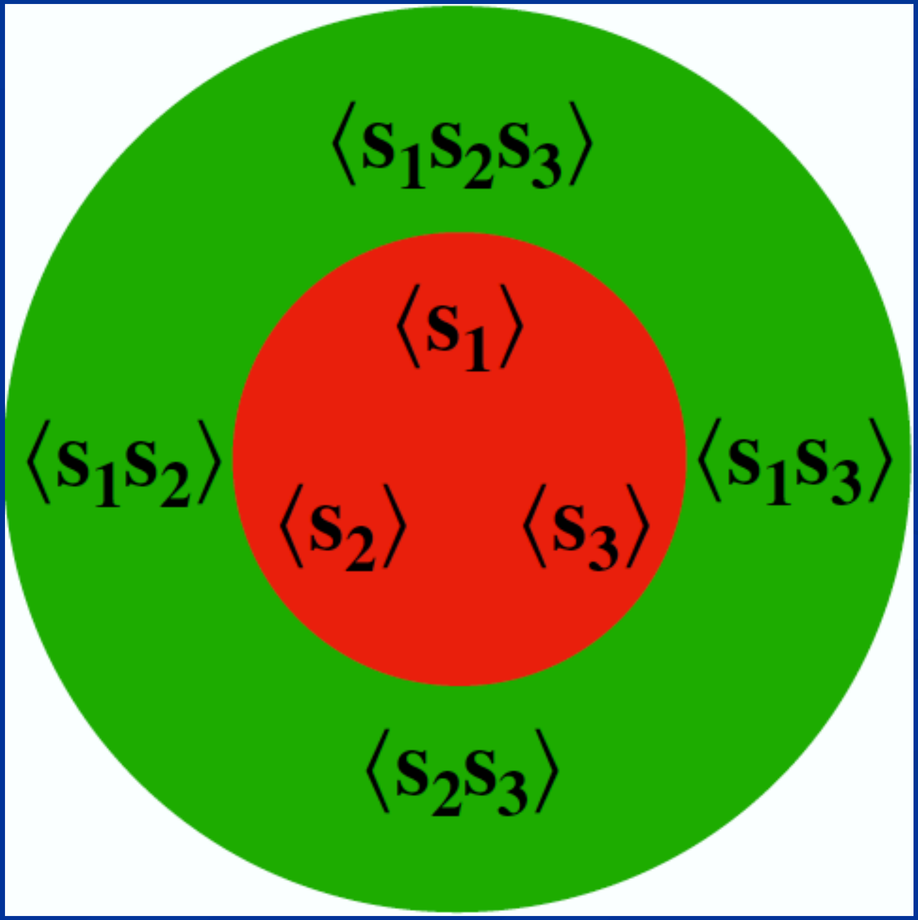
Quantum mechanics and quantum computing from classical bits



Quantum mechanics from cellular automata

Infinitely many bits : this is possible

Deterministic evolution – probabilistic interpretation

- quantum mechanics arises from

quantum subsystems

- subsystems are genuinely probabilistic
- part of information is lost by focus on subsystem
- partially ”integrating out” degrees of freedom

Determinism vs. Probabilism

" Does god throw dices ? "

… an old dispute

Gott würfelt

Gott würfelt nicht

"Es scheint hart, dem Herrgott in die Karten zu gucken. Aber dass er würfelt und sich telepatischer Mittel bedient (wie es ihm von der gegenwärtigen Quantentheorie zugemutet wird), kann ich keinen Augenblick glauben.."

Einstein: Brief an Cornelius Lanczos am 21. März 1942

not todays topic

Gott würfelt — Gott würfelt nicht

humans can only deal with probabilities

determinism vs. probabilism

my personal view :

- determinism not needed
- one can start with probabilistic theory and probabilistic evolution
- nevertheless : deterministic evolution is a possible option

Embedding of quantum mechanics in classical statistics

- it works
- quantum mechanics does not need new fundamental concepts beyond classical statistics
- raises interesting new questions

Does our brain use quantum computing ?

Do artificial neural networks employ quantum algorithms ?

Can classical statistical memory materials perform quantum operations ?

