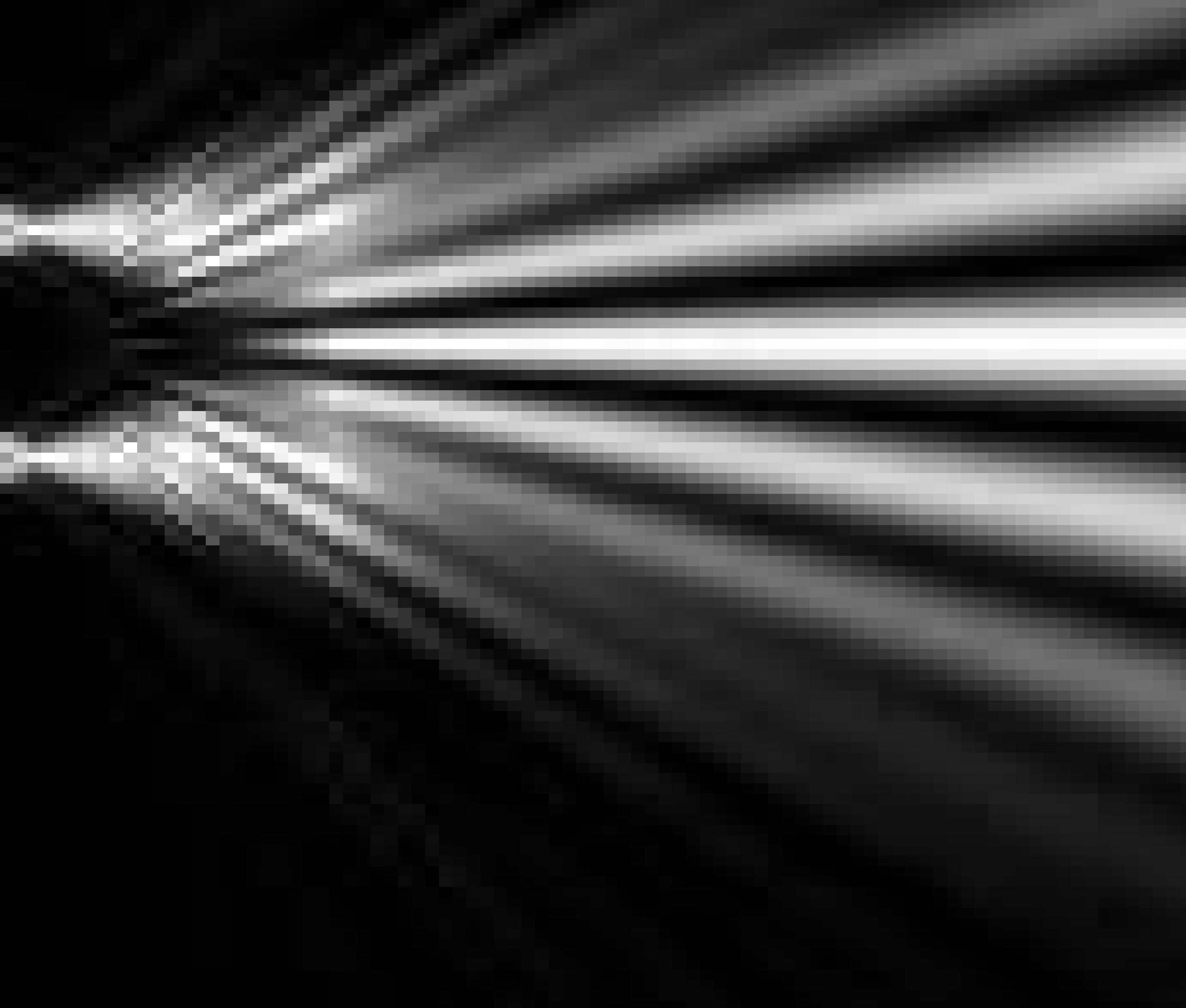
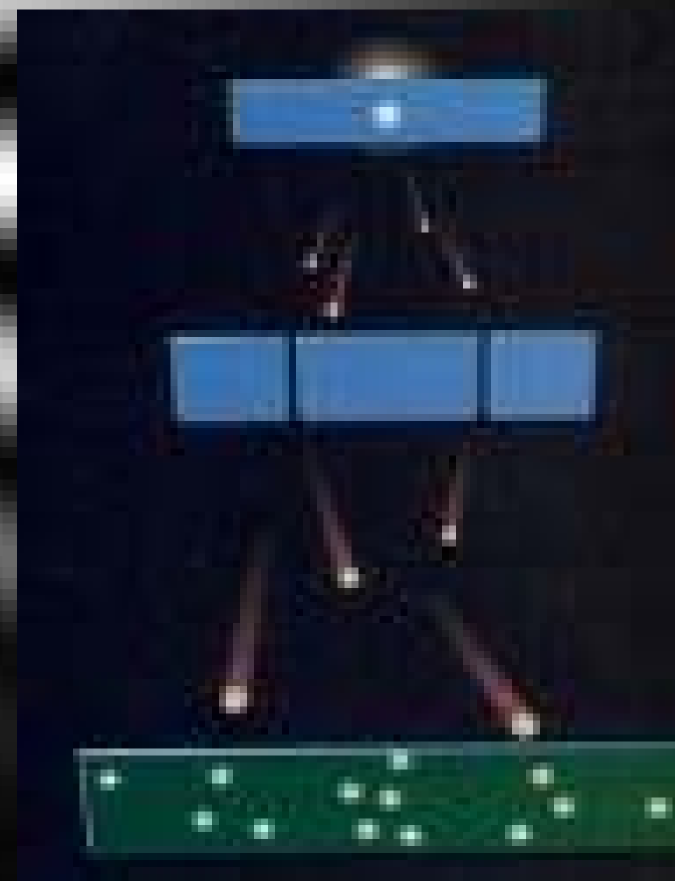
Quantum mechanics

from information transport in classical statistics

quantum mechanics can be described by classical statistics !

