Scale symmetry in particle physics and cosmology

Scale transformation

- Scale all lengths with constant factor
- Scale all masses with inverse factor

Quantum scale symmetry

No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized.

Continuous global symmetry

Scale transformation of renormalized fields

$$g'_{\mu\nu} = \alpha^{-2} g_{\mu\nu}\,, \quad \sqrt{g'} = \alpha^{-4}\sqrt{g}$$

$$A'_{\mu} = A_{\mu}\,, \quad \psi' = \alpha^{3/2}\psi\,.$$

$$\chi' = \alpha\chi$$

$$\mathcal{L}_{\chi} = \frac{1}{2} g^{\mu\nu}\partial_{\mu}\chi\partial_{\nu}\chi + \tilde{\lambda}\chi^{4}$$

Scale symmetric classical gravity

Replace Planck mass by scalar field

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \ \ldots \ \right\}$$

$$g_{\mu\nu} \to \alpha^{-2} g_{\mu\nu}$$

$$\chi \to \alpha \chi$$

$$g^{\mu\nu} \to \alpha^{2} g^{\mu\nu}$$

$$\sqrt{g} \to \alpha^{-4} \sqrt{g}$$

Scale invariance of scalar kinetic term

$$\sqrt{g}\,\partial^\mu \chi\, \partial_\mu \chi$$

$$= \sqrt{g}\, g^{\mu\nu} \partial_\mu \chi\, \partial_\nu \chi$$

$$g_{\mu\nu} \rightarrow \alpha^{-2} g_{\mu\nu}$$

$$\chi \rightarrow \alpha\, \chi$$

$$\sqrt{g} \rightarrow \alpha^{-4} \sqrt{g}$$

$$g^{\mu\nu} \rightarrow \alpha^{2} g^{\mu\nu}$$

Classical scale symmetry

No parameter with dimension of length or mass is present in the classical action.

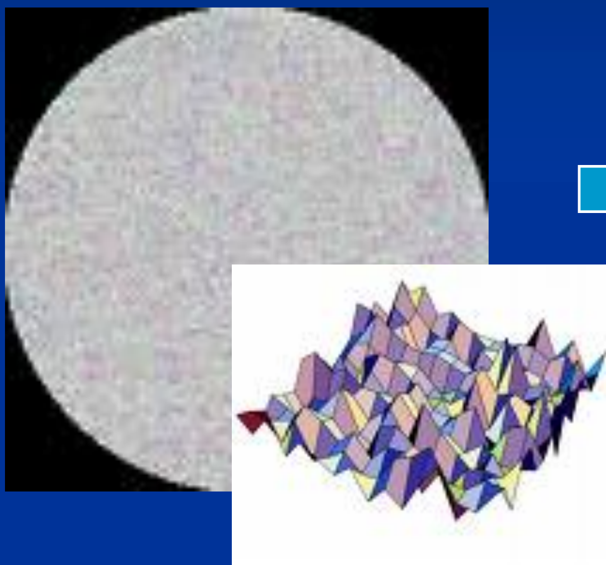
Classical scale symmetry

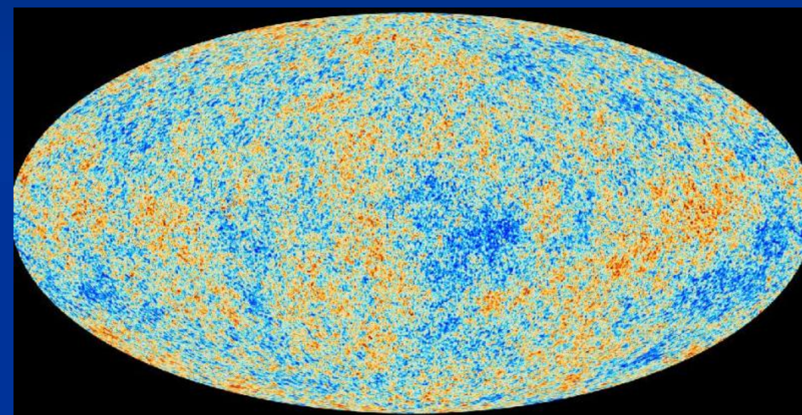
No parameter with dimension of length or mass is present in the classical action.

Examples : pure QCD,
QED with zero electron mass

Have we observed scale symmetry ?

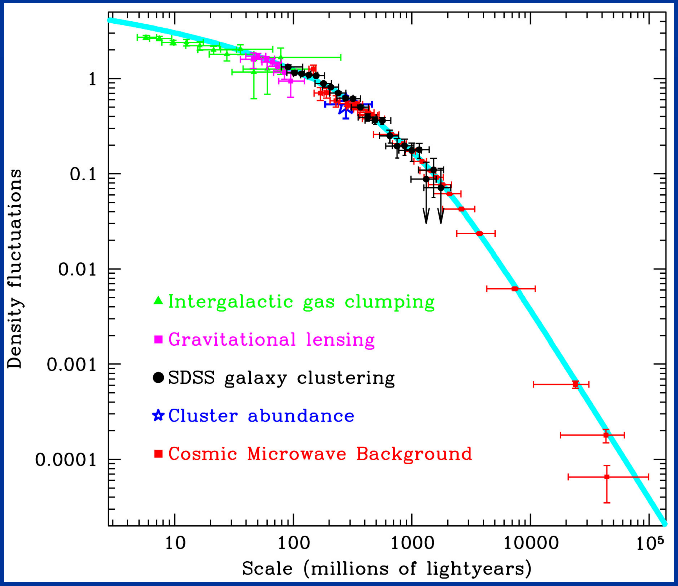
Almost scale invariant primordial fluctuation spectrum seeds all structure in the universe

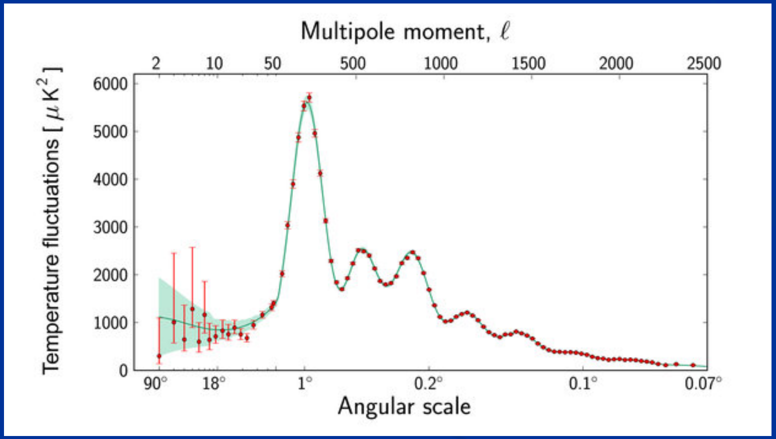




Scale symmetry in cosmology ?

Almost scale invariant primordial fluctuation spectrum





scales are present in cosmology

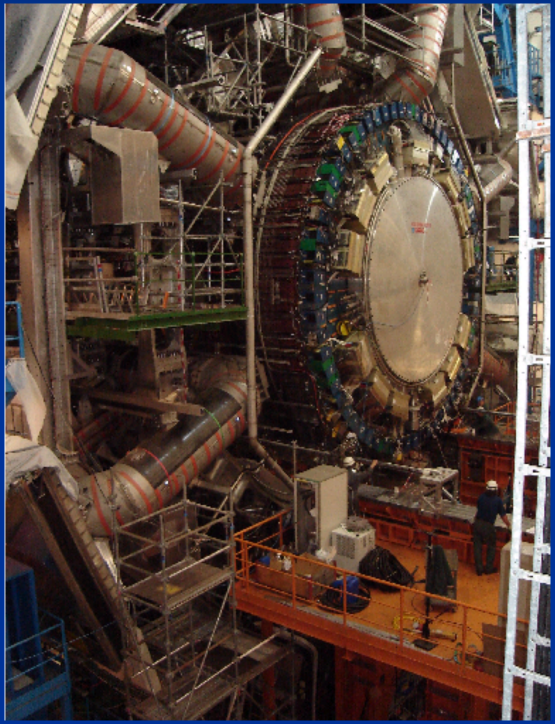
Scale symmetry in elementary particle physics ?

proton mass , electron mass

Scales are present in particle physics,
but very small as compared to Planck mass

High momentum scattering almost scale invariant

Approximate scale symmetry at highest energies at the LHC



and at the next collider ?

Quantum scale symmetry

Quantum fluctuations induce running (momentum dependent) couplings

Running fine structure constant

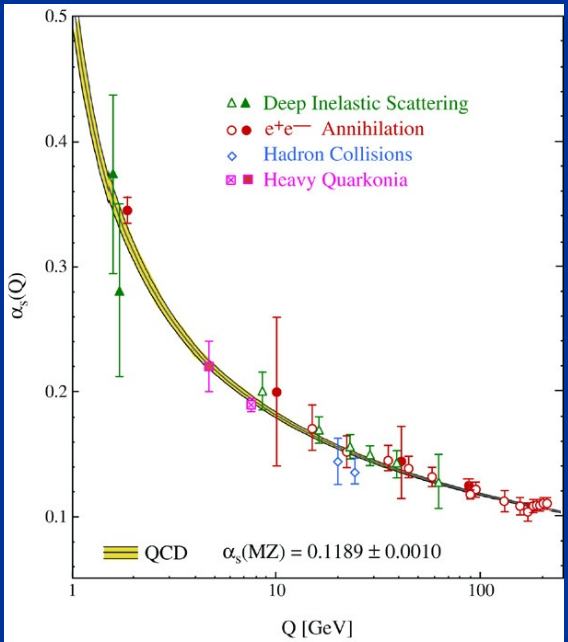
$\alpha = 1/137$ for atoms

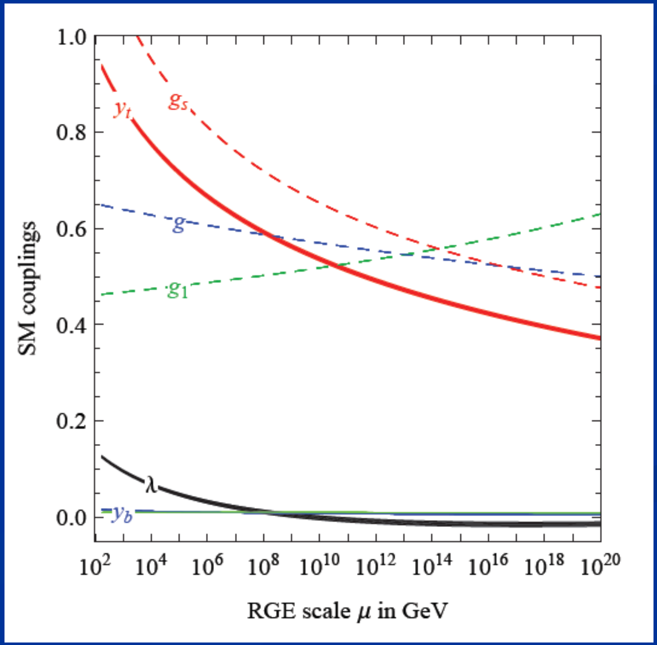
$\alpha = 1/128$ at LEP

Induced by quantum fluctuations of electrons, muons, quarks etc.