Quantum subsystem

One qubit from three classical Ising spins

Ising spin : macroscopic two-level observable

Neuron fires above or below certain level

Particle present or not

Bit in a computer

Three Ising spins (classical bits)

$$s_k(t) = \pm 1,\ k = 1, 2, 3$$

Arbitrary macroscopic two-level observables

Eight states

$$\tau = 1, \ldots, 8$$

We may number the states by $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$, where 1 denotes spin up and 0 stands for spin down. The expectation value of s_3 is then given by $\langle s_3 \rangle = -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8$, or for $A = s_1 s_2$ one has $\langle s_1 s_2 \rangle = p_1 + p_2 - p_3 - p_4 - p_5 - p_6 + p_7 + p_8$. We denote the expectation values of the three spins by ρ_z, $z = 1, \ldots, 3$,

Classical statistics

$$p_\tau \geq 0\,, \quad \sum_\tau p_\tau = 1$$

$$\langle A \rangle = \sum_\tau A_\tau p_\tau$$

One qubit from three classical bits

Expectation values : $\rho_z = \langle s_z \rangle \,, \quad -1 \leq \rho_z \leq 1$

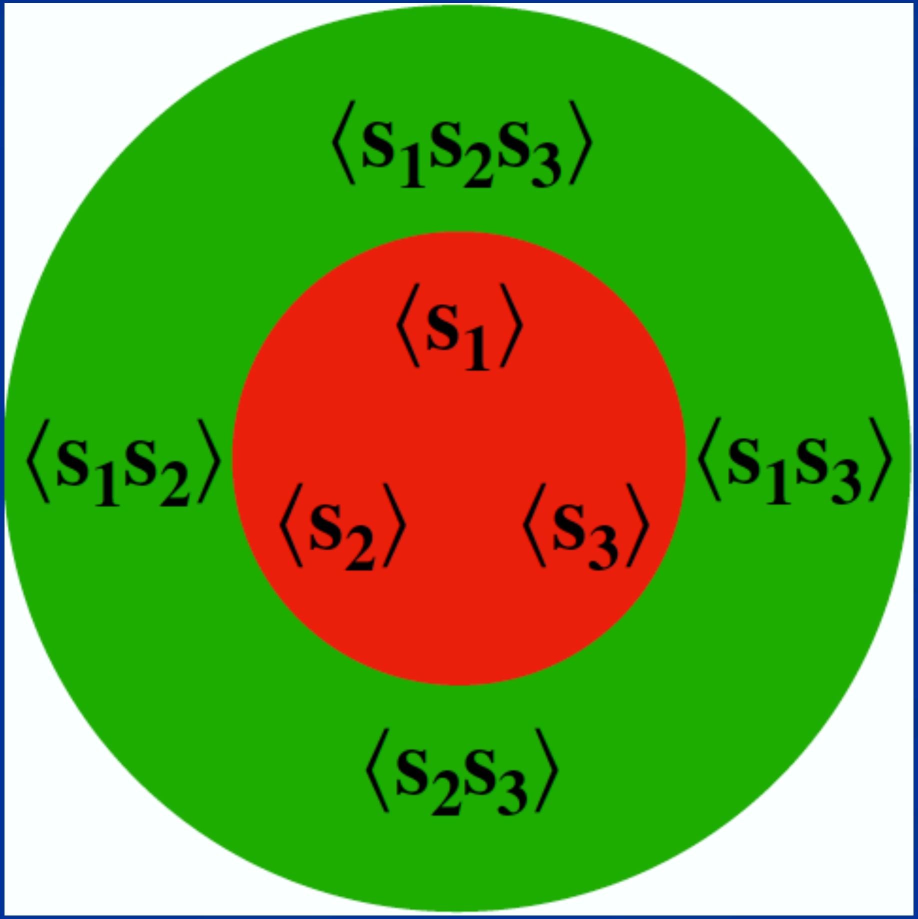
Quantum subsystem defined by density matrix

$$\rho = \frac{1}{2}\left(1 + \rho_z \tau_z\right) \,, \quad \rho^\dagger = \rho \,, \quad \mathrm{tr}(\rho) = 1$$

Only part of classical statistical information used for subsystem

$$\begin{aligned}
\rho_1 &= -p_1 - p_2 - p_3 - p_4 + p_5 + p_6 + p_7 + p_8 \\
\rho_2 &= -p_1 - p_2 + p_3 + p_4 - p_5 - p_6 + p_7 + p_8 \\
\rho_3 &= -p_1 + p_2 - p_3 + p_4 - p_5 + p_6 - p_7 + p_8 .
\end{aligned}$$

Subsystem in space of correlation functions



Thermalization of pure quantum state

- where does the information go ?
- into n – point functions with extremely high n ! (Avogadro's number)
- initial information is no longer visible in low order correlation functions, which approach thermal equilibrium values
- subsystem : low order correlation functions

Aarts, Bonini, Berges, Borsanyi…

Quantum subsystems

Quantum systems are subsystems of classical statistical systems

They use only part of the available probabilistic information

Incomplete statistics

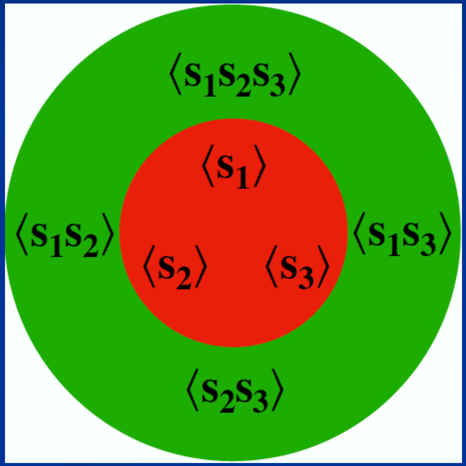
Incomplete statistics

Classical correlation function

between Ising spin 1 and Ising spin 2

cannot be computed

from information in subsystem !