quantum particle from classical probabilities





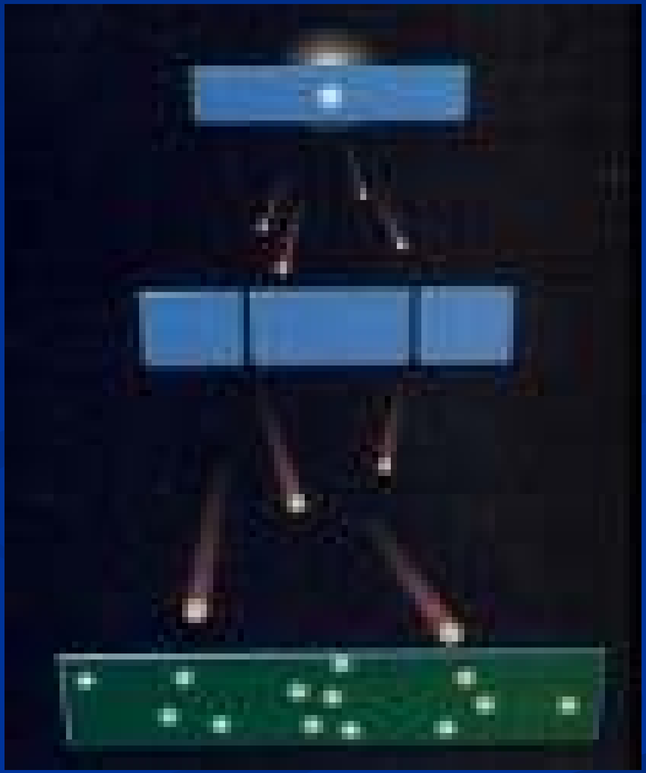
Double slit experiment

Is there a classical probability density $w(x,t)$ describing interference ?

Or hidden parameters $w(x,\alpha,t)$?
or $w(x,p,t)$?

Suitable time evolution law :
local , causal ? Yes !

Bell's inequalities ?
Kochen-Specker Theorem ?



statistical picture of the world

- basic theory is not deterministic
- basic theory makes only statements about probabilities for sequences of events and establishes correlations
- probabilism is fundamental , not determinism !

quantum mechanics from classical statistics :
not a deterministic hidden variable theory

Probabilistic realism

Physical theories and laws only describe probabilities

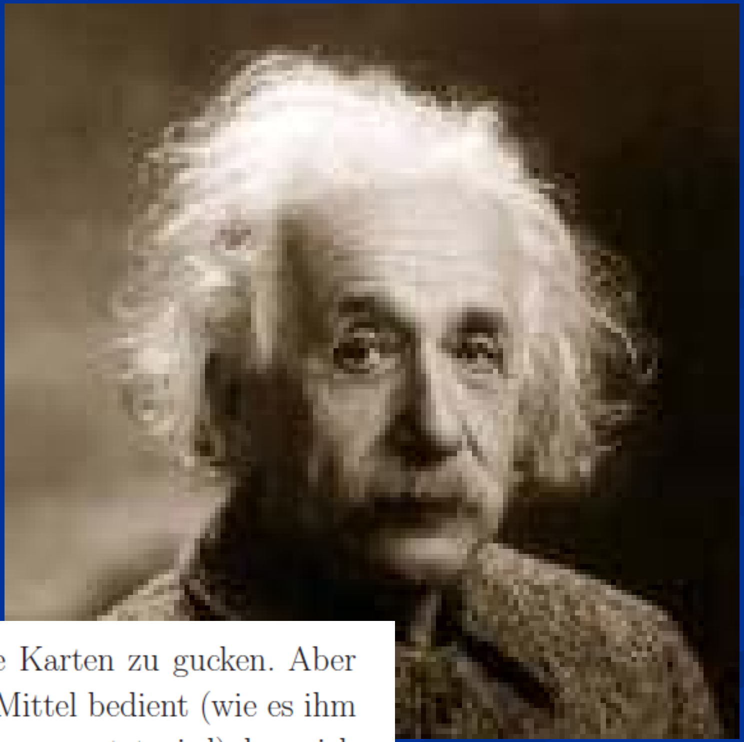
Physics only describes probabilities

Gott würfelt

Physics only describes probabilities

Gott würfelt

Gott würfelt nicht



"Es scheint hart, dem Herrgott in die Karten zu gucken. Aber dass er würfelt und sich telepatischer Mittel bedient (wie es ihm von der gegenwärtigen Quantentheorie zugemutet wird), kann ich keinen Augenblick glauben.."

