The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies and distant stars against a black background. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. Some are bright and clear, while others are faint and distant. The stars appear as small, bright points of light, some with visible diffraction patterns. The overall scene is a dense, colorful mosaic of cosmic objects.

Scale symmetry in particle physics and cosmology

Scale symmetry

No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized.

Continuous global symmetry

Scale symmetry in elementary particle physics ?

proton mass , electron mass

scales are present in particle physics,
but very small as compared to Planck mass

Scale symmetry and Fermi scale

- Vacuum electroweak phase transition is (almost) second order
- Critical surface of second order phase transition:
exact fixed point, **quantum scale symmetry**
- Scale symmetry guarantees **“naturalness”** of small φ_0/M
(gauge hierarchy)

C. Wetterich, Phys. Lett.B140(1984)215,
W. A. Bardeen, FERMILAB-CONF-95-391-T(1995)

- No fine tuning for renormalisation group improved perturbation theory

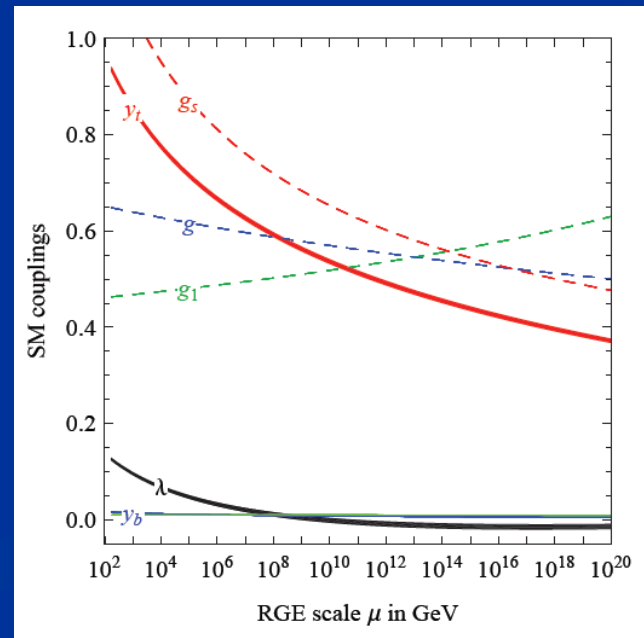
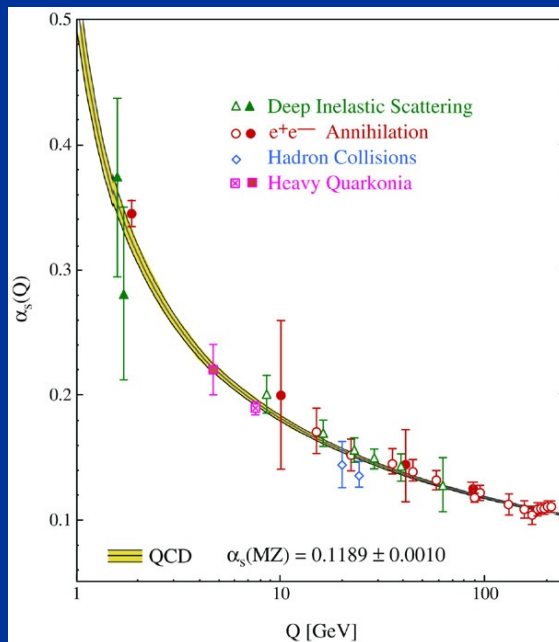
$$\mu \frac{\partial}{\partial \mu} \delta = A \delta$$

$$A = \frac{1}{16\pi^2} \left(2\lambda_H + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2 \right)$$

Quantum scale symmetry

Quantum fluctuations induce running couplings

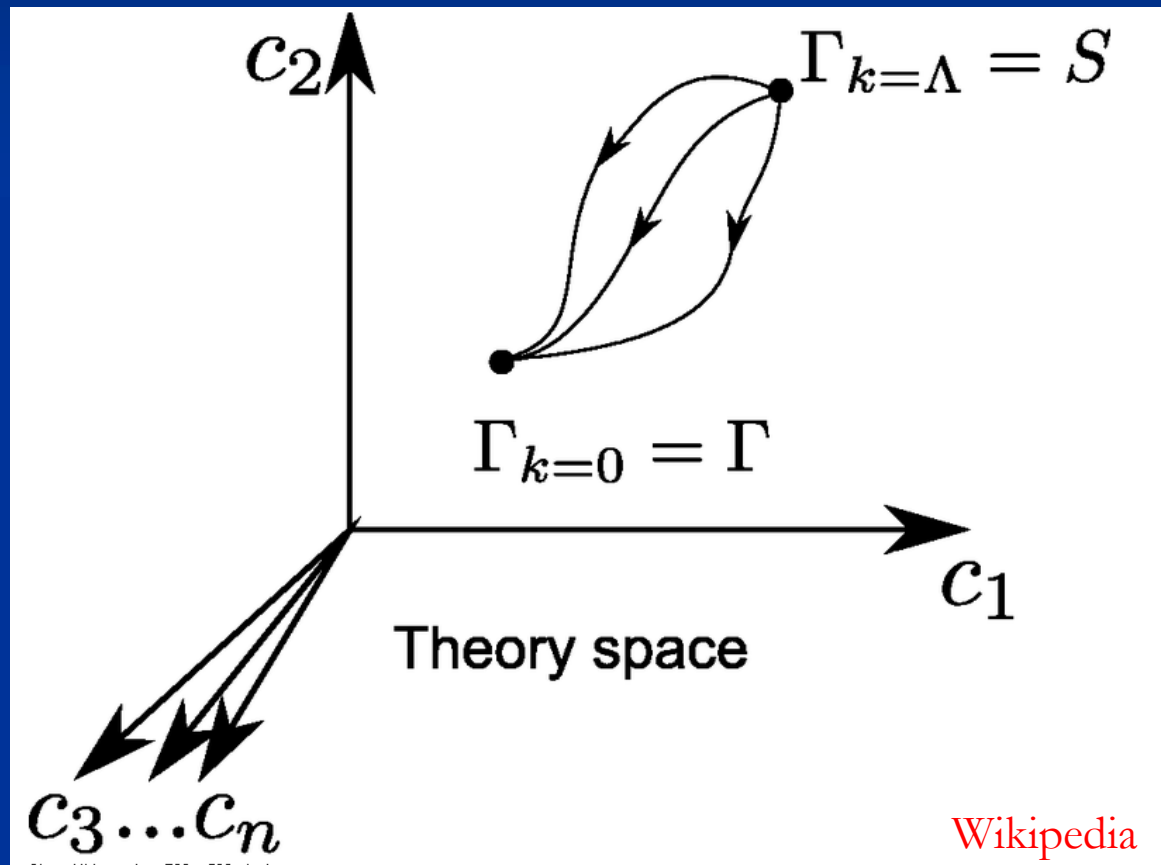
- violation of scale symmetry
- well known in QCD or standard model



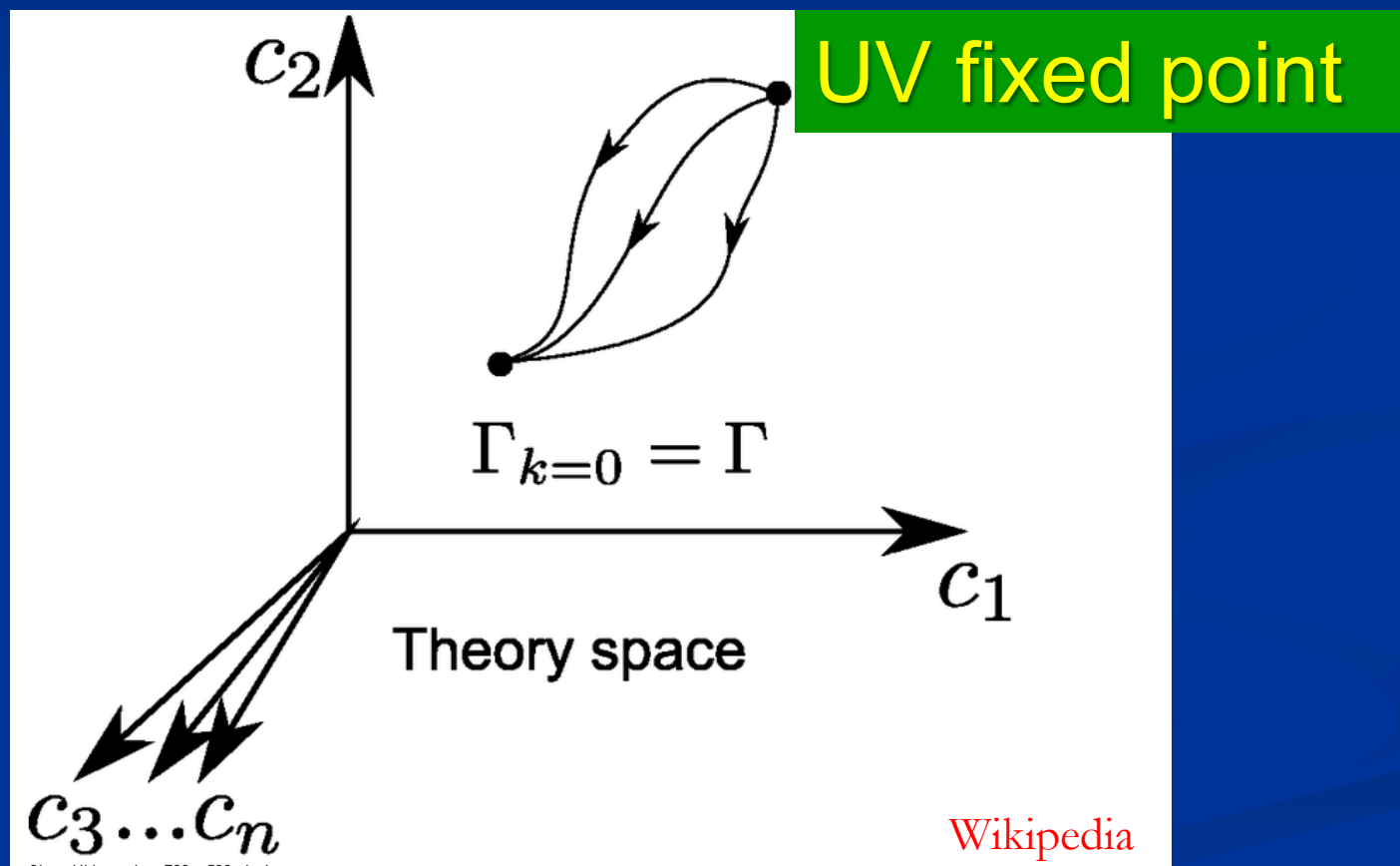
Quantum scale symmetry

- quantum fluctuations violate scale symmetry
- running dimensionless couplings
- at fixed points , scale symmetry is exact !
- quantum fluctuations can generate scale symmetry !