It involves information about environment of subsystem : probabilistic information beyond

$$\rho_z = \langle s_z \rangle$$

incomplete statistics
is origin of
representation of observables by
non-commuting operators

Non-commuting operators

$$L_z = \tau_z$$

Quantum rule for expectation values follows from classical statistical rules :

$$\langle s_z \rangle = \mathrm{tr}\left(L_z \rho\right) = \rho_z$$

$$\rho = \frac{1}{2}\left(1 + \rho_z \tau_z\right) , \quad \rho^\dagger = \rho , \quad \mathrm{tr}(\rho) = 1$$

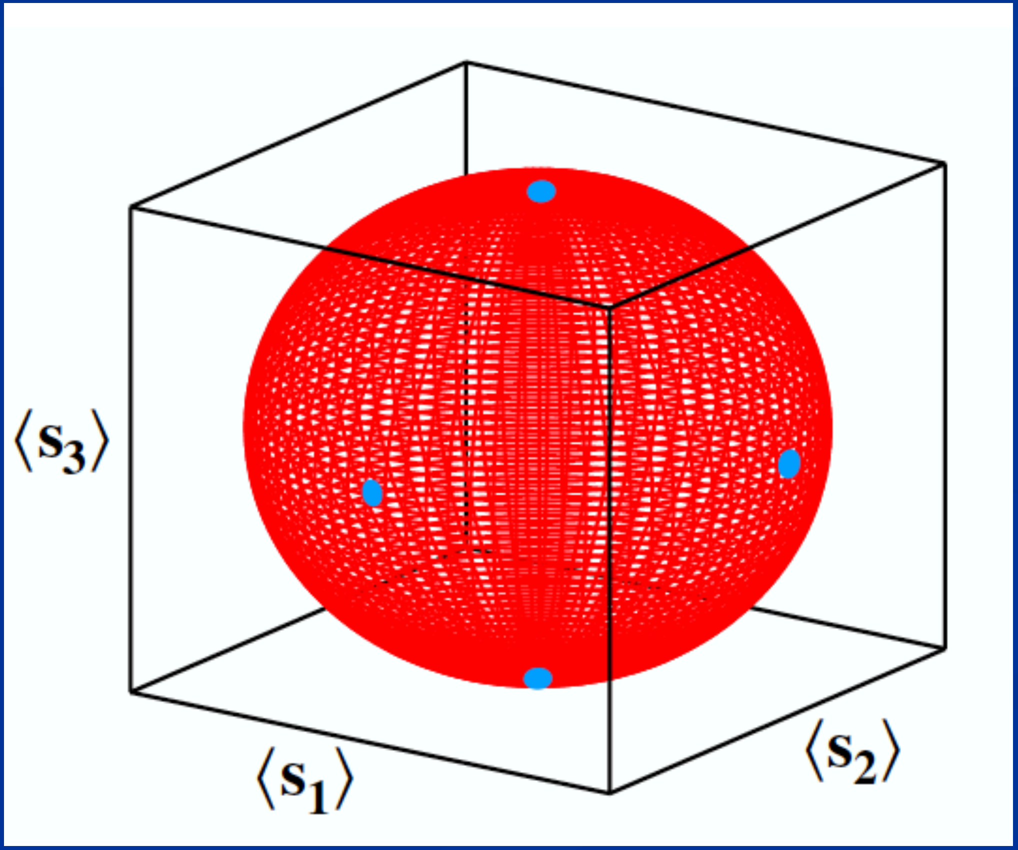
Quantum condition

Positivity of density matrix requires quantum condition :

$$\rho_z \rho_z \leq 1$$

This implies uncertainty relation !