Einstein: Brief an Cornelius Lanczos am 21. März 1942

Physics only describes probabilities

Gott würfelt Gott würfelt nicht

humans can only deal with probabilities

probabilistic Physics

- There is one reality
- This can be described only by probabilities

one droplet of water …

- 10^{20} particles
- electromagnetic field
- exponential increase of distance between two neighboring trajectories

probabilistic realism

The basis of Physics are probabilities
for predictions of real events

laws are based on probabilities

determinism as special case :

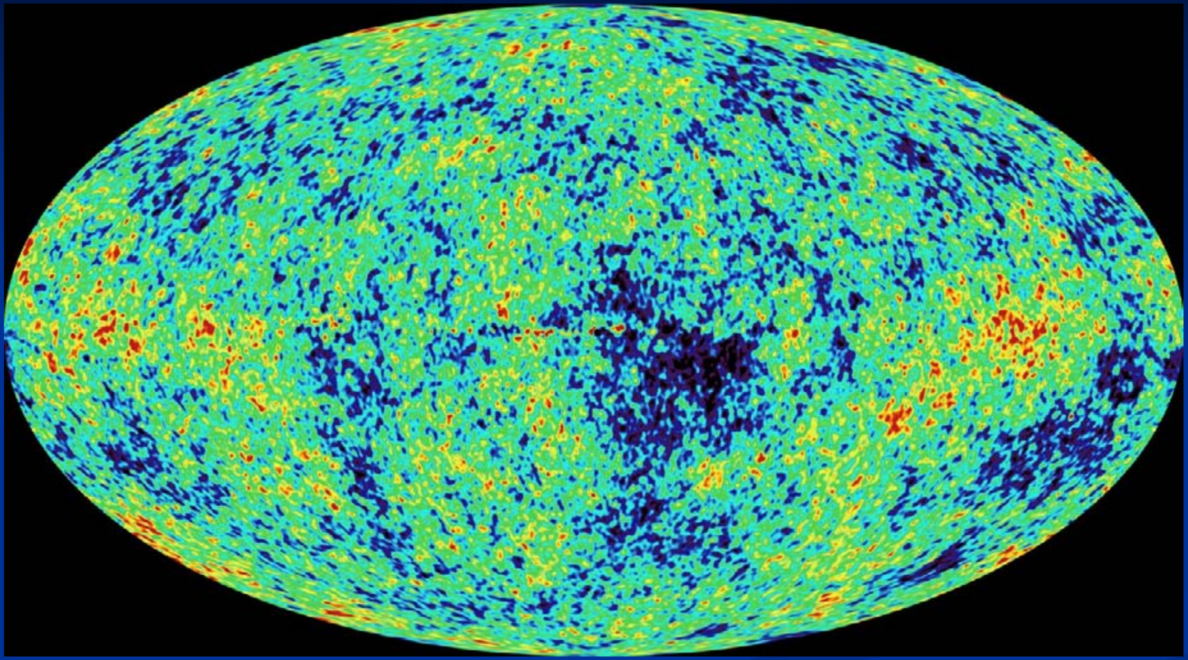
probability for event = 1 or 0

- law of big numbers
- unique ground state …

conditional probability

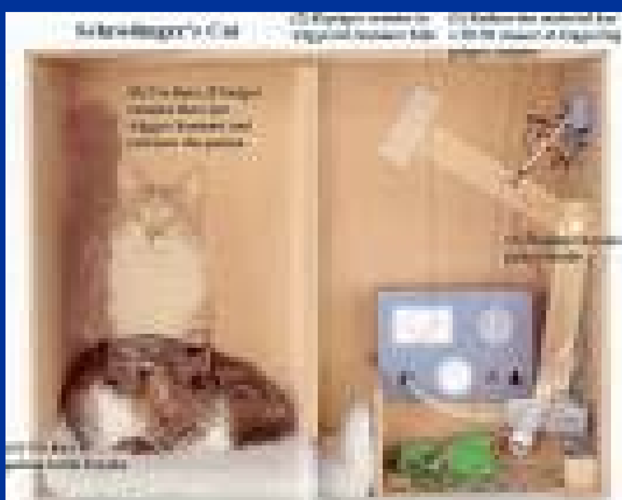
sequences of events(measurements)
are described by
conditional probabilities

both in classical statistics
and in quantum statistics

$w(t_1)$  :

not very suitable
for statement , if here and now
a pointer falls down

Schrödinger's cat



conditional probability :
if nucleus decays
then cat dead with $w_c = 1$
(reduction of wave function)

classical particle without classical trajectory

no classical trajectories

also for classical particles :

trajectories with sharp position
and momentum for each moment
in time are inadequate idealization !

still possible formally as limiting case

quantum particle

- particle-wave duality
- uncertainty
- no trajectories
- tunneling
- interference for double slit

classical particle

- particle – **wave duality**
- sharp position and momentum
- ~~classical trajectories~~
- maximal energy limits motion
- only through one slit

quantum particle

- quantum - probability - amplitude $\psi(x)$

- Schrödinger - equation

$$i\hbar\frac{\partial}{\partial t}\psi_Q(x) = -\frac{\hbar^2}{2m}\Delta\psi_Q(x) + V(x)\psi_Q(x)$$

classical particle

- classical probability in phase space w(x,p)

- Liouville - equation for w(x,p) (corresponds to Newton eq. for trajectories)

$$\frac{\partial}{\partial t}w = -Lw$$

$$L = \frac{p}{m}\frac{\partial}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

quantum formalism for classical particle

probability distribution for **one** classical particle

classical probability distribution in phase space

$$w(x, p; t)$$

wave function for classical particle

classical probability distribution in phase space

$$w(x,p;t)$$

wave function for classical particle

$$\psi_C(x,p;t)$$

depends on position and momentum !