Functional renormalization : flowing action



Ultraviolet fixed point



Quantum scale symmetry

Exactly on fixed point:

No parameter with dimension of length or mass is present in the quantum effective action.

Then invariance under dilatations or global scale transformations is realized as a quantum symmetry.

Continuous global symmetry

Scale symmetry and Fermi scale

- Vacuum electroweak phase transition is (almost) second order, **including all effects from quantum fluctuations**
- Critical surface of second order phase transition: exact fixed point, **quantum scale symmetry**
- Scale symmetry guarantees **“naturalness”** of gauge hierarchy

C. Wetterich, Phys. Lett.B140(1984)215,
W. A. Bardeen, FERMILAB-CONF-95-391-T(1995)

Quantum Gravity

*Quantum Gravity is a
renormalisable quantum field theory*

Asymptotic safety

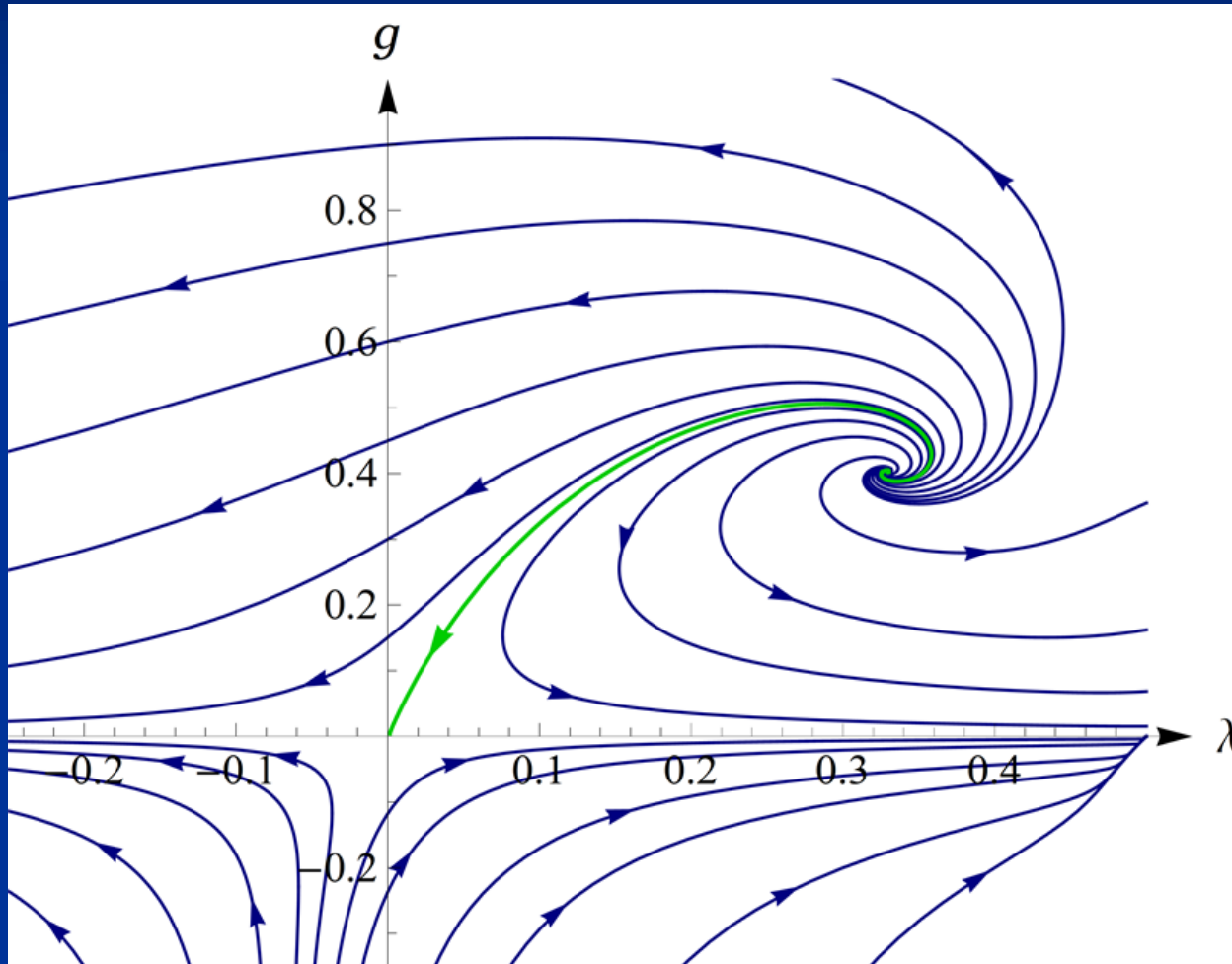
Asymptotic safety of quantum gravity

if UV fixed point exists :

*quantum gravity is
non-perturbatively renormalizable !*

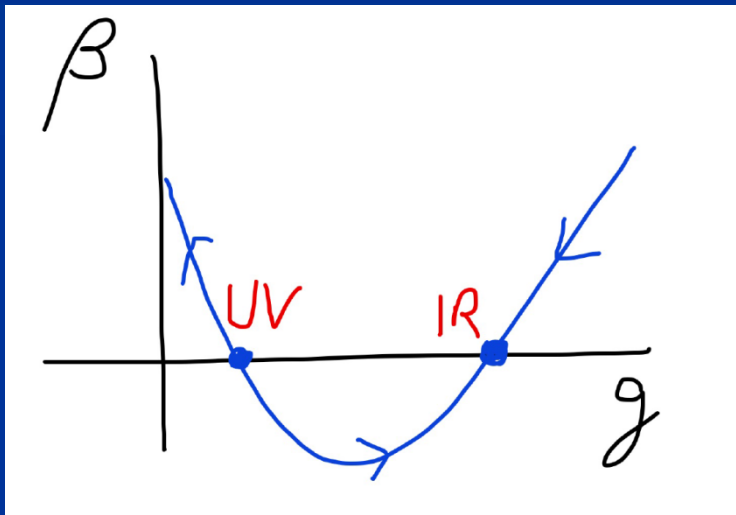
S. Weinberg , M. Reuter

UV- fixed point for quantum gravity

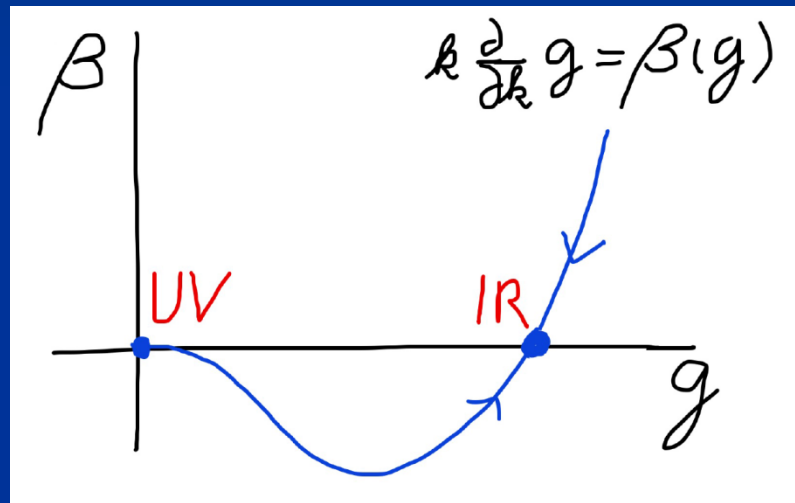


Wikipedia

Asymptotic safety



Asymptotic freedom



Relevant parameters yield undetermined couplings.
Quartic scalar coupling is not relevant and can
therefore be predicted.

a prediction...

Asymptotic safety of gravity and the Higgs boson mass

Mikhail Shaposhnikov

Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

Christof Wetterich

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

12 January 2010

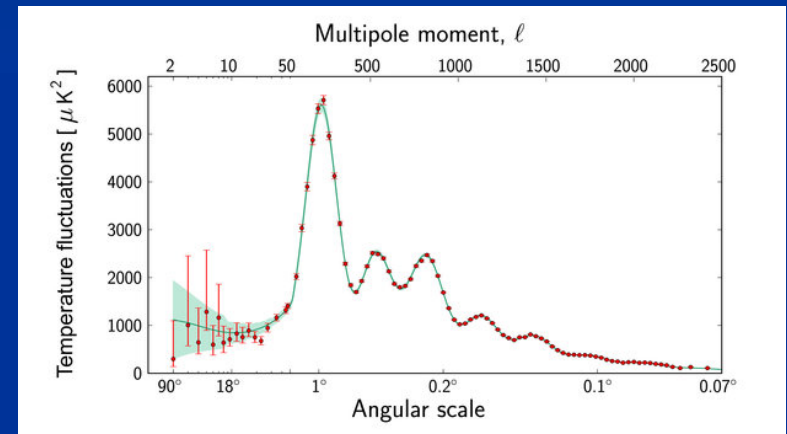
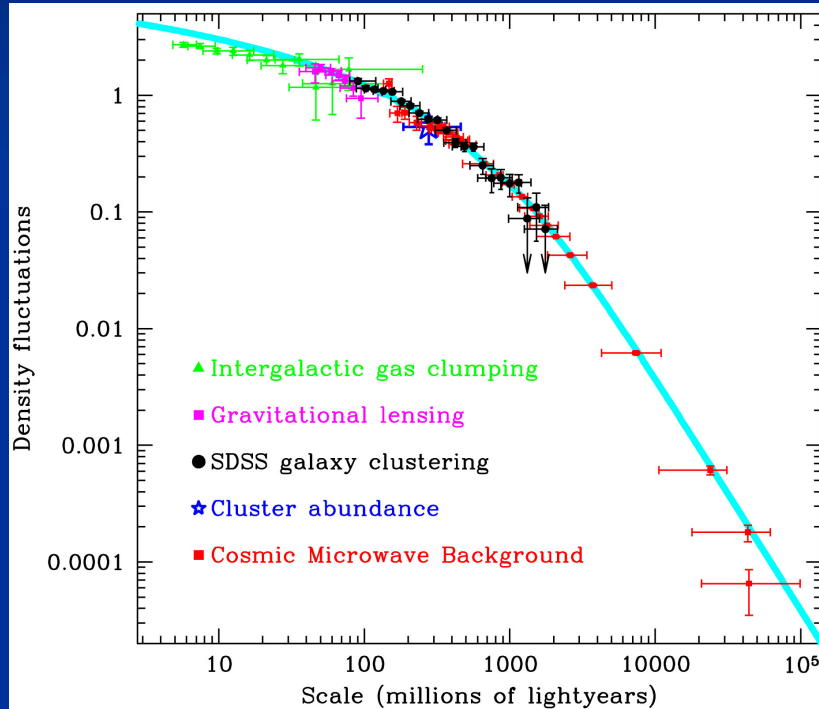
Abstract

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well.

s in $m_H = m_{\min} = 126$ GeV, with o

Quantum gravity with
scalar field –
the role of scale symmetry
for cosmology

Scale symmetry in cosmology ?