Not every classical probability distribution admits a quantum subsystem



$$\rho_z \rho_z \leq 1$$

Pure quantum states

saturation of quantum condition :

- pure state
- density matrix can be written as bilinear in complex wave function

Quantum computing

- quantum computing proceeds by quantum gates
- discrete unitary transformations of density matrix in consecutive time steps

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

- a few basic gates are sufficient

Quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Single qubit:
Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Two qubits:
CNOT gate

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}$$

Realization of quantum gates

- by deterministic manipulations of classical bits or Ising spins

cellular automata

- by changes of classical probability distribution for classical bits

probabilistic computing

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\rho_1 \to \rho_3\,, \quad \rho_2 \to -\rho_2\,, \quad \rho_3 \to \rho_1$$

$$p_1 \leftrightarrow p_3\,, \quad p_2 \leftrightarrow p_7\,, \quad p_4 \leftrightarrow p_5\,, \quad p_6 \leftrightarrow p_8$$

Can be realized by deterministic spin flip

$$s_1 \leftrightarrow s_3,\ s_2 \to -s_2$$

A few one qubit gates

$$s_1 \to s_2\,,\ s_2 \to -s_1\ :\ \rho_1 \to \rho_2\,,\ \rho_2 \to -\rho_1\ :\ U_{12} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\,,$$

$$s_3 \to s_1\,,\ s_1 \to -s_3\ :\ \rho_3 \to \rho_1\,,\ \rho_1 \to -\rho_3\ :\ U_{31} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\,,$$

$$s_1 \to -s_1\,,\ s_2 \to -s_2\ :\ \rho_1 \to -\rho_1\,,\ \rho_2 \to -\rho_2\ :\ U_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,$$

$$s_1 \to -s_1\,,\ s_3 \to -s_3\ :\ \rho_1 \to -\rho_1\,,\ \rho_3 \to -\rho_3\ :\ U_Y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,,$$

$$s_2 \to -s_2\,,\ s_3 \to -s_3\ :\ \rho_2 \to -\rho_2\,,\ \rho_3 \to -\rho_3\ :\ U_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,.$$

Bit - quantum map

evolution of
classical probabilities

classical
probability
distribution

$$
\begin{tikzcd}
\rho'(t) \arrow[r, "S(t)"'] \arrow[d, "f"'] & \rho'(t+\epsilon) \arrow[d, "f"] \\
\rho(t) \arrow[r, "U(t)"] & \rho(t+\epsilon)
\end{tikzcd}
$$

classical
Ising spins

bit quantum map

qubits

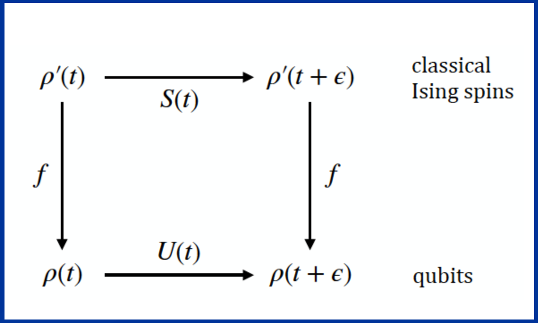
quantum
subsystem

unitary
quantum
evolution

Complete bit – quantum map

- every positive density matrix of the quantum subsystem can be realized by suitable classical probability distribution

- then all quantum operations can be performed by suitable changes of classical probability distribution



Single qubit quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

can be realized by suitable change of classical probability distribution

rotation : not deterministic

Two qubits

$$\rho = \frac{1}{4}(1 + \rho_z L_z) = \frac{1}{4}(1 + \rho_{\mu\nu} L_{\mu\nu})$$

$$\rho_z = \rho_{\mu\nu} = \langle \sigma_{\mu\nu} \rangle$$

SU(4)- generators : $L_z = L_{\mu\nu} = \tau_\mu \otimes \tau_\nu$

use fifteen classical bits

or use eight classical bits and correlations

$$\rho_{k0} = \langle s_k^{(1)} \rangle \,, \quad \rho_{0k} = \langle s_k^{(2)} \rangle \,, \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$$

different bit - quantum maps

Quantum gates

Unitary transformation of density matrix

$$\rho(t+\epsilon) = U(t)\rho(t)U^\dagger(t)$$

Single qubit:
Rotation

$$U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Two qubits:
CNOT gate

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}$$

CNOT gate for two qubits

$$\rho = \frac{1}{4}(1 + \rho_z L_z) = \frac{1}{4}(1 + \rho_{\mu\nu} L_{\mu\nu})$$ $$\rho_z = \rho_{\mu\nu} = \langle \sigma_{\mu\nu} \rangle$$

SU(4)- generators : $L_z = L_{\mu\nu} = \tau_\mu \otimes \tau_\nu$

CNOT – gate:

$$\rho_{10} \leftrightarrow \rho_{11}\,, \quad \rho_{20} \leftrightarrow \rho_{21}\,, \quad \rho_{13} \leftrightarrow -\rho_{22}\,,$$
$$\rho_{02} \leftrightarrow \rho_{32}\,, \quad \rho_{03} \leftrightarrow \rho_{33}\,, \quad \rho_{23} \leftrightarrow \rho_{12}\,,$$
$$\rho_{30},\ \rho_{01},\ \rho_{31}\ \ invariant.$$

to be achieved by
suitable change of probability distribution
for fifteen classical bits : deterministic

Entanglement

- CNOT - gate transforms product state for two qubits into entangled state

$$\psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\downarrow\rangle\right)$$
$$= \frac{1}{\sqrt{2}}(0,1,0,-1) \, ,$$

$$\psi_f = U_C \psi_{in} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}(0,1,-1,0).$$

- All unitary operations for two qubits can be constructed from Hadamard, rotation and CNOT gates.