$$w = \psi_C^2$$

wave function for one classical particle

$$\psi_C(x,p;t)$$

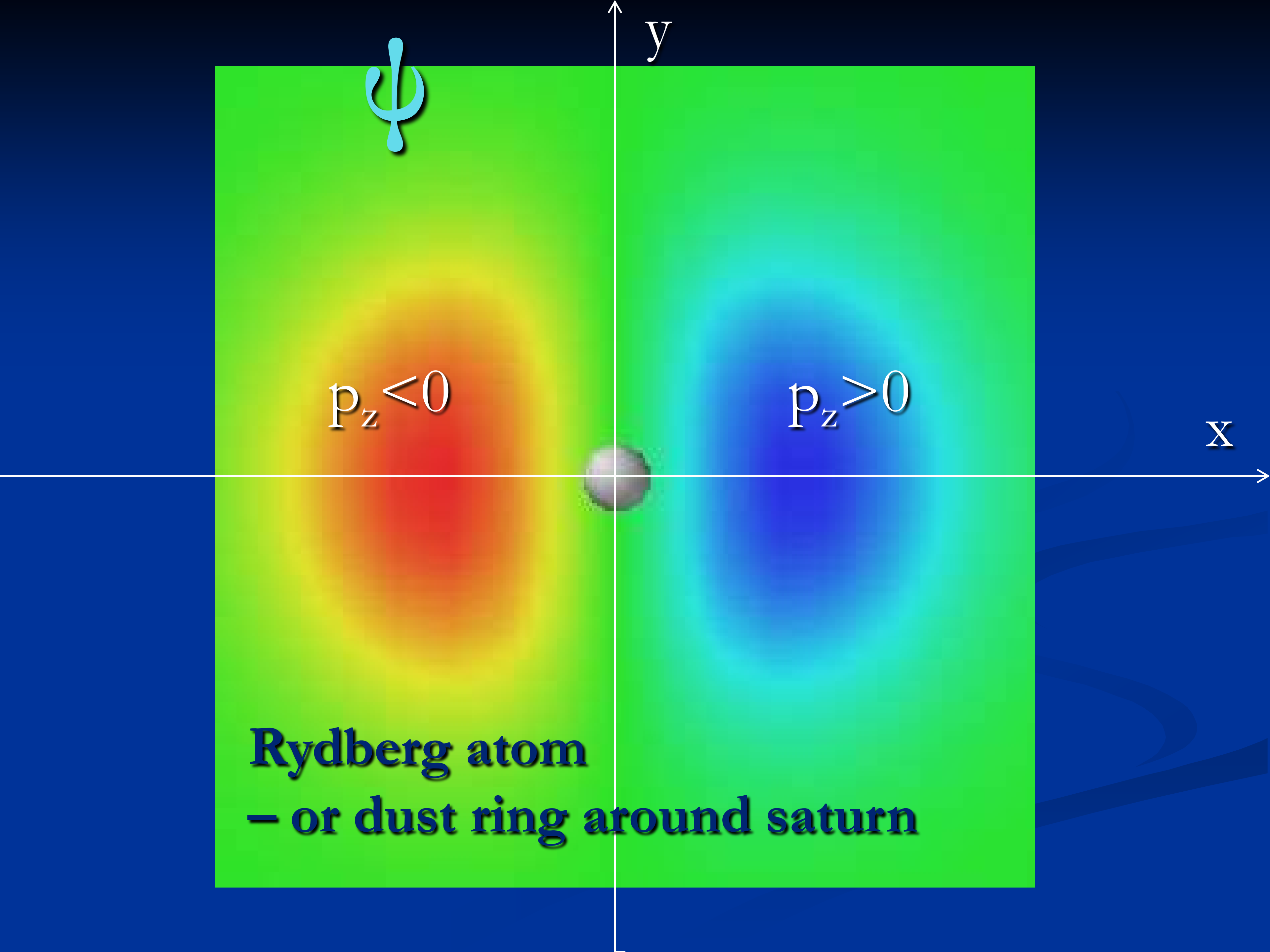
$$w = \psi_C^2$$

- real
- depends on position and momentum
- square yields probability

quantum laws for observables

$$\langle x^2 \rangle = \int_{x,p} \psi^*_{\mathbf{C}}(x,p)\, x^2\, \psi_{\mathbf{C}}(x,p)$$