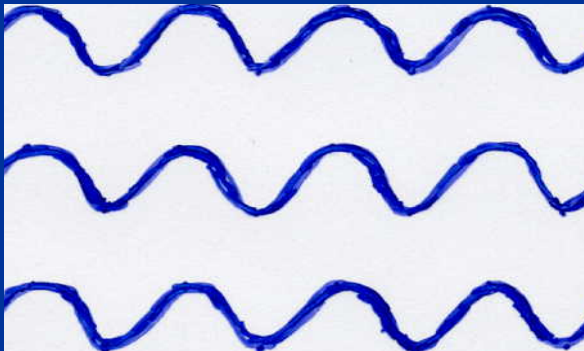
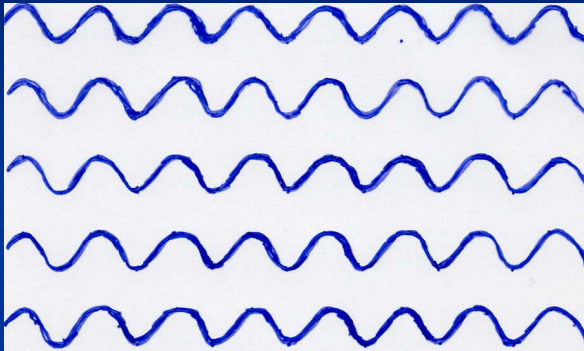


scales are present in cosmology

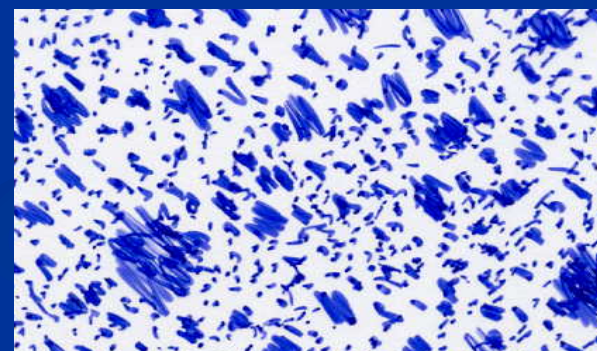
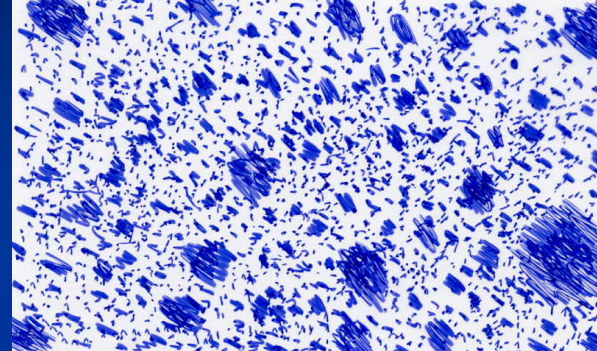
Spontaneous breaking of scale symmetry

- expectation value of scalar field breaks scale symmetry spontaneously
- massive particles are compatible with scale symmetry
- in presence of massive particles : sign of exact scale symmetry is exactly **massless Goldstone boson** — the dilaton

Scale symmetry



no scale symmetry



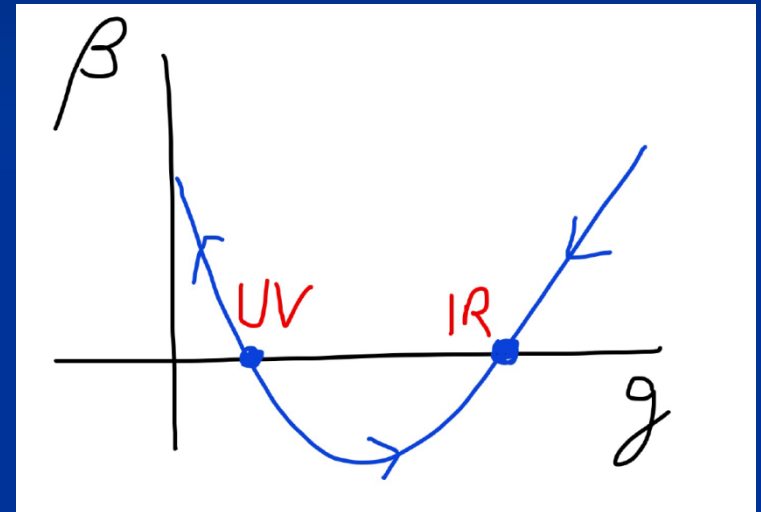
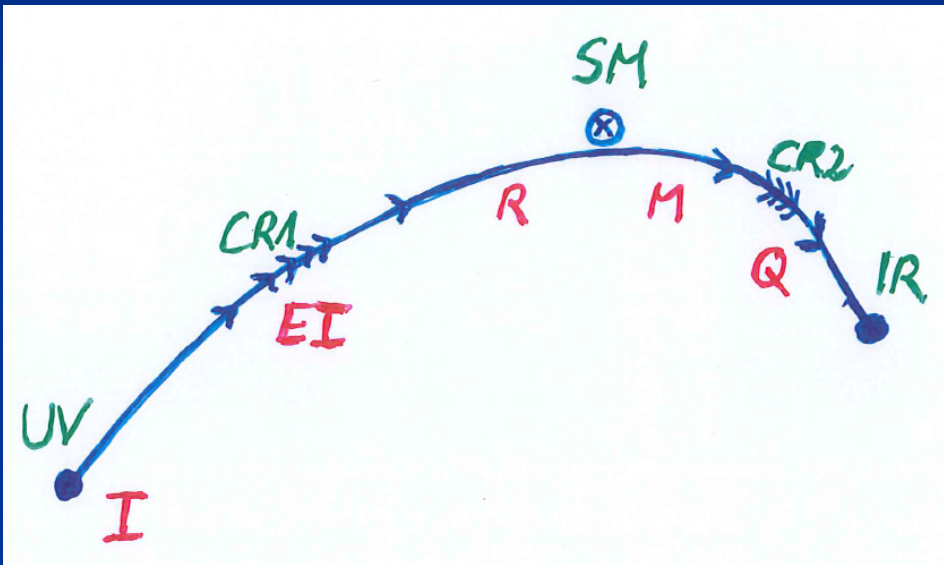
scale symmetry

only if no spontaneous symmetry breaking!

Approximate scale symmetry near fixed points

- UV : approximate scale invariance of primordial fluctuation spectrum from inflation
- IR : cosmon is pseudo Goldstone boson of spontaneously broken scale symmetry, tiny mass, responsible for dynamical Dark Energy

Possible consequences of crossover in quantum gravity



Realistic model for inflation and dark energy
with single scalar field

Scale symmetric standard model

■ Replace all mass scales by scalar field χ

(1) Higgs potential

$$U = \frac{\lambda_H}{2}(\varphi^\dagger\varphi - \epsilon\chi^2)^2 \quad \longrightarrow \quad \varphi_0^2 = \epsilon\chi^2$$

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$$g(\chi) = \bar{g} \quad \longrightarrow \quad \Lambda_{\text{QCD}} = \chi \exp\left(-\frac{1}{b_0\bar{g}^2}\right) \quad b_0 = \frac{1}{16\pi^2} \left(22 - \frac{4}{3}N_f\right)$$

(3) Similar for all dimensionless couplings

*Quantum effective action for standard model does
not involve intrinsic mass or length*

Quantum scale symmetry

For $\chi_0 \neq 0$ massless Goldstone boson

Two scale symmetries

- Gravity scale symmetry:

includes transformation of metric and scalar singlet

- Particle scale symmetry:

metric and scalar singlet kept fixed

relative scaling of momenta with respect to Planck mass

variable gravity

*“Newton’s constant is not constant –
and particle masses are not constant”*

Πάντα ρεῖ



Variable Gravity

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

Scale symmetry in variable gravity (IR – fixed point)

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \cancel{\mu^2 \chi^2} + \frac{1}{2} (\cancel{B(\chi/\mu)} - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

quantum effective action,
variation yields field equations

Einstein gravity : $\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} M^2 R \right\}$

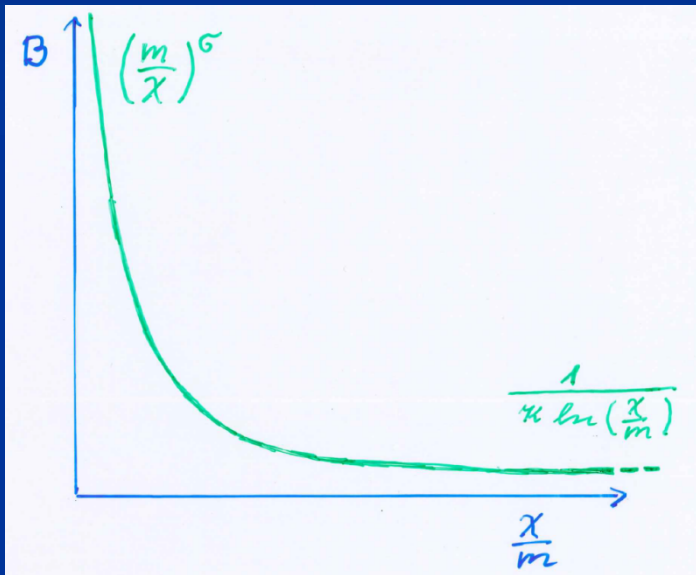
Variable Gravity

- Scalar field coupled to gravity
- Effective Planck mass depends on scalar field
- Simple quadratic scalar potential involves intrinsic mass μ
- Nucleon and electron mass proportional to dynamical Planck mass

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

Kinetic B :

Crossover between two fixed points



assumption:

running
coupling obeys
flow equation

$$\mu \frac{\partial B}{\partial \mu} = \frac{\kappa \sigma B^2}{\sigma + \kappa B}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

m : scale of crossover

can be exponentially larger than intrinsic scale μ

Four-parameter model

- model has four dimensionless parameters
- three in kinetic :
 - $\sigma \sim 2.5$
 - $\kappa \sim 0.5$
 - $c_t \sim 14$ (or m/μ)
- one parameter for growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than Λ CDM

Cosmology

Add matter and radiation

(standard model + dark matter)

Solve field equations...