Scaling for many qubits

- Independent classical spins for each entry of density matrix: $2^{2Q}-1$ classical bits needed

- Use correlations: $\rho_{k0} = \langle s_k^{(1)} \rangle\,, \quad \rho_{0k} = \langle s_k^{(2)} \rangle\,, \quad \rho_{kl} = \langle s_k^{(1)} s_l^{(2)} \rangle$

Only 3Q classical bits needed !

Probabilistic computing with static memory materials ?

- Let general equilibrium classical statistics transport information from one layer to the next

- Simulation, with D. Sexty

Static memory materials

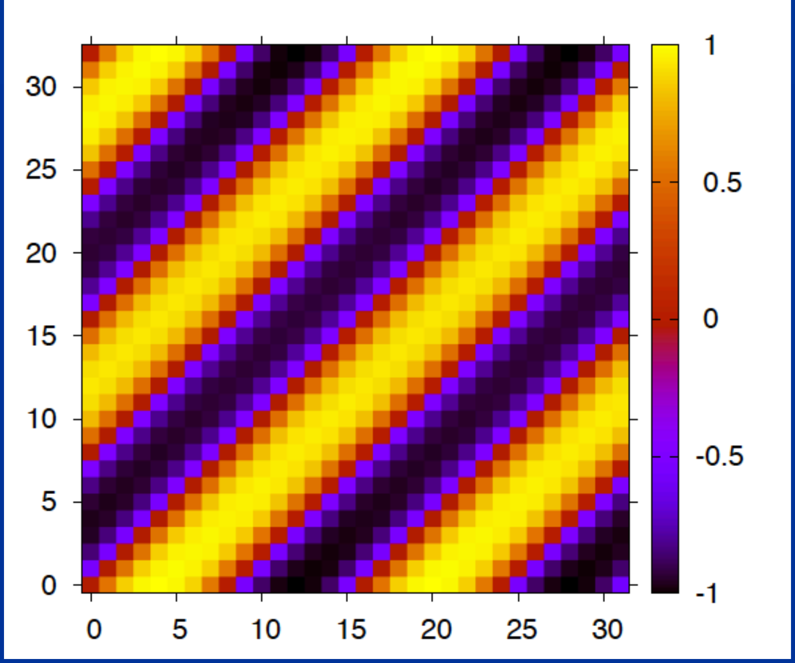
Generalized Ising model: $$w[s] = Z^{-1} \exp\big(-S[s]\big) b(s_{in}, s_f)$$

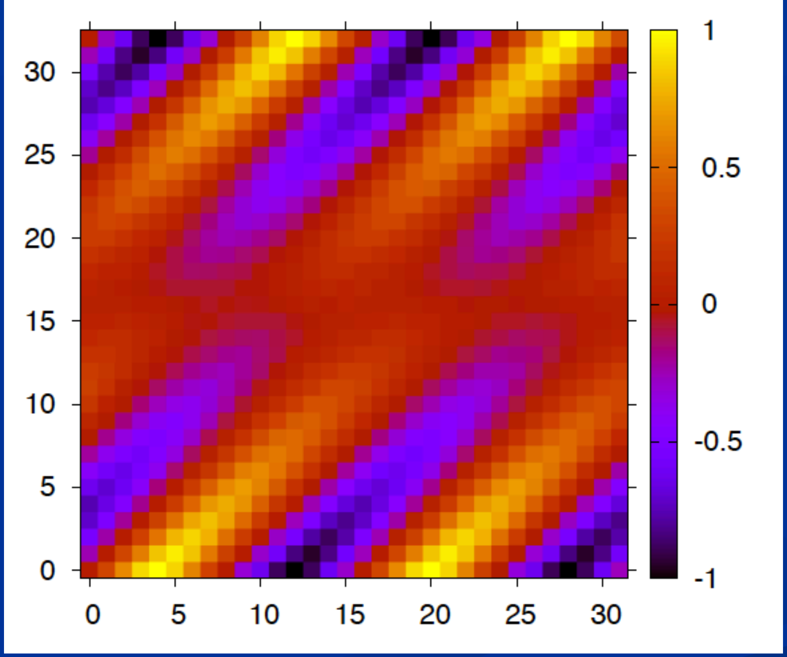
$$S = -\frac{\beta}{2} \sum_{x,t} s(t,x) \Big[s(t+1, x+1) + \sigma s(t+1, x-1) \Big]$$

