Skalares Feld auf Gitter, $d=4$ (3.1)

Kontinuum:

$$S(x_n, \dots, x_1)$$

$$= \frac{\int D\phi \, e^{-S[\phi]} \, \phi(x_n) \cdots \phi(x_1)}{\int D\phi \, e^{-S[\phi]}}$$

$$S[\phi] = \int dx \left[\frac{1}{2} \phi (-\Box) \phi + \underbrace{V(\phi(x))}_{\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4} \right]$$

Diskretisierung:

$$\Big\{ x_n = \overset{\text{Gitterkonst.}}{a} \cdot n \Big\} \quad \text{kubisches Gitter}$$

$$n = (n_1, \dots n_4), \quad n_i \in \mathbb{Z}$$

$$\phi(x) \longrightarrow \phi(x_n)$$

$$\int dx \, \psi(x) \phi(x) \dots$$

$$\to \sum_n a^4 \psi(x_n) \phi(x_n) \dots$$

$$\Box \phi(x) \to \frac{1}{a^2} \sum_{\mu} \Big(\phi(x_n + \hat{\mu} a) + \phi(x_n - \hat{\mu} a) - 2 \phi(x_n) \Big)$$

3.2

$\hat{\phi}_n : a\,\phi(x_n)$ dim los.

$$S[\phi] \to \sum_n \left(-\frac{1}{2}\hat{\phi}_n \Box \hat{\phi}_n + V(\hat{\phi}_n) \right)$$

$$V(\hat{\phi}_n) = \frac{1}{2}\hat{m}^2 \hat{\phi}_n^2 + \frac{\hat{\lambda}}{4!}\hat{\phi}_n^4 \qquad \hat{\lambda} = \lambda$$

$\hat{m} = m \cdot a$ dim-los.

$$\int \mathcal{D}[\phi]\, e^{-S[\phi]} \to \int \prod d\hat{\phi}_n\, e^{-S[\hat{\phi}]}$$

Feldvars und Param. sind dimensionslos.

Bem!

$$-\sum_n \hat{\phi}_n (\Box \hat{\phi})_n = -\sum_n \sum_\mu (\hat{\phi}_{n+\mu} + \hat{\phi}_{n-\mu} - 2\hat{\phi}_n)\hat{\phi}_n$$

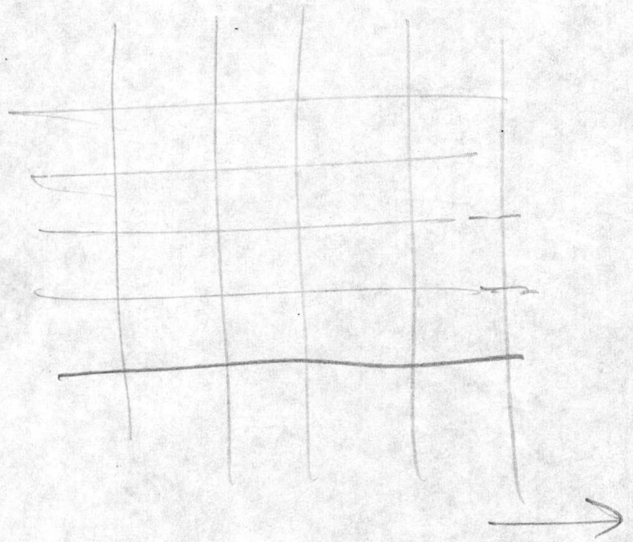
$$= \sum_{n,\mu} (\hat{\phi}_{n+\mu} - \hat{\phi}_n)^2$$

$$= 2\sum_n \left(-\sum_\mu \hat{\phi}_{n+\mu}\hat{\phi}_n + d\,\hat{\phi}_n^2 \right)$$

Transfer-Matrix

$(d=4)$

n_4



n_i

"$\wedge$" unterdrückt.

n_4 Ebenen: $\left\{ \phi(n_4, \vec{n}) = \phi_{n_4}(\vec{n}) \right\}$

- $\langle \phi_{n_4+1} | T | \phi_{n_4} \rangle$

$$\sim e^{-\sum_{\vec{n}} \left[\frac{1}{2}\left(\phi_{n_4+1}(\vec{n}) - \phi_{n_4}(\vec{n})\right)^2 + \frac{1}{2}\left(\sum_{k=1,..,3} \frac{1}{2}\left(\phi_{n_4}(\vec{n}+\hat{k}) - \phi_{n_4}(\vec{n})\right)^2 + V\left(\phi_{n_4}(\vec{n})\right)\right) + \left(n_4 \to n_4+1\right) \right]}$$

3.4

$$T \sim e^{-\frac{1}{2}H_1} e^{-H_0} e^{-\frac{1}{2}H_1}$$

$$H_0 \Psi[\phi] = \frac{1}{2}\left(\frac{1}{i}\frac{\delta}{\delta\phi(\vec{n})}\right)^2 \Psi[\phi]$$

$$H_1 \Psi[\phi] = \left(\sum_{\vec{n}} \frac{1}{2}\sum_{k=1,\ldots,3}\left(\phi(\vec{n}+\hat{k}) - \phi(\vec{n})\right)^2 + V(\phi(\vec{n}))\right)\Psi[\phi]$$

siehe S. 2.17

Skal. Produkt: $(\Psi', \Psi) = \int \prod_{\vec{n}} d\phi(\vec{n}) \; \overline{\Psi'}[\phi]\,\Psi[\phi]$

e^{-H_0} positiv Op. (H_0 bereits positiv)