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

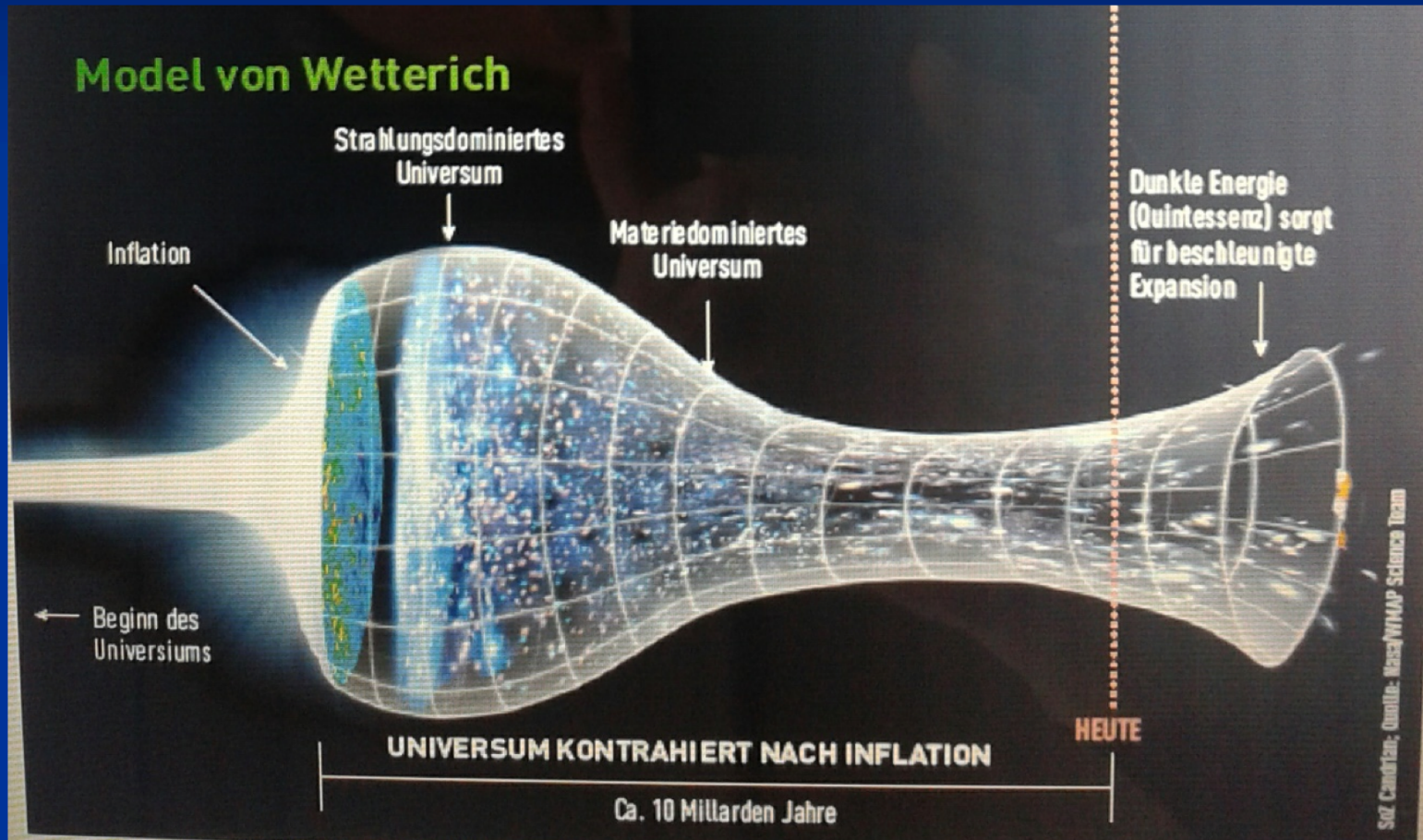
Model is compatible with present observations

Together with variation of neutrino mass over
electron mass in present cosmological epoch :
model is compatible with all present
observations, including inflation and dark energy

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

Strange evolution of Universe



Sonntagszeitung Zürich, Laukenmann

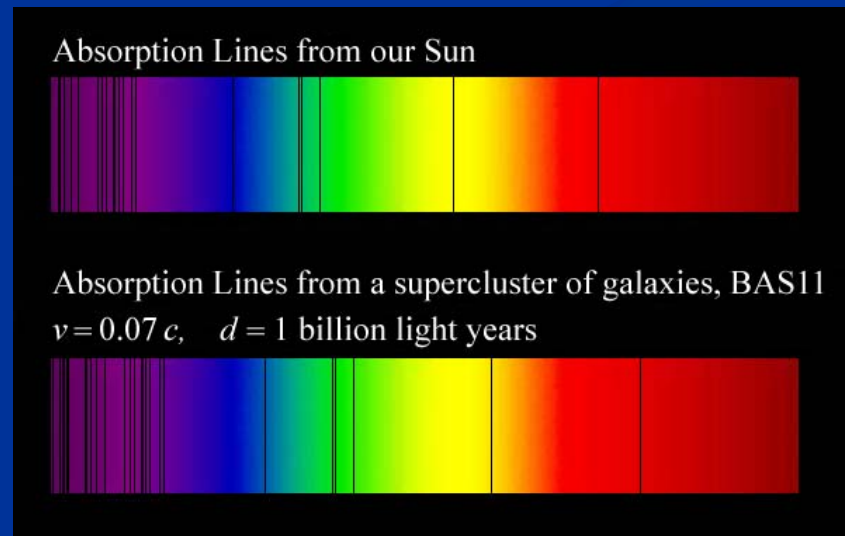
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Expanding Universe or
shrinking atoms ?

Do we know that the Universe expands ?

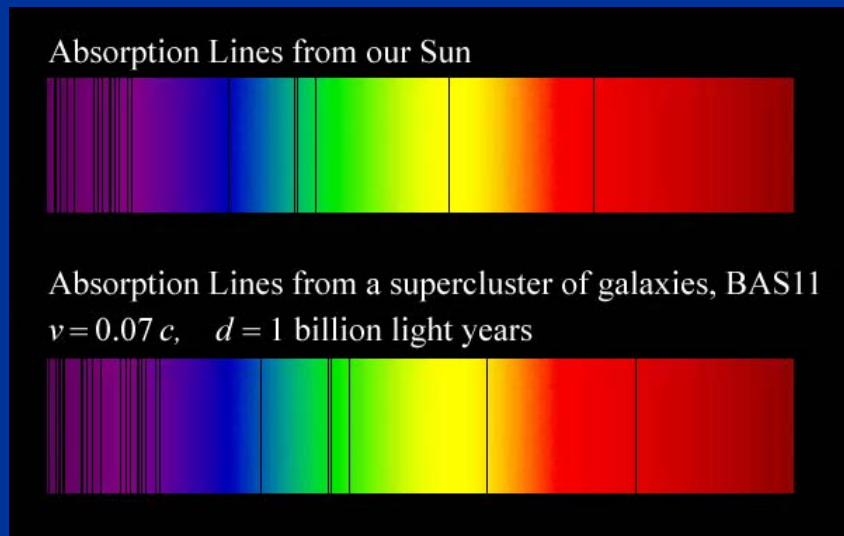
instead of redshift due to expansion :

smaller frequencies have been emitted in the past,
because electron mass was smaller !



Why do we see redshift of photons emitted in the distant past ?

photons are more red because they have been **emitted** with longer wavelength



frequency \sim mass

wavelength \sim
atoms size

What is increasing ?

Ratio of distance between galaxies
over size of atoms !

atom size constant : expanding geometry

alternative : shrinking size of atoms

How can particle masses change with time ?

- Particle masses are proportional to scalar field χ .
Similar to Higgs field.
- Scalar field varies with time.
- Ratios of particle masses are independent of χ and therefore remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Dimensionless couplings are independent of χ .

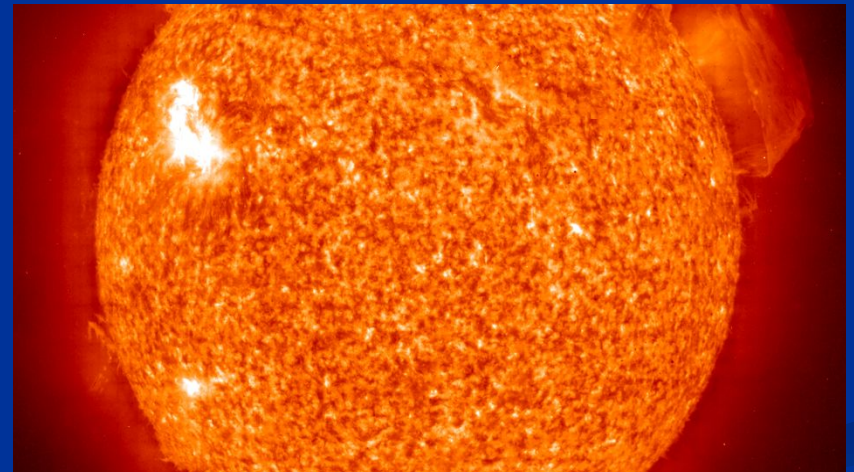
A deep-field astronomical image, likely from the Hubble Space Telescope, showing a vast field of galaxies and distant stars. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms, scattered across a black background. The text "Hot big bang or freeze ?" is overlaid in the upper center in a yellow, serif font.

Hot big bang or freeze ?

Do we know that the temperature was higher in the early Universe than now ?

Cosmic microwave radiation , nucleosynthesis

instead of
higher temperature :
smaller particle masses



Hot plasma ?

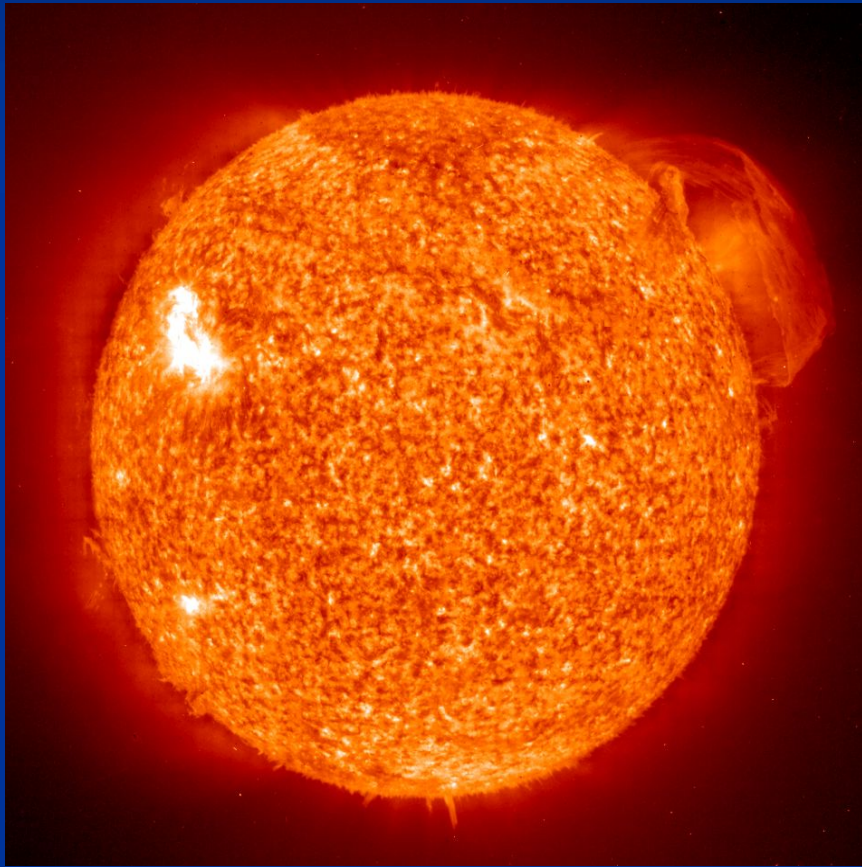
- Temperature in radiation dominated Universe :
 $T \sim \chi^{1/2}$ **smaller** than today
- Ratio temperature / particle mass :
 $T / m_p \sim \chi^{-1/2}$ **larger** than today
- T/m_p counts ! This ratio decreases with time.
- Nucleosynthesis , CMB emission as in standard cosmology !