Boundary term : $$b(s_{in}, s_f) = \bar{f}_f(s_f) f_{in}(s_{in})$$

Classical interference

Depending on boundary conditions :





Positive interference

Negative interference

Probabilistic formalism for cellular automata

Generalized Ising model:

$$w[s] = Z^{-1} \exp\big(- S[s]\big) b(s_{in}, s_f)$$

$$S = -\frac{\beta}{2} \sum_{x,t} s(t,x) \Big[s(t+1, x+1) + \sigma s(t+1, x-1) \Big]$$

limit beta to infinity , sigma to zero :
only one possibility for change , unique jump

probabilistic aspects only in boundary term :

$$b(s_{in}, s_f) = \bar{f}_f(s_f) f_{in}(s_{in})$$

Static memory material for
two dimensional Ising spins
on Euclidean square lattice
can describe propagation of Weyl fermion
in two- dimensional Minkowski space

Quantum formalism for classical statistics

- Formalism for information transport from one hypersurface to the next:
- Classical wave functions and density matrix
- Transfer matrix formalism : Heisenberg picture
- Wave functions : Schroedinger picture
- Non commuting operators for observables
- Quantum rules from classical statistical rules

Arbitrary quantum operations

- Arbitrary quantum gates for an arbitrary number of qubits can be realized by suitable changes of probability distributions
- Infinite number of classical bits or continuous Ising spins needed
- Similar to description of rotations by classical bits

Quantum Field Theories

- Continuous classical observables or fields always involve an infinite number of bits
- Bits: yes/no decisions
- Possible measurement values 1 or 0

  or 1 or -1

Discrete spectrum of observables

Artificial neural networks

- Can neural networks learn to perform quantum operations ?

Emulating quantum computation with artificial neural networks

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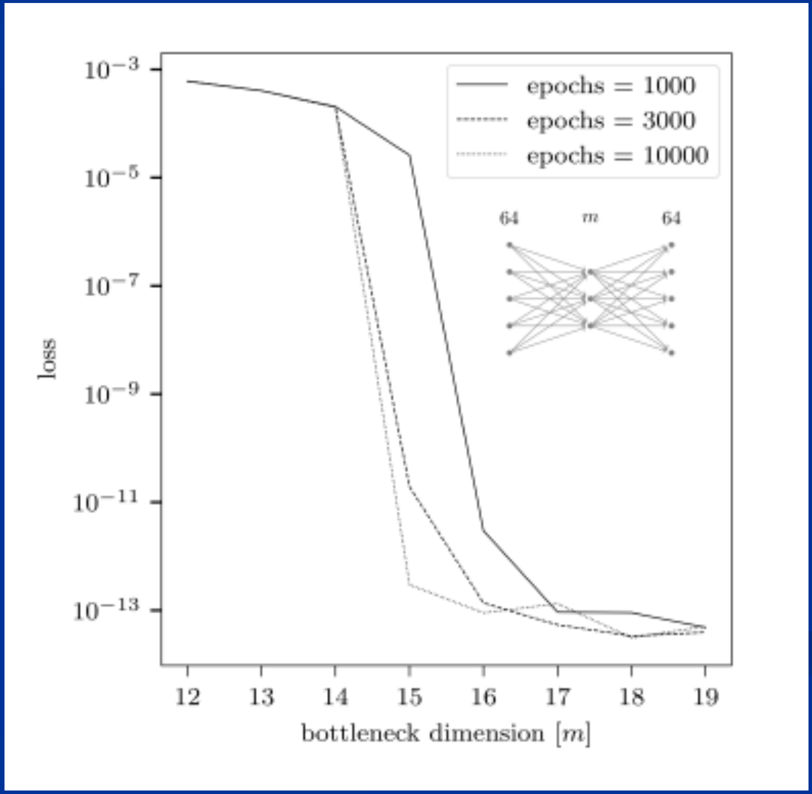
We demonstrate, that artificial neural networks (ANN) can be trained to emulate single or multiple basic quantum operations. In order to realize a quantum state, we implement a novel "quantumness gate" that maps an arbitrary matrix to the real representation of a positive hermitean normalized density matrix. We train the CNOT gate, the Hadamard gate and a rotation in Hilbert space as basic building blocks for processing the quantum density matrices of two entangled qubits. During the training process the neural networks learn to represent the complex structure, the hermiticity, the normalization and the positivity of the output matrix. The requirement of successful training