$$\langle x^2 \rangle = \int_{x,p} x^2 w(x,p)$$



ψ
y
x
p_z<0
p_z>0
Rydberg atom
– or dust ring around saturn

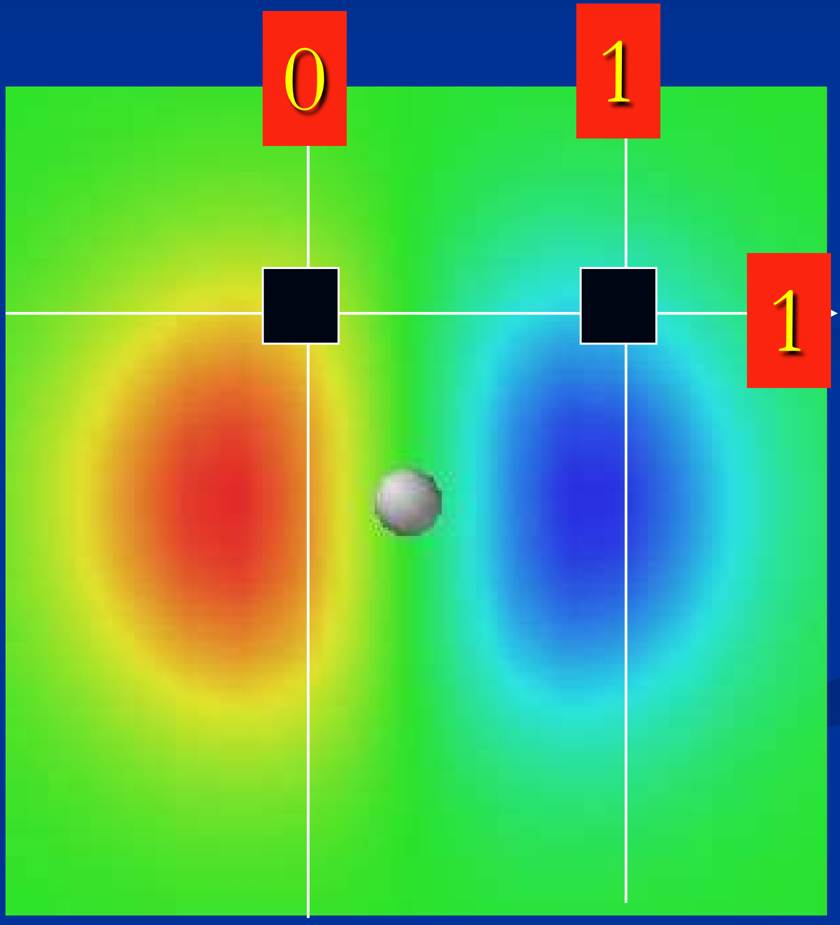
particle - wave duality

wave properties of particles :

continuous probability distribution

particle – wave duality

experiment if particle at position x – yes or no :
discrete alternative



probability distribution
for finding
particle at position x :
continuous

particle wave duality

- discreteness : observables
- continuous wave : probabilities

quantum formalism for classical particles

All statistical properties of classical *particles*

can be described in quantum formalism !

no quantum particles yet !

time evolution of classical wave function

Liouville - equation

$$\frac{\partial}{\partial t} w = -Lw$$

$$L = \frac{p}{m}\frac{\partial}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

describes classical time evolution of classical probability distribution for one particle in potential V(x)

time evolution of classical wave function

$$\frac{\partial}{\partial t} w = -Lw$$

$$w = \psi_C^2$$

$$\frac{\partial}{\partial t}\psi_C = -L\psi_C$$

$$\partial_t \psi^2 = 2\psi\partial_t\psi = -2\psi L\psi = -L\psi^2$$

wave equation

$$\frac{\partial}{\partial t}\psi = -L\psi$$

$$i\hbar\frac{\partial}{\partial t}\psi_{C} = H_L\psi_{C}$$

$$H_L = -i\hbar L = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + i\hbar\frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

modified Schrödinger - equation

wave equation

$$i\hbar \frac{\partial}{\partial t} \psi_C = H_L \psi_C$$

$$H_L = -i\hbar L = -i\hbar \frac{p}{m} \frac{\partial}{\partial x} + i\hbar \frac{\partial V}{\partial x} \frac{\partial}{\partial p}$$

fundamenal equation for classical particle in potential V(x)

replaces Newton's equations

modification of Liouville equation

modification of evolution for classical probability distribution

$$i\hbar\frac{\partial}{\partial t}\psi_C = H_L\psi_C$$

$$H_L = -i\hbar L = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + i\hbar\frac{\partial V}{\partial x}\frac{\partial}{\partial p}$$

$$H_L \longrightarrow H_W$$

$$H_W = -i\hbar\frac{p}{m}\frac{\partial}{\partial x} + V\left(x + \frac{i\hbar}{2}\frac{\partial}{\partial p}\right) - V\left(x - \frac{i\hbar}{2}\frac{\partial}{\partial p}\right)$$

quantum particle

with evolution equation

$$
\begin{aligned}
\partial_t \underset{C}{\psi}(x,p) &= -\frac{p}{m}\partial_x \underset{C}{\psi}(x,p) + K(x,\partial_p)\underset{C}{\psi}(x,p), \\
K &= -i\left[V\left(x+\frac{i}{2}\partial_p\right) - V\left(x-\frac{i}{2}\partial_p\right)\right]
\end{aligned}
$$

all expectation values and correlations for quantum – observables , as computed from classical probability distribution ,
coincide for all times precisely with predictions of quantum mechanics for particle in potential V

Schrödinger equation

- obtains by coarse graining
- integrate out variable p
- introduce complex structure

quantum particle from classical probabilities in phase space !

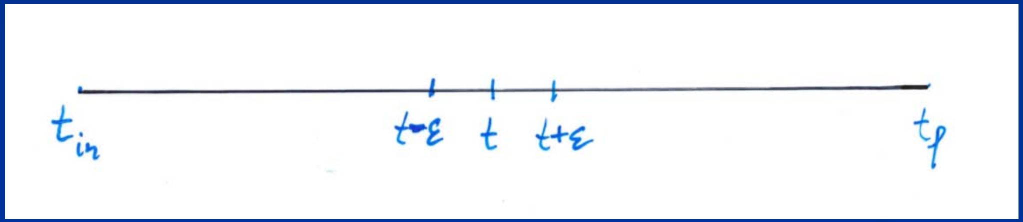
classical probabilities –
not a deterministic classical theory

quantum mechanics from information transport in classical statistics

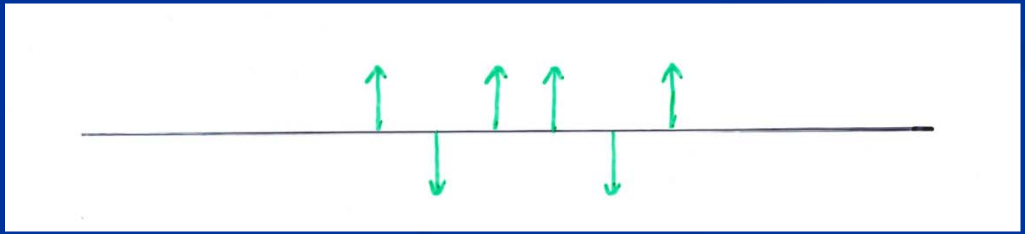
- *Why wave function ?*
- *What determines evolution equation ?*
- *Non – commuting observables ?*