Quantum fluctuations induce running couplings

- possible violation of scale symmetry
- well known in QCD or standard model





Quantum scale symmetry

- quantum fluctuations can violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !
- quantum fluctuations can generate scale symmetry !

Fixed points and scale symmetry

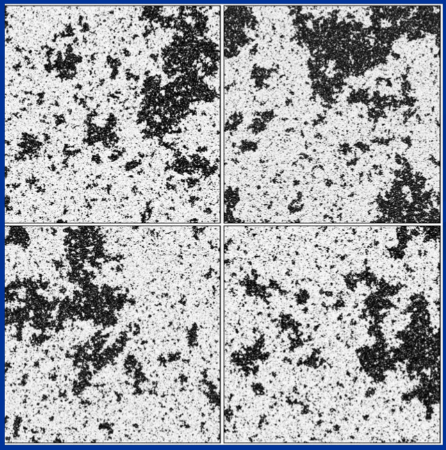
- At a fixed point, scale symmetry is exact in the presence of fluctuations (quantum scale symmetry)
- Well known in condensed matter physics :

  second order phase transition →

  fixed point or scaling solution →

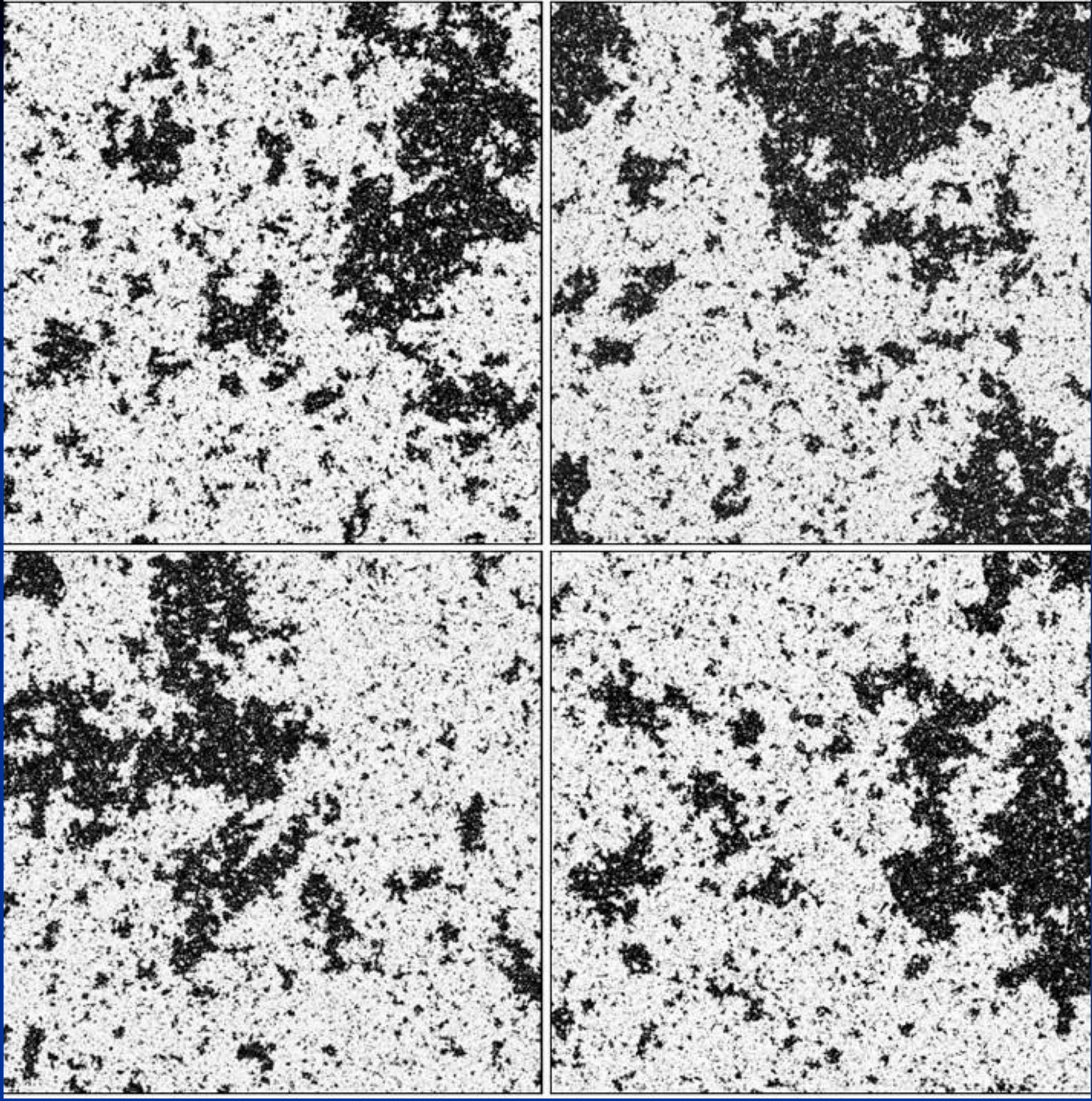
  exact scale symmetry →

  critical phenomena
- Classical or quantum Ising model

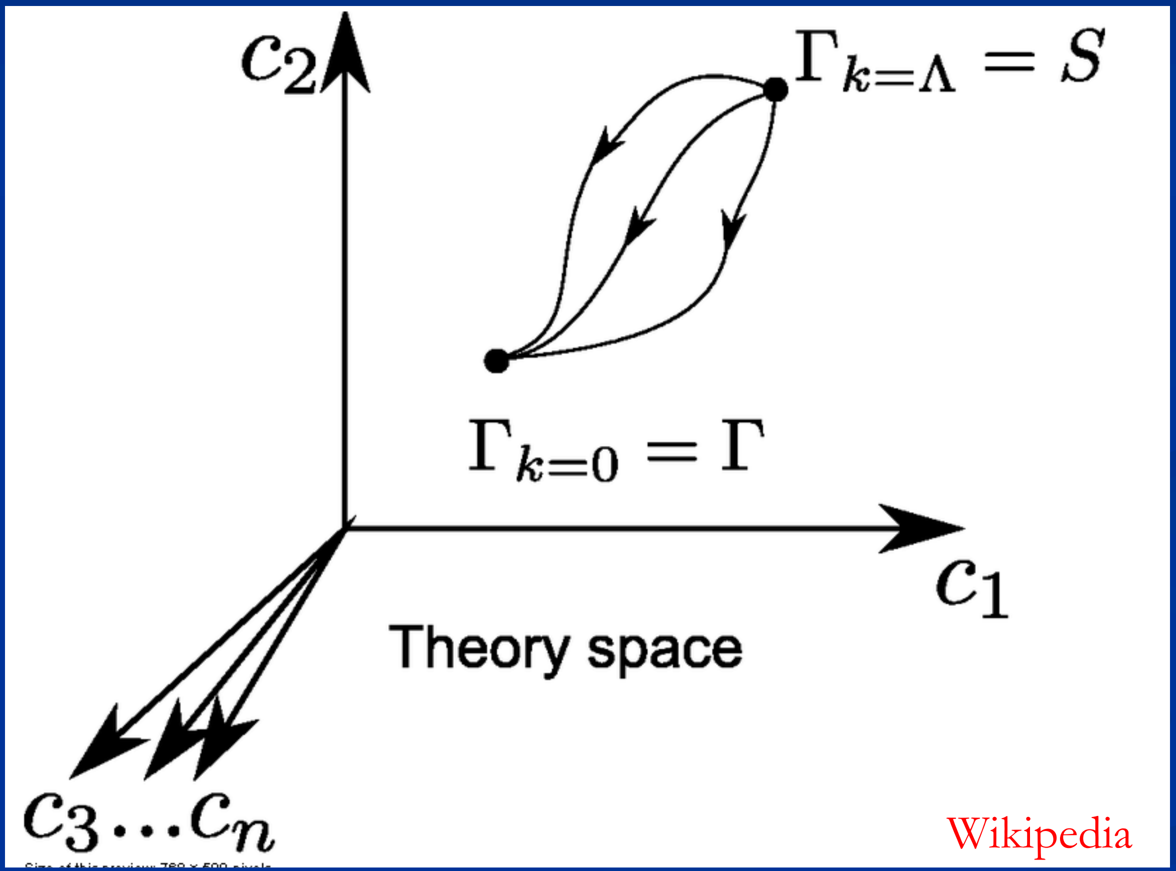


Unbroken scale symmetry :