Freeze Universe

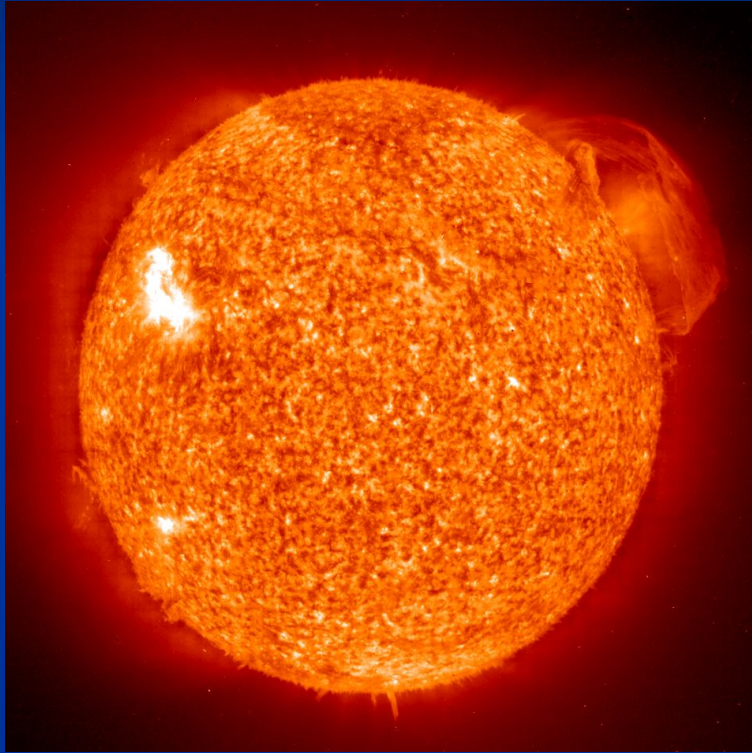
*The Universe may have started very cold,
and only later heat up.*

Freeze picture of the Universe

Big bang or freeze ?



Big bang or freeze ?



freeze picture :
only rods for measurements
(masses) are different !



Big bang is not wrong,

but alternative pictures exist !

Einstein frame

- “Weyl scaling” maps variable gravity model to Universe with fixed masses and standard expansion history.
- Exact equivalence of different frames !
“different pictures”
- Standard gravity coupled to scalar field.

Einstein frame

Weyl scaling :

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

effective action in Einstein frame :

$$\Gamma = \int_x \sqrt{g'} \left\{ -\frac{1}{2} M^2 R' + V'(\varphi) + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}$$

$$V'(\varphi) = M^4 \exp \left(-\frac{\alpha \varphi}{M} \right)$$

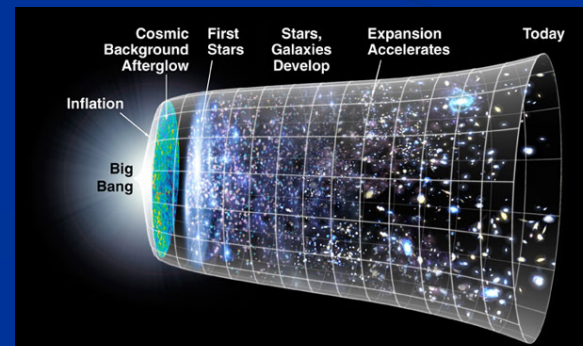
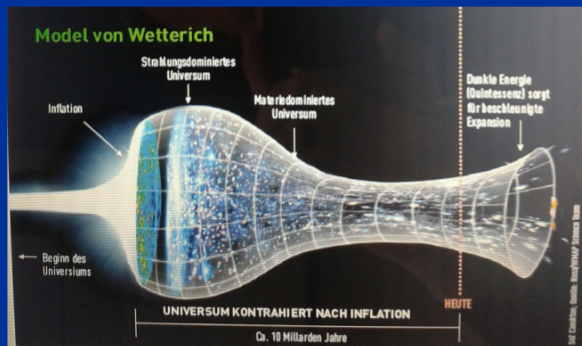
$$k^2 = \frac{\alpha^2 B}{4}$$

Field relativity

Weyl scaling :

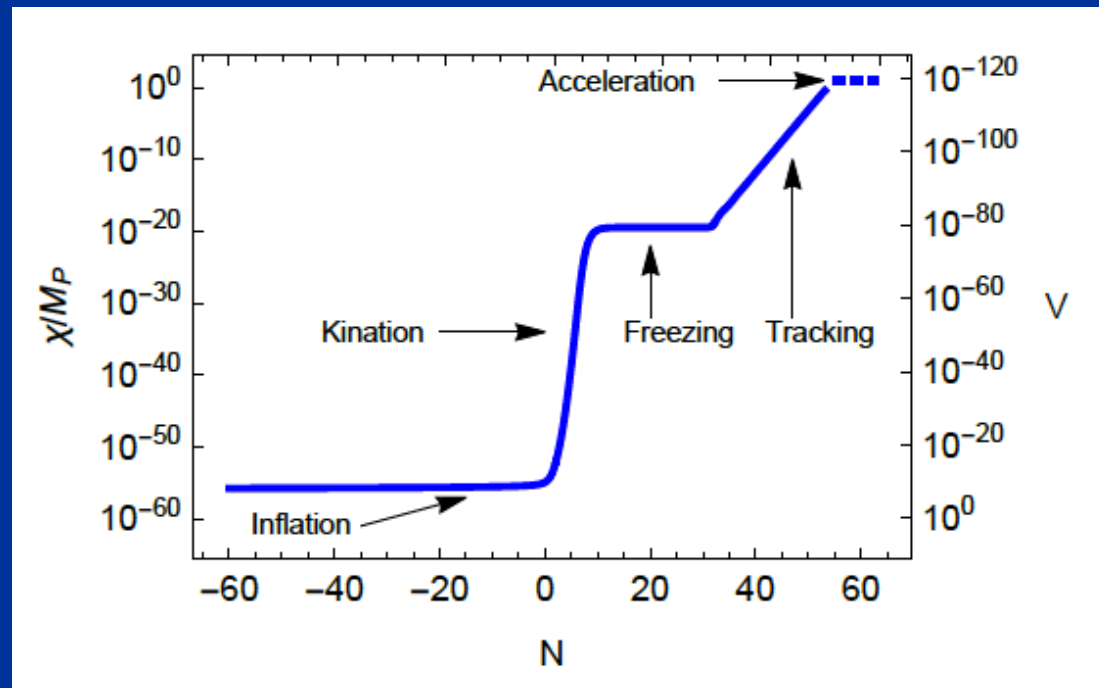
$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

changes geometry,
not a coordinate transformation



Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



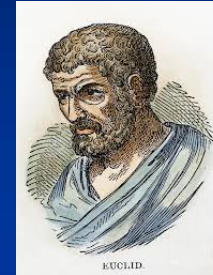
Field relativity :

different pictures of cosmology

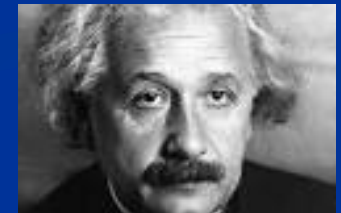
- same physical content can be described by different pictures
- related by field – redefinitions ,
e.g. Weyl scaling , conformal scaling of metric
- observables cannot depend on choice of fields
- metric is one of the fields
- which picture is usefull ?

Relativity of geometry

- Euclid ... Newton : space and time are absolute



- Special relativity : space and time depend on observer
- General relativity : space-time is influenced by matter (including radiation)
geometry is independent of coordinates
geometry is observable
- Field relativity : geometry is relative



*Space-time is a description
of correlations between “matter”.*

*Observables cannot depend on choice of
fields used to describe them.*

Different pictures for geometry exist.

Why should you care about the freeze picture of the Universe ?

Some aspects are understood easier :

- Natural tiny present dark energy
- Beginning of Universe
- Role of scale symmetry
- Range of impact of quantum gravity

What is Dark Energy ?

Dark energy is energy density of scalar field χ

$$\rho = V + \text{kinetic term}$$

$$V = \mu^2 \chi^2$$

$$p = -V + \text{kinetic term}$$

Dark energy is dynamical if χ changes with time

**No small parameter for
dark energy**

Four-parameter model

- model has four dimensionless parameters
- three in kinetic :
 - $\sigma \sim 2.5$
 - $\kappa \sim 0.5$
 - $c_t \sim 14$ (or m/μ)
- one parameter for growth rate of neutrino mass over electron mass : $\gamma \sim 8$
- + standard model particles and dark matter : sufficient for realistic cosmology from inflation to dark energy
- no more free parameters than Λ CDM

asymptotically vanishing cosmological „constant“

- What matters : Ratio of potential divided by fourth power of Planck mass

$$\frac{V}{\chi^4} = \frac{\mu^2 \chi^2}{\chi^4} = \frac{\mu^2}{\chi^2}$$

$$V = \mu^2 \chi^2$$

- vanishes for $\chi \rightarrow \infty$!

small dimensionless number ?

- needs two intrinsic mass scales
- standard approach : V and M (cosmological constant and Planck mass)
- variable gravity : Planck mass moving to infinity , with fixed or moderately increasing V
 - ➔ ratio vanishes asymptotically !