discrete variables

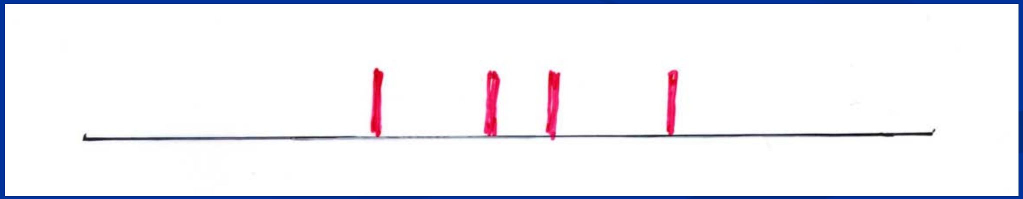
wire with
discrete points t



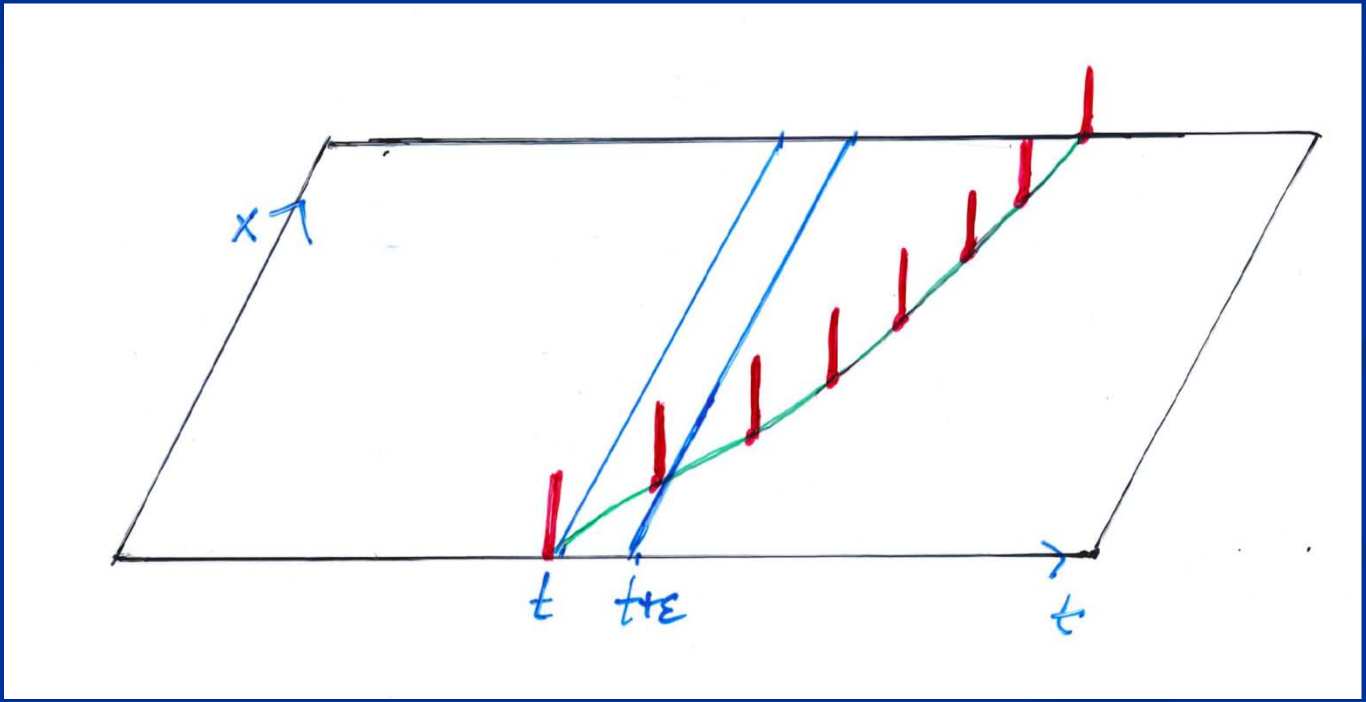
Ising spins
s = 1, -1



occupation numbers
n = 0, 1 (fermions)



occupation numbers in two dimensions



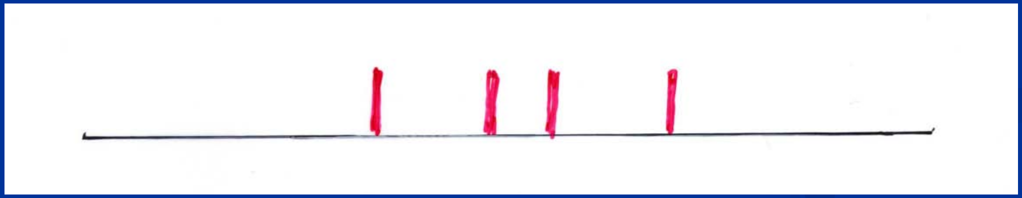
Ising-type lattice model

x : points
on lattice

n(x) =1 : particle present , n(x)=0 : particle absent

classical statistical probability distribution

- {n} : configuration of occupation numbers for all n(t)



$[\, n_s \,] = [0,0,1,0,1,1,0,1,0,1,1,1,1,0,\ldots]$

- w[n] : probability distribution

quasi-local probability distribution

$$K[n] = \exp(-S[n])$$
$$= \exp\left(-\sum_t L([n(t+\epsilon)],[n(t)])\right)$$
$$= \prod_t \mathcal{K}(t)$$