Learning unitary operations

step 1: quantumness gate
learns to map input information
on two – qubit density matrix

step 2 : learns basic
quantum gates for
unitary operations individually



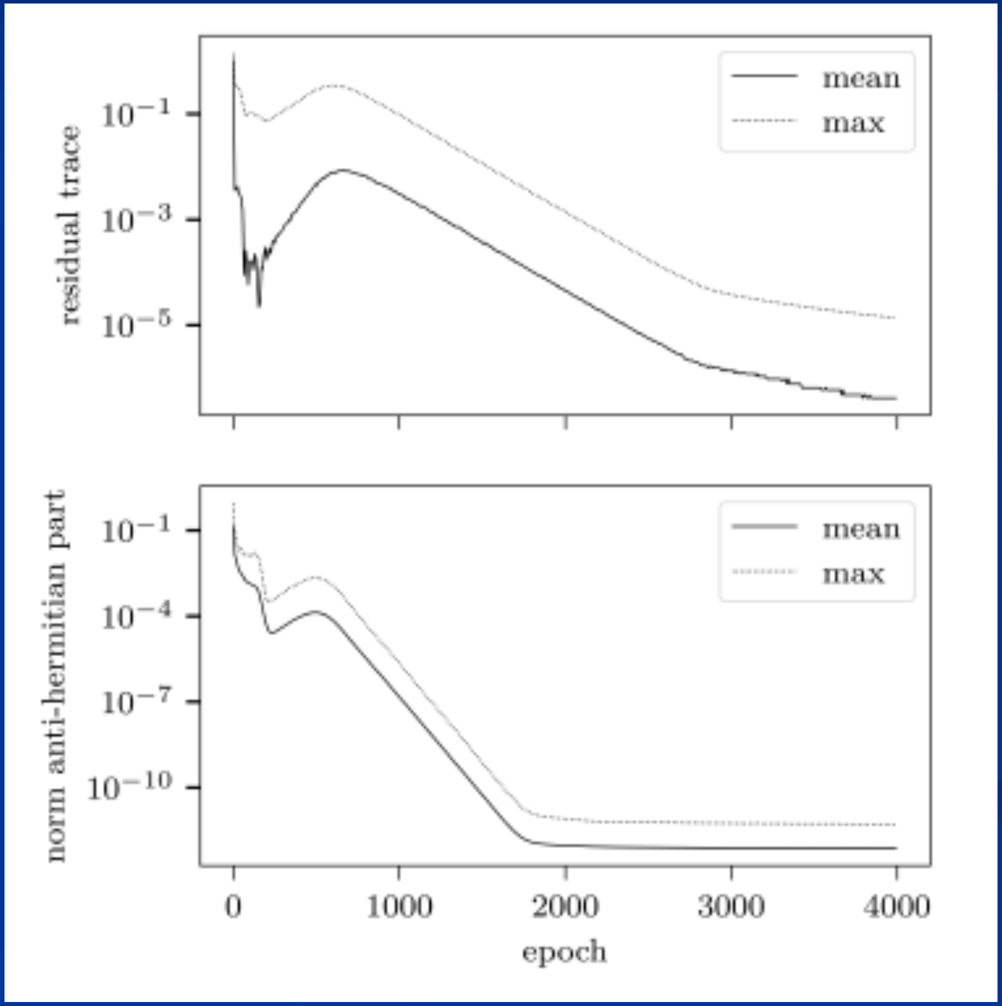
Learning unitary operations

learns :

normalization

hermiticity

of output matrix



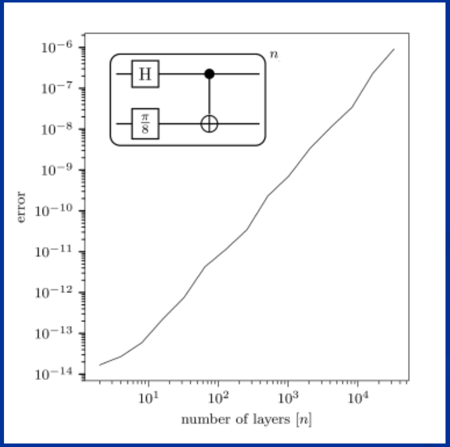
Chain of unitary transformations

After the learning phase the network can perform arbitrary chains of unitary transformations

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & \tau_1 \end{pmatrix}, \; U_{HR} = U_{H1} U_{R2},$$

$$U_{H1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, U_{R2} = \begin{pmatrix} U_T & 0 \\ 0 & U_T \end{pmatrix}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, U_T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$$\rho(t + n\epsilon) = \tilde{U}(t + n\epsilon, t)\rho(t)\tilde{U}^{\dagger}(t + n\epsilon, t)$$

$$\tilde{U}(t + n\epsilon, t) = \bar{U}(t + (n-1)\epsilon) \ldots \bar{U}(t + \epsilon)\bar{U}(t) = \bar{U}(t)^n$$

Conclusion

- Quantum operations can be performed by classical statistical systems
- Very low temperature or

  well isolated systems of microscopic qubits

  not needed !