Quintessence

Dynamical dark energy ,
generated by scalar field (cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

**homogeneous dark energy
influences recent cosmology**

- of same order as dark matter -

Original models do not fit the present observations
.... modifications
(different growth of neutrino mass)

No tiny dimensionless parameters (except gauge hierarchy)

- one mass scale $\mu = 2 \cdot 10^{-33} \text{ eV}$
- one time scale $\mu^{-1} = 10^{10} \text{ yr}$
- Planck mass does not appear as parameter
- Planck mass grows large dynamically
- Dark energy is tiny because Universe is old

Slow Universe

Asymptotic solution in
freeze frame :

$$H = \frac{\mu}{\sqrt{3}} , \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

$$\mu = 2 \cdot 10^{-33} \text{ eV}$$

Expansion or shrinking always slow ,
characteristic time scale of the order of the age of the
Universe : $t_{\text{ch}} \sim \mu^{-1} \sim 10 \text{ billion years} !$

Hubble parameter of the order of **present** Hubble
parameter for all times , including inflation and big bang !
Slow increase of particle masses !

infinite past

Infinite past : slow inflation

$\sigma = 2$: field equations

$$\ddot{\chi} + \left(3H + \frac{1}{2} \frac{\dot{\chi}}{\chi} \right) \dot{\chi} = \frac{2\mu^2 \chi^2}{m}$$

$$H = \sqrt{\frac{\mu^2}{3} + \frac{m\dot{\chi}^2}{6\chi^3}} - \frac{\dot{\chi}}{\chi}$$

approximative
solution

$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

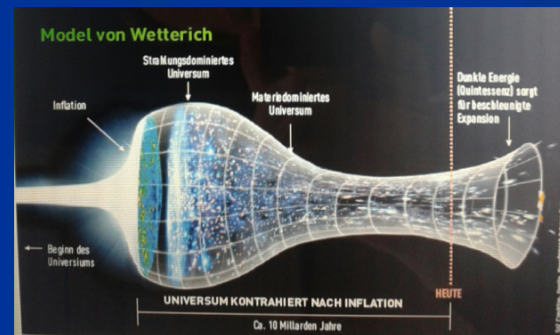
particles become massless in infinite past !

Eternal Universe

Asymptotic solution in
freeze frame :

$$H = \frac{\mu}{\sqrt{3}}, \quad \chi = \frac{3^{\frac{1}{4}} m}{2\sqrt{\mu}} (t_c - t)^{-\frac{1}{2}}$$

- solution valid back to the infinite past in physical time
- no singularity
- physical time to infinite past is infinite

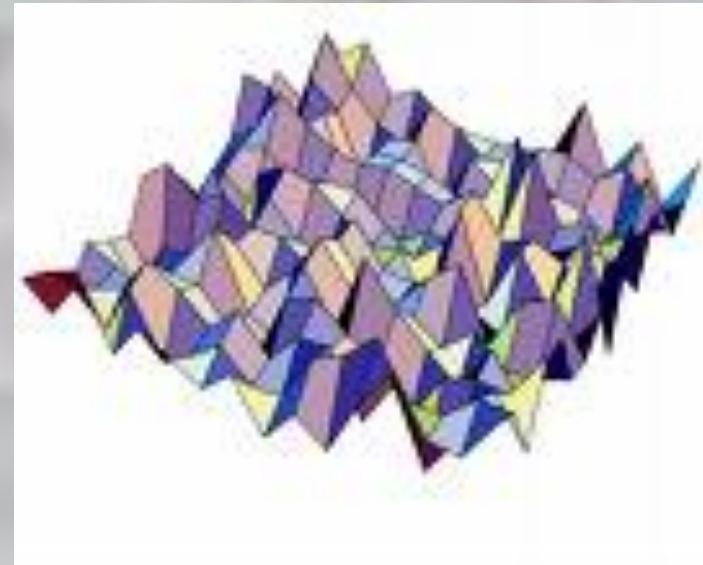


Eternal light-vacuum

Everywhere almost nothing
only fluctuations

All particles move
with light velocity,
similar to photons

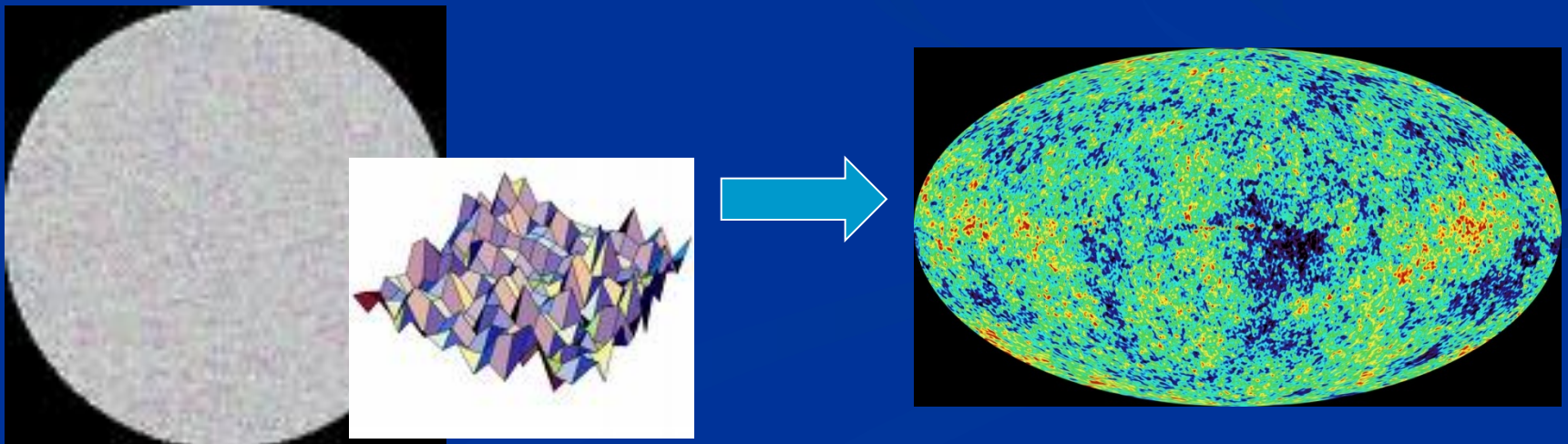
strength of gravity
much stronger than today



In the beginning was light-like emptiness.

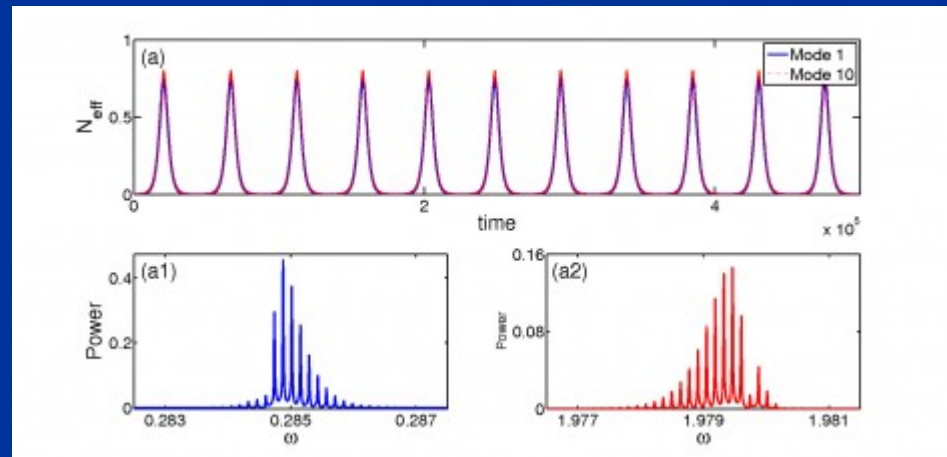
Eternal light-vacuum is unstable

- Slow increase of particle masses and weakening of gravity
- Only slow change of space-time geometry
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage 5000 billion years ago.



Physical time

count oscillations



Physical time

field equation for scalar field mode

$$(\partial_\eta^2 + 2Ha\partial_\eta + k^2 + a^2m^2)\varphi_k = 0$$

$$\varphi_k = \frac{\tilde{\varphi}_k}{a} \quad \left\{ \partial_\eta^2 + k^2 + a^2 \left(m^2 - \frac{R}{6} \right) \right\} \tilde{\varphi}_k = 0$$

determine **physical time** by counting number of oscillations

$$\tilde{t}_p = n_k$$

$$n_k = \frac{k\eta}{\pi}$$

(m=0)

Physical time

- counting : discrete
- invariant under field transformations
- same in all frames

*Big bang singularity
in Einstein frame is
field singularity !*

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu} , \quad \varphi = \frac{2M}{\alpha} \ln \left(\frac{\chi}{\mu} \right)$$

choice of frame with constant particle masses is not well suited if physical masses go to zero !

Field - singularity

- Big Bang is field - singularity
- similar (but not identical with)
coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$



Conclusions

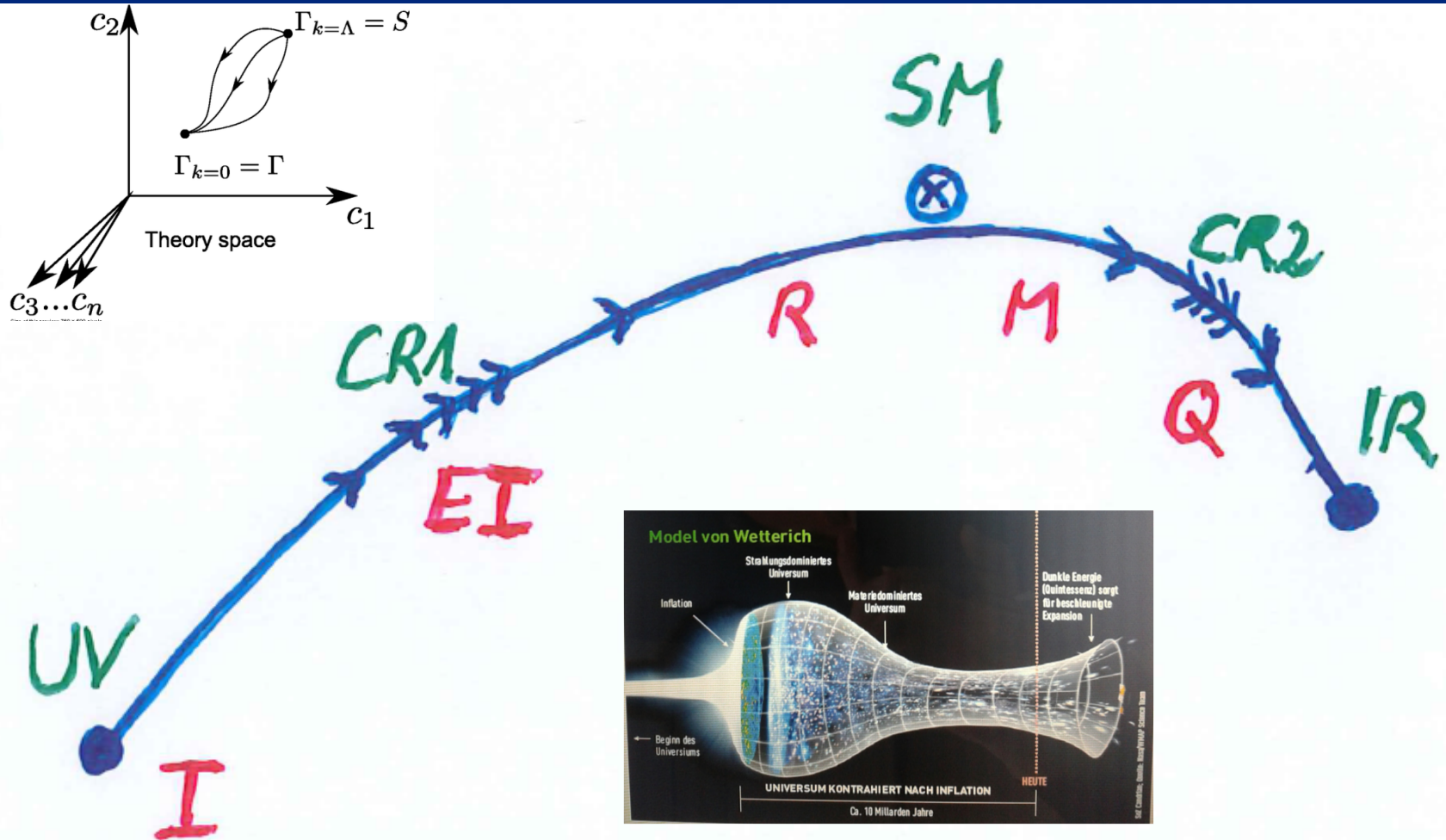
Quantum gravity may be observable in
dynamics of present Universe

