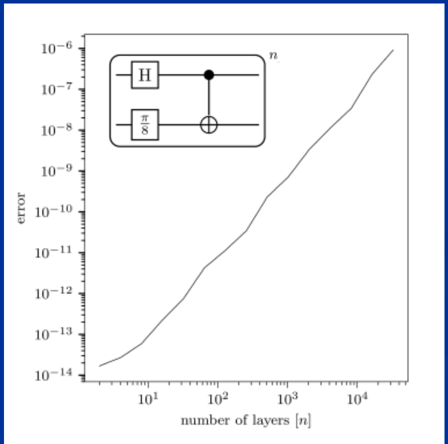
Chain of unitary transformations

After the learning phase the network can perform arbitrary chains of unitary transformations

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}, \; U_{HR} = U_{H1} U_{R2},$$

$$U_{H1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, U_{R2} = \begin{pmatrix} U_T & 0 \\ 0 & U_T \end{pmatrix}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$$\rho(t + n\epsilon) = \tilde{U}(t + n\epsilon, t)\rho(t)\tilde{U}^\dagger(t + n\epsilon, t)$$

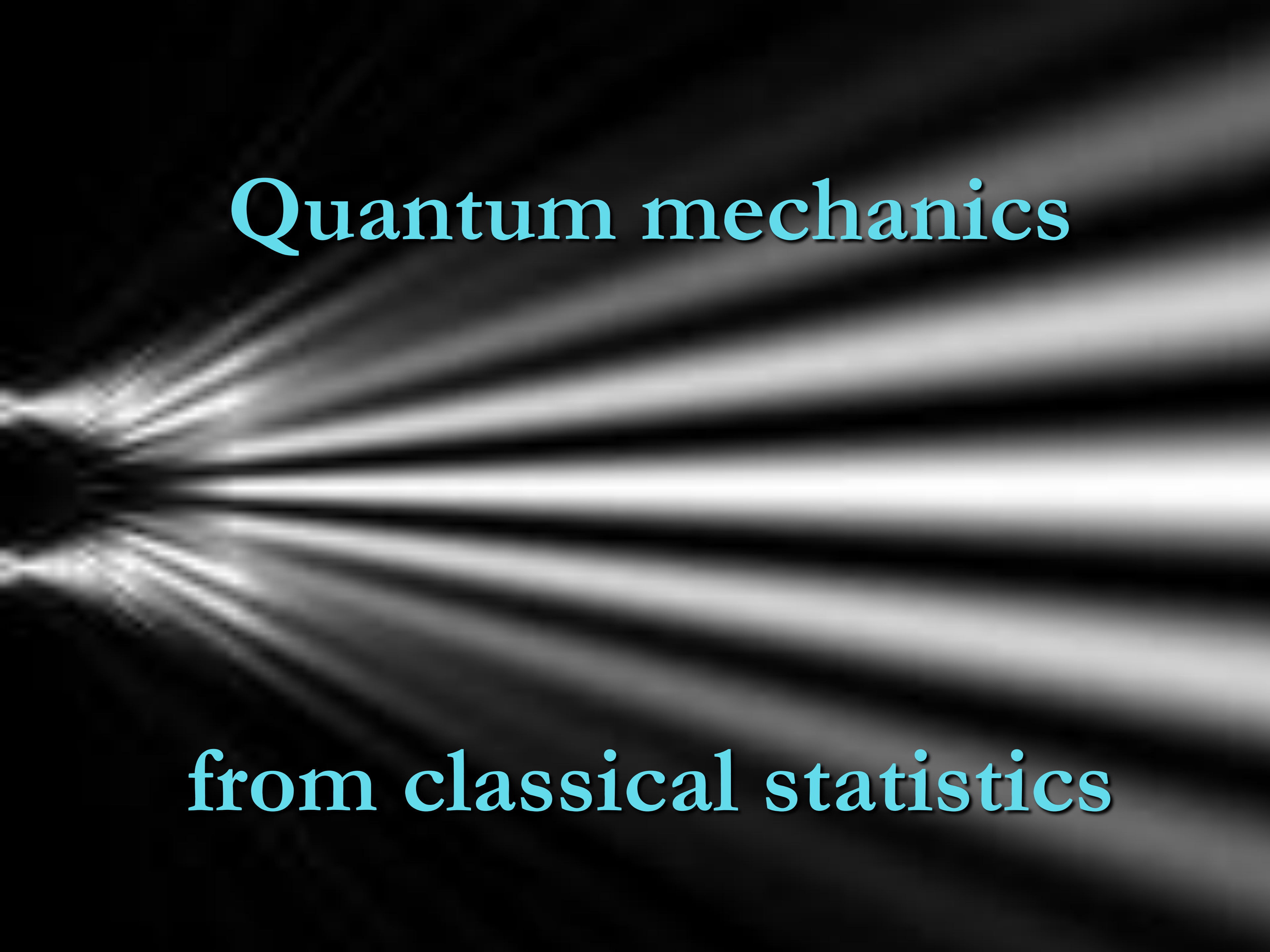
$$\tilde{U}(t + n\epsilon, t) = \bar{U}(t + (n-1)\epsilon) \ldots \bar{U}(t + \epsilon)\bar{U}(t) = \bar{U}(t)^n$$

Conclusion

- Quantum operations can be performed by classical statistical systems
- Very low temperature or

  well isolated systems of microscopic qubits

  not needed !