Physics looks the same on all scales

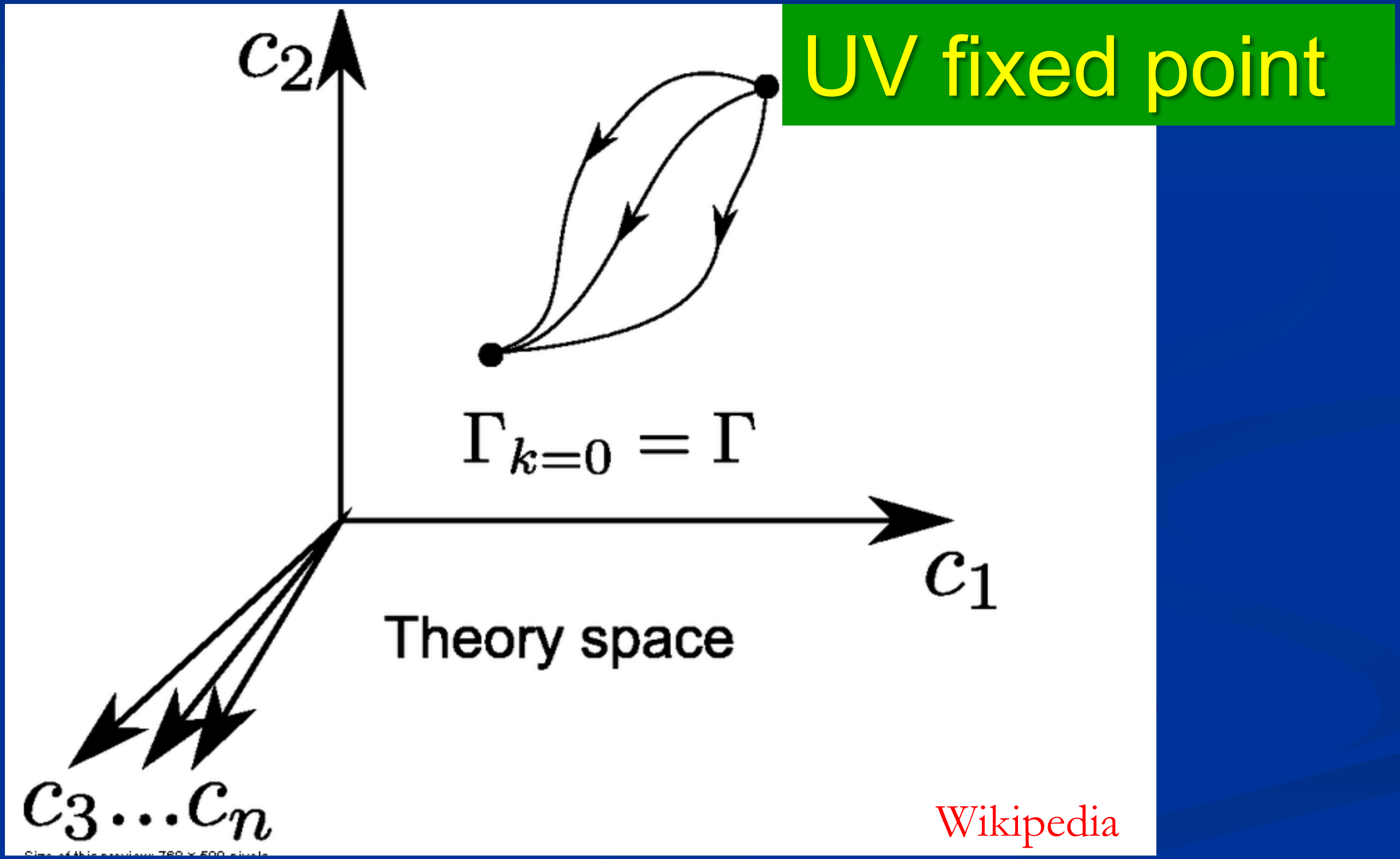


Functional renormalization : flowing action



Wikipedia

Ultraviolet fixed point



Quantum scale symmetry

Exactly on fixed point:
No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

Three scale symmetries

Gravity scale symmetry:

includes transformation of fields for particles, metric and scalar singlet **UV - fixed point**

Particle scale symmetry:

scalar singlet kept fixed (or fixed Planck mass)

relative scaling of momenta with respect to Planck mass **SM - fixed point**

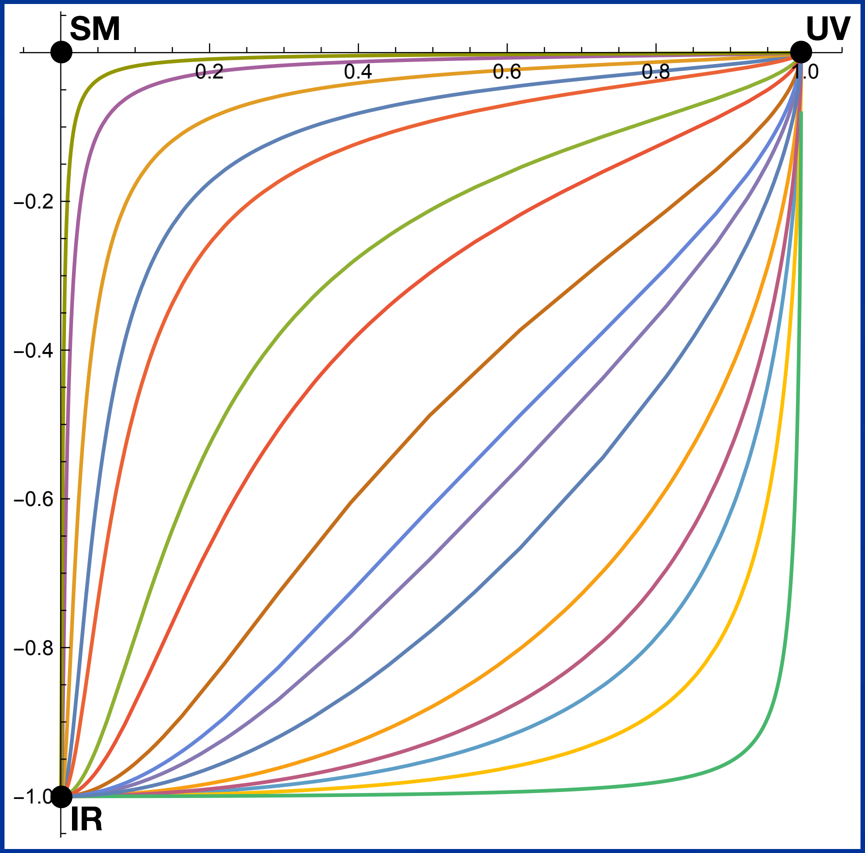
Cosmic scale symmetry :

involves metric and cosmon (pseudo Goldstone boson of spontaneously broken scale symmetry) **IR - fixed point**

Scale symmetry and fixed points

Relative strength of gravity

Particle scale symmetry

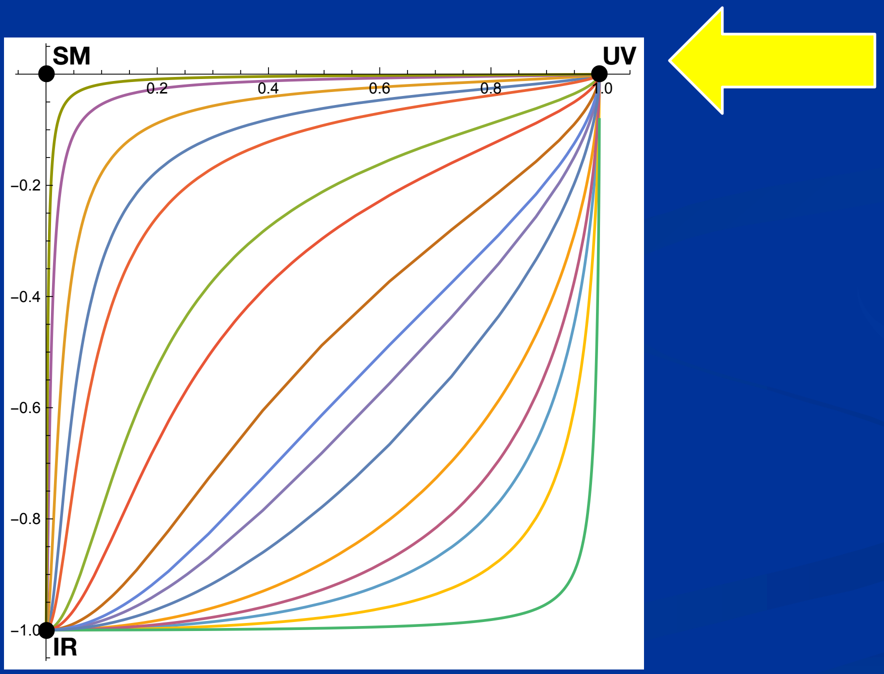


Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

Gravity scale symmetry



Gravity scale symmetry

Replace Planck mass by scalar field

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \quad \ldots \quad \right\}$$