Fixed points and scale symmetry crucial

Big bang singularity is artefact
of inappropriate choice of field variables –
no physical singularity

end

Crossover in quantum gravity and cosmology

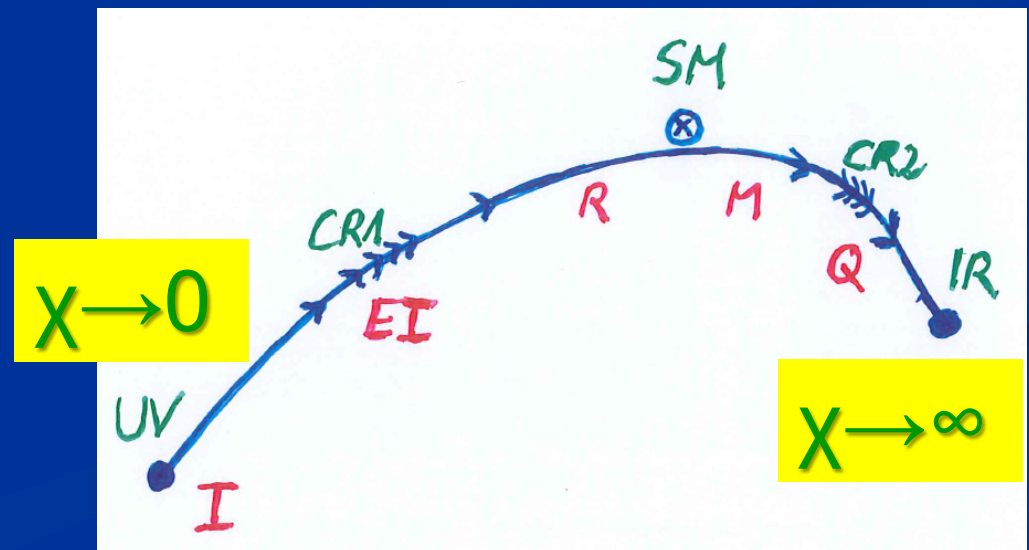


Cosmological solution : crossover from UV to IR fixed point

- Dimensionless functions as B depend only on ratio μ/χ .
- IR: $\mu \rightarrow 0$, $\chi \rightarrow \infty$
- UV: $\mu \rightarrow \infty$, $\chi \rightarrow 0$

$$\Gamma = \int_x \sqrt{g} \left\{ -\frac{1}{2} \chi^2 R + \mu^2 \chi^2 + \frac{1}{2} (B(\chi/\mu) - 6) \partial^\mu \chi \partial_\mu \chi \right\}$$

Cosmology makes
crossover between
fixed points by
variation of χ .



Renormalization flow and cosmological evolution

- renormalization flow as function of μ

is mapped by dimensionless functions to

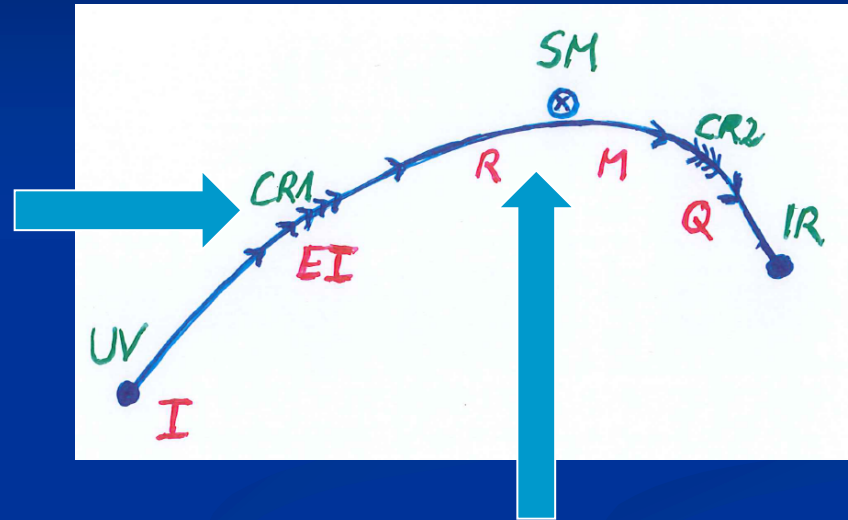
- field dependence of effective action on scalar field χ

translates by solution of field equation to

- dependence of cosmology on time t or η

Scaling solution

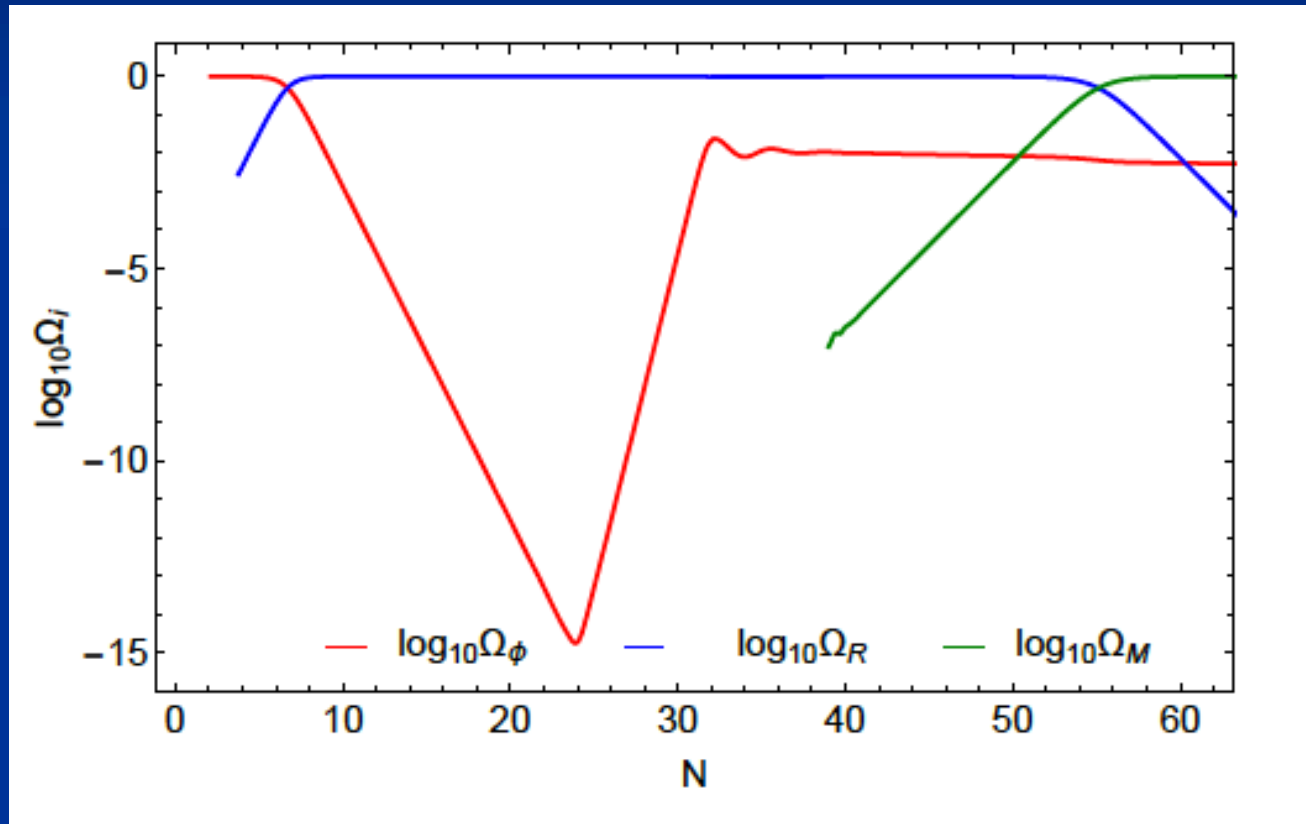
after end of inflation



Dark Energy decreases
similar to radiation and
matter

scaling solution with
few percent
of Early Dark Energy

Evolution of dark energy fraction

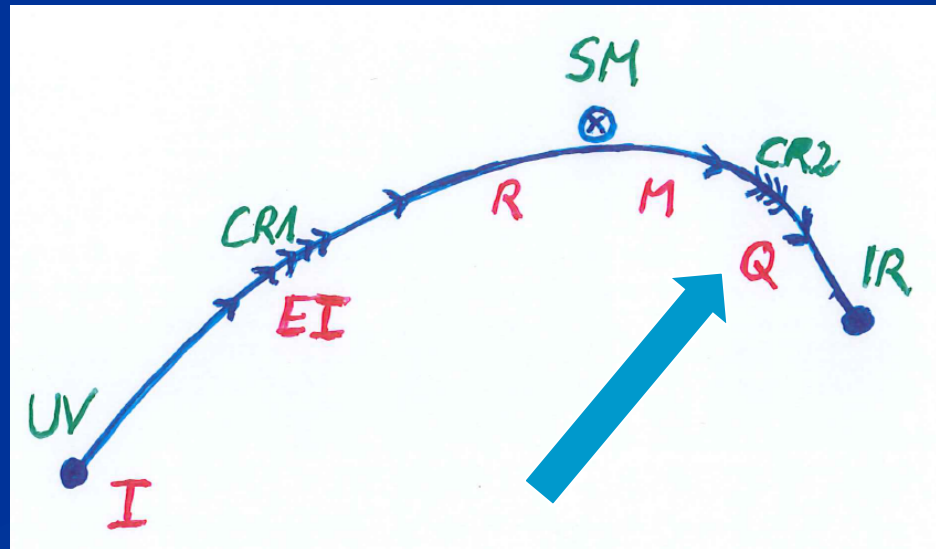


J. Rubio,...

Growing neutrino masses and quintessence

Second stage of crossover

- from SM to IR
- in sector Beyond Standard Model
- affects neutrino masses first (seesaw or cascade mechanism)



Varying particle masses at onset of second crossover

- Except for neutrinos all particle masses are proportional to χ .
- Ratios of particle masses remain constant.
- Compatibility with observational bounds on time dependence of particle mass ratios.
- Neutrino masses show stronger increase with χ , such that ratio neutrino mass over electron mass grows.