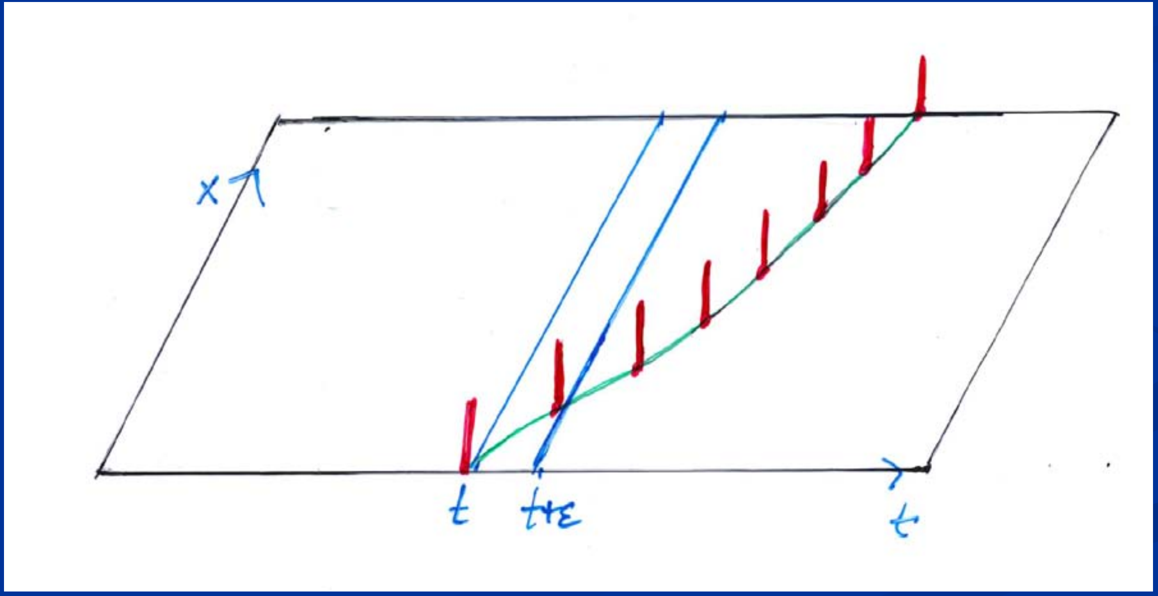
only interactions between two neighboring t

$$w[n] = \bar{f}_f[n(t_f)]\; K[n]\; f_{in}[n(t_{in})]$$

boundary conditions

information transport

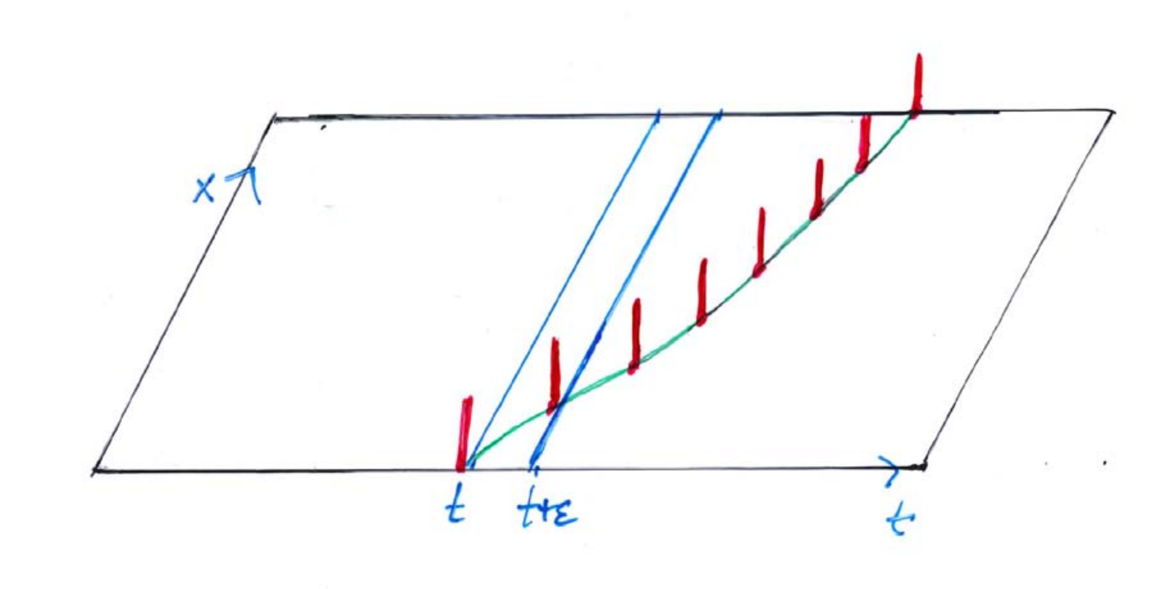
- How do boundary conditions influence expectation values in the bulk ?
- or at the other boundary ?