Gravity scale symmetry

Replace Planck mass by scalar field

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \quad \right\}$$

Cosmological solution : χ (t) differs from zero

Spontaneous breaking of scale symmetry

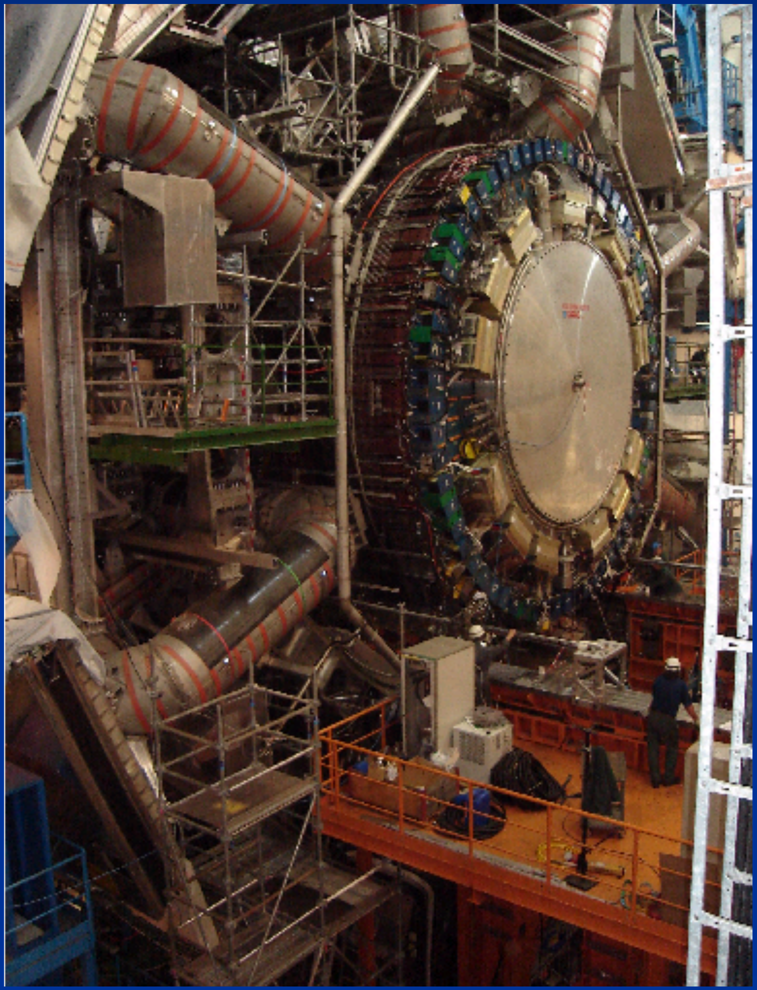
Particle masses from spontaneous scale symmetry breaking

- Electron mass proportional to χ

- $m_e \sim \varphi_0$ (expectation value of Higgs scalar)
- $\varphi_0 \sim \chi$

- $m_e = y_e \varphi_0$ (Yukawa coupling)

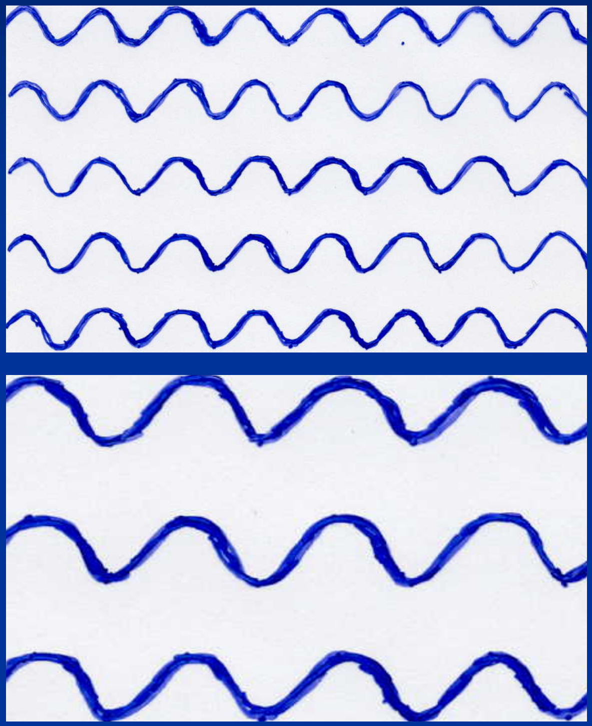
Spontaneous symmetry breaking confirmed at the LHC

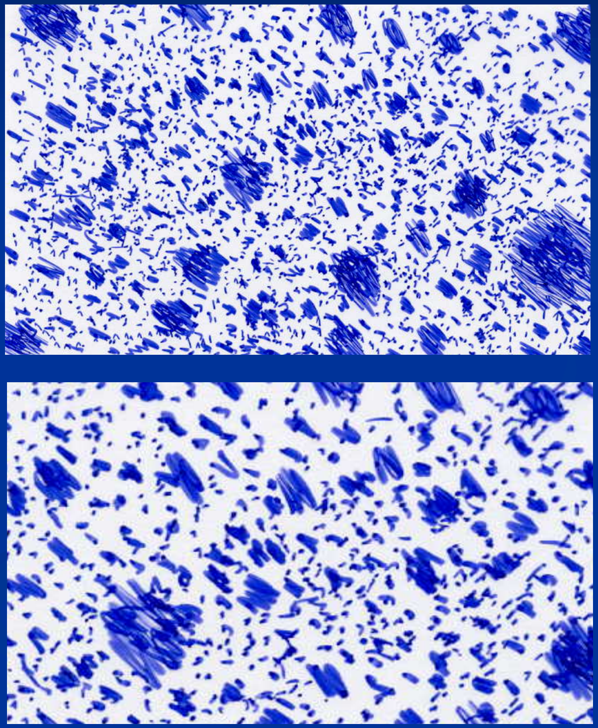


Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- particle masses can be proportional to scalar field χ
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly massless Goldstone boson – the dilaton

Scale symmetry





no scale symmetry

scale symmetry

only if no spontaneous symmetry breaking!

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$ → $\varphi_0^2 = \epsilon\chi^2$ Fujii Englert Zee

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$g(\chi) = \bar{g}$ → $\Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right)$ $b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$ CW

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

CW'87

Scale symmetric standard model

- Replace all mass scales by scalar field χ

Quantum effective action for standard model does not involve intrinsic mass or length scale

Quantum scale invariant standard model

CW'87
Shaposhnikov, Zenhausern'08

For $\chi_0 \neq 0$: massless Goldstone boson

Gravity scale symmetry does not protect small Fermi scale

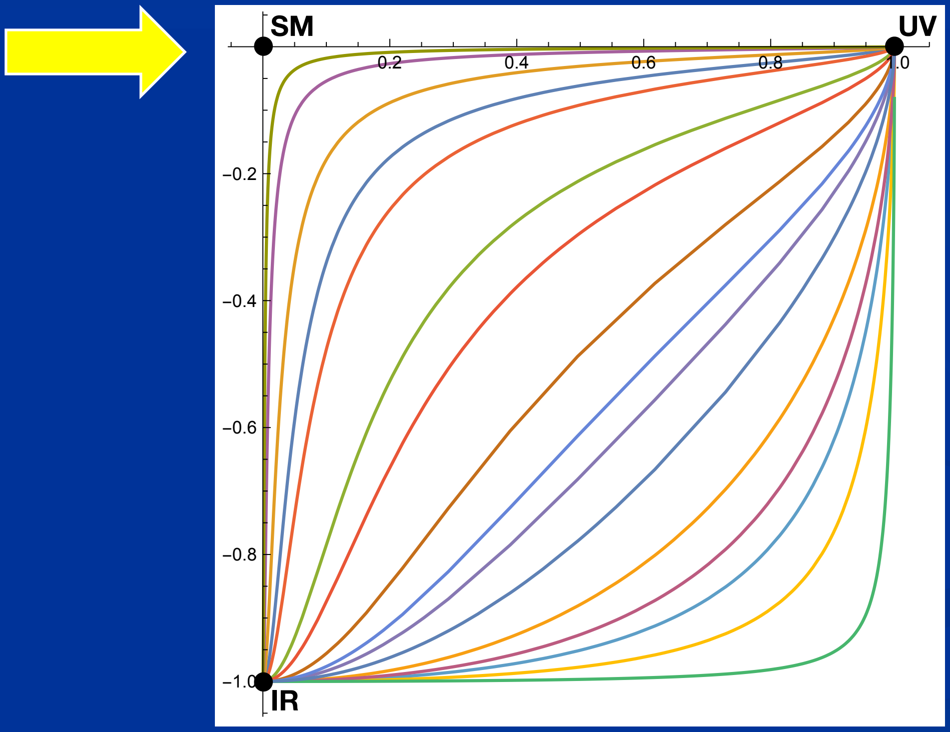
Effective potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$$

is scale invariant for arbitrary ε

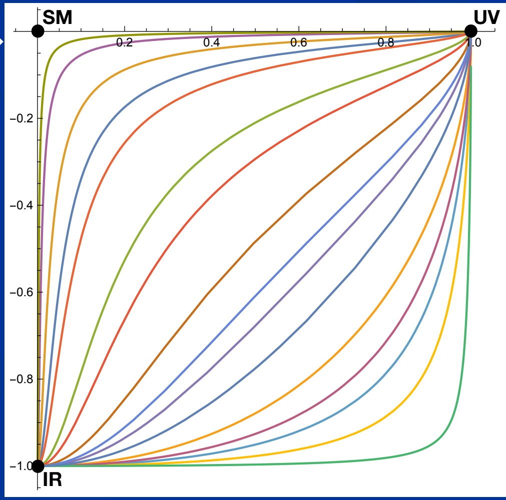
$$\varphi_0^2 = \epsilon\chi^2$$

Particle scale symmetry



Particle scale symmetry

is the scale symmetry for the
effective low energy theory below the Planck mass



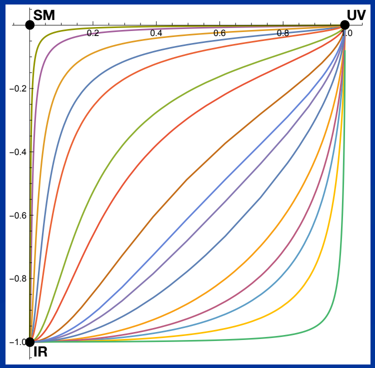
Second order vacuum electroweak phase transition

→

fixed point

→

quantum scale symmetry



Scale symmetry and Fermi scale

- Vacuum electroweak phase transition is (almost) second order, **including all effects from quantum fluctuations**
- Critical surface of second order phase transition: exact fixed point, **quantum scale symmetry**
- Scale symmetry guarantees **“naturalness”** of gauge hierarchy

C. Wetterich, Phys. Lett.B140(1984)215,
W. A. Bardeen, FERMILAB-CONF-95-391-T(1995)

Scale symmetry and Fermi scale

- Vacuum electroweak phase transition is (almost) second order
- Critical surface of second order phase transition:
  exact fixed point, **quantum scale symmetry**
- Scale symmetry guarantees **"naturalness"** of small gauge hierarchy

- No fine tuning for renormalisation group improved perturbation theory for deviation from critical surface

$$\mu \frac{\partial}{\partial \mu} \delta = A \delta$$

$$A = \frac{1}{16\pi^2} \left(2\lambda_H + 6h_t^2 - \frac{9}{2} g_2^2 - \frac{3}{2} g_1^2 \right)$$

Fine tuning

- Need to understand small parameter
- Suitable renormalized parameter needs not to be tuned: take distance from second order vacuum electroweak phase transition as one of the relevant parameters
- If distance from phase transition is small: expression in terms of other large renormalized parameters necessarily involves cancellations

Technical fine tuning ?