Quantum mechanics

from classical statistics

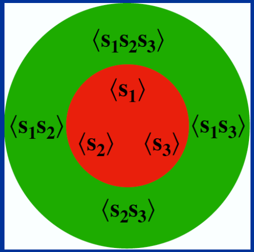
Can quantum physics be described by classical probabilities ?

“ No go “ theorems

Bell , Clauser , Horne , Shimony , Holt

implicit assumption : use of classical correlation function for correlation between measurements

not part of quantum subsystem



Kochen , Specker

assumption : unique map from quantum operators to classical observables *quantum mechanics can be described by classical statistics !*

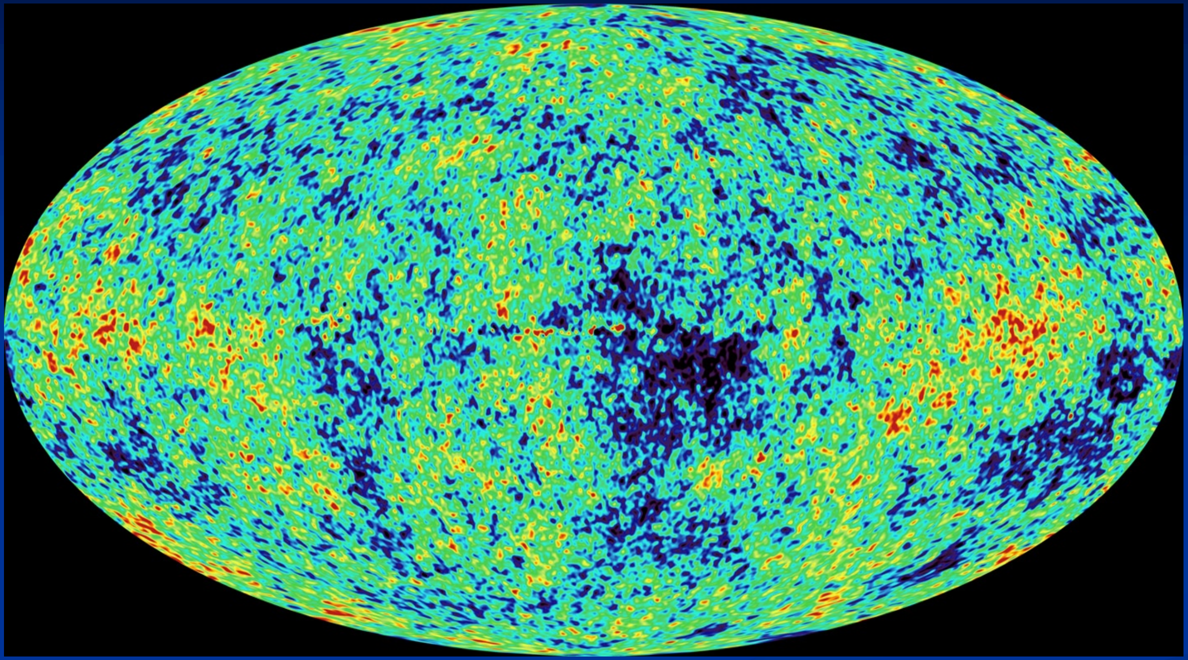
Reduction of wave function

- Reduction of wave function is a convenient technical method to describe conditional probabilities
- This must not be a physical process during the measurement

conditional probability

sequences of events(measurements)
are described by
conditional probabilities

both in classical statistics
and in quantum statistics

$\mathbf{w(t_1)}$  :

not very suitable
for statement , if here and now
a pointer falls down

Schrödinger’s cat

conditional probability :
if nucleus decays
then cat dead with $w_c = 1$
(reduction of wave function)

structural elements of quantum mechanics

unitary time evolution

$$\nu$$

h

Simple conversion factor for units

i

presence of complex structure

$$[A,B]=C$$

non – commuting operators are necessary to represent observables in incomplete statistics

correlation and operator product

- Classical statistical systems admit many product structures of observables
- Many different definitions of correlation functions possible , not only classical correlation !
- Type of measurement determines correct selection of correlation function !
- Ideal quantum measurement should be compatible with information in quantum subsystem

conclusion

- quantum statistics emerges from classical statistics

  quantum state, superposition, interference, entanglement, probability amplitude
- unitary time evolution of quantum mechanics can be described by suitable time evolution of classical probabilities
- memory materials are quantum simulators
- conditional correlations for measurements both in quantum and classical statistics

end