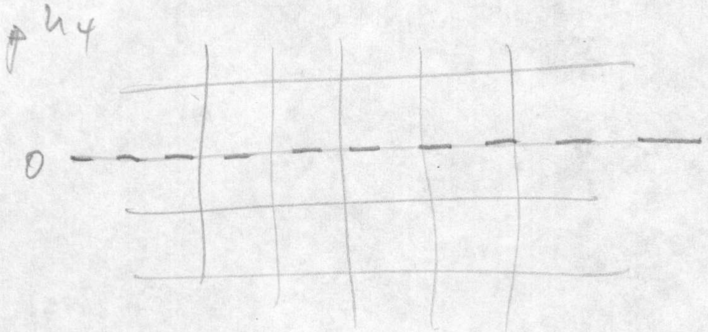
↳ T positiv

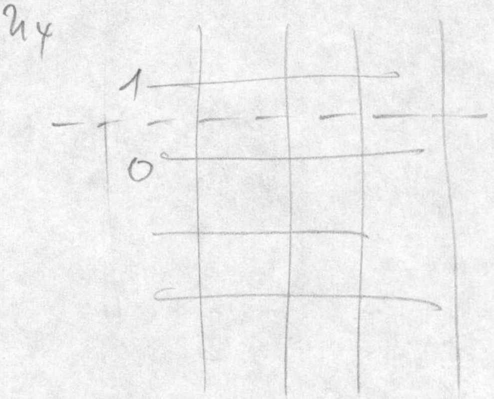
[A positiv $\Rightarrow$ $\langle\Psi|B^* A B|\Psi\rangle$

$= \langle B\Psi | A | B\Psi\rangle \geq 0$]

Reflexions Positivität

Osterwalder, Seiler





- "Site-reflection"

$$n_4 \to -n_4$$

"Link-reflection"

$$n_4 \to 1-n_4$$

"Site-refle"

- $S = S_+ + S_-$

S_+: $n_4 \geq 0$, S_-: $n_4 \leq 0$

$$S_- = \Theta S_+$$

3.6

$$\langle \Theta f_+ \; f_+ \rangle$$

$$= \int \Pi \, d\phi_n \quad \Theta\left(f_+ e^{-S_+} \right) f_+ e^{-S_+}$$

$$\psi[\phi_0] := \int_{n_4 > 0} \Pi \, \phi_n \quad f_+ e^{-S_+}$$

$\hookrightarrow$

$$\langle \Theta f_+ f_+ \rangle = \int \Pi \, \phi_{0,\vec{n}} \; e \; \left| \psi[\phi_0] \right|^2 \geq 0$$

$$\Longleftrightarrow \quad \langle \psi | \psi \rangle \geq 0$$

$f_{+,n}$:

$$\langle \Theta f_n \; f_n \rangle \hat{=} \langle \psi | T^{2n} | \psi \rangle$$

$\hookrightarrow$ T^2 positiv.

Link - Refl.

$$S = \sum_n \Big[-\kappa \sum_\mu \phi(n+\hat\mu)\,\phi(n) + \phi(n)^2 + \lambda\left(\phi^2(n)-1\right)^2 \Big]$$

$\hat{n} \Longleftrightarrow \kappa$

$$S = \underset{}{S_c} + \overset{n_4 \geq 1}{\overset{\downarrow}{S_+}} + \overset{n_4 \leq 0}{\overset{\downarrow}{S_-}} \,, \qquad S_+ = \Theta S_-$$

$$S_c = -\kappa \sum_{\vec n} \phi_1(\vec n)\,\phi_0(\vec n)$$

$\hookrightarrow$

$$\langle \Theta f_+ \, f_+ \rangle = \int \Pi d\phi_n \, e^{-S_c}\, \Theta\left(f_+ e^{-S_+}\right) f_+ e^{-S_+}$$

$$e^{-S_c} = \sum_{\ell=0}^{\infty} \kappa^\ell \sum \underbrace{c_{\ell i}}_{\geq 0} (\Theta S_i)\, S_i$$

$$\Big[(\phi_1 \cdot \phi_0)^n = \sum_{\vec n \ldots \vec n_\ell} \phi_{1,\vec n_1} \cdots \phi_{1,\vec n_\ell} \; \phi_{0,\vec n_1} \cdots \phi_{0,\vec n_\ell} \Big|$$

$\langle \Theta f_+ , f_+ \rangle$

$$= \sum_{\ell=0}^{\infty} \kappa^{\ell} \sum_i c_{n,i} \left| \int_{x_4 > 0} \prod d\phi(x) \, S_i \, f_+ \, e^{-S_+} \right|^2$$