Quadratic divergences concern bare perturbation theory for location of critical surface in coupling constant space.

not relevant for observation,

not particularly interesting,

regularization dependent, not universal,

always depends on unknown microscopic details

bare perturbation theory is bad expansion

Vacuum electroweak phase transition in quantum gravity

Quantum Gravity

Quantum Gravity can be a renormalisable quantum field theory

Asymptotic safety

Asymptotic safety of quantum gravity

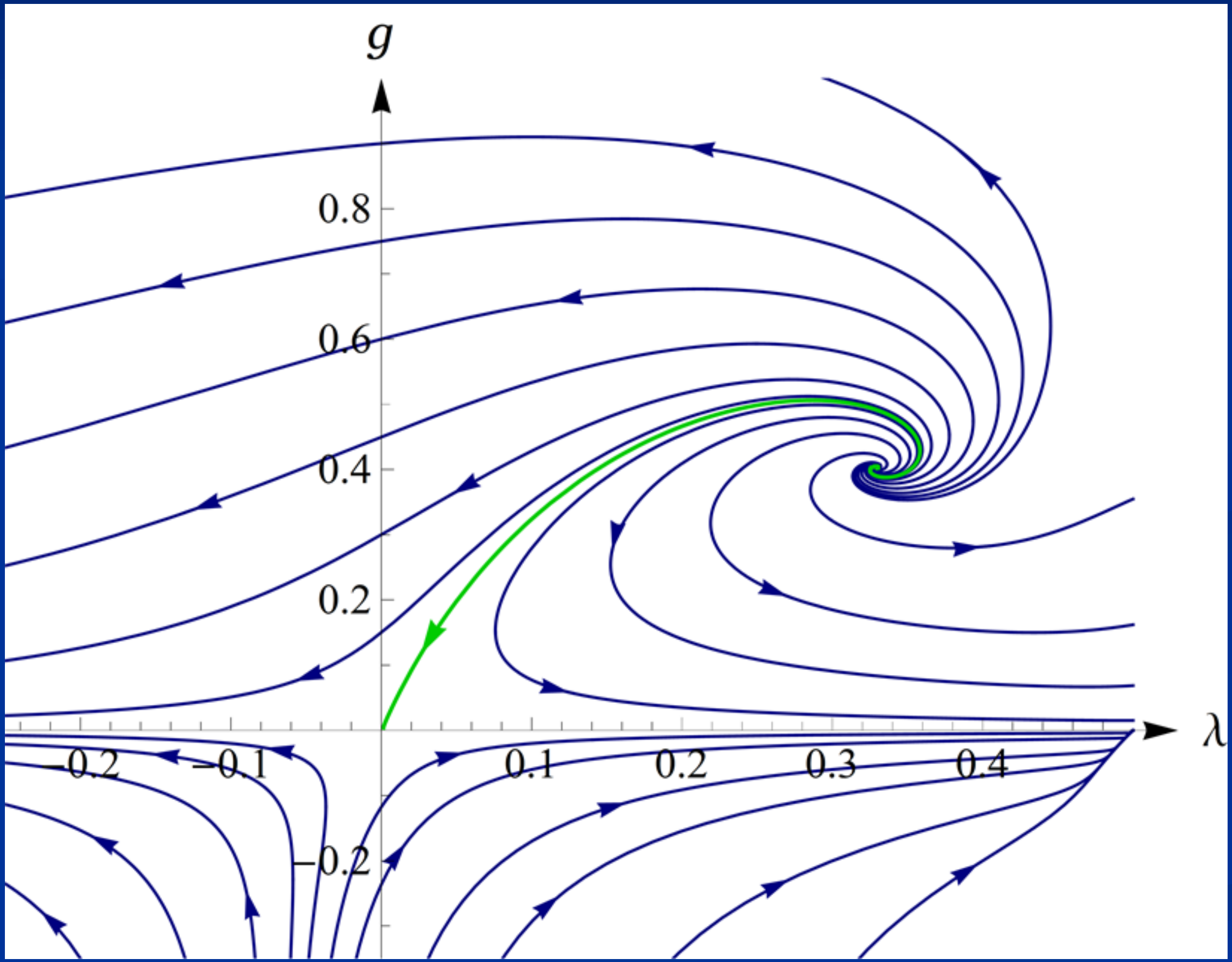
if UV fixed point exists :

quantum gravity is

non-perturbatively renormalizable !

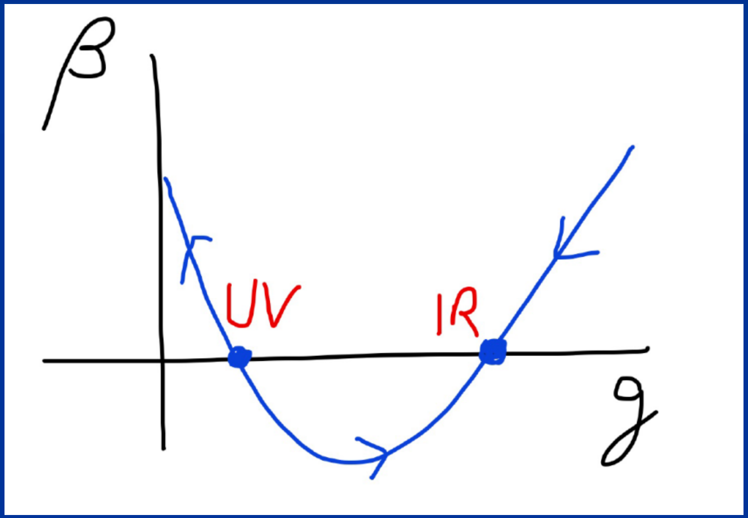
S. Weinberg , M. Reuter

UV- fixed point for quantum gravity

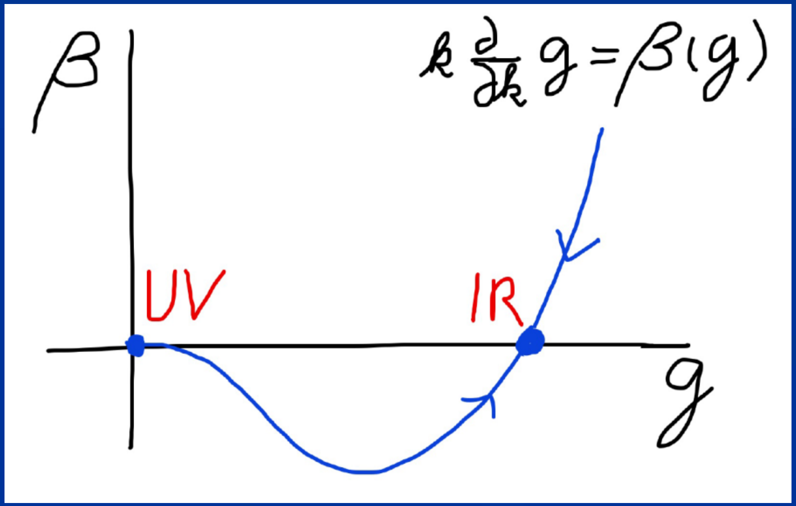


Wikipedia

Asymptotic safety



Asymptotic freedom



Relevant parameters yield undetermined couplings. Quartic scalar coupling is not relevant and can therefore be predicted.

a prediction…

Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

Christof Wetterich

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010

Abstract

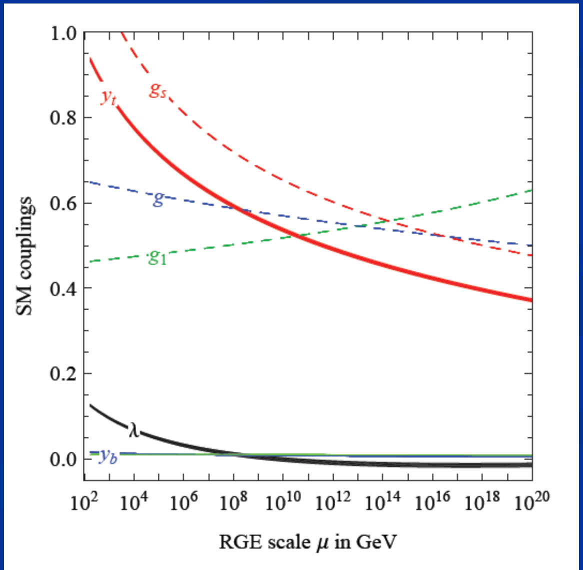
There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\text{min}} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\text{min}} = 126$ GeV, with o

Essential points for prediction of Higgs boson mass:

- More precisely : ratio Higgs boson mass over W-boson mass, or Higgs boson mass over top quark mass
- Initial value of quartic scalar coupling near Planck mass is predicted by UV- fixed point