Cosmic trigger

- Stop of evolution of scalar field when neutrinos become non-relativistic
- Transition from scaling solution to (almost) cosmological constant

connection between dark energy and neutrino properties

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.27 \left(\frac{\gamma m_\nu(t_0)}{eV} \right)^{\frac{1}{4}} 10^{-3} eV$$

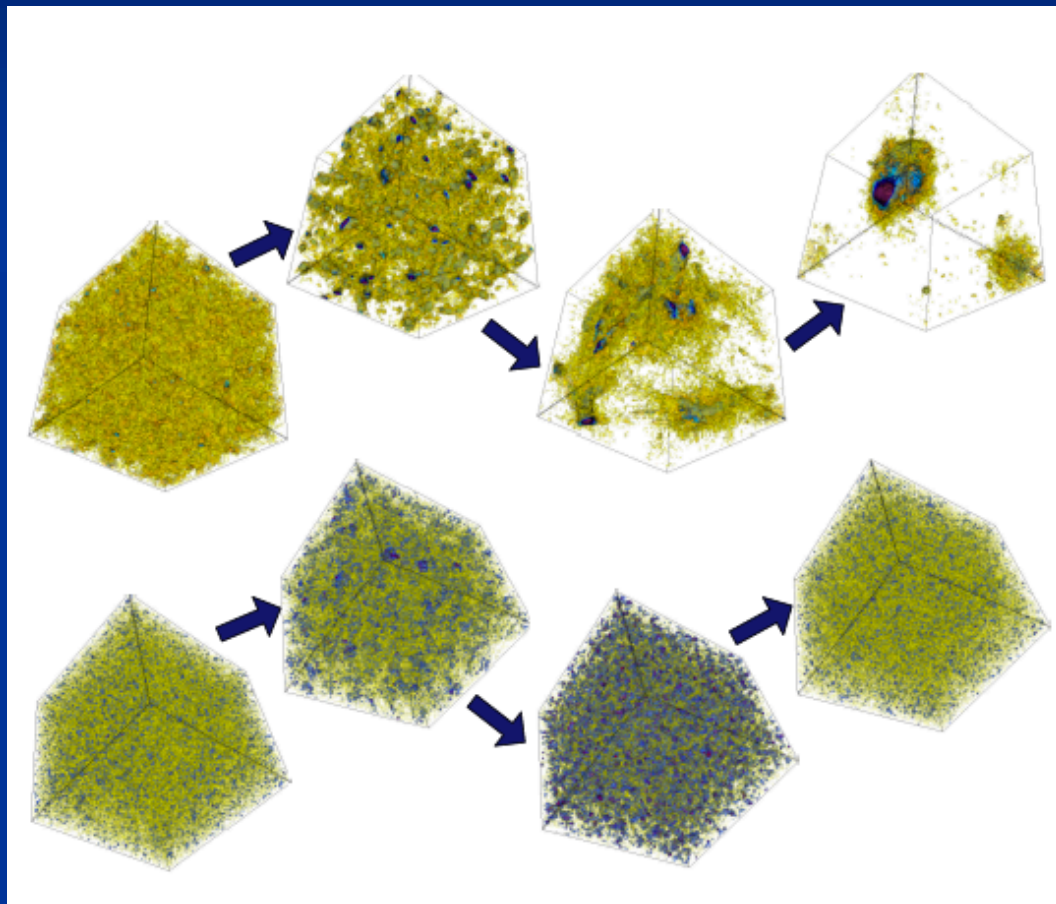
L.Amendola,
M.Baldi, ...

present dark energy density given by neutrino mass

present equation
of state given by
neutrino mass !

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12eV}$$

Neutrino lumps

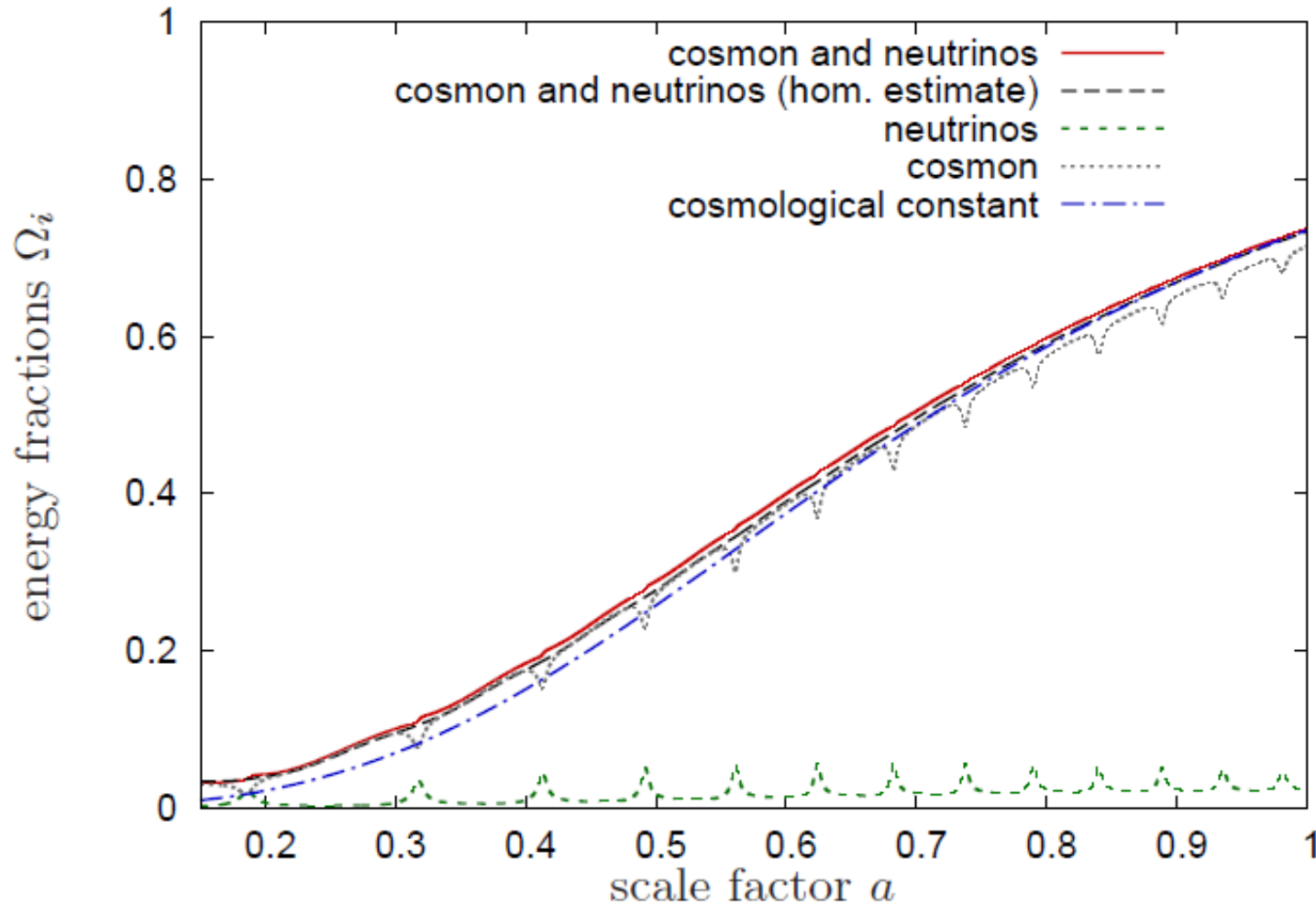


large m_ν

small m_ν

Casas, Pettorino,...

Evolution of dark energy similar to Λ CDM



Compatibility with observations and possible tests

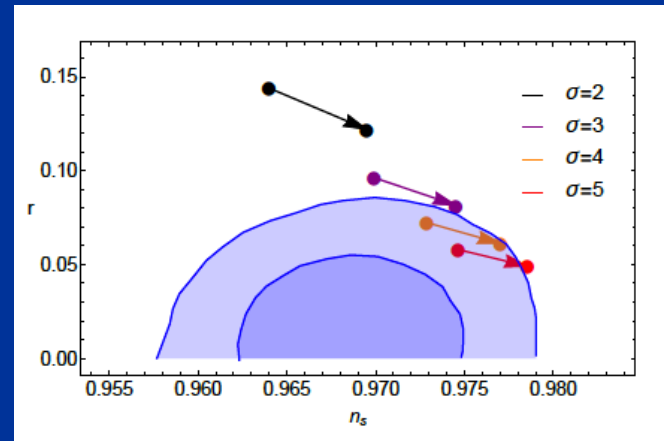
- Realistic inflation model
- Almost same prediction for radiation, matter, and Dark Energy domination as Λ CDM
- Presence of small fraction of Early Dark Energy
- Large neutrino lumps

Simplicity

simple description of **all** cosmological epochs

natural incorporation of Dark Energy :

- inflation
- Early Dark Energy
- present Dark Energy dominated epoch



J.Rubio...

*In quantum gravity,
the graviton fluctuations can
play an important role on
distances as large as the
size of the Universe*

- for long range scalar fields and dynamical dark energy
- not for all quantities

Instability of graviton propagator

effective action

$$\Gamma = \int_x \sqrt{g} \left(-\frac{M^2}{2} R + V \right)$$

flat space:

$$G^{-1} = \frac{M^2 q^2}{4} - \frac{V}{2}$$

Instability for $V > 0$: "tachyonic mass term"

$$-\frac{2V}{M^2}$$

curved space:

$$G^{-1} = \sqrt{g} \left\{ \frac{M^2}{4} \left(-D^2 + \frac{2R}{3} \right) - \frac{V}{2} \right\}$$

Graviton barrier

Quantum gravity computation :

For $\chi \rightarrow \infty$

V cannot increase stronger than M^2 !

Instability of graviton propagator is avoided

Graviton barrier and solution of the cosmological constant problem

V cannot increase stronger than M^2 !

If M increases with χ , and for cosmological solutions where χ asymptotically diverges for time going to infinity:

Effective cosmological constant vanishes in infinite future

$$M = \chi \quad : \quad V = \mu^2 \chi^2$$

Amplitude of density fluctuations

small because of logarithmic running
near UV fixed point !

$$\mathcal{A} = \frac{(N+3)^3}{4} e^{-2c_t}$$

$$c_t = \ln \left(\frac{m}{\mu} \right) = 14.1. \quad \sigma=1$$

$$\frac{m}{\mu} = \frac{(N+3)^{\frac{3}{2}}}{2\sqrt{\mathcal{A}}} = 1.32 \cdot 10^6 \left(\frac{N}{60} \right)^{\frac{3}{2}}$$

$$B^{-1} - \frac{\kappa}{\sigma} \ln B = \kappa \left[\ln \left(\frac{\chi}{\mu} \right) - c_t \right] = \kappa \ln \left(\frac{\chi}{m} \right)$$

N : number of e – foldings at horizon crossing