Quantum mechanics
from classical statistics

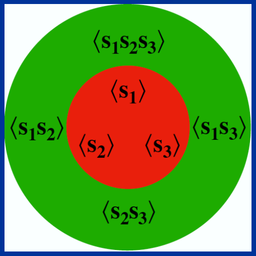
Can quantum physics be described by classical probabilities ?

" No go " theorems

Bell , Clauser , Horne , Shimony , Holt

implicit assumption : use of classical correlation function for correlation between measurements

not part of quantum subsystem



Kochen , Specker

assumption : unique map from quantum operators to classical observables *quantum mechanics can be described by classical statistics !*

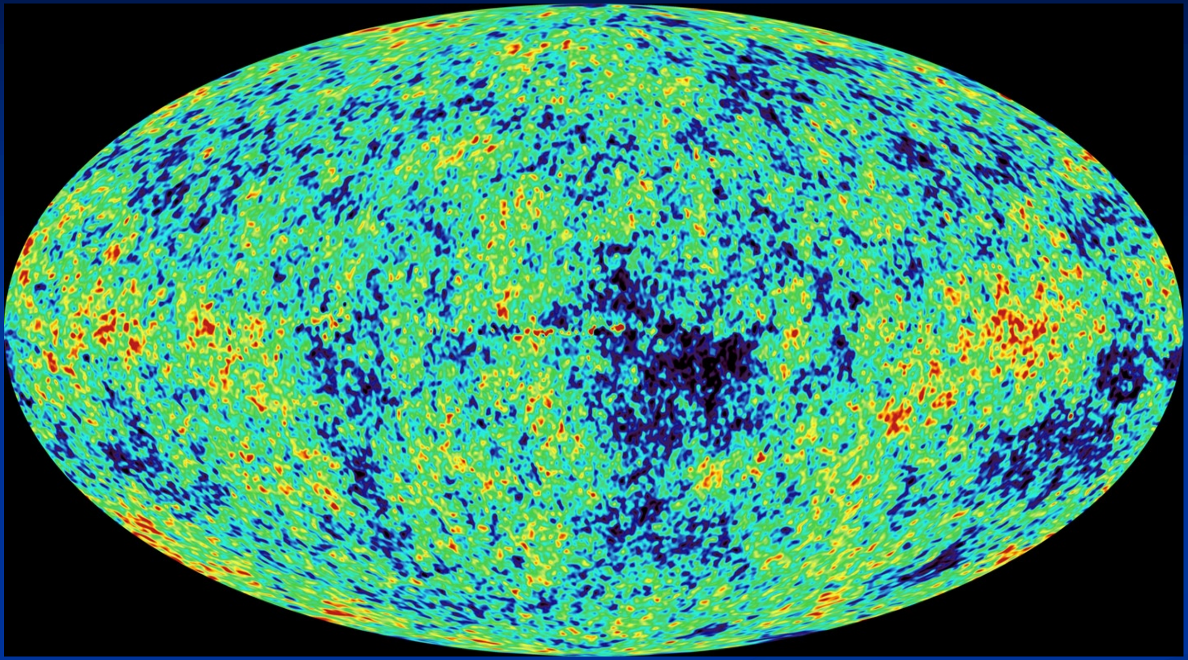
Reduction of wave function

- Reduction of wave function is a convenient technical method to describe conditional probabilities
- This must not be a physical process during the measurement

conditional probability

sequences of events(measurements)
are described by
conditional probabilities

both in classical statistics
and in quantum statistics

$w(t_1)$  :

not very suitable
for statement , if here and now
a pointer falls down

Schrödinger's cat

conditional probability :
if nucleus decays
then cat dead with $w_c = 1$
(reduction of wave function)

structural elements of quantum mechanics

unitary time evolution

ν

h

Simple conversion factor for units

i

presence of complex structure

$$[A,B]=C$$

non – commuting operators are necessary to represent observables in incomplete statistics

correlation and operator product

- Classical statistical systems admit many product structures of observables
- Many different definitions of correlation functions possible , not only classical correlation !
- Type of measurement determines correct selection of correlation function !
- Ideal quantum measurement should be compatible with information in quantum subsystem

Deterministic evolution – probabilistic interpretation

- quantum mechanics arises from

quantum subsystems

- subsystems are genuinely probabilistic
- part of information is lost by focus on subsystem
- partially ”integrating out” degrees of freedom

Determinism vs. Probabilism

" Does god throw dices ? "

… an old dispute

Gott würfelt

Gott würfelt nicht

"Es scheint hart, dem Herrgott in die Karten zu gucken. Aber dass er würfelt und sich telepatischer Mittel bedient (wie es ihm von der gegenwärtigen Quantentheorie zugemutet wird), kann ich keinen Augenblick glauben.."

Einstein: Brief an Cornelius Lanczos am 21. März 1942

not todays topic

Gott würfelt		Gott würfelt nicht

humans can only deal with probabilities

determinism vs. probabilism

my personal view :

- determinism not needed
- one can start with probabilistic theory and probabilistic evolution
- nevertheless : deterministic evolution is a possible option

conclusion

- quantum statistics emerges from classical statistics

  quantum state, superposition, interference, entanglement, probability amplitude
- unitary time evolution of quantum mechanics can be described by suitable time evolution of classical probabilities
- memory materials are quantum simulators
- conditional correlations for measurements both in quantum and classical statistics

end