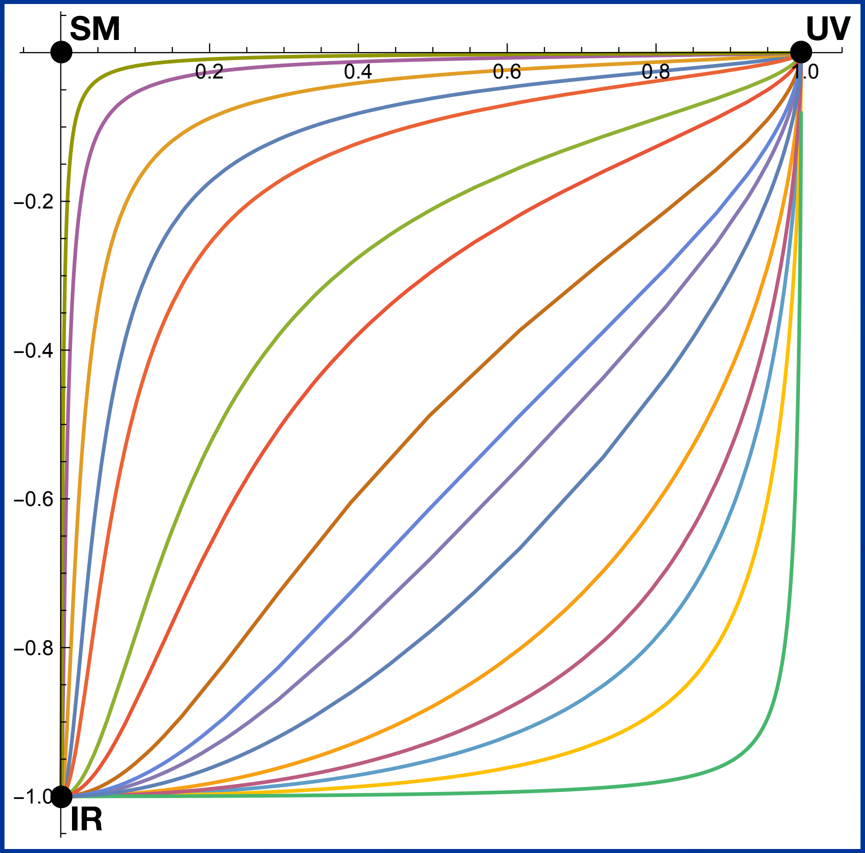
Extrapolate perturbatively to Fermi scale :



Flow diagram

Relative strength of gravity

Particle scale symmetry



Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

Flowing dimensionless Planck mass

- Renormalization scale k : Only fluctuations with momenta larger k are included

Flowing Planck mass $M^2(k)$

$$\partial_t M^2 = 4ck^2$$

$$\partial_t = k\partial_k$$

Dimensionless squared Planck mass

$$w = \frac{M^2}{2k^2}$$

$$\partial_t w = -2w + 2c$$

solution :

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Fixed point and crossover

UV - fixed point

$$w_* = c$$

approached for k → ∞

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$M^2(k) = M^2 + 2c_M k^2$$

Near UV – fixed point : M ~ k

Transition to constant M for small k,
Gravity gets weak, w^{-1} decreases to zero

Flowing dimensionless ratio for distance from electroweak phase transition

Dimensionless distance from EW-phase transition

$$\gamma = \frac{\delta}{k^2}$$

$$\partial_t \gamma = (-2 + A)\gamma$$

Particle contribution

$$A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$

Gravity contribution : A=b/w , dominates for large k, small w

Flow equation :

$$\partial_t \gamma = \left(-2 + \frac{b}{w}\right)\gamma$$

Ultraviolet (UV) fixed point

$$\partial_t w = -2w + 2c$$

$$\partial_t \gamma = \left(-2 + \frac{b}{w}\right)\gamma$$

$$w_* = c$$

$$\gamma_* = 0$$

Planck mass scales proportional to k

Exactly on electroweak phase transition

General solution of flow equations

$$w = c + \frac{\bar{M}^2}{2k^2}$$

$$\gamma = \gamma_0 (1+2c)^{-\frac{b}{2c}} \frac{\bar{M}^2}{k^2} \left(1 + 2c\frac{k^2}{\bar{M}^2}\right)^{\frac{b}{2c}}$$

Two relevant parameters
(free integration constants)

$\bar{M}$: $M^2(k=0) = \bar{M}^2$ and $\gamma_0 = \gamma(k = \bar{M})$

M(k=0) can either be intrinsic scale or field χ

Flow diagram

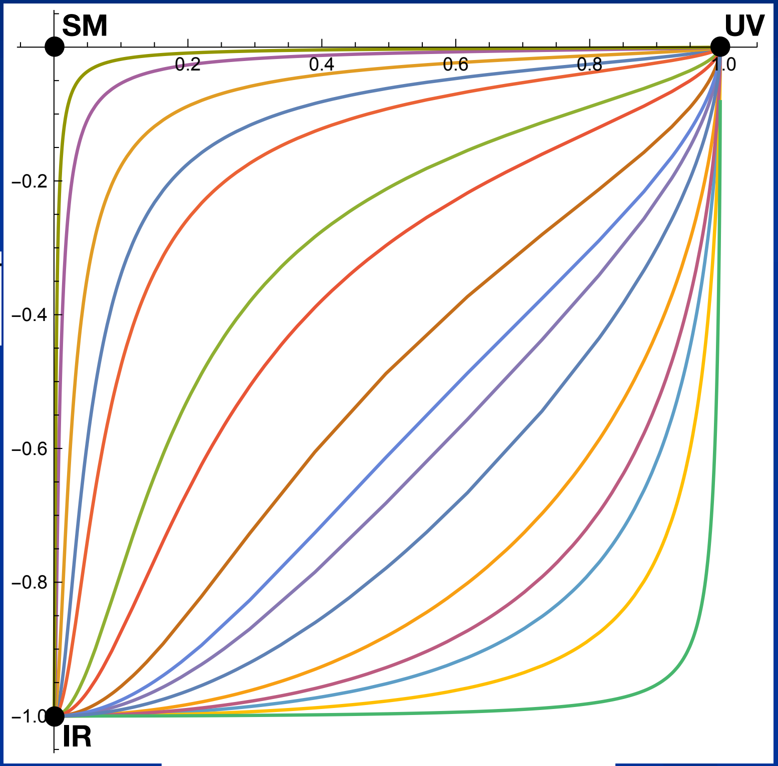
$$x = \ln\left(\frac{k^2}{\bar{M}^2}\right)$$

$$w = c + \frac{1}{2}\mathrm{e}^{-x}$$

$$\gamma = \gamma_0(1+2c)^{-\frac{b}{2c}}\,\mathrm{e}^{-x}\,(1+2c\mathrm{e}^{x})^{\frac{b}{2c}}$$

$$\gamma = 2\gamma_0(w-c)\left(\frac{w}{(1+2c)(w-c)}\right)^{\frac{b}{2c}}$$

$$f_\gamma = \gamma/\sqrt{1+\gamma^2}$$



$$f_w = 1/\sqrt{1+w^2}$$

Different trajectories: different $\gamma_0 = \gamma(k = \bar{M})$

Critical trajectory

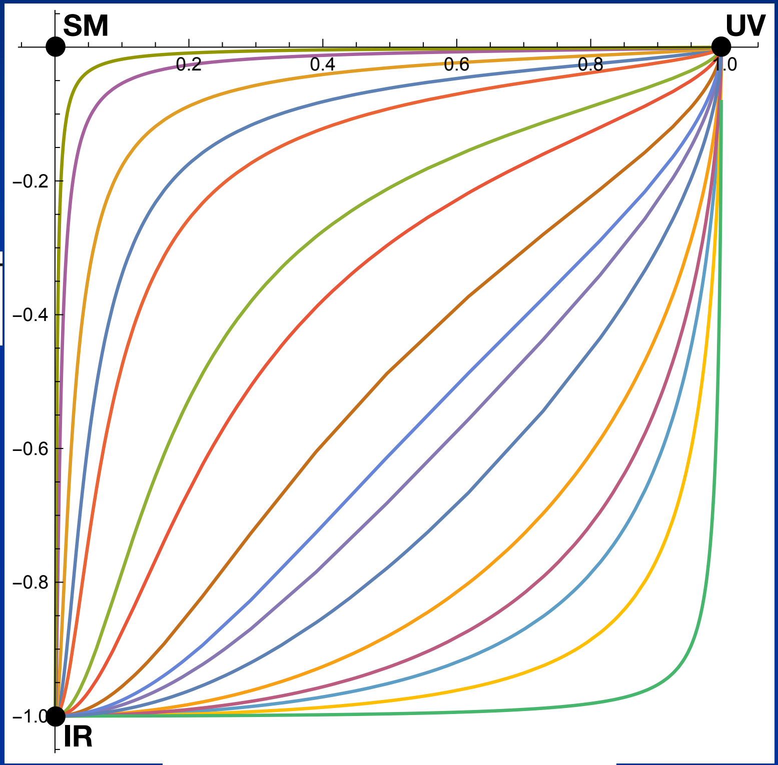
$\gamma_0 =$ 0

γ =0

not crossed
by flow !

Particle scale
symmetry

$f_\gamma = \gamma/\sqrt{1+\gamma^2}$



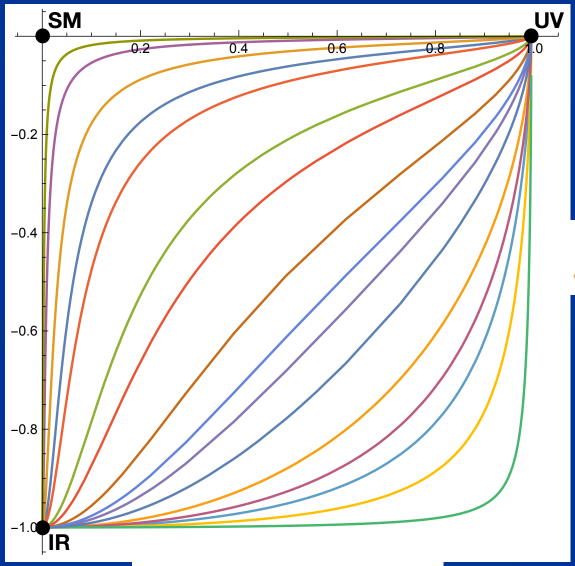
$f_w = 1/\sqrt{1+w^2}$

Fixed points

$$\text{UV:}\quad w_* = c\,,\ f_{w*} = \frac{1}{\sqrt{1+c^2}}\,,\ \gamma_* = 0\,,\ f_{\gamma*} = 0$$

$$\text{SM:}\quad w_*^{-1} = 0\,,\ f_{w*} = 0\,,\ \gamma_* = 0\,,\ f_{\gamma*} = 0$$

$$\text{IR:}\quad w_*^{-1} = 0\,,\ f_{w*} = 0\,,\ \gamma^{-1} = 0\,,\ f_{\gamma*} = -1$$



