QM

$$H = \frac{p^2}{2m} + V = H_0 + V$$

$$e^{-\tau H} \qquad \tau > 0 \, ,$$ wohldef. wenn H nach unten beschr.

$$e^{-H\tau} = \left(e^{-H\varepsilon}\right)^N \qquad \Delta\tau = \varepsilon = \frac{\tau}{N}$$

Trotter:

$$e^{-H\tau} = \lim_{N\to\infty} \left(e^{-H_0\varepsilon}\, e^{-V\cdot\varepsilon}\right)^N$$

Suzuki - Trotter

beruht auf: $$e^{-V\frac{\varepsilon}{2}}\, e^{-H_0\varepsilon}\, e^{-V\frac{\varepsilon}{2}} = e^{-(H_0 + V_\varepsilon)\varepsilon}$$

$$V_\varepsilon = V + O(\varepsilon^3)$$

$\hookrightarrow$

$$e^{-H(\varepsilon)\tau} \equiv \Big(\underbrace{e^{-V\frac{\varepsilon}{2}}\, e^{-H_0\varepsilon}\, e^{-V\frac{\varepsilon}{2}}}_{T(\varepsilon)}\Big)^N$$

$$H(\varepsilon) = H + O(\varepsilon^3)$$

$$\equiv \left(T(\varepsilon)\right)^N$$

$$T(\varepsilon) := e^{-V\frac{\varepsilon}{2}} \, e^{-H_0 \varepsilon} \, e^{-V\frac{\varepsilon}{2}}$$

$$\langle q' | T(\varepsilon) | q \rangle$$

$$= e^{-V(q')\frac{\varepsilon}{2}} \langle q' | e^{-H_0 \varepsilon} | q \rangle \, e^{-V(q)\frac{\varepsilon}{2}}$$

$$\langle q' | e^{-H_0 \varepsilon} | q \rangle = \left(\frac{m}{2\pi \varepsilon} \right)^{1/2} e^{-\frac{m}{2\varepsilon}(q - q')^2}$$

$\hookrightarrow$

$$\langle q' | e^{-H\tau} | q \rangle = \int\limits_{\substack{q(\cdot) \\ q(0) = q \, ; \; q(\tau) = q'}} \mathcal{D}[q] \, e^{-S[q(\cdot)]}$$

$$S[q(\cdot)] = \int_0^\tau d\tau' \left(\frac{m}{2} \dot{q}^2 + V \right)$$

$$\int \mathcal{D}[q(\cdot)] = \lim_{N \to \infty} \left(\frac{m}{2\pi \varepsilon} \right)^{N/2} \int dq_{N-1} \; \dots \; dq_1$$

$$q_1 = q(\varepsilon)$$

Feynman - Kac

H_0 ms. Maß

Gauss'scher Zufallsweg.

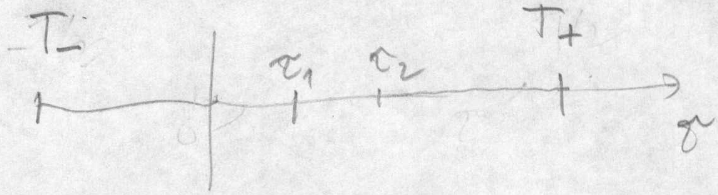
$$\langle q' | e^{-H\tau} | q \rangle$$

$$= \int \underbrace{D[q]\, e^{-S_0[q]}}_{d\mu[q]} e^{-S_1[q]}$$

$$= \int D[q]\, e^{-S_0[q]} \cdot \left(\frac{\int d\mu[q]\, e^{-S_1[q]}}{\int d\mu[q]} \right)$$

$$= \left(\frac{m}{2\pi\tau}\right)^{1/2} e^{-\frac{m}{2\tau}(q-q')^2} \cdot \left\langle e^{-\int_0^\tau d\tau' V(q(\tau'))\, d\tau'} \right\rangle$$

Korrelation



$$\langle q(\tau_2)\; q(\tau_1) \rangle \qquad \left(= \langle q(\tau_1)\; q(\tau_2) \rangle \right)$$

$$:= \int\limits_{\substack{q(T_-)=q_- \\ q(T_+)=q_+}} D[q(\cdot)]\; e^{-S[q(\cdot)]} \cdot q(\tau_2)\; q(\tau_1)$$

Schreibe als:

$$\int dq_1 \int dq_2 \int\limits_{\substack{q(\tau_2)=q_2 \\ q(T_+)=q_+}} e^{-S}\; q_2 \int\limits_{\substack{q(\tau_1)=q_1 \\ q(\tau_2)=q_2}} e^{-S}\; q_1 \int\limits_{\substack{q(T_-)=q_- \\ q(\tau)=q_1}} e^{-S}$$

$$= \int dq_1 dq_2 \langle q_+ | e^{-H(T_+-\tau_2)} | q_2 \rangle\, q_2 \langle q_2 | e^{-H(\tau_2-\tau_1)} | q_1 \rangle\, q_1 \langle q_1 | e^{-H(\tau_1-T_-)} | q_- \rangle$$

$$\int dq_i\, | q_i \rangle\, q_i \langle q_i | = Q$$

$$= \langle q_+ | e^{-HT_+}\; Q(\tau_2)\; Q(\tau_1)\, e^{+HT_-} | q_- \rangle$$

$$Q(\tau) = e^{H\tau}\, Q\, e^{-H\tau}$$