information transport

important problem in classical statistics and information theory

described by quantum formalism



quantum formalism for information transport

- go from one t to the next
- described by generalized Schrödinger equation for classical wave function

local observables

depend only on occupation numbers at given t

$$< A(t) > = \int \mathcal{D}n \quad A(t,[n(t)]) \quad w[n]$$

sum over configurations

local probabilities

local observable can be computed from local probabilities

$$p(t,[n(t)]) = \prod_{t' \neq t} \int \mathcal{D}n(t')\, w[n]$$

$$\langle A(t) \rangle = \int \mathcal{D}n(t)\, A(t,[n(t)])\, p(t,[n(t)])$$

$$\langle A(t) \rangle = \int \mathcal{D}n \quad A(t,[n(t)]) \quad w[n]$$

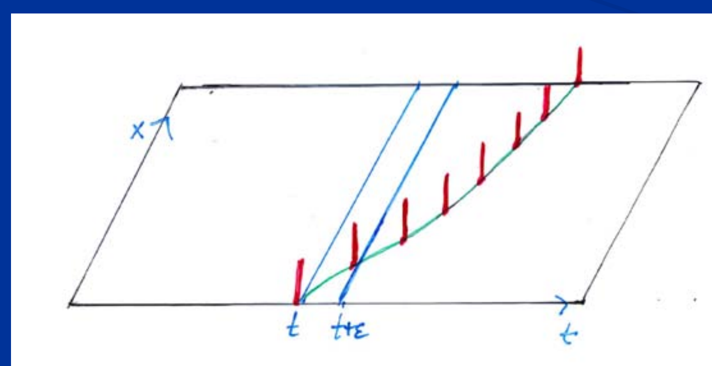
classical wave function

$$\psi(t;[n(t)]) = \int \mathcal{D}n(t'<t) \prod_{t'<t} \mathcal{K}(t')\, f_{in}(t_{in})$$

classical wave function integrates the past half

$$\bar{\psi}(t;[n(t)]) = \int \mathcal{D}n(t'>t) \prod_{t'>t} \mathcal{K}(t')\, \bar{f}_{f}(t_{f})$$

conjugate wave function integrates the future half



quantum rule for expectation value

$$\langle A(t)\rangle = \int \mathcal{D}n(t)\, \bar{\psi}(t;[n(t)])\, A(t;[n(t)])\, \psi(t;[n(t)])$$

$$\langle A\rangle = \bar{\psi}\, A'\, \psi$$

$$\langle A(t)\rangle = \int \mathcal{D}n \quad A(t,[n(t)]) \quad w[n]$$

$$w[n] = \bar{f}_f[n(t_f)] \quad K[n] \quad f_{in}[n(t_{in})]$$

$$\psi(t;[n(t)]) = \int \mathcal{D}n(t'<t) \prod_{t'<t} \mathcal{K}(t')\, f_{in}(t_{in})$$

evolution is formulated in terms of wave function

$$\psi(t+\varepsilon; [n(t+\varepsilon)]) = \int Dn(t)\, K(t)\, \psi(t; [n(t)])$$

$$\psi(t; [n(t)]) = \int Dn(t'<t) \prod_{t'<t} K(t')\, f_{in}(t_{in})$$

linear equation ,
superposition principle !

evolution is formulated in terms of wave function

$$\psi(t+\varepsilon, [n(t+\varepsilon)]) = \int Dn(t)\, \mathcal{K}(t)\, \psi(t, [n(t)])$$

$$\psi(t, [n(t)]) = \psi_\tau(t)\, f_\tau([n(t)])$$

expand in
basis functions f

$$\psi_\tau(t+\varepsilon) = S_{\tau\rho}\, \psi(t)$$

matrix equation

S : related to transfer matrix

generalized Schrödinger equation

continuum limit : $\partial_t \psi = W \psi$

complex structure : $i \partial_t \psi = G \psi$

main difference to quantum mechanics
G is not hermitean !

$$G = H + iJ \qquad H^\dagger = H \qquad J^\dagger = J$$

density matrix

$$\partial_t \rho = [W, \rho]$$

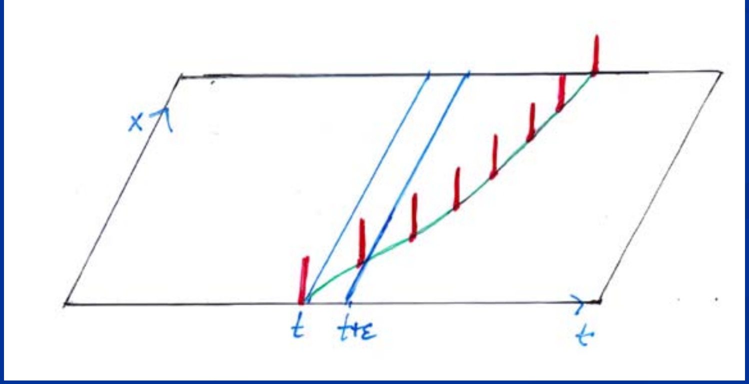
- local probabilities are diagonal elements of density matrix
- evolution of local probabilities need density matrix
- cannot be formulated in terms of local probabilities only

memory materials

- statistical systems where J plays no role

$$i\partial_t \psi = G\psi \qquad G = H + iJ \qquad H^\dagger = H \qquad J^\dagger = J$$

- bulk preserves memory of boundary conditions
- quantum simulators



structural elements of quantum mechanics

unitary time evolution

ν

h

Simple conversion factor for units

i

presence of complex structure

$$[A,B]=C$$

non-commuting observables

- classical statistical systems admit many product structures of observables
- many different definitions of correlation functions possible , not only classical correlation !
- type of measurement determines correct selection of correlation function !
- example 1 : euclidean lattice gauge theories
- example 2 : function observables

conclusion

- quantum statistics emerges from classical statistics

  quantum state, superposition, interference, entanglement, probability amplitude
- unitary time evolution of quantum mechanics can be described by suitable time evolution of classical probabilities
- memory materials are quantum simulators
- conditional correlations for measurements both in quantum and classical statistics

end

Can quantum physics be described by classical probabilities ?

“ No go “ theorems

Bell , Clauser , Horne , Shimony , Holt

implicit assumption : use of classical correlation function for correlation between measurements

Kochen , Specker

assumption : unique map from quantum operators to classical observables