f_γ

$f_w = 1/\sqrt{1+w^2}$

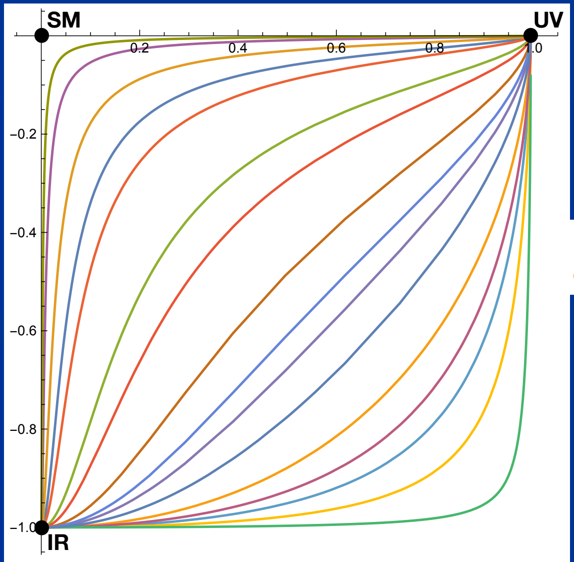
$$\gamma_0 = \gamma(k = \bar{M})$$

Particle scale symmetry

$$\text{SM:} \quad w_*^{-1} = 0\,,\; f_{w*} = 0\,,\; \gamma_* = 0\,,\; f_{\gamma*} = 0$$

Gravity is neglected

Relative scaling of momentum scales as compared to Planck mass



f_γ

$f_w = 1/\sqrt{1+w^2}$

Quantum effective action including all fluctuations

$$k \to 0$$

$$M^2(k=0) = \bar{M}^2$$

$$\delta(k=0) = \gamma_0(1+2c)^{-\frac{b}{2c}}\,\bar{M}^2$$

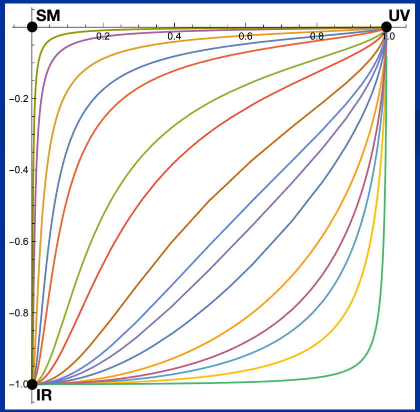
M(k=0) can either be intrinsic scale or field χ

For M(k=0) = χ :

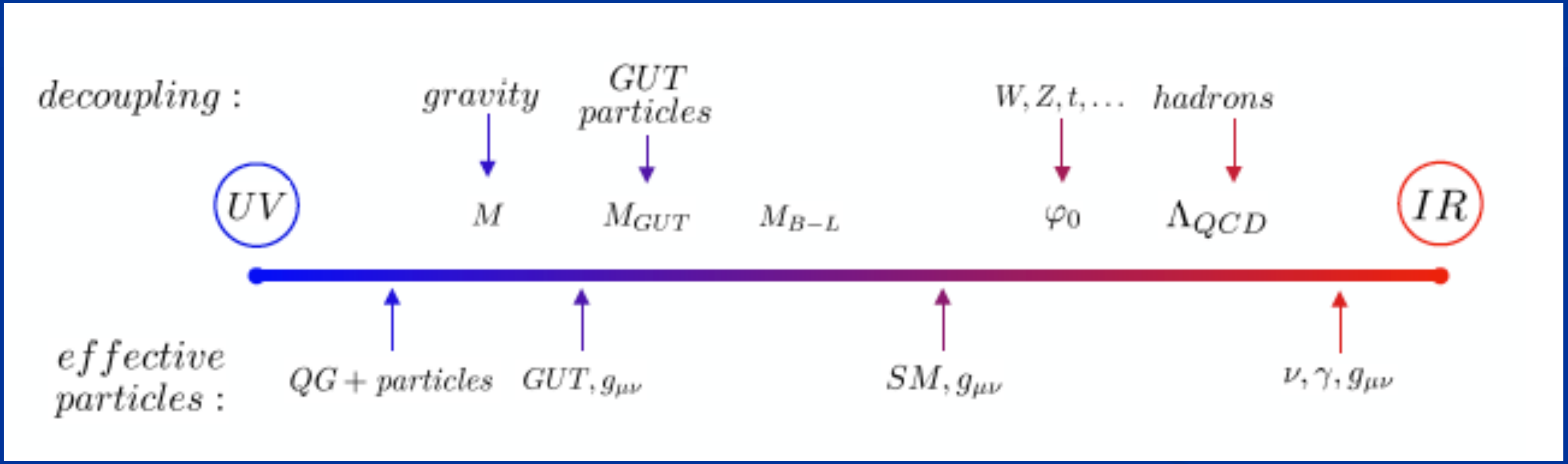
quantum scale invariant standard model

Where to solve the gauge hierarchy problem ?

- Important consequence of particle scale symmetry:
- The scale of physics which could explain the gauge hierarchy may be as high as the Planck scale
- Close vicinity of second order phase transition holds for all scales
- No need for solution at TeV-scales !



Decoupling of degrees of freedom



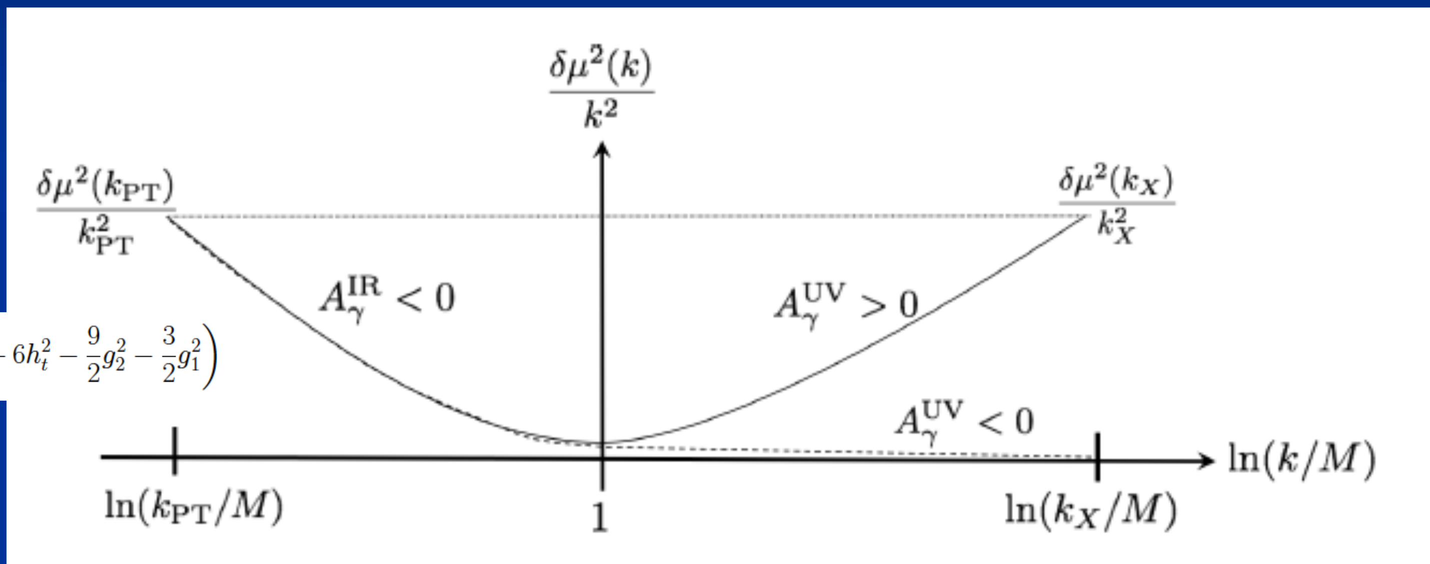
Gauge hierarchy

- Possible explanation of small parameter : distance from second order vacuum electroweak phase transition is **irrelevant parameter** at UV – fixed point

Possible explanation of gauge hierarchy

$A_\gamma^{\mathrm{IR}} = -2 + A$

$$A = \frac{1}{16\pi^2}\left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right)$$



Gauge hierarchy problem in asymptotically safe gravity –the resurgence mechanism

Christof Wetterich[1] and Masatoshi Yamada[1]

Phys.Lett. B770 (2017) 268-271

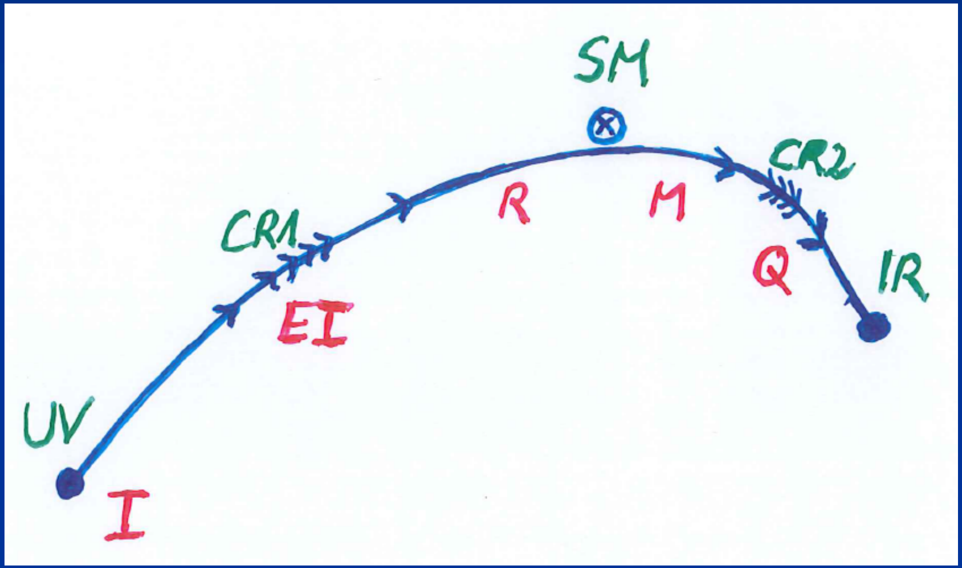
Scale symmetry in cosmology

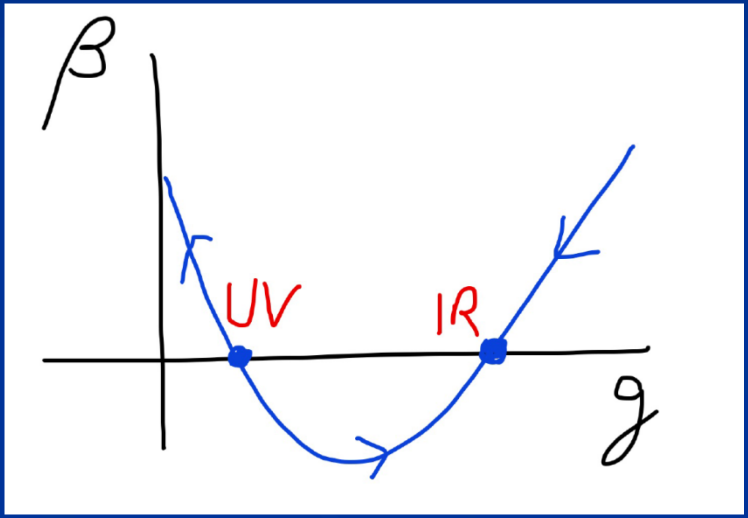
Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation

- IR : cosmon is pseudo Goldstone boson of
  spontaneously broken scale symmetry,
  tiny mass,
  responsible for dynamical Dark Energy

Possible consequences of crossover in quantum gravity





Realistic model for inflation and dark energy with single scalar field

Inflation :

the vicinity of the UV - fixed point

Starobinski inflation

$$\mathcal{L} = \sqrt{g}\left\{-\frac{C}{2}R^2 - \frac{M^2}{2}R + V\right\}$$

Scale symmetry if M^2/R (and V/R^2) go to zero.

Cosmological solution : R decreases

Early stages : very large R, close to scale symmetry

Starobinski inflation

$$\mathcal{L} = \sqrt{g}\left\{-\frac{C}{2}R^2 - \frac{M^2}{2}R + V\right\}$$

Scale symmetry for large R/M^2

End of inflation : C R near M^2
Substantial violation of scale symmetry

Primordial fluctuation spectrum:
frozen long before end of inflation
approximate scale symmetry of fluctuation spectrum

Higgs inflation

$$\mathcal{L} = -\frac{F}{2}R + D_\mu h^\dagger D^\mu h + U$$

$$U = \frac{1}{2}\lambda_H \left(h^\dagger h/M^2\right)\left(h^\dagger h\right)^2$$

$$F = M^2 + \xi_H \left(h^\dagger h/M^2\right) h^\dagger h$$

Scale symmetry if: $\left(h^\dagger h/M^2\right)$ large,

and running of dimensionless couplings slow

Higgs inflation

- Inflationary epoch : large $\xi_H \left(h^\dagger h/M^2\right)$
- End of inflation : $\xi_H \left(h^\dagger h/M^2\right)$ around one
- Standard cosmology after end of inflation : small $\xi_H \left(h^\dagger h/M^2\right)$

Cosmon inflation

$$\mathcal{L} = -\frac{F}{2}R + \frac{1}{2}\sqrt{g}K\partial^{\mu}\chi\partial_{\mu}\chi + U - \frac{C}{2}R^2$$

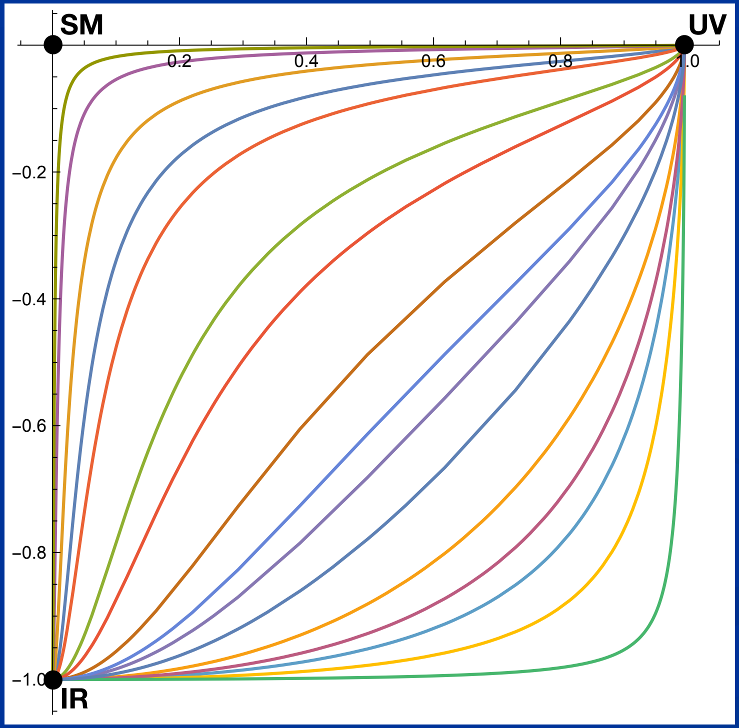
$$U = b\bar{\mu}^2\chi^2 + c\bar{\mu}^4$$

$$F = \chi^2 + d\bar{\mu}^2$$

scale invariance : small μ^2/R
or small μ/χ

UV and IR fixed point

Cosmic scale symmetry



variable gravity

"Newton's constant is not constant – and particle masses are not constant"

Variable Gravity

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2} M^2 R\right\}$

+ scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2$ → $\varphi_0^2 = \epsilon\chi^2$

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$g(\chi) = \bar{g}$ → $\Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0 \bar{g}^2}\right)$ $b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$

+ scale invariant action for dark matter

Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi\right\}$$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Scale symmetry in variable gravity (IR – fixed point)

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2\chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu\chi\partial_\mu\chi \right\}$$

IR fixed point for $\mu/\chi = 0$:
quantum scale symmetry

Tiny violation of scale symmetry
for tiny μ/χ .

Cosmic scale symmetry and the cosmological constant problem

- IR – fixed point reached for $\chi \to \infty$
- Impact of intrinsic mass scale disappears

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \mu^2 \chi^2 + \frac{1}{2}\left(B(\chi/\mu) - 6\right)\partial^\mu \chi \partial_\mu \chi \right\}$$

asymptotically vanishing cosmological „constant"

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

- vanishes for $\chi \rightarrow \infty$!

Quintessence

Dynamical dark energy ,

generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications
(different growth of neutrino mass)

Predictions of quantum gravity ?

Simple approximation for graviton contribution to scalar potential:

- Predicts mass of Higgs scalar
- Solves Gauge Hierarchy problem
- Solves cosmological constant problem

Conclusions

Quantum scale symmetry plays important role in particle physics and cosmology

- Particle scale symmetry is crucial for understanding of gauge hierarchy

  SM- fixed point

- Cosmic scale symmetry is crucial for dynamical dark energy

  IR- fixed point

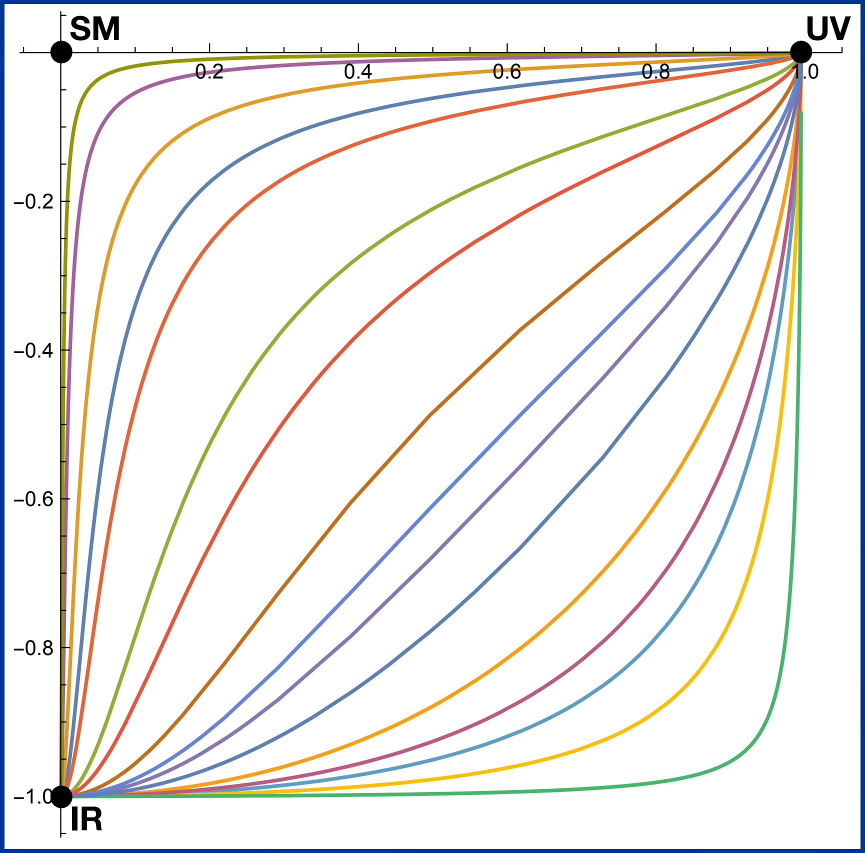
- Gravity scale symmetry rules beginning of cosmology

  UV- fixed point

Scale symmetry and fixed points

Relative strength of gravity

Particle scale symmetry



Gravity scale symmetry

Distance from electroweak phase transition

Cosmic scale symmetry

Conclusions (2)

Many incorrect statements on naturalness neglect the important consequences of quantum scale symmetry and associated fixed points.

Symmetries crucial for naturalness

Near fixed points : Individual contributions do not represent a natural value for the total effect end